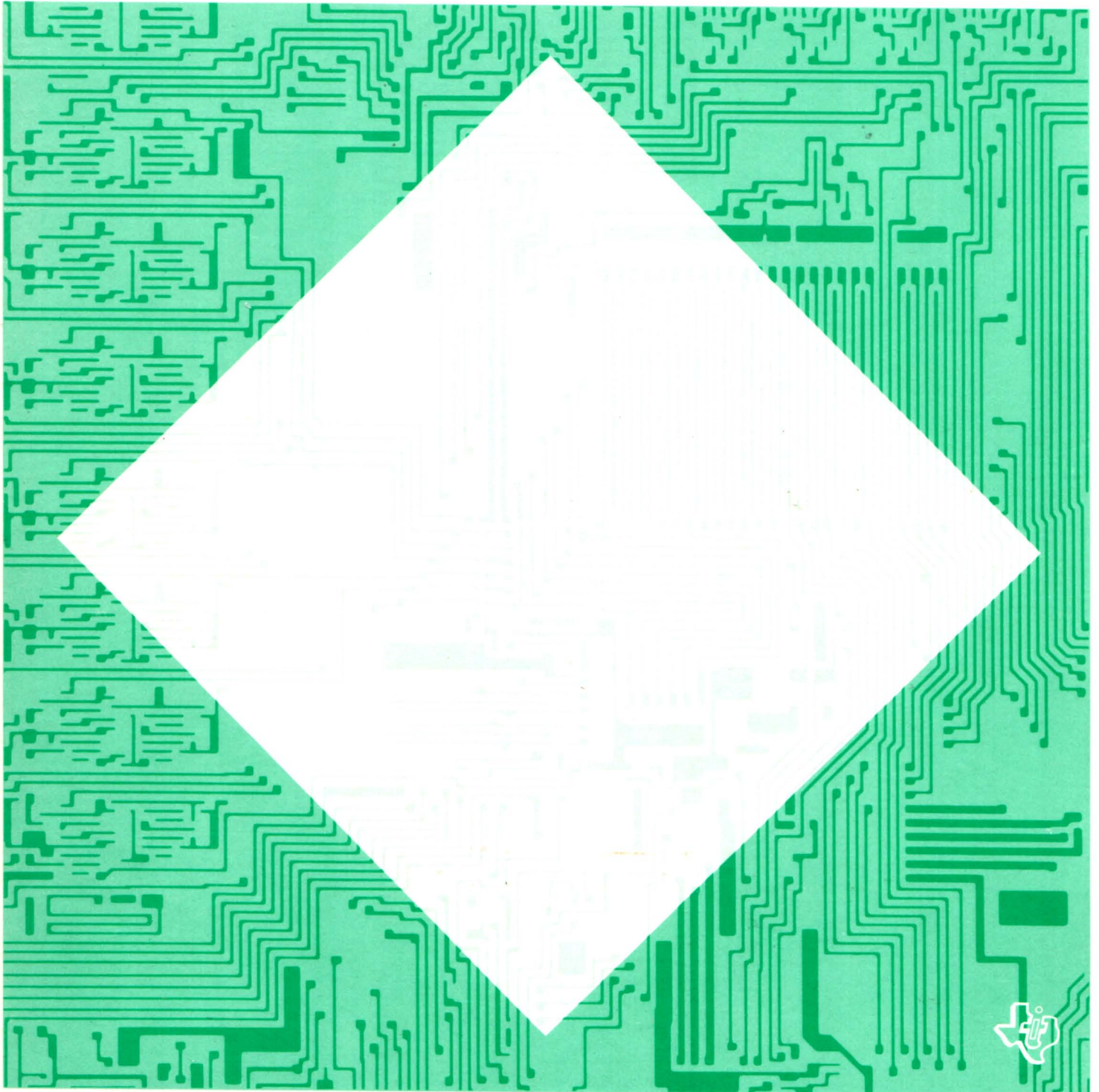


TI Programmable 58/59

Electrical Engineering

Using the power of your *Solid State Software*[™] module



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*Printer required.

**Printer and TI-59 required.

INTRODUCTION

The Electrical Engineering Library contains programming aids for the design engineer, systems engineer, and engineering technical assistant. From computing component values of filters and other electrical circuits to numerical aids for analysis and design of feedback control systems, parameter conversions to transmission line calculations, signal detection to time series transformations, the programs cover a broad range of electrical engineering and systems engineering applications.

USING THIS MANUAL

Following this brief introduction, you will find the description, user instructions, and example problems for each of the 19 programs in the Electrical Engineering Library. Each program is easily identified by the "EE" number in the upper corner of the page. This number corresponds with the call number you use to tell the calculator which program in the *Solid State Software** module you wish to use.

The primary reference point in this manual for each program is the User Instructions. These user instructions are also available for you in the handy pocket guide furnished with the library. The program description and sample problems should be used when you first run a program, to help you understand its full capabilities and limitations. Nonmagnetic label cards to identify the user-defined keys are also included in the library. Carefully remove the cards from the sheet and insert them in the card carrying case for convenient storage. Note that a special holder has been built into the case for storage of the library module.

When using the *Solid State Software* programs as subroutines to your own programs, you will also want to check Register Contents for the program and check Program Reference Data provided in Appendix A.

USING THE OPTIONAL PRINTER

If you have the optional PC-100A or PC-100C printer[†], a printed record of entries and results is automatic. The User Instructions and example problems are marked to show exactly which values are printed in addition to being displayed.

Use the Calculator Mounting procedure in the PC-100A or PC-100C Owner's Manual to mount your calculator on the printer. The switch called out in Step 2 of the PC-100A manual should be set to "OTHER" for your calculator. Always turn the calculator and printer off before mounting or removing the calculator.

TIPS FOR RUNNING PROGRAMS

Before you begin using the *Solid State Software* programs on your own, here are a few things to keep clearly in mind until you become familiar with your calculator.

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[†]Note: The TI Programmable 58 and TI Programmable 59 will not operate on the PC-100 print cradle.

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1. Press [CLR] before running a program if you are not sure of the status of the calculator. (To be completely sure of calculator status, turn it off and on again – but remember that this will clear the program memory.)
2. Some programs may leave the calculator in fix-decimal format (see Appendix A). In that event, you should press [INV] [2nd] [fix] before running another program if this format is not desired.
3. There is no visual indication of which *Solid State Software* program has been called. If you have any doubts, the safest method is to call the desired program with [2nd] [Pgm] mm, where mm is the two-digit program number. The calculator will remain at this program number until another program is called, [RST] is pressed or the calculator is turned off.
4. A flashing display normally indicates an improper key sequence or that a numerical limit has been exceeded. When this occurs, always repeat the program sequence and check that each step is performed as directed by the User Instructions. Any unusual limits of a program are given in the User Instructions or related notes. The In Case of Difficulty portion of Appendix A in *Personal Programming* may be helpful in isolating a problem.
5. Some of the *Solid State Software* programs may run for several minutes depending on input data. If you desire to halt a running program, press the [RST] key. This is considered as an emergency halt operation which returns control to the main memory. A program must be recalled to be run again.

USING SOLID STATE SOFTWARE PROGRAMS AS SUBROUTINES

Any of the *Solid State Software* programs may be called as a subroutine to your own program in the main memory. Either of two program sequences may be used: 1) [2nd] [Pgm] mm (User-Defined Key) or 2) [2nd] [Pgm] mm [SBR] (Common Label). Both will send the program control to program mm, run the subroutine sequence, and then automatically return to the main program without interruption. Following [2nd] [Pgm] mm with anything other than [SBR] or a user-defined key is not a valid key sequence and can cause unwanted results.

It is very important to consider the Program Reference Data in Appendix A for any program called as a subroutine. You must plan and write your own program such that the data registers, flags, subroutine levels, parentheses levels, T-register, angular mode, etc., used by the called subroutine are allowed for in your program. In addition, a Register Contents section of each program description provides a guide to determine where data is or must be located to run the program. A sample program that calls a *Solid State Software* program as a subroutine is provided in the PROGRAMMING CONSIDERATIONS section of *Personal Programming*.

If you need to examine and study the content of a *Solid State Software* program, you can download as described in the following paragraphs.

DOWNLOADING SOLID STATE SOFTWARE PROGRAMS

If you need to examine a *Solid State Software* program, it can be downloaded into the main program memory.* This will allow you to single step through a program in or out of the learn mode. It also allows using the program list or trace features of the optional printer. The only

*Unless the library is a protected special-purpose library.

INTRODUCTION

requirement for downloading a *Solid State Software* program is that the memory partition be set so there is sufficient space in the main program memory to receive the downloaded program. The key sequence to download a program is [2nd] [CP] [2nd] [Pgm] mm [2nd] [Op] 09 where mm is the program number to be downloaded. This procedure places the requested program into program memory beginning at program location 000. The downloaded program writes over any instructions previously stored in that part of program memory. Remember to press [RST] before running or tracing the downloaded program.

All of the EE programs can be downloaded in the TI Programmable 59 using normal power-up partitioning.

The partition must be changed from the power-up condition in the TI Programmable 58 for programs EE-02, 03, 06, 08, 09, 13, 14, 16, 17, and 19. The key sequence to repartition the main memory for EE-03, 06, 13, and 17 is 2 [2nd] [Op] 17. The sequence for EE-02, 14, 16, and 19 is 1 [2nd] [Op] 17. The sequence for EE-08 and 09 is 0 [2nd] [Op] 17.

REMOVING AND INSTALLING MODULES

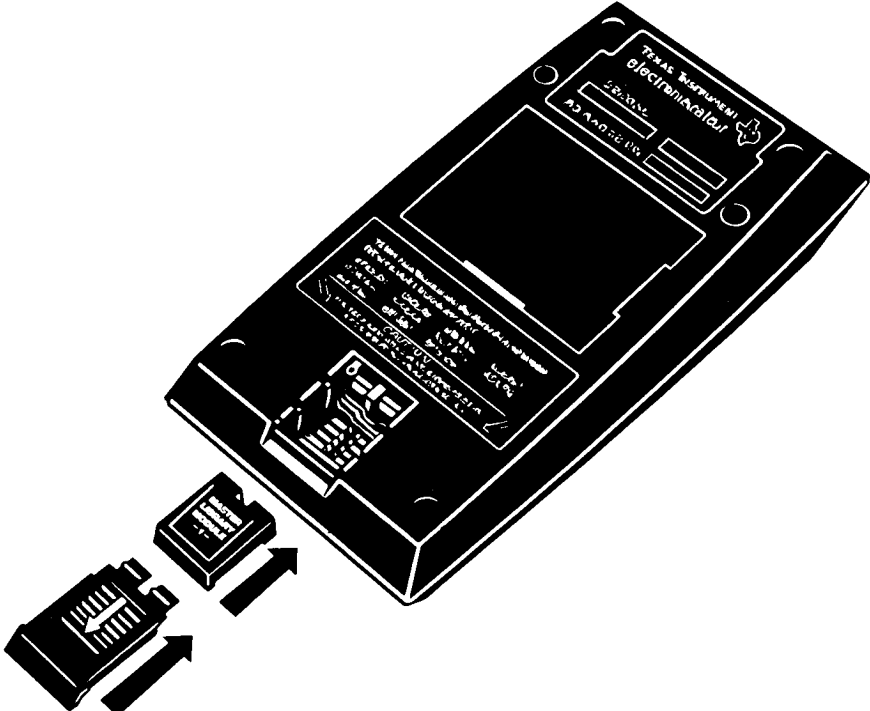
The Electrical Engineering Module can easily be installed in the calculator or replaced with another. It is a good idea to leave the module in place in the calculator except when replacing it with another module. Be sure to follow these instructions when you need to remove or replace a module.

CAUTION

Be sure to touch some metal object before handling a module to prevent possible damage by static electricity.

1. **Turn the calculator OFF.** Loading or unloading the module with the calculator ON may cause the keyboard or display to lock out. Also, shorting the contacts can damage the module or calculator.
2. Slide out the small panel covering the module compartment at the bottom of the back of the calculator. (See Diagram on following page.)
3. Remove the module. You may turn the calculator over and let the module fall out into your hand.
4. Insert the module, notched end first with the labeled side up into the compartment. The module should slip into place effortlessly.
5. Replace the cover panel, securing the module against the contacts.

INTRODUCTION



Don't touch the contacts inside the module compartment as damage can result.

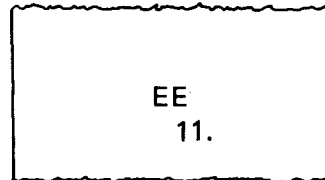
ELECTRICAL ENGINEERING MODULE CHECK

This program performs the following functions separately.

1. Library Module Check
2. Linear Regression Initialization

Library Module Check

When you want to know which of your *Solid State Software* modules is in the calculator without physically looking at it, you can call the Library Module check portion of the routine. If the Electrical Engineering Module is in the calculator, the number 11. will be displayed. This number is unique to the Electrical Engineering Library (other optional libraries use other identifying digits). If the calculator is attached to a PC-100A or PC-100C print cradle, the following will be printed:



```
EE
11.
```

If these results are not obtained, there is probably a malfunction in the calculator or the *Solid State Software* module. First, install the Master Library module and try running the diagnostic program, ML-01. If this program does not run properly, refer to Appendix A of *Personal Programming* for an explanation of the various procedures to be followed when you have difficulties.

Linear Regression Initialization

This routine initializes the calculator for linear regression by clearing data registers R_{01} through R_{06} and the T-register. It should be used whenever linear regression or other built-in statistics functions are to be started. You can also use the routine at any time to clear these registers selectively without disturbing any other registers.



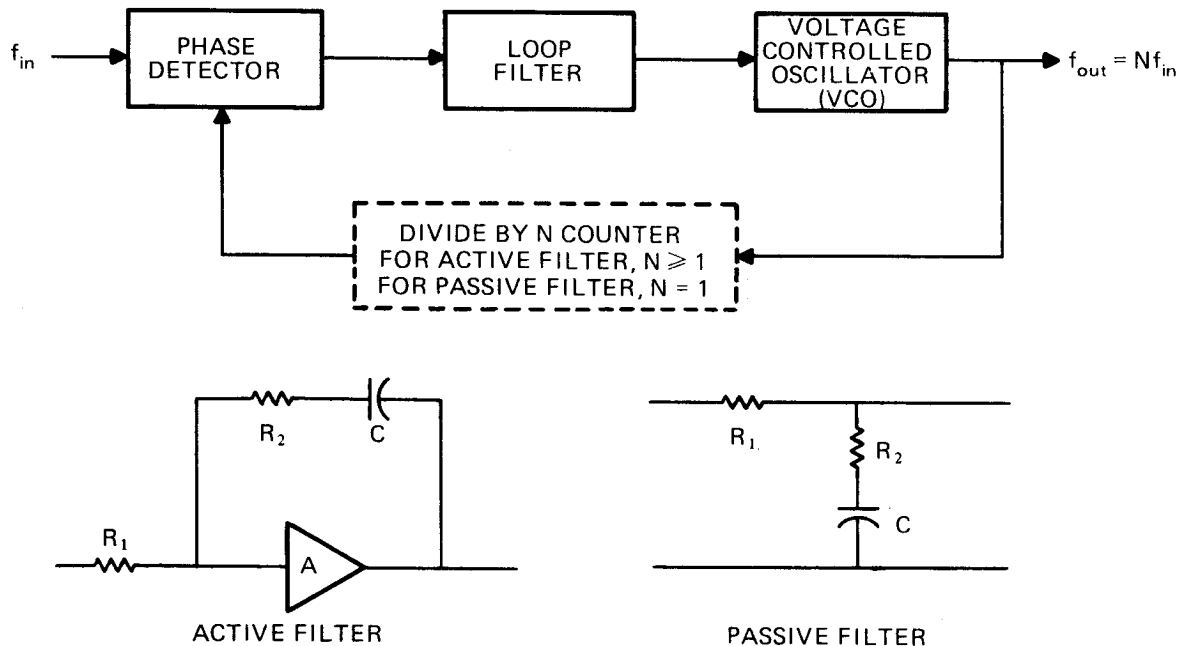
USER INSTRUCTIONS

STEP	PROCEDURE	ENTER	PRESS	DISPLAY
	Library Module Check			
A1	Select Program		[2nd] [Pgm] 01	
A2	Run Module Check		[SBR] [2nd] [R/S]	11. ¹
	Initialize Linear Regression			
B1	Select Program		[2nd] [Pgm] 01	
B2	Initialize Linear Regression		[SBR] [CLR]	0.

NOTE: 1. The number 11. indicates the Electrical Engineering Library.

PHASE-LOCKED LOOP

Given the choice of an appropriate loop filter (active or passive), this program computes the resulting design parameters for a basic phase-locked loop (PLL) as illustrated below. A type-2, second-order loop is realized with an active filter while a type-1, second-order loop results with a passive filter.



The basic PLL transfer function is $\frac{\theta_o}{\theta_i}(S) = \frac{G F(S)}{S + G F(S)}$

where

θ_i is the input phase

θ_o is the output phase

G is the loop gain

$F(S)$ is the transfer function of the loop filter.

$$G = K_p K_v$$

where


K_p is the gain of the phase detector in volts/radian

K_v is the gain of the VCO in radians/(second volt)

If the loop filter is active, a ($\div N$) counter can be added to the loop, making the output frequency, f_{out} , an integral multiple of the input or reference frequency, f_{in} . In this case, N is an integer divisor ≥ 1 and $f_{out} = N f_{in}$. This technique is used for multiple frequency generation in frequency synthesizers. If the loop filter is passive, the program ignores the ($\div N$) counter and in effect $N = 1$ making $f_{out} = f_{in}$.

EE-02

In this program, the user must input the loop gain G and a value for C in farads. If the loop filter chosen is active, then N is also required. At this point, the user has the option of either inputting ω_n , the natural angular frequency in radians/second and ζ , the damping factor and solving for the resistances R_1 and R_2 in ohms, or vice versa. With a passive loop filter, a test is made before computing R_1 and R_2 to insure that the resistance values are greater than zero. Finally, after completing the above computations, the loop-noise bandwidth can be computed.

 Solid State Software TI © 1979				
PHASE LOCKED LOOP				EE-02
N	R₁	R₂	→ B	Execute (PF)
G	ω_n	C	ζ	Execute (AF)

USER INSTRUCTIONS

STEP	PROCEDURE	ENTER	PRESS	DISPLAY
1	Select program		[2nd] [Pgm] 02	
2	Initialize		[SBR] [CLR]	0.
3	Enter loop gain ^{1,5}	G	[A]	G
4	Enter C for PLL filter in farads	C	[C]	C
5	Enter integer divisor ² (omit this step for passive PLL filter)	N	[2nd] [A']	N
6	Enter remaining PLL parameters as required ⁵			
	a. Natural frequency in radians/second	ω _n	[B]	ω _n
	b. Damping factor	ζ	[D]	ζ
	c. R ₁ in ohms	R ₁	[2nd] [B']	R ₁
	d. R ₂ in ohms	R ₂	[2nd] [C']	R ₂
7	Compute desired PLL (active filter) parameters			
	a. Natural frequency in radians/second		[E] [B]	ω _n
	b. Damping factor		[E] [D]	ζ
	c. R ₁ in ohms		[E] [2nd] [B']	R ₁
	d. R ₂ in ohms		[E] [2nd] [C']	R ₂
8	Compute desired PLL (passive filter) parameters			
	a. Natural frequency in radians/second		[2nd] [E'] [B]	ω _n
	b. Damping factor		[2nd] [E'] [D]	ζ
	c. R ₁ in ohms ³		[2nd] [E'] [2nd] [B']	R ₁
	d. R ₂ in ohms ³		[2nd] [E'] [2nd] [C']	R ₂
9	Compute loop-noise bandwidth in		[2nd] [D']	B

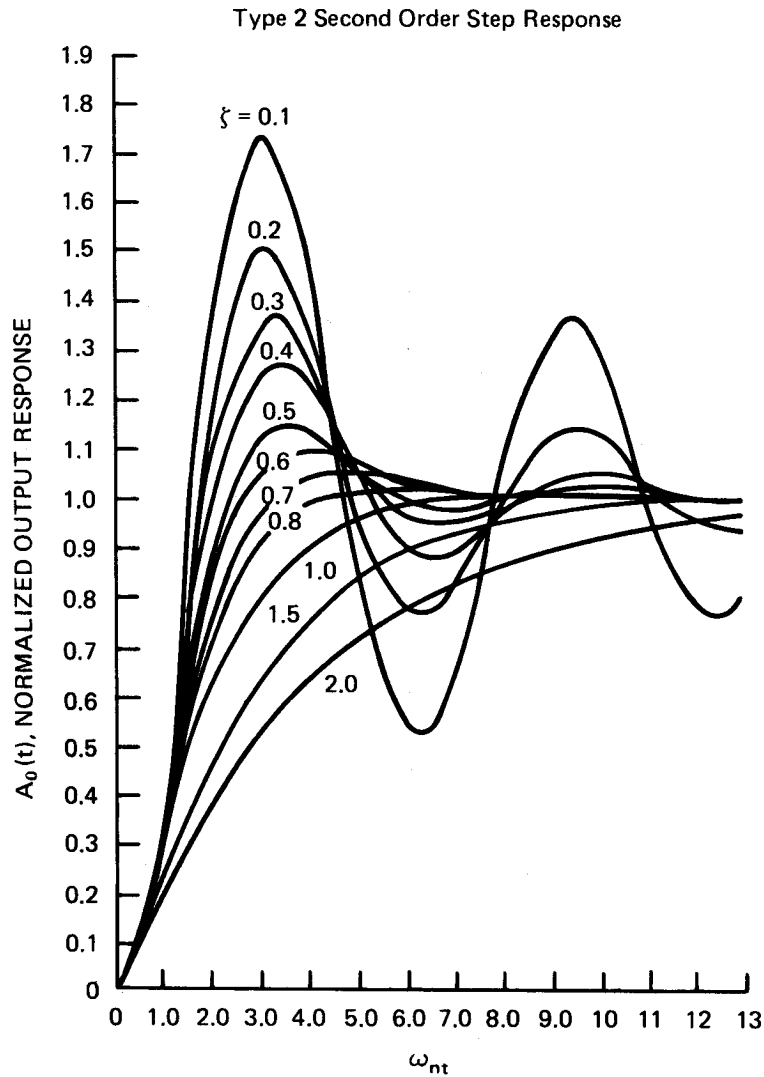
- NOTES:**
- The dimension for the loop gain is 1/second since $G = K_p K_v$ where K_p = phase detector gain in volts/radian and K_v = VCO gain in radians/second volt.
 - Input must be an integer ≥ 1 . Enter 1 for N when working with basic PLL design ($f_{in} = f_{out}$).
 - Flashing 9's in the display indicates that the input quantities G, ω_n , or ζ need to be modified so that

$$0 < \frac{2\zeta\omega_n G - \omega_n^2}{G^2} < 1.$$

This insures that the resistances will be greater than zero.

- Loop-noise bandwidth is one-sided.
- Inputs must be greater than zero.

Example 1: Determine the active filter component values for a phase-locked loop with the following specifications. The loop must achieve a lock so that $A_o(t)$ is within 10% of its final value after 500 milliseconds with a damping factor (ζ) of 0.5. From the curves below, where the normalized output response, $A_o(t)$, is plotted as a function of the normalized time ($\omega_n t$), $\zeta = 0.5$ is within 10% at $\omega_n t = 4.5$. Thus, $\omega_n = 4.5/0.5 = 9$ radians/second.



Data sheet specifications for the phase detector give a value of 0.111 volts/radian for the gain constant. The measured value for the gain of the VCO is 9.5×10^6 radians/second volt. The output frequency is 10 megahertz with a channel spacing of 250 hertz. Thus, $N = 10 \text{ MHz} / 250 \text{ Hz} = 40000$ and loop gain = $(0.111) (9.5 \times 10^6) = 1054500$. The value for C is chosen to be 10 microfarads.

ENTER	PRESS	DISPLAY	COMMENTS
	[2nd] [Pgm] 02		Select program
	[SBR] [CLR]	0.	Initialize
1054500	[A]	1054500.	Enter loop gain
.00001	[C]	0.00001	Enter C in farads
40000	[2nd] [A']	40000.	Enter integer divisor
9	[B]	9.	Enter natural frequency
.5	[D]	0.5	Enter damping factor
	[E] [2nd] [B']	32546.2963	R ₁
	[E] [2nd] [C']	11111.11111	R ₂
	[2nd] [D']	4.5	Loop-noise bandwidth

Example 2: A phase-locked loop has an overall loop gain of 20000. If the passive filter component values are: $R_1 = 5\text{ k}\Omega$, $R_2 = 500\Omega$, and $C = 50\mu\text{F}$, what is the natural frequency damping factor, and noise bandwidth for the loop?

ENTER	PRESS	DISPLAY	COMMENTS
	[2nd] [Pgm] 02		Select program
	[SBR] [CLR]	0.	Initialize
20000	[A]	20000.	Enter loop gain
.00005	[C]	0.00005	Enter C in farads
5000	[2nd] [B']	5000.	Enter R ₁ in ohms
500	[2nd] [C']	500.	Enter R ₂ in ohms
	[2nd] [E'] [B]	269.679945	Compute natural frequency
	[2nd] [E'] [D]	3.377741311	Compute damping factor
	[2nd] [D']	465.4345854	Compute loop-noise bandwidth

Register Contents

R ₀₀	R ₀₅	R ₁₀ G	R ₁₅	R ₂₀ N	R ₂₅
R ₀₁	R ₀₆	R ₁₁ R ₁	R ₁₆	R ₂₁	R ₂₆
R ₀₂	R ₀₇	R ₁₂ R ₂	R ₁₇ ω_n	R ₂₂	R ₂₇
R ₀₃	R ₀₈	R ₁₃ C	R ₁₈ ζ	R ₂₃	R ₂₈
R ₀₄	R ₀₉	R ₁₄	R ₁₉	R ₂₄	R ₂₉

Method Used

$F(S) = \frac{S\tau_2 + 1}{S\tau_1}$ for an active filter provided that the amplifier gain is very large and

$F(S) = \frac{S\tau_2 + 1}{S(\tau_1 + \tau_2) + 1}$ for a passive filter where $\tau_1 = R_1 C$, $\tau_2 = R_2 C$.

Using the transfer function representative $F(S)$ for the passive filter, the basic PLL transfer function becomes

$$\frac{\theta_o}{\theta_i}(S) = \frac{G(S\tau_2 + 1)/(\tau_1 + \tau_2)}{S^2 + S(1 + G\tau_2)/(\tau_1 + \tau_2) + G/(\tau_1 + \tau_2)}$$

In order to preserve the analogy with second-order servo controls, one can substitute for the time constants τ_1 and τ_2 , the parameters of ω_n , the natural angular frequency, and ζ , the damping factor. Making these substitutions,

$$\frac{\theta_o}{\theta_i}(S) = \frac{S\omega_n(2\zeta - \omega_n/G) + \omega_n^2}{S^2 + 2\zeta\omega_n S + \omega_n^2}$$

where, by definition

$$\omega_n = \sqrt{\frac{G}{\tau_1 + \tau_2}} \quad \text{and} \quad \zeta = \frac{\omega_n}{2} \left(\tau_2 + \frac{1}{G} \right)$$

A similar argument applies in the case where the loop filter is active except that the ($\div N$) counter changes the form of the PLL transfer function,

$$\frac{\theta_o}{\theta_i}(S) = \frac{G F(S)/N}{S + G F(S)/N}$$

Using the transfer function representation $F(S)$ for the active filter, the PLL transfer function becomes

$$\frac{\theta_o}{\theta_i}(S) = \frac{G(S\tau_2 + 1)/N\tau_1}{S^2 + S(G\tau_2/N\tau_1) + G/N\tau_1} \quad \text{or} \quad \frac{\theta_o}{\theta_i}(S) = \frac{2\zeta\omega_n S + \omega_n^2}{S^2 + 2\zeta\omega_n S + \omega_n^2}$$

where, by definition

$$\omega_n = \sqrt{\frac{G}{N\tau_1}} \quad \text{and} \quad \zeta = \frac{\omega_n\tau_2}{2}$$

The loop filter components R_1 and R_2 are computed using the defining equations for ω_n and ζ .

Active filter calculations for R_1 and R_2

$$\omega_n = \sqrt{\frac{G}{N\tau_1}} \Rightarrow R_1 = \frac{G}{N\omega_n^2 C}$$

$$\zeta = \frac{\omega_n}{2} \tau_2 \Rightarrow R_2 = \frac{2\zeta}{\omega_n C}$$

Passive filter calculations for R_1 and R_2

$$\omega_n = \sqrt{\frac{G}{\tau_1 + \tau_2}} \Rightarrow R_1 = \frac{G}{\omega_n^2 C} - R_2$$

$$\zeta = \frac{\omega_n}{2} \left(\tau_2 + \frac{1}{G} \right) \Rightarrow R_2 = \frac{2\zeta}{\omega_n C} - \frac{1}{GC}$$

NOTE: In order to insure $R_1, R_2 > 0$ in the passive case,

$$0 < \frac{2\zeta\omega_n G - \omega_n^2}{G^2} < 1$$

Finally, for both active and passive loops, the one-sided loop-noise bandwidth B in hertz is computed

$$B = \frac{\omega_n}{2} \left(\zeta + \frac{1}{4\zeta} \right)$$

Reference: *Phaselock Techniques*, Floyd M. Gardner, John Wiley & Sons 1966.

S \rightleftharpoons Y PARAMETER CONVERSIONS

Small-signal, two-port "black boxes" are often characterized in terms of complex scattering (S) parameters normalized to a common input and output transmission line impedance Z_0 . The translation of these parameters to admittance (Y) parameters is necessary to create equivalent circuit models. This program converts S parameters to Y parameters or vice versa.

The matrix equation relating these parameters is

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} \frac{1 - S_{11} + S_{22} - \Delta S}{1 + S_{11} + S_{22} + \Delta S} & \frac{-2S_{12}}{1 + S_{11} + S_{22} + \Delta S} \\ \frac{-2S_{21}}{1 + S_{11} + S_{22} + \Delta S} & \frac{1 + S_{11} - S_{22} - \Delta S}{1 + S_{11} + S_{22} + \Delta S} \end{bmatrix}$$

where $\Delta S = S_{11} S_{22} - S_{12} S_{21}$ and the S parameters are input in polar form ($R, < \theta$).

The relationship for converting from Y parameters to S parameters may be obtained by interchanging Y and S in the above matrix equation. The angle θ representation for the respective parameters as output from this program is in degrees where $-90^\circ \leq \theta < 270^\circ$.

When converting from S to Y parameters, once the matrix elements are calculated, the Y matrix is multiplied by the real scalar Z_0^{-1} to give the actual Y parameter.

When converting from Y to S parameters, the Y parameters are multiplied by Z_0 before the matrix elements are calculated to give the actual S parameters.

Solid State Software TI © 1979				
S ↔ Y PARAMETER CONVERSIONS				EE-03
→ S ₁₁ ; ∠ ₁₁	→ S ₁₂ ; ∠ ₁₂	→ S ₂₁ ; ∠ ₂₁	→ S ₂₂ ; ∠ ₂₂	S ← Y
S ₁₁ ; ∠ ₁₁	S ₁₂ ; ∠ ₁₂	S ₂₁ ; ∠ ₂₁	S ₂₂ ; ∠ ₂₂	S → Y

USER INSTRUCTIONS

STEP	PROCEDURE	ENTER	PRESS	DISPLAY
1	Select program		[2nd] [Pgm] 03	
2	Initialize ¹		[SBR] [CLR]	0.
3	Enter polar form of S (or Y) matrix ²			
	a. Magnitude S ₁₁ (or Y ₁₁)	S ₁₁ (Y ₁₁)	[x≧t]	0.
	b. Angle S ₁₁ (or Y ₁₁)	∠S ₁₁ (Y ₁₁)	[A]	Real S ₁₁ (or Y ₁₁)
	c. Magnitude S ₁₂ (or Y ₁₂)	S ₁₂ (Y ₁₂)	[x≧t]	Imag S ₁₁ (or Y ₁₁)
	d. Angle S ₁₂ (or Y ₁₂)	∠S ₁₂ (Y ₁₂)	[B]	Real S ₁₂ (or Y ₁₂)
	e. Magnitude S ₂₁ (or Y ₂₁)	S ₂₁ (Y ₂₁)	[x≧t]	Imag S ₁₂ (or Y ₁₂)
	f. Angle S ₂₁ (or Y ₂₁)	∠S ₂₁ (Y ₂₁)	[C]	Real S ₂₁ (or Y ₂₁)
	g. Magnitude S ₂₂ (or Y ₂₂)	S ₂₂ (Y ₂₂)	[x≧t]	Imag S ₂₁ (or Y ₂₁)
	h. Angle S ₂₂ (or Y ₂₂)	∠S ₂₂ (Y ₂₂)	[D]	Real S ₂₂ (or Y ₂₂)
	i. Display imag S ₂₂ (or Y ₂₂), optional		[x≧t]	Imag S ₂₂ (or Y ₂₂)
4	Enter real characteristic impedance in ohms ³ and press one of the following	Z ₀		Z ₀
	Convert S → Y		[E]	0.
	Convert Y → S		[2nd] [E']	0.
5.	Display conversion results			
	a. Magnitude Y ₁₁ (or S ₁₁)		[2nd] [A']	Y ₁₁ (S ₁₁) [†]
	b. Angle Y ₁₁ (or S ₁₁) ⁴		[x≧t]	∠Y ₁₁ (S ₁₁) [†]
	c. Magnitude Y ₁₂ (or S ₁₂)		[2nd] [B']	Y ₁₂ (S ₁₂) [†]
	d. Angle Y ₁₂ (or S ₁₂)		[x≧t]	∠Y ₁₂ (S ₁₂) [†]
	e. Magnitude Y ₂₁ (or S ₂₁)		[2nd] [C']	Y ₂₁ (S ₂₁) [†]
	f. Angle Y ₂₁ (or S ₂₁)		[x≧t]	∠Y ₂₁ (S ₂₁) [†]
	g. Magnitude Y ₂₂ (or S ₂₂)		[2nd] [D']	Y ₂₂ (S ₂₂) [†]
	h. Angle Y ₂₂ (or S ₂₂)		[x≧t]	∠Y ₂₂ (S ₂₂) [†]

- NOTES:**
1. Default value for Z₀ set at 50 ohms.
 2. Angle input is in degrees.
 3. Enter 0 if default value of 50 ohms or last-entered value for Z₀ is desired.
 4. Angle output θ is in degrees where -90° ≤ θ < 270°.

[†]These values are printed if printer is connected.

EE-03

Example: Convert the following S parameters to Y parameters:

$$S_{11} = .1\angle 50^\circ, S_{12} = .2\angle 55^\circ, S_{21} = .3\angle 60^\circ, \text{ and } S_{22} = .4\angle 65^\circ.$$

$$Z_0 = 50 \text{ ohms}$$

ENTER	PRESS	DISPLAY	COMMENTS
	[2nd] [Pgm] 03		Select program
	[SBR] [CLR]	0.	Initialize
.1	[x ≧ t]	0.	Enter S ₁₁
50	[A]	0.064278761	Enter ∠S ₁₁
.2	[x ≧ t]	.0766044443	Enter S ₁₂
55	[B]	.1147152873	Enter ∠S ₁₂
.3	[x ≧ t]	.1638304089	Enter S ₂₁
60	[C]	0.15	Enter ∠S ₂₁
.4	[x ≧ t]	.2598076211	Enter S ₂₂
65	[D]	.1690473047	Enter ∠S ₂₂
	[x ≧ t]	.3625231148	Display imag S ₂₂
0		0.	Use default value for Z ₀
	[E]	0.	Convert S→Y
Display conversion results			
	[2nd] [A']	.0173534572 [†]	Y ₁₁
	[x ≧ t]	-3.227794565 [†]	∠Y ₁₁
	[2nd] [B']	.0061012628 [†]	Y ₁₂
	[x ≧ t]	216.2717472 [†]	∠Y ₁₂
	[2nd] [C']	.0091518942 [†]	Y ₂₁
	[x ≧ t]	221.2717472 [†]	∠Y ₂₁
	[2nd] [D']	.0141294793 [†]	Y ₂₂
	[x ≧ t]	-35.53169433 [†]	∠Y ₂₂

[†]These values are printed if printer is connected.

Register Contents

R ₀₀	R ₀₅ I ₁₁	R ₁₀ ∠ ₂₁	R ₁₅ ΔS, ΔY (real)	R ₂₀	R ₂₅
R ₀₁ Used	R ₀₆ ∠ ₁₁	R ₁₁ I ₂₂	R ₁₆ ΔS, ΔY (imag)	R ₂₁	R ₂₆
R ₀₂ Used	R ₀₇ I ₁₂	R ₁₂ ∠ ₂₂	R ₁₇ Z ₀	R ₂₂	R ₂₇
R ₀₃ Used	R ₀₈ ∠ ₁₂	R ₁₃ S ₂₂ -S ₁₁ Y ₂₂ -Y ₁₁ (real)	R ₁₈	R ₂₃	R ₂₈
R ₀₄ Used	R ₀₉ I ₂₁	R ₁₄ S ₂₂ -S ₁₁ Y ₂₂ -Y ₁₁ (imag)	R ₁₉	R ₂₄	R ₂₉

COMPLEX ARITHMETIC

For two given complex numbers $X = a + bi$ or $r_1 \angle \theta_1$ and $Y = c + di$ or $r_2 \angle \theta_2$, this program calculates the following:

$$X + Y$$

$$X - Y$$

$$X \times Y$$

$$X \div Y$$


$$Y^X$$

$$\sqrt[X]{Y}$$

$$\log_Y X$$

These operations may be chained in the following manner. Initially, two complex numbers are entered with the first number being X and the second number Y . After a function has been performed, the result becomes the new X , and a new Y may be entered. X and Y may be interchanged when necessary.

Also, a result from this program is stored in R_{01} and R_{02} and may be used (without reentering it) as the input for this program or for programs EE-05 and EE-06. In other words, Steps 2a and 2b or 2c, 2d and 2e may be omitted in sequential calculations using these programs.

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COMPLEX ARITHMETIC				EE-04
Y	→ X-Y	→ X+Y	→ log _y X	X $\frac{\rightarrow}{\leftarrow}$ Y
X	→ X+Y	→ X×Y	→ Y ^x	→ $\sqrt[x]{Y}$

USER INSTRUCTIONS

STEP	PROCEDURE	ENTER	PRESS	DISPLAY
1	Select program.		[2nd] [Pgm] 04	
	If rectangular form:			
2a	Enter real part of X	a	[A]	a
2b	Enter imaginary part of X (2a and 2b must be performed in sequence)	b	[A]	b
	OR			
	If polar form:			
2c	Enter modulus of X	r ₁	[A]	r ₁
2d	Enter argument of X in degrees	θ ₁	[A]	θ ₁
2e	Convert polar to rectangular (2c, 2d, and 2e must be performed in sequence)		[SBR] [2nd] [P→R]	0.
	If rectangular form:			
3a	Enter real part of Y	c	[2nd] [A']	c
3b	Enter imaginary part of Y (3a and 3b must be performed in sequence)	d	[2nd] [A']	d
	OR			
	If polar form:			
3c	Enter modulus of Y	r ₂	[2nd] [A']	r ₂
3d	Enter argument of Y in degrees	θ ₂	[2nd] [A']	θ ₂
3e	Convert polar to rectangular (3c, 3d, and 3e must be performed in sequence)		[SBR] [2nd] [P→R]	0.
	Perform either Step 4, 5, 6, 7, 8, 9, or 10.			
4	Calculate X + Y		[B] [x≧t]	real part imaginary part
5	Calculate X - Y		[2nd] [B'] [x≧t]	real part imaginary part
6	Calculate X × Y		[C] [x≧t]	real part imaginary part
7	Calculate X ÷ Y		[2nd] [C'] [x≧t]	real part imaginary part
8	Calculate Y ^x		[D] [x≧t]	real part imaginary part
9	Calculate log _y X		[2nd] [D'] [x≧t]	real part imaginary part
10	Calculate $\sqrt[x]{Y}$		[E] [x≧t]	real part imaginary part
	After a calculation, the result becomes the new X.			
11	To swap X and Y		[2nd] [E']	0.

Example 1: $X = 2 + 3i$, $Y = 2 \angle 30^\circ$

Find $X + Y$

ENTER	PRESS	DISPLAY	COMMENTS
	[2nd] [Pgm] 04		Select program
2	[A]	2.	a
3	[A]	3.	b
2	[2nd] [A']	2.	r
30	[2nd] [A']	30.	θ
	[SBR] [2nd] [P→R]	0.	Convert polar to rectangular
	[B]	3.732050808	Real (X + Y)
	[x≥t]	4.	Imaginary (X + Y)

Calculate polar form of $X + Y$

	[2nd] [Pgm] 05		Select program
	[B]	5.470667531	r
	[x≥t]	46.98475372	θ

Example 2: Solve the following expression.

$$[(2 + 3i)(1 - i)]^{(1 + i)}$$

ENTER	PRESS	DISPLAY	COMMENTS
	[2nd] [Pgm] 04		Select program
2	[A]	2.	a
3	[A]	3.	b
1	[2nd] [A']	1.	c
1	[+/-] [2nd] [A']	-1.	d
	[C]	5.	Real (X X Y)
	[x≥t]	1.	Imaginary (X X Y)
	[2nd] [E']	0.	$X \neq Y$
1	[A]	1.	a
1	[A]	1.	b
	[D]	-1.058423508	Real (Y ^X)
	[x≥t]	4.049577726	Imaginary (Y ^X)

Register Contents

R ₀₀	R ₀₅	R ₁₀	R ₁₅	R ₂₀	R ₂₅
R ₀₁ a	R ₀₆	R ₁₁	R ₁₆	R ₂₁	R ₂₆
R ₀₂ b	R ₀₇	R ₁₂	R ₁₇	R ₂₂	R ₂₇
R ₀₃ Used	R ₀₈	R ₁₃	R ₁₈	R ₂₃	R ₂₈
R ₀₄ Used	R ₀₉	R ₁₄	R ₁₉	R ₂₄	R ₂₉

EE-04

Method Used

$$X = a + bi \quad Y = c + di$$

$$X + Y = (a + c) + (b + d)i$$

$$X - Y = (a - c) + (b - d)i$$

$$X \times Y = (ac - bd) + (ad + bc)i$$

$$X \div Y = \frac{ac + bd}{c^2 + d^2} + \frac{bc - ad}{c^2 + d^2} i$$

$$Y^X = e^{X \ln Y}, Y \neq 0$$

$$\sqrt[X]{Y} = \frac{\ln Y}{e^{X/Y}}, X \neq 0, Y \neq 0$$

$$\log_Y X = \frac{\ln X}{\ln Y}, X \neq 0, Y \neq 0$$


For calculation of e^X , $\ln X$, and $1/X$ for complex numbers, see program EE-05.

COMPLEX FUNCTIONS

The following functions are calculated for the complex number $X = a + bi$ or $r\angle\theta$.

Polar representation (r, θ) of X , or
Rectangular representation $(a + bi)$ of X
 X^2
 \sqrt{X}
 $1/X$
 e^X
 $\ln X$

After a function has been performed, the result is stored in R_{01} and R_{02} and becomes the new X . Therefore, a result may be used without reentering it in this program and in programs EE-04 and EE-06.

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COMPLEX FUNCTIONS				EE-05
→ ln X	→ e ^x		→ a+bi	
X	→ r,θ	→ X ²	→ √X	→ 1/X

USER INSTRUCTIONS

STEP	PROCEDURE	ENTER	PRESS	DISPLAY
1	Select program. If rectangular form:		[2nd] [Pgm] 05	
2a	Enter real part of X	a	[A]	a
2b	Enter imaginary part of X (2a and 2b must be performed in sequence)	b	[A]	b
	OR			
	If polar form:			
2c	Enter modulus of X	r	[A]	r
2d	Enter argument of X in degrees	θ	[A]	θ
2e	Convert polar to rectangular (2c, 2d, and 2e must be performed in sequence)		[SBR] [2nd] [P→R]	0.
3	Calculate polar form of X, if desired		[B] [x≧t]	r θ
4	Calculate rectangular form of X, if desired		[2nd] [D'] [x≧t]	a b
	Perform either Step 5, 6, 7, 8, or 9			
5	Calculate X ²		[C] [x≧t]	real part imaginary part
6	Calculate √X		[D] [x≧t]	real part imaginary part
7	Calculate 1/X		[E] [x≧t]	real part imaginary part
8	Calculate ln X		[2nd] [A'] [x≧t]	real part imaginary part
9	Calculate e ^x		[2nd] [B'] [x≧t]	real part imaginary part
	After a calculation, the result becomes the new X.			

Example 1: Find X^2 if $X = 2\angle 60^\circ$

ENTER	PRESS	DISPLAY	COMMENTS
	[2nd] [Pgm] 05		Select program
2	[A]	2.	r
60	[A]	60.	θ
	[SBR] [2nd] [P→R]	0.	Convert polar to rectangular
	[C]	-2.	Real (X^2)
	[x≥t]	3.464101615	Imaginary (X^2)

Calculate polar form of X^2

[B]	4.	r
[x≥t]	120.	θ

Example 2: Find $\ln X^2$ if $X = 2 + 3i$

ENTER	PRESS	DISPLAY	COMMENTS
	[2nd] [Pgm] 05		Select program
2	[A]	2.	a
3	[A]	3.	b
	[C]	-5.	Real (X^2)
	[x≥t]	12.	Imaginary (X^2)
	[2nd] [A']	2.564949357	Real ($\ln X^2$)
	[x≥t]	1.965587446	Imaginary ($\ln X^2$)

Register Contents

R ₀₀	R ₀₅	R ₁₀	R ₁₅	R ₂₀	R ₂₅
R ₀₁ a	R ₀₆	R ₁₁	R ₁₆	R ₂₁	R ₂₆
R ₀₂ b	R ₀₇	R ₁₂	R ₁₇	R ₂₂	R ₂₇
R ₀₃ Used	R ₀₈	R ₁₃	R ₁₈	R ₂₃	R ₂₈
R ₀₄ Used	R ₀₉	R ₁₄	R ₁₉	R ₂₄	R ₂₉

EE-05

Method Used

$$X = a + bi$$

$$\text{Magnitude of } X = r = \sqrt{a^2 + b^2}$$

$$\text{Angle of } X \text{ (degrees)} = \theta, \text{ where } -90^\circ \leq \theta < 270^\circ **$$

$$\theta = \begin{cases} \tan^{-1} b/a & \text{if } a \neq 0 \\ 90^\circ & \text{if } a = 0, b > 0 \\ -90^\circ & \text{if } a = 0, b < 0 \end{cases}$$

$$X^2 = r^2 (\cos 2\theta + i \sin 2\theta)$$

$$\sqrt{X} = \sqrt{r} \left(\cos \frac{\theta}{2} + i \sin \frac{\theta}{2} \right)$$

$$\frac{1}{X} = \frac{1}{a + bi}$$

$$e^X = e^a \cos b + i e^a \sin b$$

$$\ln X = \ln r + i\theta, X \neq 0$$

**See page V-31 of *Personal Programming*.

COMPLEX TRIGONOMETRIC FUNCTIONS

This program calculates the value of trigonometric functions for a complex number $X = a + bi$ or $r\angle\theta$. The following functions are evaluated:

$\sin X$

$\cos X$

$\tan X$

$\sin^{-1} X$

$\cos^{-1} X$

$\tan^{-1} X$

The result of any function is stored in rectangular form in R_{01} and R_{02} and becomes the new value of X . Therefore a result may be used without reentering it in this program and in programs EE-04 and EE-05.

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COMPLEX TRIGONOMETRIC FUNCTIONS				EE-06
	→ sin ⁻¹ X	→ cos ⁻¹ X	→ tan ⁻¹ X	
X	→ sin X	→ cos X	→ tan X	

USER INSTRUCTIONS

STEP	PROCEDURE	ENTER	PRESS	DISPLAY
1	Select program. If rectangular form:		[2nd] [Pgm] 06	
2a	Enter real part of X	a	[A]	a
2b	Enter imaginary part of X (2a and 2b must be performed in sequence)	b	[A]	b
	OR			
	If polar form:			
2c	Enter modulus of X	r	[A]	r
2d	Enter argument of X in degrees	θ	[A]	θ
2e	Convert polar to rectangular (2c, 2d, and 2e must be performed in sequence)		[SBR] [2nd] [P→R]	0.
	Perform either Step 3, 4, 5, 6, 7, or 8			
3	Calculate sin X		[B] [x≥t]	real part imaginary part
4	Calculate cos X		[C] [x≥t]	real part imaginary part
5	Calculate tan X		[D] [x≥t]	real part imaginary part
6	Calculate sin ⁻¹ X		[2nd] [B'] [x≥t]	real part imaginary part
7	Calculate cos ⁻¹ X		[2nd] [C'] [x≥t]	real part imaginary part
8	Calculate tan ⁻¹ X		[2nd] [D'] [x≥t]	real part imaginary part

- NOTES:** 1. After a calculation, the result becomes the new X.
 2. X is expressed in radians. Program leaves calculator in radian mode.

Example 1: Find $\tan X$ if $X = 3\angle 56^\circ$

ENTER	PRESS	DISPLAY	COMMENTS
	[2nd] [Pgm] 06		Select program
3	[A]	3.	r
56	[A]	56.	θ
	[SBR] [2nd] [P→R]	0.	Convert polar to rectangular
	[D]	-.0029707247	Real ($\tan X$)
	[x \rightrightarrows t]	1.013601147	Imaginary ($\tan X$)

Use Pgm 05 to convert to polar form of $\tan X$ if desired.

	[2nd] [Pgm] 05		Select program
	[B]	1.0136055	r
	[x \rightrightarrows t]	90.16792552	θ

Example 2: Find $\sin X$ if $X = 2 + 3i$

ENTER	PRESS	DISPLAY	COMMENTS
	[2nd] [Pgm] 06		Select program
2	[A]	2.	a
3	[A]	3.	b
	[B]	9.154499147	Real ($\sin X$)
	[x \rightrightarrows t]	-4.16890696	Imaginary ($\sin X$)

Register Contents

R ₀₀ Used	R ₀₅	R ₁₀	R ₁₅	R ₂₀	R ₂₅
R ₀₁ a	R ₀₆	R ₁₁	R ₁₆	R ₂₁	R ₂₆
R ₀₂ b	R ₀₇	R ₁₂	R ₁₇	R ₂₂	R ₂₇
R ₀₃ Used	R ₀₈	R ₁₃	R ₁₈	R ₂₃	R ₂₈
R ₀₄ Used	R ₀₉	R ₁₄	R ₁₉	R ₂₄	R ₂₉

EE-06

Method Used

$$\sin X = \frac{e^{iX} - e^{-iX}}{2i}$$

$$\cos X = \frac{e^{iX} + e^{-iX}}{2}$$

$$\tan X = \sin X / \cos X$$

$$\sin^{-1} X = \sin^{-1} (a \pm bi) = \sin^{-1} B + i \ln [A \pm (A^2 - 1)^{1/2}]$$

$$\cos^{-1} X = \cos^{-1} (a \pm bi) = \pm \cos^{-1} B - i \ln [A + (A^2 - 1)^{1/2}]$$

where

$$A = \frac{1}{2} [(a + 1)^2 + b^2]^{1/2} + \frac{1}{2} [(a - 1)^2 + b^2]^{1/2}$$

$$B = \frac{1}{2} [(a + 1)^2 + b^2]^{1/2} - \frac{1}{2} [(a - 1)^2 + b^2]^{1/2}$$

$$\tan^{-1} X = \frac{1}{2} \tan^{-1} \left[\frac{2a}{1 - a^2 - b^2} \right] + \frac{i}{4} \ln \left[\frac{a^2 + (b + 1)^2}{a^2 + (b - 1)^2} \right]$$

Note: X is in radians.

DECIBELS, NEPERS, POWER, VOLTAGE CURRENT RATIO CONVERSIONS

Power, voltage, and current ratios are often expressed in decibels (dB) or nepers (Np). This program converts between any of these when one is given. Input and output voltages and currents are assumed to be measured at points having the same impedance. If this is not the case, then decibel and neper values may be calculated from the output-to-input voltage or current ratio if V_2/V_1 is replaced by $V_2\sqrt{Z_1}/V_1\sqrt{Z_2}$ and I_2/I_1 with $I_2\sqrt{Z_2}/I_1\sqrt{Z_1}$ where Z_1 is the input impedance and Z_2 is the output impedance.

Solid State Software TI © 1979				
dB,Np,P,V,I RATIO CONVERSIONS				EE-07
P_2/P_1	$V_2/V_1 = I_2/I_1$	Np	dB	→ Execute

USER INSTRUCTIONS

STEP	PROCEDURE	ENTER	PRESS	DISPLAY
1	Select program.		[2nd] [Pgm] 07	
2	Initialize		[SBR] [CLR]	0.
3	Input any one of the following: Power ratio ¹ Voltage or Current ratio ¹ Nepers Decibels	P_2/P_1 V_2/V_1 or I_2/I_1 Np dB	[A] [B] [C] [D]	P_2/P_1 V_2/V_1 or I_2/I_1 Np dB
4	Calculate any of the following: Power ratio Voltage and Current ratio Nepers ² Decibels ²		[E] [A] [E] [B] [E] [C] [E] [D]	P_2/P_1 $V_2/V_1 = I_2/I_1$ Np dB

- Notes:**
1. P_2/P_1 , V_2/V_1 , I_2/I_1 must be greater than zero.
 2. Negative values denote an Np or dB loss.

EE-07

Example 1: For a circuit with a voltage ratio of 3.1623, calculate the power ratio, dB and Np values. Assume equal impedances at measuring points.

ENTER	PRESS	DISPLAY	COMMENTS
	[2nd] [Pgm] 07		Select program
	[SBR] [CLR]	0.	Initialization
3.1623	[B]	3.1623	Voltage ratio
	[E] [A]	10.00014129	Power ratio
	[E] [C]	1.151299611	Nepers
	[E] [D]	10.00006136	Decibels

Example 2: What is the loss in decibels for an attenuator which has 8 volts at the input and 2 volts at the output? The input and output impedances are equal and terminated properly.

ENTER	PRESS	DISPLAY	COMMENTS
	[2nd] [Pgm] 07		Select program
	[SBR] [CLR]	0.	Initialization
.25	[B]	0.25	Voltage ratio
	[E] [D]	-12.04119983	Decibels

Register Contents

R ₀₀	R ₀₅	R ₁₀ P ₂ /P ₁	R ₁₅	R ₂₀	R ₂₅
R ₀₁	R ₀₆	R ₁₁ V ₂ /V ₁ = I ₂ /I ₁	R ₁₆	R ₂₁	R ₂₆
R ₀₂	R ₀₇	R ₁₂ Np	R ₁₇	R ₂₂	R ₂₇
R ₀₃	R ₀₈	R ₁₃ dB	R ₁₈	R ₂₃	R ₂₈
R ₀₄	R ₀₉	R ₁₄	R ₁₉	R ₂₄	R ₂₉

Method Used

- Given the power ratio P₂/P₁ :

$$\text{dB} = 10 \log (P_2/P_1) \quad N_p = .5 \ln (P_2/P_1) \quad V_2/V_1 = e^{N_p}$$

- Given the voltage or current ratio V₂/V₁ = I₂/I₁ :

$$\text{dB} = 20 \log (V_2/V_1) \quad P_2/P_1 = 10^{\text{dB}/10} \quad N_p = \ln (V_2/V_1)$$

- Given the decibels dB:

$$P_2/P_1 = 10^{\text{dB}/10} \quad N_p = .5 \ln (P_2/P_1) \quad V_2/V_1 = e^{N_p}$$

- Given the nepers Np:

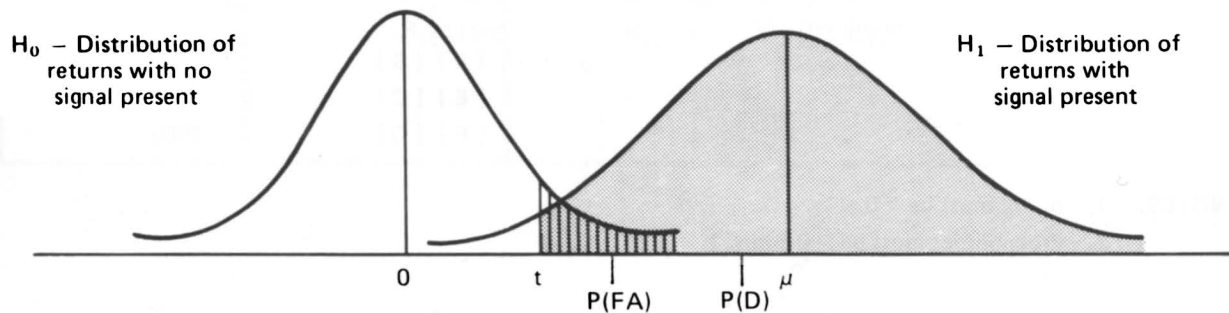
$$P_2/P_1 = e^{2N_p} \quad \text{dB} = 10 \log (P_2/P_1) \quad V_2/V_1 = e^{N_p}$$

SIGNAL DETECTION

There are many circumstances in which one must decide if a signal is present, even though noise and other variations cloud the issue. This program relates areas under two overlapping normal curves and determines the probability of falsely declaring a signal present when none exists (probability of a false alarm), as well as the probability of properly declaring a signal present (probability of detection).

This program is stated in terms of radar detection; however, this procedure applies to all problems of this type. The radar return to a given range cell is subject to both internally and externally generated noise. Under many conditions this return is approximately normally distributed about a mean we may take to be 0 with a standard deviation σ_0 . We'll call this normal curve H_0 .

The presence of a point target in this range cell adds energy to the distribution and alters the return to a normal distribution with mean μ and standard deviation σ_1 . We'll call this normal curve H_1 .




Let t represent a threshold value which one may vary. When the signal strength is greater than t , a signal is declared present. When the signal strength is less than t , no signal is declared. The area under H_0 to the right of t represents the probability of a false alarm $P(FA)$, and the area under H_1 to the right of t represents the probability of detection, $P(D)$.

The signal-to-noise ratio (in dB) may be defined as

$$\text{SNR} = 20 \log_{10} (\mu/\sigma_0)$$

Once σ_1/σ_0 has been entered, this program calculates

- 1) SNR given $P(FA)$ and the $P(D)$
- 2) $P(FA)$ given $P(D)$ and SNR
- 3) $P(D)$ given $P(FA)$ and SNR

 Solid State Software TI © 1979				
SIGNAL DETECTION				EE-08
σ_1/σ_0	SNR	P(FA)	P(D)	→ Execute

USER INSTRUCTIONS

STEP	PROCEDURE	ENTER	PRESS	DISPLAY
1	Select program		[2nd] [Pgm] 08	
2	Initialize		[SBR] [CLR]	0.
3	Enter ratio of standard deviations	σ_1/σ_0^1	[A]	σ_1/σ_0
4	Enter two of the following in any order		[B]	
	a. Signal-to-noise ratio (dB)	SNR	[B]	$\mu/\sigma_0 = 10^{\text{SNR}/20}$
	b. Probability of declaring a signal present when none exists	P(FA) ²	[C]	P(FA)
	c. Probability of detecting a signal	P(D) ²	[D]	P(D)
5	Calculate the remaining variable			
	SNR		[E] [B]	SNR
	P(FA)		[E] [C]	P(FA)
	P(D)		[E] [D]	P(D)

- NOTES:**
- σ_1/σ_0 must be > 0 .
 - Probabilities must be > 0 and < 1 .

Example 1: For $\sigma_1 = \sqrt{2}\sigma_0$, what is the false alarm probability when the threshold is set for a detection probability of 0.9 and the signal-to-noise ratio is 10 dB.

ENTER	PRESS	DISPLAY	COMMENTS
	[2nd] [Pgm] 08		Select program
	[SBR] [CLR]	0.	Initialize
2	[√x] [A]	1.414213562	Enter σ_1/σ_0
.9	[D]	0.9	Enter P(D)
10	[B]	3.16227766	Enter SNR
	[E] [C]	.0885568121	Compute P(FA)

Under the same conditions, what signal-to-noise ratio (in dB) is required to achieve a false alarm probability of 0.001. Note that σ_1/σ_0 and P(D) need not to be reentered.

.001	[C]	0.001	Enter P(FA)
	[E] [B]	13.80952205	Compute SNR

Example 2: For $\sigma_1 = 2\sigma_0$ and a signal-to-noise ratio of 20 dB, what is the probability of detection, if the probability of a false alarm is set at 1×10^{-9} .

ENTER	PRESS	DISPLAY	COMMENTS
	[2nd] [Pgm] 08		Select program
	[SBR] [CLR]	0.	Initialize
2	[A]	2.	Enter σ_1/σ_0
9	[+/-] [INV] [2nd] [log] [C]	0.000000001	Enter P(FA)
20	[B]	10.	Enter SNR
	[E] [D]	.9773102447	Compute P(D)

Register Contents

R ₀₀	R ₀₅	R ₁₀ σ_1/σ_0	R ₁₅ Used	R ₂₀	R ₂₅
R ₀₁	R ₀₆	R ₁₁ μ/σ_0	R ₁₆ Used	R ₂₁	R ₂₆
R ₀₂	R ₀₇	R ₁₂ P(FA)	R ₁₇ Used	R ₂₂	R ₂₇
R ₀₃	R ₀₈	R ₁₃ P(D)	R ₁₈ Used	R ₂₃	R ₂₈
R ₀₄	R ₀₉	R ₁₄ Used	R ₁₉	R ₂₄	R ₂₉

Method Used

SIGNAL DETECTION FORMULAS

Let U represent the normalized values of X for the normal curve in question.

$$1. \quad P(D) = \frac{1}{\sqrt{2\pi}} \int_{U_{P(D)}}^{\infty} e^{-t^2/2} dt$$

$$2. \quad P(FA) = \frac{1}{\sqrt{2\pi}} \int_{U_{P(FA)}}^{\infty} e^{-t^2/2} dt$$

$$3. \quad U_{P(FA)} = \frac{X_{P(FA)}}{\sigma_0} = \frac{X_{P(D)}}{\sigma_0}$$

$$4. \quad U_{P(D)} = \frac{\frac{X_{P(D)}}{\sigma_0} - \frac{\mu}{\sigma_0}}{\frac{\sigma_1}{\sigma_0}}$$

$$5. \quad U_{P(FA)} = \frac{X_{P(FA)}}{\sigma_0} = U_{P(D)} \frac{\sigma_1}{\sigma_0} + \frac{\mu}{\sigma_0}$$

$$6. \quad \text{SNR} = 20 \log_{10} \left(\frac{\mu}{\sigma_0} \right)$$

$$7. \quad \frac{\mu}{\sigma_0} = 10^{\text{SNR}/20}$$

Find SNR given $P(FA)$ and $P(D)$

1. Calculate $U_{P(D)}$ from Formula 1.
2. Calculate $U_{P(D)} \frac{\sigma_1}{\sigma_0}$.
3. Calculate $U_{P(FA)}$ from Formula 2.
4. Calculate $\frac{\mu}{\sigma_0}$ from Formula 4 using relationship in Formula 3.
5. Calculate SNR from Formula 6.

Find P(FA) given P(D) and SNR

1. Calculate $U_{P(D)}$ from Formula 1.
2. Calculate $U_{P(D)} \frac{\sigma_1}{\sigma_0}$.
3. Calculate $\frac{\mu}{\sigma_0}$ from Formula 7.
4. Calculate $U_{P(FA)}$ from Formula 5.
5. Calculate P(FA) from Formula 2.

Find P(D) given P(FA) and SNR

1. Calculate $U_{P(FA)}$ from Formula 2.
2. Calculate $\frac{\mu}{\sigma_0}$ from Formula 7.
3. Calculate $\frac{X_{P(D)}}{\sigma_0} - \frac{\mu}{\sigma_0}$ using relationship in Formula 3.
4. Calculate $U_{P(D)}$ from Formula 4.
5. Calculate P(D) from Formula 1.

METHOD USED FOR INTEGRAL EVALUATION

$$P(U) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^U e^{-t^2/2} dt$$

$$P(U) \approx 1 - Z(U) (a_1 t + a_2 t^2 + a_3 t^3) + \epsilon(U)$$

$$Q(U) = \frac{1}{\sqrt{2\pi}} \int_U^{\infty} e^{-t^2/2} dt = 1 - P(U)$$

$$Q(U) \approx Z(U) (a_1 t + a_2 t^2 + a_3 t^3) + \epsilon(U)$$

where:

$$Z(U) = \frac{1}{\sqrt{2\pi}} e^{-U^2/2}$$

$$t = (1 + pU)^{-1}$$

$$p = .33267$$

$$a_1 = .4361836$$

$$a_2 = -.1201676$$

$$a_3 = .9372980$$

$$|\epsilon(U)| < 1 \times 10^{-5}$$

EE-08

METHOD USED FOR FINDING U_p WHEN THE PROBABILITY (P) IS GIVEN

$$P = Q(U_p) = \frac{1}{\sqrt{2\pi}} \int_{U_p}^{\infty} e^{-t^2/2} dt$$

$$U_p \approx t - \frac{c_0 + c_1 t + c_2 t^2}{1 + d_1 t + d_2 t^2 + d_3 t^3} + \epsilon(P)$$

where:

$$t = \left(\ln \frac{1}{p^2} \right)^{1/2}$$

$$c_0 = 2.515517$$

$$c_1 = .802853$$

$$c_2 = .010328$$

$$d_1 = 1.432788$$

$$d_2 = .189269$$

$$d_3 = .001308$$

$$|\epsilon(P)| < 4.5 \times 10^{-4}$$


Reference: *Handbook of Mathematical Functions*, Abramowitz and Stegun, National Bureau of Standards, 1964.

ROOTS OF A POLYNOMIAL

The general form of a polynomial of degree n is $P_n(x) = a_1 x^n + a_2 x^{n-1} + \dots + a_n x + a_{n+1}$. This program uses the Lin-Bairstow method to find all roots, real and complex, of up to a 21st degree polynomial in one variable with real coefficients. This method is based on the fact that the polynomial can be represented as a product of quadratic and linear factors. An initial guess is made of a trial quadratic factor which, for purposes of convergence, is hopefully near the desired factor of the polynomial. The trial factor is divided into the polynomial and a remainder results which is generally non-zero, assuming the initial guess is not an exact factor. The trial factor is transformed into an "exact" factor of the polynomial by an iterative process which modifies the coefficients of the quadratic term to make the remainder less than or equal to ϵ (error factor). The resulting roots can be found by using the quadratic formula. The original polynomial can now be reduced by dividing out the quadratic factor. The remaining roots of the reduced polynomial are computed in the same manner as above, with the program using the exact quadratic factor which was just found as the new guess or allowing the user the option of entering a quadratic factor for the new guess. If, in the course of finding the root pairs by dividing out the quadratic factors, the reduced polynomial has degree = 2, the remaining roots are found using the quadratic formula; if degree = 1, the obvious root is computed.

GENERAL NOTES REGARDING PROGRAM OPERATION

1. An initial estimate of $u = v = 0$ is recommended in searching for all roots. In general, there is no advantage in using previously computed values of u and v .
2. A plot of $f(x)$ may be used to find the approximate location of real roots. Then, Master Library program ML-08 may be used to find the exact location (within ϵ) of real roots. The order of $f(x)$ may then be lowered by dividing $f(x)$ by factors of the form $(x - R_i)$ where R_i are the real roots.
3. If $f(x)$ has multiple sets of equal roots, Δu and Δv will appear to be converging but then may never get smaller than the stated ϵ . The solution to this pathological case is to define a larger ϵ and thus accept a less precise approximation of the equal roots.
4. Do not assume divergence of Δu , Δv simply because monotonic convergence does not occur. Δu and Δv will often oscillate with large amplitude prior to convergence.

 Solid State Software TI © 1979	
ROOTS OF A POLYNOMIAL EE-09	
	→ roots(EG)
n	→ roots(CG) → Reduce P _n (x) Control

USER INSTRUCTIONS

STEP	PROCEDURE	ENTER	PRESS	DISPLAY
1	Select program		[2nd] [Pgm] 09	
2	Initialize		[SBR] [CLR]	0.
3	Repartition (TI-58 only)	6	[2nd] [Op] 17	0.59
4	Enter degree of polynomial	n ¹	[A]	n
5	Enter coefficient of x ⁿ , repeat until all coefficients have been entered.	a ₁ ² ⋮ a _{n+1}	[R/S]	(n-1) ⋮ -1.
6	Enter allowable error	ε	[B]	ε
7	Enter control digit	CTRL ³	[E]	CTRL
8	Enter initial estimate of u, then enter initial estimate of v in display	u v	[x≥t]	v
9	Calculate first two roots		[2nd] [C'] ⁴	Degree ^{5, †}
10	Calculate discriminant of quadratic		[R/S]	Disc. ^{6, †}
11	Calculate 1st root		[R/S]	Root 1 ^{6, †}
12	Calculate 2nd root		[R/S]	Root 2 ^{6, †}
13	Execute exit logic		[R/S]	2. ⁷
14	Reduce degree of polynomial by 2 by dividing P(x) by x ² - ux - v		[D]	1.
15	Calculate next pair ⁸ of roots		[C] ⁹	Degree ^{5, †}
16	Repeat steps 10 through 15 until all roots have been found			

- NOTES:**
- 2 ≤ n ≤ 21
 - All coefficients must be entered in sequence starting with the coefficient of xⁿ. Missing terms have coefficients = 0.
 - CTRL = 1: PRINT successive values of Δu and Δv and final values of u and v.
 CTRL = 0: Suppress output of Δu and Δv and final values of u and v.
 CTRL = -1: DISPLAY (FLASHING) successive values of Δu and Δv (use [R/S] to clear flashing and proceed with calculations of next Δu and Δv and final values of u and v, and degree of polynomial).
 CTRL = 1 or -1 is recommended if user is uncertain of convergence.
 - [2nd] [C'] is used to calculate a pair of roots when the user desires to specify initial estimates of u and v.
 - See Note 3 describing output which may occur prior to display of Degree = n.
 - If discriminant < 0, the following root calculations give the imaginary and real parts, respectively, of the complex conjugate roots. If discriminant ≥ 0, the following roots are both real.
 - Exit logic performs initialization of flags.
 - If reduced polynomial is of first degree, only one root exists.
 - [C] is used to calculate roots using previously calculated values of u and v.

[†]These values are printed if printer is connected.

Example: Find all roots of

$$x^5 - 17.8x^4 + 99.41x^3 - 261.218x^2 + 352.611x - 134.106$$

Set error limit (ϵ) at 1×10^{-9} , use 0 as initial guess for u and v.

The roots are: $x = 3.619868415$
 $x = 0.5801315846$
 $x = 10.3$
 $x = 1.65 \pm j 1.86480562$

ENTER	PRESS	DISPLAY	COMMENTS
	[2nd] [Pgm] 09		Select program
	[SBR] [CLR]	0.	Initialize
6	[2nd] [Op] 17	0.59	Repartition (TI-58 only)
5	[A]	5.	Enter degree of polynomial
1	[R/S]	4.	Enter coefficient of x^5
17.8	[+/-] [R/S]	3.	Enter coefficient of x^4
99.41	[R/S]	2.	Enter coefficient of x^3
261.218	[+/-] [R/S]	1.	Enter coefficient of x^2
352.611	[R/S]	0.	Enter coefficient of x
134.106	[+/-] [R/S]	-1.	Enter constant term
9	[+/-] [INV] [2nd] [log] [B]	0.000000001	Enter ϵ
1	[E]	1.	Enter CTRL (output Δu & Δv on printer)
0	[x \geq t]		Enter initial guess of u
0		0.	Enter initial guess of v
	[2nd] [C']		Find first two roots (see tape)
		5. [†]	
	[R/S]	9.24 [†]	Print discriminant value
	[R/S]	3.619868415 [†]	Print root 1 (real)
	[R/S]	.5801315846 [†]	Print root 2 (real).
	[R/S]	2.	Execute exit logic
	[D]	1.	Divide P(x) by $x^2 - ux - v$
0	[E]	0.	Change CTRL to suppress output of Δu & Δv
	[C]	3. [†]	Find next two roots using current values of u and v
	[R/S]	-13.91 [†]	Print discriminant value
	[R/S]	1.86480562 [†]	Print imaginary part of roots 3 and 4
	[R/S]	1.65 [†]	Print real part of roots 3 and 4
	[R/S]	2.	Execute exit logic
	[D]	1.	Divide P(x) by $x^2 - ux - v$
	[C]	1. [†]	Find final root
	[R/S]	10.3 [†]	Print root 5 (real)
	[R/S]	10.3	Execute exit logic

[†]These values are printed if printer is connected.

2.374099678	Δu
2.69134463	Δv
1.909294944	Δu
.0146947731	Δv
-.1559301479	Δu
-3.23729809	Δv
.0289236121	Δu
-1.250626589	Δv
.0395701278	Δu
-.2990634176	Δv
.0040246833	Δu
-.0189731566	Δv
.0000171029	Δu
-.0000781489	Δv
.0000000003	Δu
-.0000000013	Δv
-.0000000001	Δu
4.3203682-11	Δv
4.2	u
-2.1	v
5.	Degree
9.24	Discriminant
3.619868415	Real 1
.5801315846	Real 2
3.	Degree
-13.91	Discriminant
1.86480562	Imaginary Part 3 and 4
1.65	Real Part 3 and 4
1.	Degree
10.3	Real 5

Register Contents

R_{00}	a_{n+1} (constant term)
R_{01}	a_n (coefficient of x term)
↓	↓
R_{21}	a_1 (coefficient of x^{2^1} term)
R_{22}	Used
R_{23}	b_1
R_{24}	b_2
↓	↓
R_{44}	b_{22}
R_{45}	c_{i-2}
R_{46}	c_{i-1}
R_{47}	c_i
R_{48}	η
R_{49}	ϵ
R_{50}	Used
R_{51}	Used
R_{52}	u
R_{53}	v
R_{54}	Used
R_{55}	Used
R_{56}	Used
R_{57}	Used
R_{58}	Not used
R_{59}	Used

Method Used

The following is an outline of the Lin-Bairstow method.

Given a polynomial of degree n : $P_n(x) = a_1 x^n + a_2 x^{n-1} + \dots + a_n x + a_{n+1}$ and trial quadratic factor: $x^2 - ux - v$

$$\begin{aligned}
 P_n(x) &= (x^2 - ux - v) Q_{n-2}(x) + \text{remainder} \\
 &= (x^2 - ux - v)(b_1 x^{n-2} + b_2 x^{n-3} + \dots + b_{n-2} x + b_{n-1}) + b_n(x - u) + b_{n+1}
 \end{aligned}
 \tag{1}$$

Multiplying out and equating coefficients of like powers of x :

$$\begin{array}{ll}
 a_1 = b_1 & b_1 = a_1 \\
 a_2 = b_2 - ub_1 & b_2 = a_2 + ub_1 \\
 a_3 = b_3 - ub_2 - vb_1 & b_3 = a_3 + ub_2 + vb_1 \\
 \vdots & \vdots \\
 \vdots & \text{or} \quad \vdots \\
 \vdots & \vdots \\
 a_i = b_i - ub_{i-1} - vb_{i-2} & b_i = a_i + ub_{i-1} + vb_{i-2} \\
 \text{for } i = 4, \dots, n+1 & \text{for } i = 4, \dots, n+1
 \end{array}$$

Note: Remainder term from equation (1) is $b_n(x - u) + b_{n+1}$, and b_n, b_{n+1} both zero implies $(x^2 - ux - v)$ is an exact quadratic factor. Normally, this is not the case. Thus, values of u and v must be modified so that b_n and b_{n+1} both approach zero.

Since b_n and b_{n+1} are functions of parameters u and v , $b_n(u, v)$ and $b_{n+1}(u, v)$ can be expanded as a Taylor series for a function of two variables in terms of $(u^* - u) = \Delta u$ and $(v^* - v) = \Delta v$ where $\Delta u, \Delta v$ are presumed small so that terms of higher order than the first are negligible.

$$\begin{aligned}
 b_n(u^*, v^*) &= b_n(u, v) + \frac{\partial b_n}{\partial u} \Delta u + \frac{\partial b_n}{\partial v} \Delta v + \dots \\
 b_{n+1}(u^*, v^*) &= b_{n+1}(u, v) + \frac{\partial b_{n+1}}{\partial u} \Delta u + \frac{\partial b_{n+1}}{\partial v} \Delta v + \dots
 \end{aligned}$$

Taking (u^*, v^*) as the point where the remainder is zero, (Δu and Δv being the increments added the original u and v to get the new values u^* and v^*),

$$b_n(u^*, v^*) = 0 \approx b_n + \frac{\partial b_n}{\partial u} \Delta u + \frac{\partial b_n}{\partial v} \Delta v \tag{2}$$

$$b_{n+1}(u^*, v^*) = 0 \approx b_{n+1} + \frac{\partial b_{n+1}}{\partial u} \Delta u + \frac{\partial b_{n+1}}{\partial v} \Delta v \tag{3}$$

The problem now is to solve equations (2) and (3) simultaneously for Δu and Δv . Bairstow showed that the required partial derivatives can be obtained from the b 's in the same way that the b 's were obtained from the a 's.

Define a set of c 's as follows:

$$\begin{array}{lll}
 c_1 = b_1 & \frac{\partial b_1}{\partial u} = \frac{\partial a_1}{\partial u} = 0 & \frac{\partial b_1}{\partial v} = \frac{\partial a_1}{\partial v} = 0 \\
 c_2 = b_2 + uc_1 & \frac{\partial b_2}{\partial u} = \frac{\partial b_1}{\partial u} u + b_1 = b_1 = c_1 & \frac{\partial b_2}{\partial v} = \frac{\partial a_2}{\partial v} + \frac{\partial b_1}{\partial v} u = 0 \\
 c_3 = b_3 + uc_2 + vc_1 & \frac{\partial b_3}{\partial u} = \frac{\partial b_2}{\partial u} u + b_2 = c_2 & \frac{\partial b_3}{\partial v} = \frac{\partial b_2}{\partial v} u + \frac{\partial b_1}{\partial v} v + b_1 = b_1 = c_1 \\
 \vdots & \vdots & \vdots \\
 \vdots & \vdots & \vdots
 \end{array}$$

The above three columns can be generalized by the following recurrence relation:

$$\begin{aligned}
 c_i &= b_i + uc_{i-1} + vc_{i-2} \\
 &\text{for } i = 1, n
 \end{aligned}$$

where $\frac{\partial b_i}{\partial u} = \frac{\partial b_{i+1}}{\partial v} = c_{i-1}$

Equations (2) and (3) can be rewritten as:

$$\begin{aligned}
 -b_n &= c_{n-1} \Delta u + c_{n-2} \Delta v \\
 -b_{n+1} &= c_n \Delta u + c_{n-1} \Delta v
 \end{aligned}$$

Then, using Cramer's rule

$$\begin{aligned}
 \Delta u &= \frac{\begin{vmatrix} -b_n & c_{n-2} \\ -b_{n+1} & c_{n-1} \end{vmatrix}}{\begin{vmatrix} c_{n-1} & c_{n-2} \\ c_n & c_{n-1} \end{vmatrix}} \\
 \Delta v &= \frac{\begin{vmatrix} c_{n-1} & -b_n \\ c_n & -b_{n+1} \end{vmatrix}}{\begin{vmatrix} c_{n-1} & c_{n-2} \\ c_n & c_{n-1} \end{vmatrix}}
 \end{aligned}$$

EE-09

If denominator = 0, $\Delta u = 1 + \Delta u'$, $\Delta v = 1 + \Delta v'$ where $\Delta u'$, $\Delta v'$ are values of Δu , Δv computed from previous iteration.

If $|\Delta u| \leq \epsilon$ and $|\Delta v| \leq \epsilon$ where ϵ is the allowable error factor, then $(x^2 - ux - v)$ is a factor of $P(x)$. Roots of $(x^2 - ux - v)$ are found by using the quadratic formula.

$$\frac{P_n(x)}{(x^2 - ux - v)} = Q_{n-2}(x), \text{ the reduced polynomial used to find the remaining roots.}$$

Reference:

Applied Numerical Analysis, Curtis F. Gerald, Addison-Wesley, 1970.

CHAINED MULTIPLICATION OF POLYNOMIALS

This program performs the multiplication of two polynomials $P_1 \times P_2 = P_3$. The polynomials (P) are of the form $a_0 + a_1x + a_2x^2 + \dots + a_nx^n$ where the coefficients are real numbers. The resulting product P_3 may subsequently be multiplied by another polynomial. This chained multiplication process $(P_1 \times P_2 \times P_3 \times \dots \times P_k) = P_{\text{product}}$ can be repeated until the limits of the calculator are reached. The degree of the final product polynomial must be ≤ 25 for the TI-58 and ≤ 40 for the TI-59. The intermediate products may be output, if desired.

Solid State Software TI © 1979				
CHAINED MULTIPLICATION OF POLYNOMIALS EE-10				
Exit			No Print	TI-58
Initialize	Multiply	→ Product	Print	TI-59

USER INSTRUCTIONS

STEP	PROCEDURE	ENTER	PRESS	DISPLAY
1	Select program		[2nd] [Pgm] 10	
2a	Printer used		[D]	
2b	Printer not used		[2nd] [D']	
3a	TI-59 used ¹		[E]	
3b	TI-58 used ¹		[2nd] [E']	
4	Initialize		[A]	0. ²
5	Enter the coefficients of P_1 starting with a_0 . Repeat for all coefficients.		[R/S]	1. ²
6	Prepare to multiply		[B]	0. ²
7	Enter the coefficients of P_2 starting with a_0 . Repeat for all coefficients. ^{3,4}		[R/S]	1. ²
8a	Output product with printer ⁵		[C]	a_0, a_1, \dots are printed
8b	1. Output product without printer ⁵		[C]	No. of coefficients in product
	2. Display a_0 (Repeat for a_1, a_2, \dots, a_n)		[R/S]	a_0, a_1, \dots, a_n
9	Exit the program ⁶		[2nd] [A']	479.59 or 239.29

- NOTES:**
1. A TI-59 will be partitioned 239.89, for products of up to 40th degree.
A TI-58 will be partitioned 0.59, for products of up to 25th degree.
 2. The number in the display shows the power of x for the coefficient to be entered. If this power of x is missing in the polynomial, a 0 must be entered as its coefficient.
 3. Wait until the next power of x is displayed (when actual multiplication is completed).
 4. If another polynomial is to be multiplied, repeat steps 6 and 7.
 5. Intermediate products may be output.
 6. [2nd] [A'] must be used to return to standard partitioning and to reset flags.

EE-10

Example: Multiply the three polynomials $(3 + 2x - 5x^2)(x)(4 - x^2)$ in order from left to right. Rewriting these polynomials to include zero coefficients, we have $(3 + 2x - 5x^2)(0 + x)(4 + 0x + x^2)$.

ENTER	PRESS	DISPLAY	COMMENTS	PRINTOUT
	[2nd] [Pgm] 10		Select program	
	[D]		Assume printer	
	[E]		Assume TI-59	
	[A]	0.	Initialize	
3	[R/S]	1.	Enter constant term	
2	[R/S]	2.	Enter coefficient of x	
5	[+/-] [R/S]	3.	Enter coefficient of x ²	
	[B]	0.	Prepare to multiply	
0	[R/S]	1.	Enter constant term	
1	[R/S]	2.	Enter coefficient of x	
	[C]		Print intermediate product	0.
				3.
				2.
		-5.		-5.
	[B]	0.	Prepare to multiply	
4	[R/S]	1.	Enter constant term	
0	[R/S]	2.	Enter coefficient of x	
1	[+/-] [R/S]	3.	Enter coefficient of x ²	
	[C]		Print final product	0.
				12.
				8.
				-23.
				-2.
		5.		5.
	[2nd] [A']	479.59	Exit program	

Register Contents

R₀₀ Pointer to product/input area

R₀₁ Number of coefficients in product area

R₀₂ Next power of x to be input

R₀₃ Used

R₀₄ Used

R₀₅ Used

R₀₆ Used

R₀₇ New Coefficient

R₀₈ Coefficients

↓ of

R₄₈ Product

R₄₉ Coefficients

↓ of

R₈₉ Multiplicand

TI-59

R₀₈ Coefficients

↓ of

R₃₃ Product

R₃₄ Coefficients

↓ of

R₅₉ Multiplicand

TI-58

REACTANCE CHART

This program simulates a reactance chart by computing the capacitance C in farads, capacitive reactance X_C in ohms, inductance L in henrys and the inductive reactance X_L in ohms at an applied frequency f in hertz. The equations used are:

$$X_C = \frac{1}{\omega C} \quad C = \frac{1}{\omega X_C} \quad X_L = \omega L \quad L = \frac{X_L}{\omega} \quad \omega = 2\pi f$$


$$f = \frac{1}{2\pi X_C C} = \frac{X_L}{2\pi L}$$

If $f = f_0$, where f_0 is the frequency of resonance ($X_C = X_L$), the following relationships apply:

$$f_0 = \frac{1}{2\pi\sqrt{LC}} \quad C = \frac{1}{\omega_0^2 L} \quad L = \frac{1}{\omega_0^2 C}$$

Given any two of the elements in the sets $\{f, X_C, C\}$, $\{f, X_L, L\}$, or $\{f_0, L, C\}$ which are comprised of the quantities used in the defining equations for frequency, this program computes the remaining element:

Given	Compute
C, f	X_C
X_C, f	C
L, f	X_L
X_L, f	L
C, X_C	f
L, X_L	f
C, f_0	L
L, f_0	C
C, L	f_0

 Solid State Software TI © 1979	
REACTANCE CHART	
f_0	X_L
f	X_C
EE-11	
→ Execute	

USER INSTRUCTIONS

STEP	PROCEDURE	ENTER	PRESS	DISPLAY
1	Select program		[2nd] [Pgm] 11	
2	Initialize		[SBR] [CLR] [2nd] [Eng]	0
3	Enter any two of the following:			
	a. Applied frequency or Resonant frequency in hertz	f f_0	[A] [2nd] [A']	f f_0
	b. Inductance in henrys	L	[B]	L
	c. Inductive reactance in ohms	X_L	[2nd] [B']	X_L
	d. Capacitance in farads	C	[C]	C
	e. Capacitive reactance in ohms	X_C	[2nd] [C']	X_C
4	Compute remaining quantity in the frequency equation ¹			
	a. Applied frequency or Resonant frequency in hertz		[E] [A] [E] [2nd] [A']	f f_0
	b. Inductance in henrys		[E] [B]	L
	c. Inductive reactance in ohms		[E] [2nd] [B']	X_L
	d. Capacitance in farads		[E] [C]	C
	e. Capacitive reactance in ohms		[E] [2nd] [C']	X_C

NOTES: 1. Use computation chart as a guide.

Example 1: What is the reactance of 15 picofarads at 27 megahertz? What is the reactance of 10 microhenrys at 27 megahertz?

ENTER	PRESS	DISPLAY	COMMENTS
	[2nd] [Pgm] 11		Select program
	[SBR] [CLR] [2nd] [Eng]*	0. 00	Initialize
15	[EE] 12 [+/-] [C]	15.-12	Enter C in farads
27	[EE] 6 [A]	27. 06	Enter f in hertz
	[E] [2nd] [C']	392.97517 00	Compute X_C in ohms
10	[EE] 6 [+/-] [B]	10.-06	Enter L in henrys
	[E] [2nd] [B']	1.69646 03	Compute X_L in ohms
	[INV] [2nd] [Eng]*	1696.460033	Remove engineering mode

Example 2: With $L = 10$ microhenrys, what value of C is required to resonate at 27 megahertz? With $C = 15$ picofarads, what value of L is required to resonate at 27 megahertz?

	[2nd] [Pgm] 11		Select program
	[SBR] [CLR] [2nd] [Eng]*	0. 00	Initialize
27	[EE] 6 [2nd] [A']	27. 06	Enter f_0 in hertz
10	[EE] 6 [+/-] [B]	10.-06	Enter L in henrys
	[E] [C]	3.4746634-12	Compute C in farads
15	[EE] 12 [+/-] [C]	15.-12	Enter C in farads
	[E] [B]	2.3164422-06	Compute L in henrys
	[INV] [2nd] [Eng]*	.0000023164	Remove engineering mode

Example 3: With $C = 15$ picofarads and $L = 10$ microhenrys, what is the resonant frequency? If $C = 17$ picofarads and $X_C = 400$ ohms, what is the applied frequency, f ?

	[2nd] [Pgm] 11		Select program
	[SBR] [CLR] [2nd] [Eng]*	0. 00	Initialize
15	[EE] 12 [+/-] [C]	15.-12	Enter C in farads
10	[EE] 6 [+/-] [B]	10.-06	Enter L in henrys
	[E] [2nd] [A']	12.994947 06	Compute f_0 in hertz
17	[EE] 12 [+/-] [C]	17.-12	Enter C in farads
400	[2nd] [C']	400. 00	Enter X_C in ohms
	[E] [A]	23.405139 06	Compute f in hertz
	[INV] [2nd] [Eng]*	23405138.69	Remove engineering mode

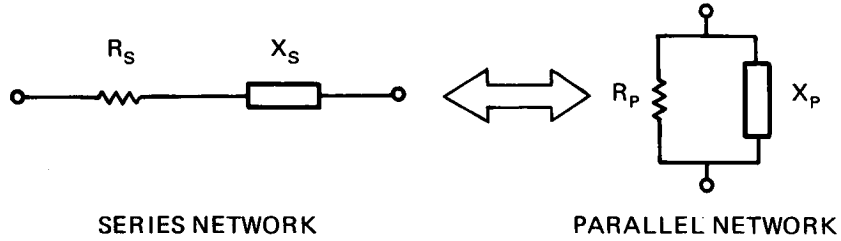
Register Contents

R_{00}	R_{05}	R_{10} C	R_{15}	R_{20}	R_{25}
R_{01}	R_{06}	R_{11} L	R_{16}	R_{21}	R_{26}
R_{02}	R_{07}	R_{12} X_C	R_{17}	R_{22}	R_{27}
R_{03}	R_{08}	R_{13} X_L	R_{18}	R_{23}	R_{28}
R_{04}	R_{09}	R_{14} $f \rightarrow \omega$	R_{19}	R_{24}	R_{29}

*These examples show optional use of the engineering mode. Be sure to remove engineering mode when computations are completed.

SERIES/PARALLEL IMPEDANCE CONVERSIONS

This program converts a two-element series impedance network to an equivalent two-element parallel impedance network, or vice versa. The conversion is illustrated by the following diagram where R_S , X_S , R_P , and X_P refer to the series resistance, series reactance, parallel resistance, and parallel reactance in ohms, respectively.



Any two-element impedance network can be represented equivalently as a series network or a parallel network.

The transformation equations are:

Given R_S and X_S


$$R_P = R_S + \frac{X_S^2}{R_S}$$

$$X_P = \frac{R_P R_S}{X_S}$$

Given R_P and X_P

$$R_S = \left(\frac{1}{R_P} + \frac{R_P}{X_P^2} \right)^{-1}$$

$$X_S = \frac{R_S R_P}{X_P}$$

 Solid State Software TI © 1979				
SERIES/PARALLEL IMPEDANCE CONVERSIONS EE-12				
R_S	X_S	R_P	X_P	→ Execute

USER INSTRUCTIONS

STEP	PROCEDURE	ENTER	PRESS	DISPLAY
1	Select program		[2nd] [Pgm] 12	
2	Initialize		[SBR] [CLR]	
	Do either Step 3 or Step 4			
3	Enter the following in any order			
	a. Series resistance in ohms	R_S	[A]	R_S
	b. Series reactance in ohms	X_S	[B]	X_S
	Compute the following in any order			
	c. Parallel resistance in ohms		[E] [C]	R_P
	d. Parallel reactance in ohms		[E] [D]	X_P
4	Enter the following in any order			
	a. Parallel resistance in ohms	R_P	[C]	R_P
	b. Parallel reactance in ohms	X_P	[D]	X_P
	Compute the following in any order			
	c. Series resistance in ohms		[E] [A]	R_S
	d. Series reactance in ohms		[E] [B]	X_S

EE-12

Example: Using an RX impedance meter, R_p of 75 ohms and C_p of 25 picofarads is measured at 125 megahertz. In order to solve our circuit application for purposes of power dissipation, an equivalent R_s and X_s would be easier to work with. Program EE-11 is used to solve for X_p before making the conversion.

ENTER	PRESS	DISPLAY	COMMENTS
	[2nd] [Pgm] 11		Select program
	[SBR] [CLR] [2nd] [Eng]*	0. 00	Initialize
125	[EE] 6 [A]	125. 06	Enter frequency in hertz
25	[EE] 12 [+/-] [C]	25. -12	Enter capacitance in farads
	[E] [2nd] [C']	50.929582 00	Compute X_p in ohms
	[2nd] [Pgm] 12	50.929582 00	Select program
	[SBR] [CLR]	0. 00	Initialize
75	[C]	75. 00	Enter R_p in ohms
50.929582	[D]	50.929582 00	Enter X_p in ohms
	[E] [A]	23.669653 00	Compute R_s in ohms
	[E] [B]	34.856441 00	Compute X_s in ohms

At this point X_s can be converted to C_s using program EE-11.

	[2nd] [Pgm] 11	34.856441 00	Select program
	[SBR] [CLR]	0. 00	Initialize
125	[EE] 6 [A]	125. 06	Enter frequency
34.856441	[2nd] [C']	34.856441 00	Enter X_s
	[E] [C]	36.528099 -12	Compute C_s

The results can be checked by converting R_s and X_s back to R_p and X_p

	[2nd] [Pgm] 12	36.528099 -12	Select program
	[SBR] [CLR]	0. 00	Initialize
23.669653	[A]	23.669653 00	Enter R_s
34.856441	[B]	34.856441 .00	Enter X_s
	[E] [C]	74.999999 00	Compute R_p
	[E] [D]	50.929581 00	Compute X_p
	[INV] [2nd] [Eng]*	50.929581	Remove engineering mode

Register Contents

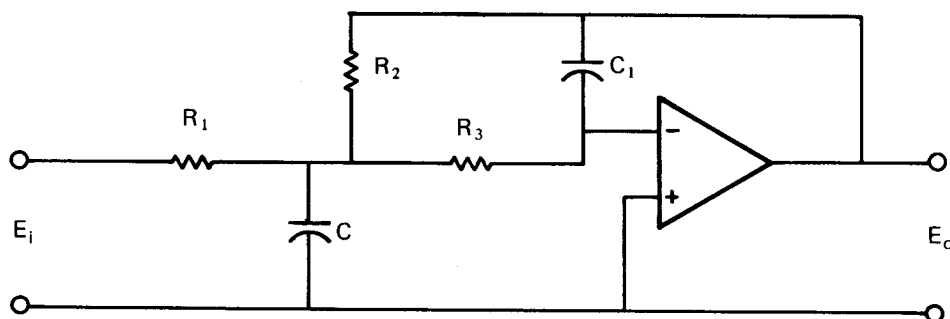
R_{00}	R_{05}	R_{10}	R_s	R_{15}	R_{20}	R_{25}
R_{01}	R_{06}	R_{11}	X_s	R_{16}	R_{21}	R_{26}
R_{02}	R_{07}	R_{12}	R_p	R_{17}	R_{22}	R_{27}
R_{03}	R_{08}	R_{13}	X_p	R_{18}	R_{23}	R_{28}
R_{04}	R_{09}	R_{14}		R_{19}	R_{24}	R_{29}

*This example shows optional use of engineering mode. Be sure to remove engineering mode when computations are completed.

ACTIVE LP, HP, BP FILTERS

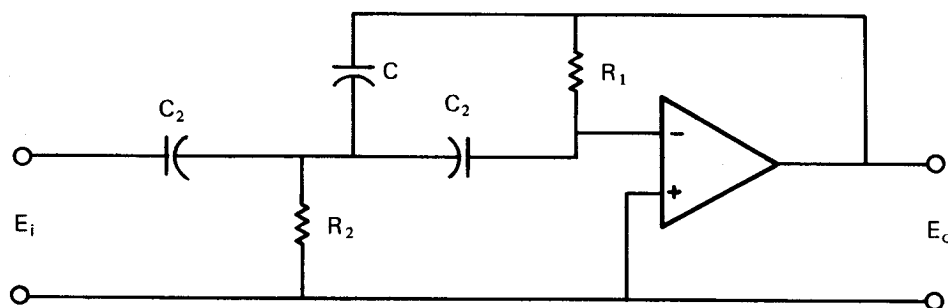
This program computes the necessary component values for use in the design of active lowpass, highpass, and bandpass filters as illustrated.

Active Lowpass Filter



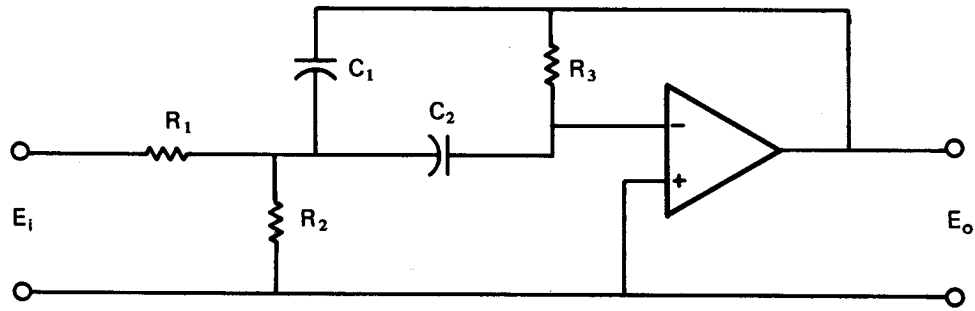
Given the peaking factor, the passband voltage gain in dB, the cutoff frequency in Hz, and C_1 in farads, the program calculates values of C , R_1 , R_2 , and R_3 .

Active Highpass Filter



Given the peaking factor, the passband voltage gain in dB, the cutoff frequency in Hz, and C_2 in farads, the program calculates values for C , R_1 , and R_2 .

Active Bandpass Filter



Given the 3-dB bandwidth in Hz, the midband voltage gain in dB, the center frequency in Hz, and C_1 and C_2 in farads, the program computes values for R_1 , R_2 , and R_3 .

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ACTIVE LP,HP,BP FILTERS				EE-13
C_1	C_2	→ BP	→ LP	→ HP
α	A	F	B	

USER INSTRUCTIONS

STEP	PROCEDURE	ENTER	PRESS	DISPLAY
1	Select program		[2nd] [Pgm] 13	
2	Initialize		[SBR] [CLR]	0.
3	Enter the following as needed:			
	a. peaking factor (LP, HP) ¹	α	[A]	α^\dagger
	b. passband voltage gain (LP, HP) or midband voltage gain (BP) in dB ²	A	[B]	A [†]
	c. cutoff frequency (LP, HP) or center frequency (BP) in hertz ¹	F	[C]	F [†]
	d. 3-dB bandwidth (BP) in hertz ¹	B	[D]	B [†]
	e. C_1 for (BP, LP) in farads ¹	C_1	[2nd] [A']	C_1^\dagger
	f. C_2 for (BP, HP) in farads ¹	C_2	[2nd] [B']	C_2^\dagger
4	Compute component values:			
	a. Bandpass ^{3,5}	R_1 in ohms	[2nd] [C']	R_1^\dagger
		R_2 in ohms	[R/S]	R_2^\dagger
		R_3 in ohms	[R/S]	R_3^\dagger
	b. Lowpass ^{4,5}	C in farads	[2nd] [D']	C [†]
		R_2 in ohms	[R/S]	R_2^\dagger
		R_1 in ohms	[R/S]	R_1^\dagger
		R_3 in ohms	[R/S]	R_3^\dagger
	c. Highpass ⁴	C in farads	[2nd] [E']	C [†]
		R_1 in ohms	[R/S]	R_1^\dagger
		R_2 in ohms	[R/S]	R_2^\dagger

- NOTES:**
- Input value must be greater than zero.
 - Because operational amplifiers are non-ideal, A should be chosen to insure H_0 is less than 10 when α is about 0.1. H_0 may be increased if α is also increased. A maximum H_0 of 100 is acceptable with $\alpha=1$ for an operational amplifier with an open-loop gain of at least 80 dB.
 - If $Q \leq (H_0/2)^{1/2}$, program execution stops with a flashing nines display. To insure that all resistances are greater than zero, either $Q = F/B$, or H_0 should be adjusted so that $Q > (H_0/2)^{1/2}$. See steps 3b, 3c, and 3d.
 - A 12-dB per octave roll-off exists in the lowpass, highpass cases.
 - The value of R_1 calculated includes source resistance, i.e., actual $R_1 = \text{Calculated } R_1 - \text{Source } R$.

[†]These values are printed if printer is connected.

EE-13

Example 1: Find the component values for an active bandpass filter with a center frequency of 150 hertz, a midband voltage gain of 30 dB, a 3-dB bandwidth of 16 hertz and $C_1=C_2=0.1\mu F$.

ENTER	PRESS	DISPLAY	COMMENTS	PRINTOUT
	[2nd] [Pgm] 13		Select program	
	[SBR] [CLR]	0.	Initialize	
16	[D]	16.	Enter 3-dB bandwidth	16.
30	[B]	30.	Enter midband voltage gain	30.
150	[C]	150.	Enter center frequency	150.
.0000001	[2nd] [A']	0.0000001	Enter C_1	0.0000001
.0000001	[2nd] [B']	0.0000001	Enter C_2	0.0000001
	[2nd] [C']	3145.575757	R_1	3145.575757 R1
	[R/S]	690.017292	R_2	690.017292 R2
	[R/S]	198943.6789	R_3	198943.6789 R3

Example 2: Find the component values for an active highpass filter with cutoff frequency = 400 hertz, passband voltage gain = 6 dB, peaking factor = 0.5 and $C_2 = 0.047 \mu F$.

	[2nd] [Pgm] 13		Select program	
	[SBR] [CLR]	0.	Initialize	
.5	[A]	0.5	Enter peaking factor	0.5
6	[B]	6.	Enter passband gain	6.
400	[C]	400.	Enter cutoff frequency	400.
.000000047	[2nd] [B']	0.000000047	Enter C_2	0.000000047
	[2nd] [E']	.0000000236	C	.0000000236 C
	[R/S]	84496.45356	R_1	84496.45356 R1
	[R/S]	1692.334014	R_2	1692.334014 R2

Example 3: Find the component values for an active lowpass filter with peaking factor = $\sqrt{2}$, passband gain = 20 dB, cutoff frequency = 1000 hertz and $C_1 = 0.02 \mu F$.

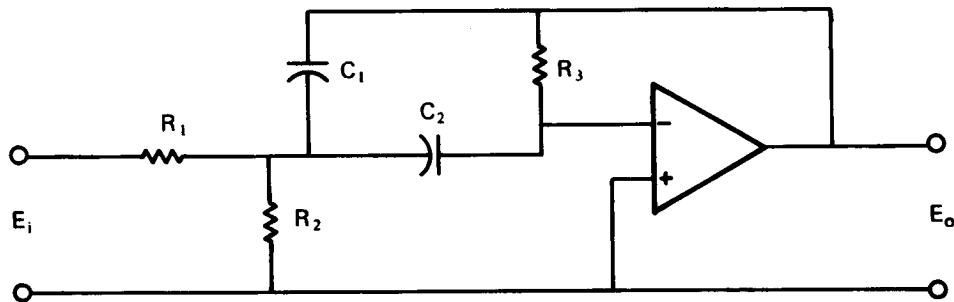
	[2nd] [Pgm] 13		Select program	
	[SBR] [CLR]	0.	Initialize	
2	[\sqrt{x}] [A]	1.414213562	Enter peaking factor	1.414213562
20	[B]	20.	Enter passband gain	20.
1000	[C]	1000.	Enter cutoff frequency	1000.
.00000002	[2nd] [A']	0.00000002	Enter C_1	0.00000002
	[2nd] [D']	0.00000044	C	0.00000044 C
	[R/S]	5626.976976	R_2	5626.976976 R2
	[R/S]	562.6976976	R_1	562.6976976 R1
	[R/S]	511.5433615	R_3	511.5433615 R3

Register Contents

R_{00}	R_{05}	R_{10} α	R_{15} Q	R_{20}	R_{25}
R_{01}	R_{06}	R_{11} $10^{A/20}$	R_{16} B	R_{21}	R_{26}
R_{02}	R_{07}	R_{12} F	R_{17} ω_0	R_{22}	R_{27}
R_{03}	R_{08}	R_{13} C_1	R_{18} R_2	R_{23}	R_{28}
R_{04}	R_{09}	R_{14} C_2	R_{19}	R_{24}	R_{29}

Method Used

ACTIVE BANDPASS FILTER



The voltage transfer function is:

$$\frac{E_o}{E_i}(s) = \frac{-(1/R_1 C_1) s}{s^2 + \left(\frac{C_1 + C_2}{R_3 C_1 C_2}\right) s + \frac{R_1 + R_2}{R_1 R_2 R_3 C_1 C_2}}$$

and the corresponding bandpass network function is:

$$H(s) = \frac{-H_0 \alpha \omega_0 s}{s^2 + \alpha \omega_0 s + \omega_0^2}$$

where

$H_0 = 10^{A/20}$, A = midband voltage gain in dB

$\alpha = 1/Q$

$Q = F/B$, quality factor measure of selectivity of filter

F = Center frequency of passband in hertz

B = 3-dB bandwidth in hertz

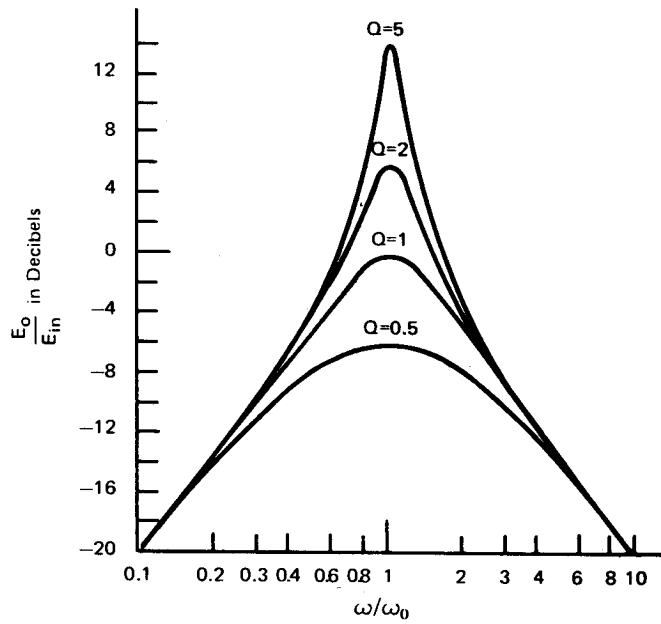
$\omega_0 = 2\pi F$

Given B , A , F , C_1 and C_2 in farads, the program calculates R_1 , R_2 and R_3 using the following equations:

$$R_1 = \frac{Q}{2\pi F H_0 C_1}$$

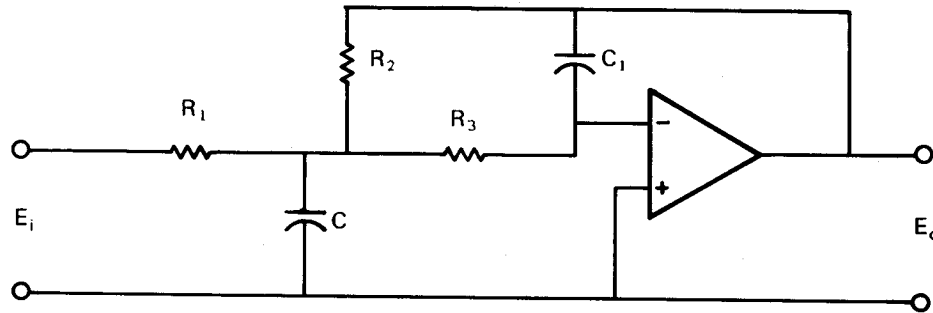
$$R_2 = [Q(C_1 + C_2)2\pi F - (1/R_1)]^{-1}$$

$$R_3 = \frac{Q}{2\pi F} \left(\frac{1}{C_1} + \frac{1}{C_2} \right)$$



Magnitude response of second order bandpass filter showing effect of Q on network voltage transfer function.

ACTIVE LOWPASS FILTER



The voltage transfer function is:

$$\frac{E_o}{E_i}(s) = \frac{-1/R_1 R_3 C C_1}{s^2 + \left(\frac{1}{R_1} + \frac{1}{R_3} + \frac{1}{R_2}\right) \left(\frac{1}{C}\right)s + \frac{1}{R_2 R_3 C C_1}}$$

The corresponding lowpass network function is:

$$H(s) = \frac{-H_0 \omega_0^2}{s^2 + \alpha \omega_0 s + \omega_0^2}$$

where

$H_0 = 10^{A/20}$, A = passband gain in dB
 $\omega_0 = 2\pi F$
 F = cutoff frequency in hertz

$\alpha = \text{peaking factor} = 2\zeta$
 $\zeta = \text{damping ratio}$

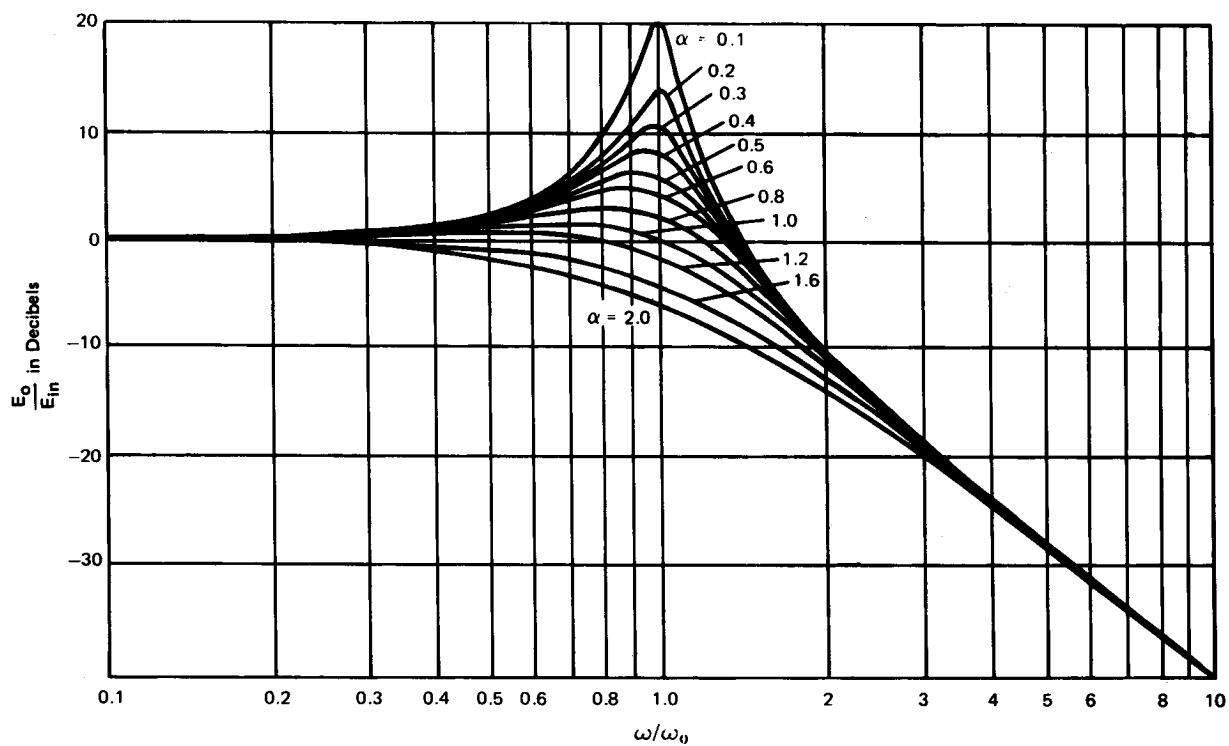
Given A , F , α and C_1 in farads, the program calculates R_1 , R_2 , R_3 and C using the following equations:

$$C = \frac{4(1 + H_0)C_1}{\alpha^2}$$

$$R_1 = \frac{R_2}{H_0}$$

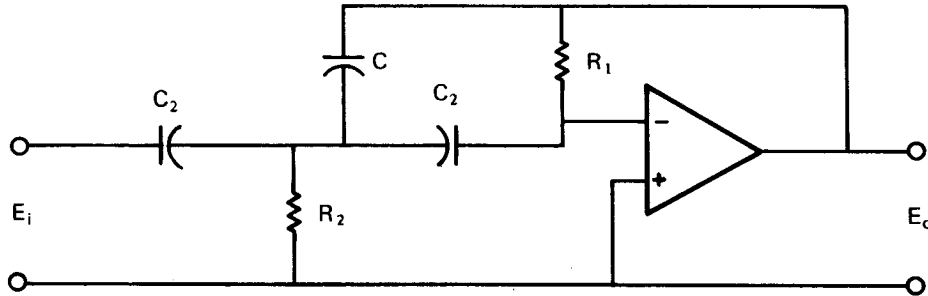
$$R_2 = \frac{\alpha}{4\pi F C_1}$$

$$R_3 = \frac{R_2}{H_0 + 1}$$



Magnitude response of second-order lowpass filters for several values of $\alpha \leq 2$.

ACTIVE HIGHPASS FILTER



The voltage transfer function is:

$$\frac{E_o}{E_i}(s) = \frac{-(C_2/C)s^2}{s^2 + \left(\frac{2}{C} + \frac{1}{C_2}\right) \left(\frac{1}{R_1}\right)s + \frac{1}{R_1 R_2 C_2 C}}$$

and the corresponding highpass network function is:

$$H(s) = \frac{-H_0 s^2}{s^2 + \alpha \omega_0 s + \omega_0^2}$$

where

$$H_0 = 10^{A/20}, \quad A = \text{passband gain in dB}$$

$$\omega_0 = 2\pi F$$

$$F = \text{cutoff frequency in hertz}$$

$$\alpha = \text{peaking factor} = 2\zeta$$

$$\zeta = \text{damping ratio}$$

Given A , F , α and C_2 , the capacitance in farads, the program calculates R_1 , R_2 and C using the following equations:

$$C = \frac{C_2}{H_0}$$

$$R_1 = \frac{2H_0 + 1}{2\pi F \alpha C_2}$$

$$R_2 = \frac{\alpha}{2\pi F C_2 (2 + 1/H_0)}$$

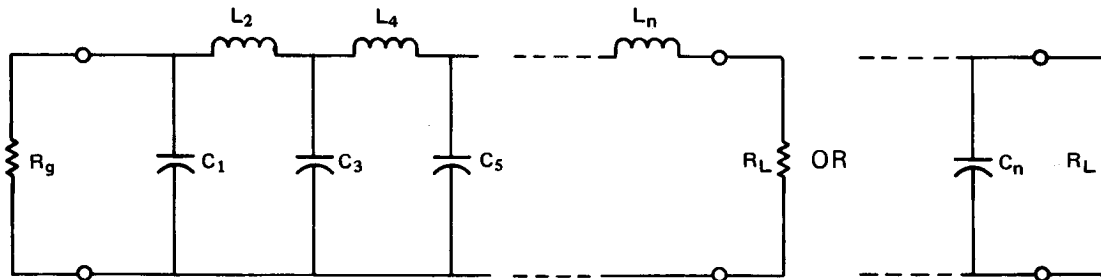
References: *Operational Amplifiers Design and Applications*, Graeme, Tobey and Huelsman, McGraw-Hill, 1971.

Theory and Design of Active RC Circuits, Huelsman, McGraw-Hill, 1968

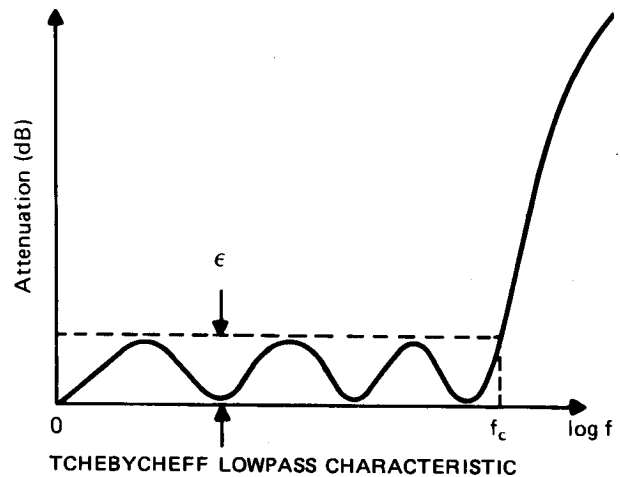
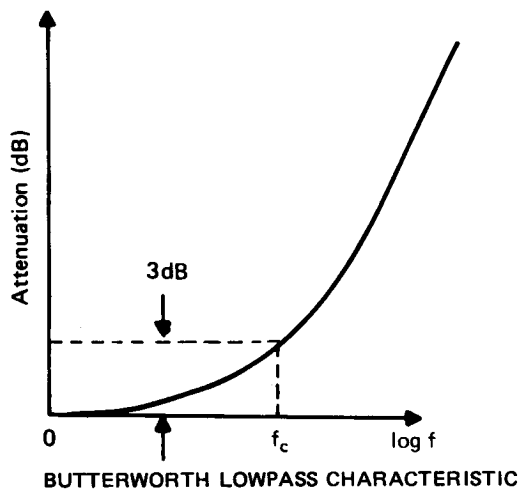
Electronics, pp 86-89, June 7, 1971

PASSIVE LOWPASS FILTERS

This program computes component values for passive lowpass Butterworth and Tchebycheff filters of the following form.



Required inputs are the filter order n (n , an integer ≥ 1) which defines the number of reactive components, allowable ripple ϵ in dB ($\epsilon = 0$ for Butterworth filters), terminal resistance R in ohms, and the cutoff frequency f_c in hertz.



For Butterworth filters with response of the form shown, the component values are computed as follows:

$$C_i = \frac{1}{\pi f_c R} \sin \left[\frac{(2i - 1)\pi}{2n} \right] \quad i = 1, 3, 5, \dots$$

$$L_i = \frac{R}{\pi f_c} \sin \left[\frac{(2i - 1)\pi}{2n} \right] \quad i = 2, 4, 6, \dots$$

EE-14

For Tchebycheff filters with response of the form shown, the component values are computed as follows:

$$\beta = \ln \left[\coth \left(\frac{\epsilon}{40 \log e} \right) \right]$$

$$\gamma = \sinh \left(\frac{\beta}{2n} \right)$$

$$a_i = \sin \left[\frac{(2i-1)\pi}{2n} \right] \quad i = 1, 2, 3, \dots, n$$

$$b_i = \gamma^2 + \sin^2 \left(\frac{i\pi}{n} \right) \quad i = 1, 2, 3, \dots, n$$

$$g_1 = \frac{2a_1}{\gamma}$$

$$g_i = \frac{4a_{i-1} a_i}{b_{i-1} g_{i-1}} \quad i = 2, 3, 4, \dots, n$$

$$C_i = \frac{g_i}{2\pi f_c R} \quad i = 1, 3, 5, \dots$$

$$L_i = \frac{R g_i}{2\pi f_c} \quad i = 2, 4, 6, \dots$$

NOTE: This program assumes that the generator resistance R_g is equal to the load resistance R_L for Butterworth filters and odd-order Tchebycheff filters. Thus, the termination resistance $R = R_g = R_L$. In the case of even-order Tchebycheff filters, $R_L = R_g / \coth^2(\beta/4)$ which, for small ripple results in the generator resistance R_g being approximately equal to the load resistance R_L . Thus, the termination resistance $R = R_g \approx R_L$.

References: *Handbook of Filter Synthesis*, Zverev, John Wiley and Sons, 1967

Microwave Filters, Impedance-Matching Networks, and Coupling Structures, Matthai, Young, Jones, McGraw-Hill, 1964

Solid State Software TI © 1979			
PASSIVE LOWPASS FILTERS			EE-14
			→ No Print
n	ε	R	→ Print

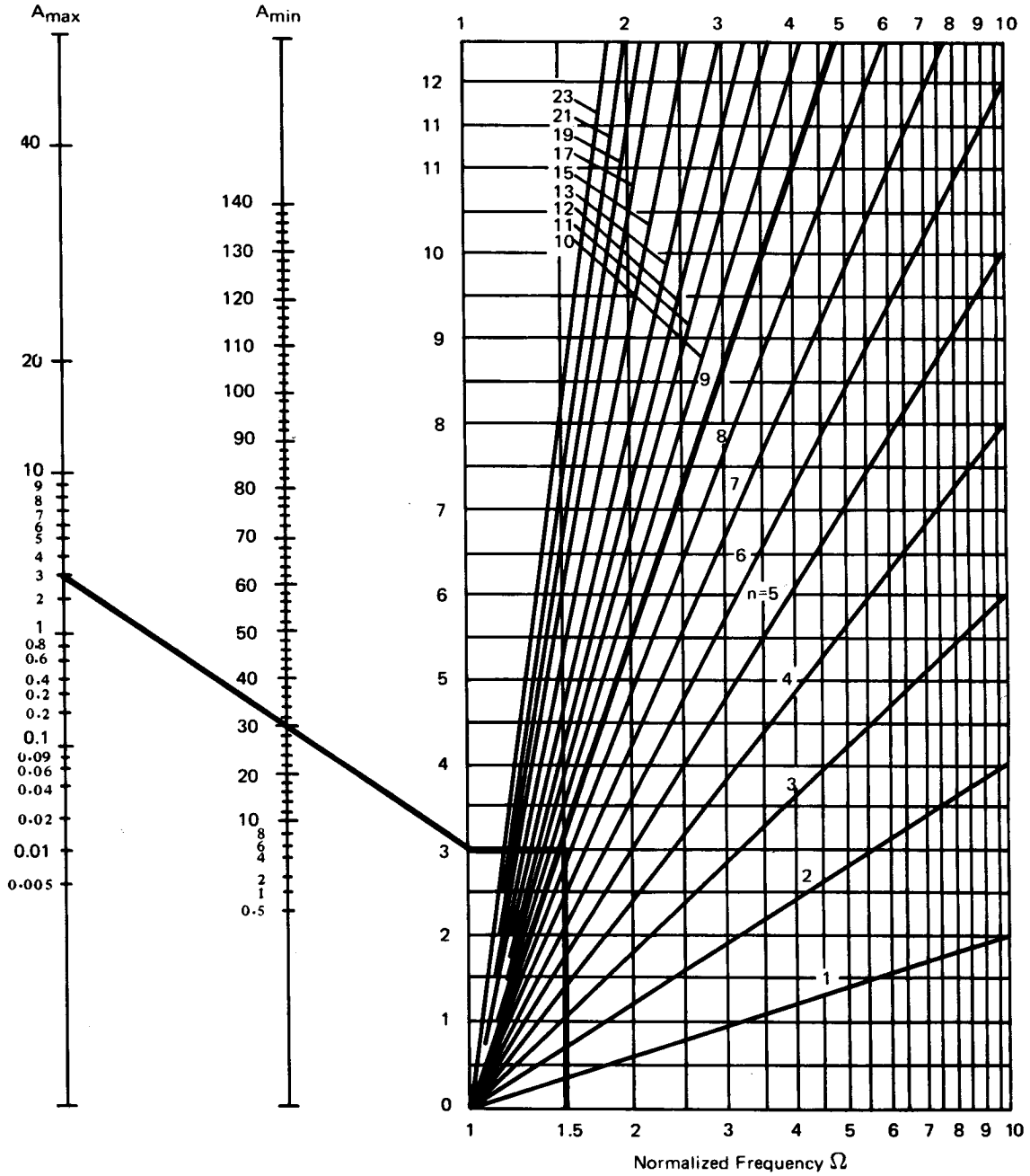
USER INSTRUCTIONS

STEP	PROCEDURE	ENTER	PRESS	DISPLAY
1	Select program		[2nd] [Pgm] 14	
2	Initialize		[SBR] [CLR]	0.
3	Enter the following in any order:			
	a. Order of the filter ¹	n	[A]	n
	b. Allowable ripple in dB ³	ε	[B]	ε
	c. Termination resistance in ohms ²	R	[C]	R
	d. Cutoff frequency in hertz ²	f _c	[D]	f _c
4	Compute and display component values. C is in farads. L is in henrys.		[2nd] [E']	"1." ^{4,5}
	OR		[R/S]	C ₁
			[R/S]	"2."
			[R/S]	L ₂
			[R/S]	"3."
			[R/S]	C ₃
			[R/S]	⋮
5	Compute and print component values.		[E]	1. ^{4,6†}
				C ₁ [†]
				2. [†]
				L ₂ [†]
				3. [†]
				C ₃ [†]
				⋮

- NOTES:**
1. Input must be an integer ≥ 1 .
 2. Input > 0 .
 3. Input ≥ 0 . For Butterworth, enter 0.
 4. If $\epsilon < 0$, display will flash.
 5. " " indicates that the component number is displayed for about 2.5 seconds.
 6. Last filter component value remains in the display.
- [†]These values are printed only.

EE-14

Example 1: Compute the component values for a Butterworth filter ($\epsilon = 0$), having equal generator and load resistances of 2000 ohms (R). The filter must pass frequencies between 0 and 10 kilohertz (f_c), the maximum attenuation at 10 kilohertz being 3 dB. At least 30 dB attenuation must be provided at 15 kilohertz and above. The order of the filter can be determined by using a nomograph as follows.



NOMOGRAPH FOR BUTTERWORTH FILTERS

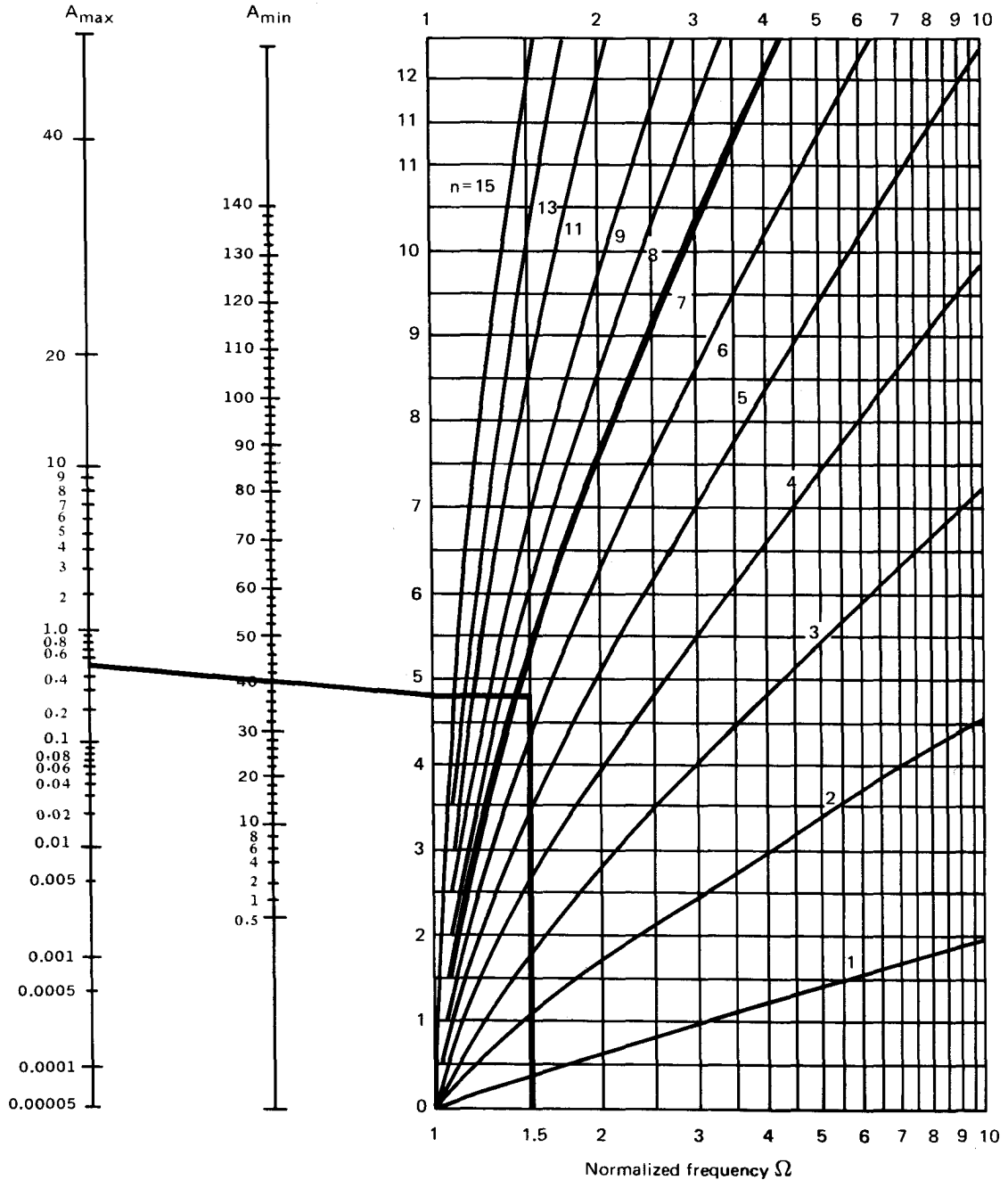
$A_{max} = 3$ dB attenuation of passband
 $A_{min} = 30$ dB attenuation of stopband
 $\Omega = (15 \text{ kilohertz}) / (10 \text{ kilohertz}) = 1.5$

} \Rightarrow at least a 9th order filter must be chosen.

ENTER	PRESS	DISPLAY	COMMENTS	PRINTOUT
	[2nd] [Pgm] 14		Select program	
	[SBR] [CLR]	0.	Initialize	
9	[A]	9.	Enter filter order	
0	[B]	0.	Enter ripple	
2000	[C]	2000.	Enter termination R	
10000	[D]	10000.	Enter f_c	
	[E]		Compute and print	1.
			Component values C_1	.0000000028
				2.
			Component values L_2	.0318309886
				3.
			Component values C_3	.0000000122
				4.
			Component values L_4	.0598226902
				5.
			Component values C_5	.0000000159
				6.
			Component values L_6	.0598226902
				7.
			Component values C_7	.0000000122
				8.
			Component values L_8	.0318309886
				9.
		.0000000028	Component values C_9	.0000000028

EE-14

Example 2: Compute the component values for a Tchebycheff filter having equal generator and load resistances of 1000 ohms (R). The filters passband must range from 0 to 3500 hertz (f_c) with an attenuation of 40 dB required at 5250 hertz. The passband ripple is allowed to be 0.5 dB with 3500 hertz being the cutoff frequency. Again a nomograph is used to determine the order of the filter.



$$\left. \begin{aligned}
 A_{\max} &= .5 \text{ dB passband ripple} \\
 A_{\min} &= 40 \text{ dB attenuation of stopband} \\
 \Omega &= \frac{5250 \text{ hertz}}{3500 \text{ hertz}} = 1.5
 \end{aligned} \right\} \Rightarrow \text{at least a 7th order filter must be chosen.}$$

ENTER	PRESS	DISPLAY	COMMENTS	PRINTOUT
	[2nd] [Pgm] 14		Select program	
	[SBR] [CLR]	0.	Initialize	
7	[A]	7.	Enter filter order	
.5	[B]	0.5	Enter ripple	
1000	[C]	1000.	Enter termination R	
3500	[D]	3500.	Enter f_c	
	[2nd] [E']	"1."	Compute component values	
	[R/S]	0.00000079	C ₁	
	[R/S]	"2."		
	[R/S]	.0572155871	L ₂	
	[R/S]	"3."		
	[R/S]	0.00000012	C ₃	
	[R/S]	"4."		
	[R/S]	0.06113069	L ₄	
	[R/S]	"5."		
	[R/S]	0.00000012	C ₅	
	[R/S]	"6."		
	[R/S]	.0572155871	L ₆	
	[R/S]	"7."		
	[R/S]	0.00000079	C ₇	

" " Indicate value is displayed for approximately 2.5 seconds.

Register Contents

R ₀₀ Dsz index	R ₀₅ b _{K-1}	R ₁₀ n	R ₁₅	R ₂₀	R ₂₅
R ₀₁ K (Used)	R ₀₆ g _{K-1}	R ₁₁ ϵ	R ₁₆	R ₂₁	R ₂₆
R ₀₂ Used (γ)	R ₀₇ g _K	R ₁₂ R	R ₁₇	R ₂₂	R ₂₇
R ₀₃ a _{K-1}	R ₀₈	R ₁₃ f _c	R ₁₈	R ₂₃	R ₂₈
R ₀₄ a _K	R ₀₉	R ₁₄	R ₁₉	R ₂₄	R ₂₉

CONVOLUTION

Given the impulse response for a linear system, one can use the convolution integral to find the system's output for a specified input waveform. The program generates outputs at intervals of Δt using the trapezoidal rule.

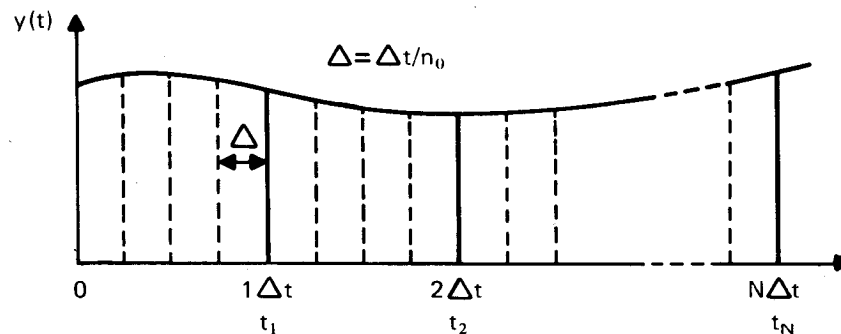
To calculate the output $y(t)$ of a linear system, the convolution integral integrates the product of the input $x(t)$ and the impulse response $h(t)$ in the form:

$$y(t) = \int_0^t x(\lambda)h(t-\lambda) d\lambda$$

Applying the trapezoidal rule to this integral yields:

$$y(t) \approx \frac{\Delta}{2} \left[x(0)h(t) + 2 \sum_{k=1}^{n-1} x(k\Delta)h(t-k\Delta) + x(t)h(0) \right]$$


The user specifies the integration step size (Δ) by the input of n_0 . Then $\Delta = \Delta t/n_0$. Note that Δ is held constant for each successive evaluation of $y(t)$.



The user specifies the number of equi-spaced arguments (N) for which the integral is to be evaluated. After the output of N values of $y(t)$, one may generate additional values (in increments of Δt).

The functions $x(t)$ and $h(t)$ are entered in program memory as a series of keystrokes.

Reference: *Linear Systems in Communication and Control*, Frederick and Carlson, John Wiley and Sons 1971.

 Solid State Software TI © 1979				
CONVOLUTION			EE-15	
$x(t)$	$h(t)$		$\rightarrow y(t) \text{ dsp}$	$\rightarrow \text{Dsp out}$
No. panels	N	Δt	$\rightarrow y(t) \text{ prt}$	$\rightarrow \text{Prt out}$

USER INSTRUCTIONS

STEP	PROCEDURE	ENTER	PRESS	DISPLAY
1	Initialize		[RST]	
2	Select learn mode.		[LRN]	
3	Use A' as the label for x(t) subroutine.		[2nd] [Lbl] [2nd] [A']	
4	Enter x(t) as a series of keystrokes. Assume t is in the display. Do not use [=] or [CLR] or any used registers.			
5	End subroutine with [INV] [SBR]		[INV] [SBR]	
6	Use B' as the label for h(t) subroutine.		[2nd] [Lbl] [2nd] [B']	
7	Enter h(t) as in steps 4 and 5			
8	Exit learn mode		[LRN]	
9	Select program 15		[2nd] [Pgm] 15	
10	Enter number of panels in each Δt	n_0	[A]	n_0
11	Number of values of y(t) desired	N	[B]	N
12	Enter increment Δt	Δt	[C]	Δt
13	Execute without printer		[2nd] [E']	t_1 (flashing) $y(t_1)$
	Repeat until N values have been displayed		[R/S]	t_2 (flashing) $y(t_2)$
14	Execute with printer		[E]	$t_1, y(t_1)$ through $t_N, y(t_N)$ are printed
15	If additional values of y(t) are desired without printer ¹		[2nd] [D']	t_{N+1} (flashing) $y(t_{N+1})$
16	If additional values of y(t) are desired with printer ¹		[D]	$t_{N+1}, y(t_{N+1})$ are printed

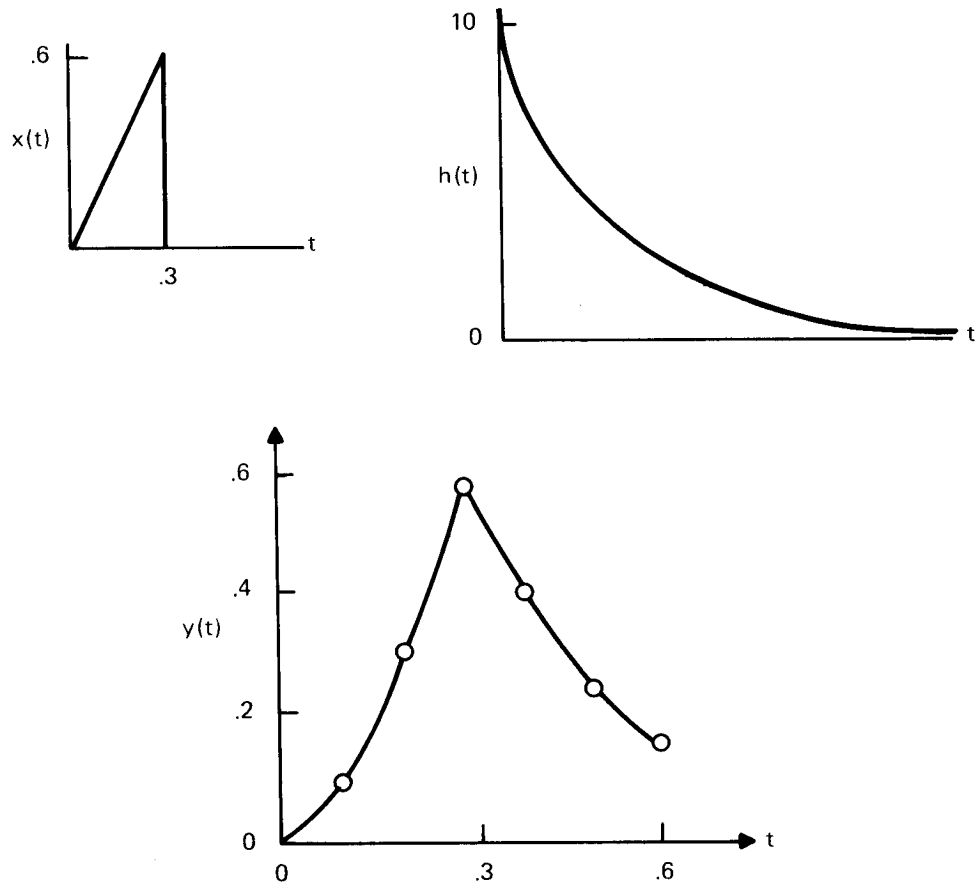
NOTES: 1. Step 15 or 16 may be repeated as many times as desired.

EE-15

Example: Given $x(t) = 2t$ for $t \leq .3$
 $= 0$ for $t > .3$
 $h(t) = 10e^{-5t}$

Find $y(t)$ at $t = .1, .2, \dots, .6$ using $n_0 = 4$. Then find $y(t)$ at $t = .7$

The problem may be graphed as follows:



ENTER	PRESS	DISPLAY	COMMENTS
	[RST]		Initialize
	[LRN]	000 00	Select learn mode
	[2nd] [Lbl] [2nd] [A']		Enter $x(t)$ subroutine
	[() [CE] [X] [2] [()]		
	[x ≥ t] .6 [2nd] [x ≥ t] [2nd] [Nop]		
	[2nd] [CP] [2nd] [Lbl]		
	[2nd] [Nop] [x ≥ t]		
	[INV] [SBR]	017 00	

ENTER	PRESS	DISPLAY	COMMENTS	PRINTOUT
	[2nd] [Lbl] [2nd] [B']		Enter h(t) subroutine	
	[() [() [CE] [X] 5 [+/-] [)] [INV] [Inx] [X] 10 [)] [INV] [SBR] [LRN]	033 00	Leave learn mode	
	[2nd] [Pgm] 15		Select program 15	
4	[A]	4.	Enter n_0	
6	[B]	6.	Enter N	
.1	[C]	.1	Enter Δt	
	[E]		Execute	0.1 T .0861547694 Y(T)
				0.2 T .2960028933 Y(T)
				0.3 T 0.580874829 Y(T)
				0.4 T .3978081927 Y(T)
				0.5 T .2412828656 Y(T)
		.1463454556		0.6 T .1463454556 Y(T)
	[D]		Execute for t=.7	0.7 T .0887630057 Y(T)
		.0887630057		

Register Contents

R ₀₀	R ₀₅	R ₁₀ n_0	R ₁₅ Used	R ₂₀	R ₂₅
R ₀₁	R ₀₆	R ₁₁ Used	R ₁₆ Used	R ₂₁	R ₂₆
R ₀₂	R ₀₇	R ₁₂ N	R ₁₇ y(t)	R ₂₂	R ₂₇
R ₀₃	R ₀₈ Used	R ₁₃ Used	R ₁₈	R ₂₃	R ₂₈
R ₀₄	R ₀₉ Used	R ₁₄ Δt	R ₁₉	R ₂₄	R ₂₉

ROOT LOCUS CALCULATIONS

Given the open-loop poles and zeros of a linear feedback system, this program calculates the following root-locus parameters: asymptote intersection point, asymptote angles, departure angles from complex poles, and arrival angles at complex zeros.

Given

$$F(s) = \frac{\prod_{i=1}^m (s - z_i)}{\prod_{k=1}^n (s - p_k)} \quad m \leq n - 1$$

where z_i are the zeros and p_k the poles of $F(s)$, the root-locus has $n - m$ asymptotes intersecting at

$$\sigma_0 = \frac{1}{n - m} \left(\sum_{k=1}^n p_k - \sum_{i=1}^m z_i \right)$$

at the angles $\psi_\nu = \frac{1}{n - m} (180^\circ + (\nu - 1) 360^\circ) \quad \nu = 1, 2, \dots, n - m$

The departure angle at any complex pole p_j is

$$\phi_j = \sum_{i=1}^m \angle(p_j - z_i) - \sum_{\substack{k=1 \\ k \neq j}}^n \angle(p_j - p_k) + 180^\circ \quad j = 1, 2, \dots, n$$


The arrival angle at any complex zero z_k is

$$\theta_k = -\sum_{\substack{i=1 \\ i \neq k}}^m \angle(z_k - z_i) + \sum_{k=1}^n \angle(z_k - p_k) - 180^\circ \quad k = 1, 2, \dots, m$$

The following program computes σ_0 , ψ_ν , ϕ_j , and θ_k for systems no greater than 13th order ($n \leq 13$, $m \leq 12$). We assumed a real system in the program so that complex poles or zeros always have complex-conjugate mates. Thus

$$\sum_{k=1}^n p_k = \sum_{k=1}^n \operatorname{Re}(p_k) \quad \text{and} \quad \sum_{i=1}^m z_i = \sum_{i=1}^m \operatorname{Re}(z_i)$$

Reference: *Linear Systems in Communication and Control*, Frederick and Carlson, John Wiley and Sons, 1971.

 Solid State Software TI © 1979	
ROOT LOCUS CALCULATIONS EE-16	
n (poles)	m (zeros)
Real; Imag	→ Asym. Par. → Dep ∠ → Arr ∠

USER INSTRUCTIONS

STEP	PROCEDURE	ENTER	PRESS	DISPLAY
1	Select program		[2nd] [Pgm] 16	
2	Initialize		[SBR] [CLR]	0.
3	Repartition (TI-58 only)	6	[2nd] [Op] 17	0.59
4	Enter number of poles	n^2	[2nd] [A']	n
5	Enter real part of 1st pole	$\text{Re}(p_1)$	[A]	1.
6	Enter imag. part of 1st pole	$\text{Im}(p_1)$	[R/S]	1.
7	Repeat steps 5 and 6 until n poles have been entered. Steps 5 and 6 must be performed in sequence.	$\text{Re}(p_i)$ $\text{Im}(p_i)$	[A] [R/S]	i i
8	Enter number of zeros	$m^{1,2}$	[2nd] [B']	m
9	Enter real part of 1st zero	$\text{Re}(z_1)$	[A]	1.
10	Enter imag. part of 1st zero	$\text{Im}(z_1)$	[R/S]	1.
11	Repeat steps 9 and 10 until m zeros have been entered. Steps 9 and 10 must be performed in sequence	$\text{Re}(z_i)$ $\text{Im}(z_i)$	[A] [R/S]	i i
12	Calculate asymptote intersection			σ_0^\dagger
13	Calculate asymptote angles			ψ_1^\dagger
14	Repeat step 13 until (n-m) angles have been calculated.			
15	Calculate departure angles from poles. Repeat for J = 1, n			J^\dagger ϕ_J^\dagger
16	Calculate arrival angles at zeros. Repeat for K = 1, m			K^\dagger θ_K^\dagger

- NOTES:**
1. If $m > (n-1)$, error condition will result at execution time.
 2. $n \leq 13, m \leq 12$

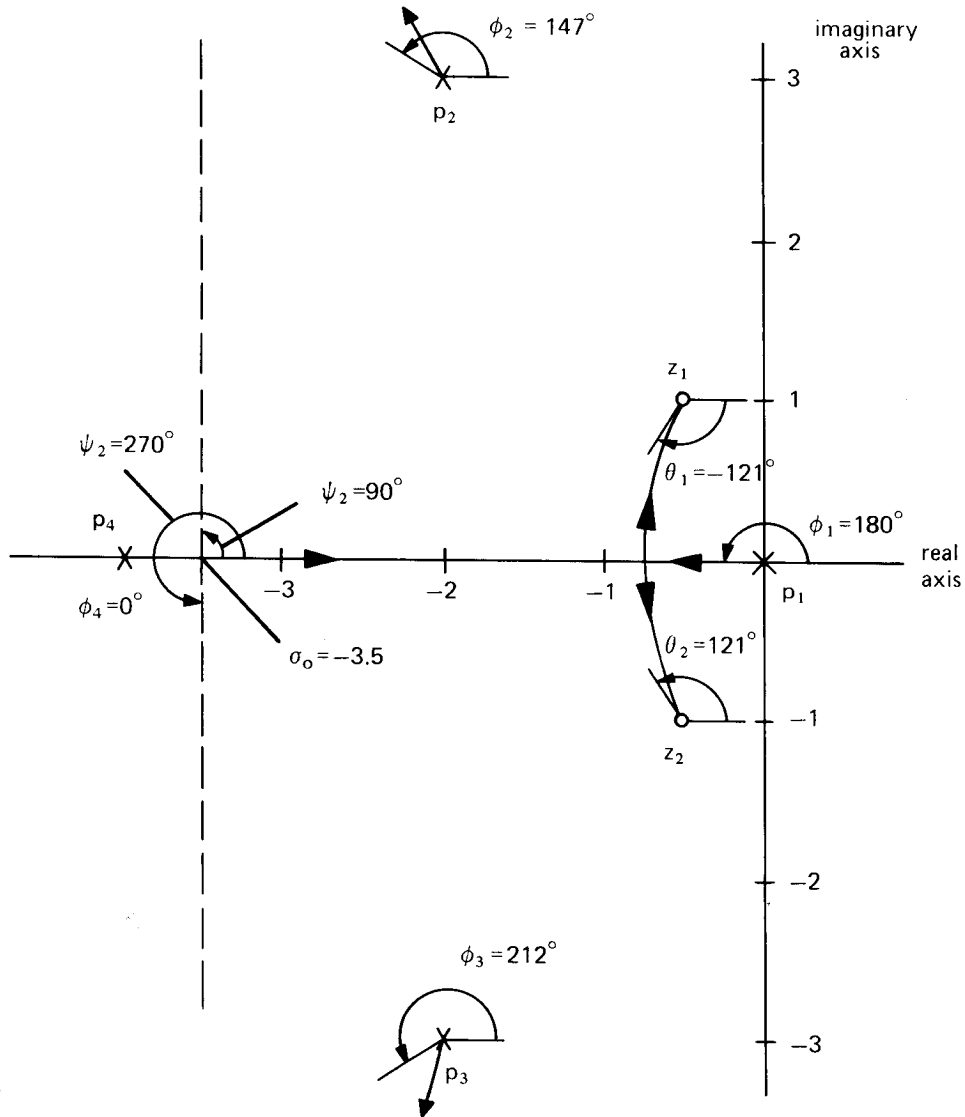
[†]These values are printed if printer is connected.

EE-16

Example: Find root locus parameters for the following transfer function.

$$F(s) = \frac{[s - (-.5 + j)] [s - (-.5 - j)]}{(s - 0) [s - (-2 + 3j)] [s - (-2 - 3j)] [s - (-4)]}$$

GRAPH OF ROOT LOCUS



ENTER	PRESS	DISPLAY	COMMENTS	PRINTOUT
	[2nd] [Pgm] 16		Select program	
	[SBR] [CLR]	0.	Initialize	
6	[2nd] [Op] 17	0.59	Repartition TI-58	
4	[2nd] [A']	4.	Enter no. of poles	
0	[A]	1.	Enter real of p ₁	
0	[R/S]	1.	Enter imag. of p ₁	
2	[+/-] [A]	2.	Enter real of p ₂	
3	[R/S]	2.	Enter imag. of p ₂	

ENTER	PRESS	DISPLAY	COMMENTS	PRINTOUT
2	[+/-] [A]	3.	Enter real of p_3	
3	[+/-] [R/S]	3.	Enter imag. of p_3	
4	[+/-] [A]	4.	Enter real of p_4	
0	[R/S]	4.	Enter imag. of p_4	
2	[2nd] [B']	2.	Enter no. of zeros	
.5	[+/-] [A]	1.	Enter real of z_1	
1	[R/S]	1.	Enter imag. of z_1	
.5	[+/-] [A]	2.	Enter real of z_2	
1	[+/-] [R/S]	2.	Enter imag. of z_2	
	[C]	-3.5	σ_0	-3.5
	[R/S]	90.	ψ_1	90.
	[R/S]	270.	ψ_2	270.
1	[D]	180.	ϕ_1	1.
				180.
2	[D]	147.4259429	ϕ_2	2.
				147.4259429
3	[D]	212.5740571	ϕ_3	3.
				212.5740571
4	[D]	.0000000004	ϕ_4	4.
				.0000000004
1	[E]	-121.1757005	θ_1	1.
				-121.1757005
2	[E]	121.1757005	θ_2	2.
				121.1757005

Register Contents

R ₀₀	Used	R ₁₂	Re (p_2)
R ₀₁	Used	R ₁₃	Im (p_2)
R ₀₂	Used	⋮	
R ₀₃	Used	R ₃₄	Re (p_{13})
R ₀₄	Used	R ₃₅	Im (p_{13})
R ₀₅	Used	R ₃₆	Re (z_1)
R ₀₆	Used	R ₃₇	Im (z_1)
R ₀₇	(n-m)	R ₃₈	Re (z_2)
R ₀₈	n(poles)	R ₃₉	Im (z_2)
R ₀₉	m(zeros)	⋮	
R ₁₀	Re(p_1)	R ₅₈	Re (z_{12})
R ₁₁	Im(p_1)	R ₅₉	Im (z_{12})

DISCRETE FOURIER TRANSFORM

This program transforms a real time series to the frequency domain (DFT) and performs the inverse transform (IDFT) from the frequency domain back to the time domain. The frequency representation of the transform is given in terms of magnitude in dB and phase in degrees. The program allows a DFT sampling rate of up to 88 points with the TI-59 and 48 points with the TI-58. For IDFT's, the user can generate a time-domain image of a signal by taking up to a 32-point sample (16-point, TI-58) of its symmetric frequency domain representation. It is assumed that the time series reflects a periodic function over some finite interval so that the frequency domain behavior can also be expressed as a discrete series representing a periodic function. The following equations are used.

Discrete Fourier Transform (DFT)

$$F_{re}(X) = \sum_{n=0}^{N-1} f(n) \cos\left(\frac{2\pi nX}{N}\right)$$

$$F_{im}(X) = -\sum_{n=0}^{N-1} f(n) \sin\left(\frac{2\pi nX}{N}\right) \quad X = 0, 1, 2, \dots, N-1$$

$$|F(X)| = \sqrt{[F_{re}(X)]^2 + [F_{im}(X)]^2}$$

$$\text{magnitude in dB} = 20 \log |F(X)|$$

$$\text{phase in degrees, } \angle F(X) = \tan^{-1} \left[\frac{F_{im}(X)}{F_{re}(X)} \right], \quad -90^\circ \leq \angle F(X) < 270^\circ$$

(see page V-31, *Personal Programming*)

where

$F(X) = F_{re}(X) + jF_{im}(X)$ is the transform of the sample at X .

$F_{re}(X)$ = real part of the transform

$F_{im}(X)$ = imaginary part of the transform

$f(n)$ = value of the waveform sampled at n

N = total number of samples

Inverse Discrete Fourier Transform (IDFT)

$$f(X) = \frac{1}{N} \sum_{n=0}^{N-1} F_{re}(n) \cos\left(\frac{2\pi nX}{N}\right) - F_{im}(n) \sin\left(\frac{2\pi nX}{N}\right)$$

where

$f(X)$ = the value of the waveform sampled at X

$F_{re}(n)$ = real part of the transform at sample n

$F_{im}(n)$ = imaginary part of the transform at sample n

N = total number of samples

The program assumes that the time function f and the frequency function F are approximated by N equally spaced samples at intervals of ΔT (DFT) or $1/\Delta T$ (IDFT). These N samples define the Discrete Fourier Transform pair $f \leftrightarrow F$ because the N time and frequency values give a discrete representation of the time and frequency domain waveforms, respectively. To minimize the effects of aliasing, choose N such that the DFT sample interval is $\Delta T \leq 1/2f_c$ where f_c is the highest frequency component of the transform and the IDFT sample interval is $1/\Delta T \geq 2f_c$ which is the Nyquist sampling rate.

Data Input

Data is input through User Instruction steps 4 and 5 or steps 8 and 9. Alternately the data can be input directly into the following data registers.

For DFT:	$f(0) \rightarrow R_{12}$	For IDFT:	$F_{re}(0) \rightarrow R_{12}; F_{im}(0) \rightarrow R_{44}$
	$f(1) \rightarrow R_{13}$		$F_{re}(1) \rightarrow R_{13}; F_{im}(1) \rightarrow R_{45}$
	\vdots		\vdots
	$f(n) \rightarrow R_{n+12}$		$F_{re}(n) \rightarrow R_{n+12}; F_{im}(n) \rightarrow R_{n+44}$
	\vdots		\vdots
	$f(47) \rightarrow R_{59}, 48 \text{ samples}$		$F_{re}(15) \rightarrow R_{27}; F_{im}(15) \rightarrow R_{59}, 16 \text{ samples}$
	\vdots TI-58 limit		\vdots TI-58 limit
	$f(87) \rightarrow R_{99}, 88 \text{ samples}$		$F_{re}(31) \rightarrow R_{43}; F_{im}(31) \rightarrow R_{75}, 32 \text{ samples}$
	\vdots TI-59 limit		\vdots TI-59 limit

- References:**
1. *The Fast Fourier Transform*, E. Oran Brigham, Prentice Hall, 1974.
 2. *An Introduction to Discrete Systems*, Kenneth Steiglitz, John Wiley and Sons, 1974.

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DISCRETE FOURIER TRANSFORM				EE-17
				→ IDFT
n	f(n)	 F(n) 	∠F(n)	→ DFT

USER INSTRUCTIONS

STEP	PROCEDURE	ENTER	PRESS	DISPLAY
1	Select program		[2nd] [Pgm] 17	
2	Initialize		[SBR] [CLR]	0.
3	Repartition (6 TI-58, 10 TI-59)	6 or 10	[2nd] [Op] 17	0.59 or 159.99
	For DFT			
	Enter sample number ($0 \leq n \leq N-1$) ¹	n	[A]	n^\dagger
4	Enter sample number ($0 \leq n \leq N-1$) ¹	n	[A]	n^\dagger
5	Enter sample value ¹	f(n)	[B]	f(n) [†]
6	Repeat steps 4 and 5 until all data is entered ¹			
7	Enter total number of samples and compute DFT ⁴	N^2	[E]	
	sample number			n^\dagger
	real part of transform			$F_{re}(n)^\dagger$
	imaginary part of transform			$F_{im}(n)^\dagger$
	magnitude of transform ³			$ F(n) ^\dagger$
	magnitude of transform in dB			$20 \log F(n) ^\dagger$
	phase of transform in degrees			$\angle F(n)^\dagger$
	This six-part sequence is repeated for all values of n.			
	For IDFT			
8	Enter sample number ($0 \leq n \leq N-1$) ¹	n	[A]	n^\dagger
9a	Enter magnitude of transform at n^1	F(n)	[C]	$ F(n) ^\dagger$
9b	Enter phase in deg of transform at n^1	∠F(n)	[D]	$\angle F(n)^\dagger$
10	Repeat steps 8 and 9 until all data is entered ¹			
11	Enter total number of samples and compute IDFT	N^2	[2nd] [E']	
	sample number			n^\dagger
	time value at n			f(n) [†]
	This two-part sequence is repeated for all values of n.			

- NOTES:**
- Steps 4, 5, and 6 or 8, 9, and 10 can be omitted if data is entered directly into registers. Step 2 clears memories, so zero values don't need to be entered.
 - For DFT, $N \leq 88$ for TI-59; $N \leq 48$ for TI-58.
For IDFT, $N \leq 32$ for TI-59; $N \leq 16$ for TI-58.
 - When magnitude approaches zero, the value of the phase angle can be ignored as the phase becomes indeterminate. If magnitude = 0, the indeterminate state is indicated by flashing nines for magnitude in dB and 45. for phase in degrees.
 - Because DFT's for real time series have magnitudes and phases which are symmetric and antisymmetric respectively, about the sample point corresponding to $\text{Int}(N/2)$, program execution may be terminated using [RST] after data from point $\text{Int}(N/2)$ has been printed.
- [†]These values are printed if a printer is connected. The output values are displayed only briefly. Therefore, for most applications a printer will be desirable.

Example 1: (Requires TI-59) Recover the original sinusoid S which has a

$$\text{DFT} = \begin{cases} -j16 & \text{when } n = 2 \\ +j16 & \text{when } n = 30 \\ 0 & \text{elsewhere} \end{cases}$$

where n = sample number, $N = 32$, the total number of samples. Enter the data directly into the data registers.

The original signal $S = \sin(n\pi/8)$ $n = 0, \dots, N - 1$ which is a sine wave at a frequency of $1/8$, the Nyquist frequency, is verified by the output listing.

For further details, see reference 2, chapter 6.

ENTER	PRESS	DISPLAY	COMMENTS
	[2nd] [Pgm] 17		Select program
	[SBR] [CLR]	0.	Initialize
10	[2nd] [Op] 17	159.99	Repartition (TI-59 required)
16	[+/-] [STO] 46	-16.	Enter data directly into registers
16	[STO] 74	16.	
32	[2nd] [E']		See tape printout, output values are displayed only briefly.

OUTPUT TAPE FOR EXAMPLE 1

0. n	11.	22.
0. f(n)	-. 9238795325	. 7071067812
1.	12.	23.
. 3826834324	-1.	. 3826834323
2.	13.	24.
. 7071067812	-. 9238795325	-2. 8-11
3.	14.	25.
. 9238795325	-. 7071067812	-. 3826834324
4.	15.	26.
1.	-. 3826834323	-. 7071067812
5.	16.	27.
. 9238795325	3. 05-11	-. 9238795325
6.	17.	28.
. 7071067812	. 3826834324	-1.
7.	18.	29.
. 3826834323	. 7071067812	-. 9238795325
8.	19.	30.
-1. 5-12	. 9238795325	-. 7071067812
9.	20.	31.
-. 3826834324	1.	-. 3826834324
10.	21.	
-. 7071067812	. 9238795325	

Example 2a: A problem which is inherent in the Discrete Fourier Transform because of the required time domain truncation is leakage. Truncation of a periodic function at other than a multiple of the period results in a sharp discontinuity in the time domain, or equivalently results in side-lobes in the frequency domain. These side-lobes are responsible for the additional frequency components which are called leakage. The following example illustrates the effect of leakage. A cosine waveform is sampled at $N = 32$ points with $\Delta T = 1$ and $f_0 = 1/9.143$. The waveform is defined by $f(t) = \cos(2\pi f_0 t)$. The definition of f_0 results in the time truncation interval not chosen equal to a multiple of the period, which causes leakage in the DFT. (Note DFT output showing non-zero frequency components at all discrete frequencies of the discrete transform.) For further information see Reference 1, Chapter 9.

ENTER	PRESS	DISPLAY	COMMENTS
	[2nd] [Pgm] 17		Select program
	[SBR] [CLR]	0.	Initialize
6	[2nd] [Op] 17	0.59	Repartition (TI-58)
0	[A]	0. [†]	Enter sample number
1	[B]	1. [†]	Enter sample value
1	[A]	1. [†]	Repeat for all samples. See input tape for values.
.773	[B]	0.773 [†]	
⋮	⋮	⋮	
31	[A]	31. [†]	
.7727	[+/-] [B]	-0.7727 [†]	
32	[E]	0.*	n
		1.0006*	$F_{re}(n)$
		0.*	$F_{im}(n)$
		1.0006*	$ F(n) $
		.0052099709*	$20 \log F(n) $
		0.*	$\angle F(n)$
		⋮	See output tape
		31.*	
		1.000639549*	
		-.9390590304*	
		1.372265051*	
		2.748760053*	
		-43.18161483*	

[†]These displayed values are printed.

*These values are displayed briefly and printed.

INPUT TAPE FOR EXAMPLE 2a

0. n	12.	24.
1. f(n)	-0.3825	-0.7072
1.	13.	25.
0.773	-0.8818	-0.0982
2.	14.	26.
0.1951	-0.9808	0.5553
3.	15.	27.
-0.4713	-0.6345	0.9568
4.	16.	28.
-0.9238	-0.0001	0.9239
5.	17.	29.
-0.9569	0.6342	0.4716
6.	18.	30.
-0.5556	0.9807	-0.1947
7.	19.	31.
0.0979	0.882	-0.7727
8.	20.	
0.707	0.3828	
9.	21.	
0.9951	-0.29	
10.	22.	
0.8315	-0.8313	
11.	23.	
0.2903	-0.9952	

OUTPUT TAPE FOR EXAMPLE 2a

0. n	5.	10.
1.0006 $F_e(n)$.9995790734	.9999412478
0. $F_m(n)$	-3.82334953	-.7990914541
1.0006 $ F(n) $	3.951855255	1.280011582
.0052099709 $20 \log F(n) $	11.93602058	2.144277989
0. $\angle F(n)$	-75.34849318	-38.62969454
1.	6.	11.
1.000639549	.9996930668	.9995972063
.9390590304	-2.366527645	-.6261383867
1.372265051	2.569015244	1.179510006
2.748760053	8.195333626	1.434032588
43.18161484	-67.09932919	-32.06264597
2.	7.	12.
1.0008672	0.999613047	1.000077817
2.536411113	-1.696769076	-.4777605807
2.726741	1.969327688	1.108336958
8.712877768	5.886359738	.8934363079
68.46582011	-59.49649849	-25.534898
3.	8.	13.
1.001877504	0.9999	.9999119019
9.503991626	-1.2937	-.3462773071
9.556652937	1.635071772	1.058173797
19.60611629	4.270736419	0.491140064
83.98230479	-52.29958375	-19.10136951
4.	9.	14.
.9935221826	.9997629142	.9998984859
-10.72776058	-1.012977911	-.2257526962
10.77413076	1.423253362	1.025066467
20.64764484	3.065644368	.2150405341
-84.68232514	-45.37617991	-12.72267462

OUTPUT TAPE FOR EXAMPLE 2a (continued)

15. n .9998188047 $F_o(n)$ -.1110298813 $F_{im}(n)$	20. 1.000077817 .4777605808	25. .9996130469 1.696769076
1.005964849 $ F(n) $.0516561098 $20 \log F(n) $ -6.336733188 $\angle F(n)$	1.108336958 .8934363074 25.53489801	1.969327688 5.886359737 59.49649849
16. 1. 2.57135-11	21. .9995972063 .6261383867	26. .9996930666 2.366527645
1. 0. .0000000015	1.179510006 1.434032588 32.06264598	2.569015244 8.195333626 67.09932919
17. .9998188047 .1110298815	22. .9999412478 .7990914542	27. .9995790734 3.82334953
1.005964849 .0516561101 6.336733197	1.280011582 2.14427799 38.62969454	3.951855255 11.93602058 75.34849318
18. .9998984859 .2257526963	23. .9997629142 1.012977911	28. 0.998522182 10.72776058
1.025066467 .2150405343 12.72267462	1.423253362 3.065644368 45.37617991	10.77413076 20.64764484 84.68232514
19. .9999119019 .3462773072	24. .9998999999 1.2937	29. 1.001877504 -9.503991627
1.058173797 .4911400638 19.10136951	1.635071772 4.270736419 52.29958375	9.556652938 19.60611629 -83.98230478

OUTPUT TAPE FOR EXAMPLE 2a (continued)

30. n	31.
1.0008672 $F_{re}(n)$	1.000639549
-2.536411112 $F_{im}(n)$	-.9390590304
2.726741 $ F(n) $	1.372265051
8.712877768 $20 \log F(n) $	2.748760053
-68.46582011 $\angle F(n)$	-43.18161483

Example 2b: A technique for reducing leakage employs a time domain truncation function

$$X(t) = \frac{1}{2} - \frac{1}{2} \cos\left(\frac{2\pi t}{N}\right)$$

where N = total number of samples.

X(t) is known as the Hanning function. The cosine waveform of example 2a will be multiplied by the Hanning function and the resulting waveform

$$f(t) \cdot X(t) = \cos(2\pi f_o t) \left[\frac{1}{2} - \frac{1}{2} \cos(2\pi t/N) \right]$$

will be sampled at N = 32 points with $\Delta T = 1$ and $f_o = 1/9.143$. The effect of the Hanning function is to reduce the discontinuity which results from the time domain truncation and, consequently, minimize the leakage. Note the DFT outputs, particularly for the discrete frequencies from sample 6 to sample 26, which are now closer to zero.

ENTER	PRESS	DISPLAY	COMMENTS
	[2nd] [Pgm] 17		Select program
	[SBR] [CLR]	0.	Initialize
6	[2nd] [Op] 17	0.59	Repartition (TI-58)
0	[A]	0.†	Enter sample number
0	[B]	0.†	Enter sample value
:	:	:	Repeat for all samples. See input tape for values.
31	[A]	31.†	
.0074	[+/-] [B]	-0.0074†	
32	[E]	0.*	n
		0.0002*	$F_{re}(n)$
		0.*	$F_{im}(n)$
		0.0002*	$ F(n) $
		-73.97940009*	$20 \log F(n) $
		0.*	$\angle F(n)$
		:	
		31.*	See output tape
		.0000007759*	
		.1646458942*	

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ENTER	PRESS	DISPLAY	COMMENTS
		.1646458942*	
		-15.6689819*	
		89.99973*	

†These displayed values are printed.

*These values are displayed briefly and printed.

INPUT TAPE FOR EXAMPLE 2b

0. n	11.	22.
0. f(n)	0. 2258	-0. 5747
1.	12.	23.
0. 0074	-0. 3265	-0. 5946
2.	13.	24.
0. 0074	-0. 8075	-0. 3536
3.	14.	25.
-0. 0397	-0. 9434	-0. 0395
4.	15.	26.
-0. 1352	-0. 6284	0. 1714
5.	16.	27.
-0. 2126	-0. 0001	0. 2126
6.	17.	28.
-0. 1714	0. 6281	0. 1353
7.	18.	29.
0. 0394	0. 9434	0. 0397
8.	19.	30.
0. 3535	0. 8076	-0. 0074
9.	20.	31.
0. 5946	0. 3268	-0. 0074
10.	21.	
0. 5748	-0. 2256	

OUTPUT TAPE FOR EXAMPLE 2b

0. n	5.	10.
0.0002 $F_r(n)$	0.000291618	.0000548594
0. $F_m(n)$	1.362082295	.0101008511
0.0002 $ F(n) $	1.362082327	0.010101
-73.97940009 $20 \log F(n) $	2.684067158	-39.91271254
0. $\angle F(n)$	89.98773314	89.68882002
1.	6.	11.
.0000007758	.0000865619	.0000064488
-.1646458943	0.19686324	.0063063648
.1646458943	0.196863259	.0063063681
-15.66898189	-14.11670659	-44.00441363
-89.99973001	89.9748067	89.94141006
2.	7.	12.
-.0002937549	.0000427157	-.0001050253
-1.342759838	.0667184439	.0041008017
1.34275987	.0667184576	.0041021464
2.559967068	-23.51508005	-47.73977698
269.9874654	89.96331706	91.46707649
3.	8.	13.
.0008544733	0.0001	-.0001868547
6.799605919	0.0307	.0025290107
6.799605973	0307001629	.0025359042
16.64967493	-30.25718641	-51.91734328
89.99279992	89.81336947	94.22559234
4.	9.	14.
-.0010949747	-.0000490217	.0001523336
-6.783699198	.0167893998	0.001677773
6.783699287	.0167894714	.0016846744
16.62933176	-35.49925956	-55.46968039
269.9907517	90.16729186	84.8120484

OUTPUT TAPE FOR EXAMPLE 2b (continued)

15. n
 -.0001601552 $F_r(n)$
 .0009240838 $F_m(n)$

 .0009378596 $|F(n)|$
 -60.55724333 $20 \log |F(n)|$
 99.8324027 $\angle F(n)$

16.
 0.0004
 1.3272-12

 0.0004
 -67.95880017
 .0000001901

17.
 -.0001601552
 -.0009240838

 .0009378596
 -60.55724384
 260.1675979

18.
 .0001523336
 -0.001677773

 .0016846744
 -55.46968041
 -84.8120479

19.
 -.0001868547
 -.0025290107

 .0025359041
 -51.91734335
 265.7744077

20.
 -.0001050253
 -.0041008017

 .0041021464
 -47.73977699
 268.5329236

21.
 .0000064488
 -.0063063648

 .0063063681
 -44.00441362
 -89.94141006

22.
 .0000548594
 -.0101008511

 0.010101
 -39.91271254
 -89.68882015

23.
 -.0000490217
 -.0167893999

 .0167894714
 -35.49925952
 269.8327081

24.
 0.0001
 -.0306999999

 .0307001628
 -30.25718643
 -89.81336954

25.
 .0000427157
 -.0667184439

 .0667184576
 -23.51508005
 -89.96331706

26.
 .0000865619
 -0.19686324

 0.196863259
 -14.11670659
 -89.97480673

27.
 .0002916181
 -1.362082295

 1.362082327
 2.684067158
 -89.98773313

28.
 -.0010949751
 6.783699198

 6.783699287
 16.62933176
 90.00924827

29.
 .0008544736
 -6.799605919

 6.799605973
 16.64967493
 -89.99279992

OUTPUT TAPE FOR EXAMPLE 2b (continued)

```

          30. n
-0.000293755 Fre(n)
  1.342759838 Fim(n)

  1.34275987 |F(n)|
  2.559967069 20 log |F(n)|
  90.01253457 LF(n)
    
```

```

          31.
.0000007759
.1646458942

.1646458942
-15.6689819
  89.99973
    
```

Register Contents

- R₀₀ Used
 - R₀₁ Used
 - R₀₂ Used
 - R₀₃ Used
 - R₀₄ N - 1
 - R₀₅ F_{re}(X) [DFT], f(X) [IDFT]
 - R₀₆ F_{im}(X) [DFT], f(X) [IDFT]
 - R₀₇ X
 - R₀₈ 2πnX/N
 - R₀₉ Used
 - R₁₀ Used
 - R₁₁ N
 - R₁₂
- ↓

Data, TI-58

↓

R₅₉

R₁₂

↓

Data, TI-59

↓

R₉₉

SMITH CHART CALCULATIONS

Graphical constructions on the Smith Chart require some special techniques, and it can be difficult to obtain accurate results. The program given here performs various transmission-line calculations equivalent to the graphical constructions on the Smith Chart. The program makes provisions for lines with attenuation and complex characteristic impedance. Thus you can obtain accurate results quickly and easily with your programmable calculator.

Consider a transmission line with characteristic impedance Z_0 (possibly complex). The normalized impedance (z) at any point will be the impedance Z at that point divided by Z_0 ($z = Z/Z_0$). Let the normalized distance in wavelengths from the load end of the line be $x = d/\lambda$. Then the per-wavelength phase constant is given by

$$b = \beta\lambda = 2\pi \text{ (radians)}$$

and the attenuation constant is

$$a = 8.686\alpha\lambda \text{ (dB per wavelength)}$$

where α is the attenuation factor (nepers per unit length).

This program has subroutines which will perform different calculations depending on the various inputs. The following terms are used in many equations and will be defined here for convenience.

- Z_0 = characteristic impedance
- Z = impedance*
- z = normalized impedance*
- Y = admittance*
- P = reflection factor* (rectangular coordinates)
- $|P|/\angle P$ = reflection factor* (polar coordinates)
- σ = voltage standing wave ratio (VSWR)

*Note that a subscript of "L" indicates values at termination, and a subscript of "i" indicates values at input.

The program uses rectangular representation $a + bj$ for entry of complex quantities. If polar to rectangular conversion is desired, see page V-31 of *Personal Programming* or use Program EE-05.

The program allows calculation of the input impedance Z_i , the reflection factor P , and the VSWR σ when the characteristic impedance Z_0 , the termination resistance Z_L , the attenuation a , and the distance in wavelengths X from the termination are entered. Provisions are made to allow conversion from admittance to impedance.

This program is designed for use with the printer.

Solid State Software TI © 1979				
SMITH CHART CALCULATIONS				EE-18
Z_0	Z	Y		
a	σ	X	P	→ Execute

STEP	PROCEDURE	ENTER	PRESS	DISPLAY
1	Select program		[2nd] [Pgm] 18	
2	Initialize		[SBR] [CLR]	0.
3	Enter complex characteristic impedance. Omit for $Z_0 = 50 + 0j$ ohms ¹	Re(Z_0) Im(Z_0)		Re(Z_0) [†] Im(Z_0) [†]
4	Enter termination impedance Z_L in ohms. If admittance given, see step 10.	Re(Z_L) Im(Z_L)	[2nd] [B'] [R/S]	Re(Z_L) Re(P_L) [*] Im(P_L) [†]
5	Enter attenuation in dB. Omit for $a = 0$. ¹	a	[A]	a [†]
6	Enter distance X in wavelengths from termination	X	[C]	X [†]
7	Compute input impedance Z_i		[E]	Re(Z_i) [*] Im(Z_i) [†]
8	Display magnitude of P_i ² Display angle of P_i		[D] [R/S]	P_i [†] $\angle P_i$ [†]
9	Enter magnitude of P and calculate VSWR ³	P	[B]	σ [†]
10	Perform $Y \rightleftharpoons Z$ conversions	Re(Y)	[2nd] [C']	Re(Y)
		Im(Y)	[R/S]	Re(Z) [*] Im(Z) [†]
		Re(Z)	[2nd] [C']	Re(Z)
		Im(Z)	[R/S]	Re(Y) [*] Im(Y) [†]

- NOTES:**
1. Values for Z_0 and a are set at $50 + 0j$ and 0, respectively, if they are not entered.
 2. This calculation is performed during execution of input impedance at step 7. Thus step 8 must immediately follow step 7 (Display of P_i is optional).
 3. If $|P_i|$ is desired for VSWR, enter $|P_i|$ using [D] in step 8. If $|P_L|$ is desired for VSWR, enter $|P_L|$ by pressing [RCL] 10 after execution of step 7.

[†]These displayed values are printed if printer is connected.

^{*}These values are printed only.

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Example: Given a transmission line with a characteristic impedance (Z_0) of $50 + 0j$ ohms, an attenuation (a) of 0.4 dB, and a termination impedance of $10 + 40j$ ohms, find the input impedance (Z_i) and the VSWR (σ) at 1, 2, and 3 wavelengths (X) from the termination.

ENTER	PRESS	DISPLAY	COMMENTS	PRINTOUT
	[2nd] [Pgm] 18		Select program	
	[SBR] [CLR]	0.	Initialize	
10	[2nd] [B']	10.	Enter Z_L in ohms	
40	[R/S]		Real P_L	-.1538461538
		.7692307692	Imag P_L	.7692307692
.4	[A]	0.4	Enter a in dB	0.4
1	[C]	1.	Enter X_1 in wavelengths	1.
	[E]		Real Z_{i1}	13.61653082
		39.1384787	Imag Z_{i1}	39.1384787
	[D]	.7154401641	Display $ P_{i1} $.7154401641
	[B]	6.028398767	Calculate σ_1	6.028398767
2	[C]	2.	Enter X_2 in wavelengths	2.
	[E]		Real Z_{i2}	17.07404104
		38.04661406	Imag Z_{i2}	38.04661406
	[D]	.6524891845	Display $ P_{i2} $.6524891845
	[B]	4.75521656	Calculate σ_2	4.75521656
3	[C]	3.	Enter X_3 in wavelengths	3.
	[E]		Real Z_{i3}	20.34245021
		36.75665428	Imag Z_{i3}	36.75665428
	[D]	.5950772089	Display $ P_{i3} $.5950772089
	[B]	3.939213212	Calculate σ_3	3.939213212

Register Contents

R ₀₀	R ₀₅ X	R ₁₀ $ P_L $	R ₁₅ $\angle P$	R ₂₀ a	R ₂₅
R ₀₁ Re X	R ₀₆ Re Z_0	R ₁₁	R ₁₆ Re Z_L	R ₂₁	R ₂₆
R ₀₂ Im X	R ₀₇ Im Z_0	R ₁₂	R ₁₇ Im Z_L	R ₂₂	R ₂₇
R ₀₃ Re Y	R ₀₈ Re P_L	R ₁₃	R ₁₈	R ₂₃	R ₂₈
R ₀₄ Im Y	R ₀₉ Im P_L	R ₁₄ $ P_i $	R ₁₉	R ₂₄	R ₂₉

Method Used

Assuming a low-loss line, the following computations are performed.

Reflection Factor at Termination

Given the termination impedance

$$Z_L = \text{Re}(Z_L) + j \text{Im}(Z_L)$$

or admittance

$$Y_L = \text{Re}(Y_L) + j \text{Im}(Y_L)$$

the complex reflection factor is calculated as follows.

$$Z_L = \frac{1}{Y_L} \quad (\text{if } Y_L \text{ entered})$$

$$z_L = \frac{Z_L}{Z_0} \quad P_L = \frac{z_L - 1}{z_L + 1}$$

Input Impedance

Given Z_L and P_L from a previous calculation, the input impedance

$$Z_i = \text{Re}(Z_i) + j \text{Im}(Z_i)$$

at distance X from the termination is computed by the following sequence.

$$|P_i| = |P_L| (10^{-0.1\alpha})^X \quad \angle P_i = \angle P_L - 4\pi X$$

$$z_i = \frac{1 + P_i}{1 - P_i} \quad Z_i = Z_0 z_i$$

Note that this sequence applies in the reverse direction (from Z_i to Z_L) if X is entered as a negative quantity.

Admittance and VSWR

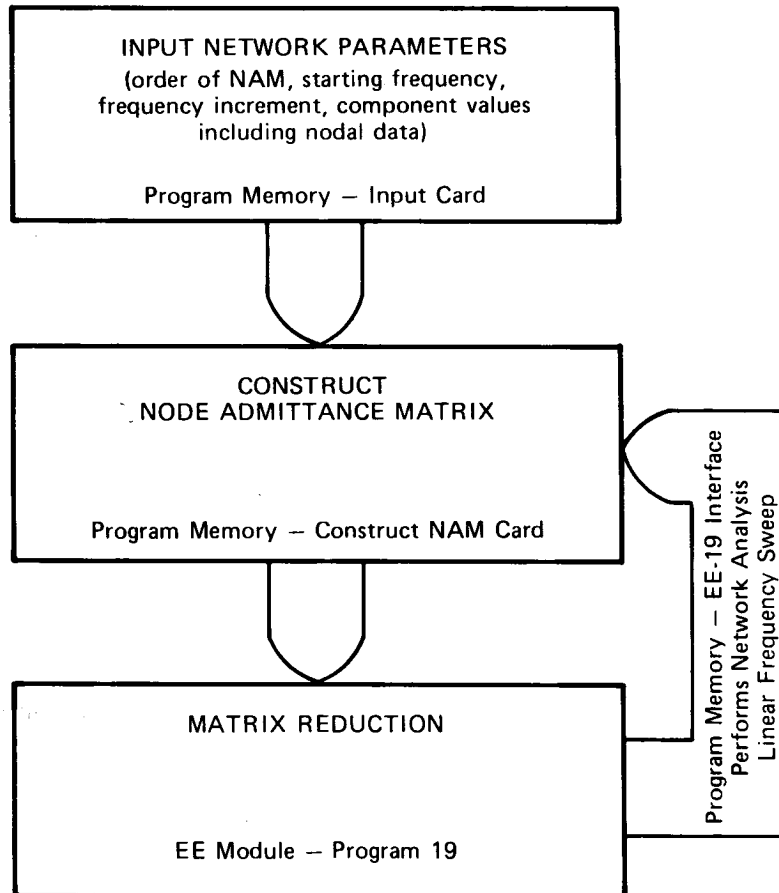
The following calculations are the same for either input or termination values.

$$Y = \frac{1}{Z} \quad \sigma = \frac{1 + |P|}{1 - |P|}$$

Reference: *Electric Transmission Lines*, Skilling, McGraw-Hill 1951.

NETWORK ANALYSIS

This program is designed for use with the TI-59 and the printer. The program computes frequency response of a general linear network made up of resistors, capacitors, inductors, and voltage controlled dependent current sources. Networks of up to 5 nodes (exclusive of ground) and 9 components, 4 nodes and 14 components, or 3 or 2 nodes and 24 components can be analyzed. The analysis is accomplished by interfacing three blocks of code as illustrated below.



The first block consists of the input code and should be keyed into program memory and recorded on the designated magnetic card per specified instructions. Execution of this code in User Instructions 1 through 6 specifies the order of the NAM (node admittance matrix), starting frequency and increment in hertz to initiate a linear frequency sweep, and remaining network parameters by entering component values and node numbers associated with each component. The number of a component is printed following input of its data to provide easy editing reference. (See User Instructions, Note 3.)

The second block consisting of the Construct NAM code should be keyed into program memory and recorded on the designated card per specified instructions. Execution of this code in User Instructions 7 through 9 constructs the network node admittance matrix and initiates the loop between program memory and the third block of code, EE module program 19, which performs the matrix reduction. (See Method Used section for explanation of NAM construction and matrix reduction.)

Program output is in the form of a linear frequency sweep which depends on starting frequency and increment specifications. Printout consists of the frequency in hertz, magnitude of the output voltage in dB and the phase in degrees.

Two magnetic cards are used in conjunction with this program. Instructions for coding and recording these cards are included in the following sections.

Input Card Preparation

PROCEDURE	PRESS	DISPLAY
1. Repartition	9 [2nd] [Op] 17	239.89
2. Clear program memory	[2nd] [CP]	239.89
3. Enter learn mode	[LRN]	000 00
4. Key in input code (see Listing)		
5. Check input code against listing	[CLR] [2nd] [List]	
6. Prepare to record Input Card	1 [2nd] [Write]	Blank
7. Insert card		1.

Label this card Input Card and place it in the card case for storage.

See *Personal Programming*, pp. V-44, 45 and VII-2, 3 for information on keying in programs and recording magnetic cards.

INPUT CODE LISTING

000	76	LBL	050	02	02	100	89	89
001	11	A	051	00	0	101	99	PRT
002	99	PRT	052	72	ST*	102	92	RTN
003	32	X:T	053	01	01	103	22	INV
004	02	2	054	61	GTD	104	86	STF
005	32	X:T	055	00	00	105	00	00
006	22	INV	056	64	64	106	43	RCL
007	67	EQ	057	72	ST*	107	04	04
008	00	00	058	01	01	108	42	STD
009	11	11	059	32	X:T	109	02	02
010	03	3	060	98	ADV	110	43	RCL
011	42	STD	061	99	PRT	111	03	03
012	87	87	062	72	ST*	112	42	STD
013	42	STD	063	02	02	113	01	01
014	05	05	064	92	RTN	114	92	RTN
015	92	RTN	065	22	INV	115	76	LBL
016	99	PRT	066	52	EE	116	15	E
017	42	STD	067	58	FIX	117	86	STF
018	78	78	068	01	01	118	00	00
019	42	STD	069	99	PRT	119	98	ADV
020	06	06	070	65	x	120	99	PRT
021	92	RTN	071	01	1	121	94	+/-
022	99	PRT	072	00	0	122	65	x
023	42	STD	073	95	=	123	02	2
024	79	79	074	74	SM*	124	85	+
025	42	STD	075	01	01	125	07	7
026	07	07	076	58	FIX	126	09	9
027	92	RTN	077	09	09	127	75	-
028	76	LBL	078	87	IFF	128	42	STD
029	14	D	079	00	00	129	02	02
030	32	X:T	080	01	01	130	01	1
031	93	.	081	03	03	131	95	=
032	03	3	082	02	2	132	42	STD
033	61	GTD	083	94	+/-	133	01	01
034	00	00	084	44	SUM	134	92	RTN
035	57	57	085	01	01	135	76	LBL
036	76	LBL	086	44	SUM	136	10	E'
037	16	A'	087	02	02	137	22	INV
038	32	X:T	088	43	RCL	138	86	STF
039	93	.	089	01	01	139	00	00
040	01	1	090	42	STD	140	22	INV
041	61	GTD	091	03	03	141	52	EE
042	00	00	092	43	RCL	142	07	7
043	57	57	093	02	02	143	07	7
044	76	LBL	094	42	STD	144	42	STD
045	12	B	095	04	04	145	02	02
046	98	ADV	096	01	1	146	07	7
047	99	PRT	097	44	SUM	147	06	6
048	35	1/X	098	89	89	148	42	STD
049	72	ST*	099	43	RCL	149	01	01

INPUT CODE LISTING (continued)

150	98	ADV	195	03	3
151	69	OP	196	00	0
152	00	00	197	69	OP
153	02	2	198	02	02
154	04	4	199	04	4
155	03	3	200	00	0
156	01	1	201	03	3
157	69	OP	202	07	7
158	02	02	203	03	3
159	03	3	204	02	2
160	03	3	205	00	0
161	04	4	206	00	0
162	01	1	207	69	OP
163	03	3	208	03	03
164	07	7	209	69	OP
165	00	0	210	05	05
166	00	0	211	98	ADV
167	00	0	212	92	RTN
168	00	0	213	76	LBL
169	69	OP	214	17	B'
170	03	03	215	98	ADV
171	69	OP	216	43	RCL
172	05	05	217	05	05
173	04	4	218	42	STD
174	02	2	219	87	87
175	01	1	220	99	PRT
176	03	3	221	43	RCL
177	69	OP	222	06	06
178	02	02	223	42	STD
179	02	2	224	78	78
180	07	7	225	99	PRT
181	04	4	226	43	RCL
182	01	1	227	07	07
183	01	1	228	42	STD
184	07	7	229	79	79
185	00	0	230	99	PRT
186	00	0	231	92	RTN
187	00	0	232	76	LBL
188	00	0	233	13	C
189	69	OP	234	32	X:T
190	03	03	235	93	.
191	69	OP	236	02	2
192	05	05	237	61	GTO
193	02	2	238	00	00
194	01	1	239	57	57

Construct NAM Card Preparation

PROCEDURE	PRESS	DISPLAY
1. Repartition	9 [2nd] [Op] 17	239.89
2. Clear program memory	[2nd] [CP]	239.89
3. Enter learn mode	[LRN]	000 00
4. Key in Construct NAM code (see listing)		
5. Check construct NAM code against listing	[CLR] [2nd] [List]	
6. Prepare to record Construct NAM Card	1 [2nd] [Write]	Blank
7. Insert card		1.

Label this card Construct NAM Card and place it in the card case for storage.

See *Personal Programming*, pp. V-44, 45 and VII-2, 3 for information on keying in programs and recording magnetic cards.

Note: The DEG and NOP instructions which appear at various places from location 160 to location 239 form a necessary part of the code since this part of program memory is recovered from program data in EE-19 to afford maximum register availability for analysis.

CONSTRUCT NAM CODE LISTING

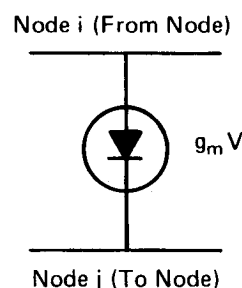
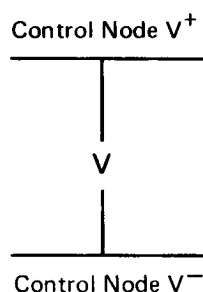
000	76	LBL	050	73	RC*	100	71	SBR
001	11	A	051	01	01	101	01	01
002	09	9	052	59	INT	102	54	54
003	09	9	053	55	+	103	43	RCL
004	42	STD	054	01	1	104	05	05
005	05	05	055	00	0	105	42	STD
006	01	1	056	95	=	106	07	07
007	00	0	057	42	STD	107	73	RC*
008	42	STD	058	06	06	108	02	02
009	03	03	059	59	INT	109	94	+/-
010	08	8	060	42	STD	110	42	STD
011	07	7	061	05	05	111	09	09
012	42	STD	062	22	INV	112	43	RCL
013	04	04	063	44	SUM	113	06	06
014	73	RC*	064	06	06	114	71	SBR
015	04	04	065	01	1	115	01	01
016	63	EX*	066	00	0	116	54	54
017	05	05	067	49	PRD	117	43	RCL
018	72	ST*	068	06	06	118	06	06
019	04	04	069	43	RCL	119	42	STD
020	69	DP	070	05	05	120	07	07
021	34	34	071	42	STD	121	43	RCL
022	69	DP	072	07	07	122	05	05
023	35	35	073	73	RC*	123	71	SBR
024	97	DSZ	074	02	02	124	01	01
025	03	03	075	42	STD	125	54	54
026	00	00	076	09	09	126	61	GTD
027	14	14	077	93	.	127	01	01
028	92	RTN	078	01	1	128	34	34
029	76	LBL	079	32	XIT	129	43	RCL
030	15	E	080	73	RC*	130	06	06
031	06	6	081	01	01	131	71	SBR
032	69	DP	082	22	INV	132	01	01
033	17	17	083	59	INT	133	54	54
034	47	CMS	084	67	EQ	134	02	2
035	09	9	085	01	01	135	94	+/-
036	69	DP	086	29	29	136	44	SUM
037	17	17	087	43	RCL	137	01	01
038	07	7	088	05	05	138	44	SUM
039	07	7	089	71	SBR	139	02	02
040	42	STD	090	01	01	140	97	DSZ
041	02	02	091	54	54	141	00	00
042	07	7	092	29	CP	142	00	00
043	06	6	093	43	RCL	143	50	50
044	42	STD	094	06	06	144	01	1
045	01	01	095	67	EQ	145	00	0
046	43	RCL	096	01	01	146	69	DP
047	89	89	097	34	34	147	17	17
048	42	STD	098	42	STD	148	11	A
049	00	00	099	07	07	149	36	PGM

CONSTRUCT NAM CODE LISTING (continued)

150	19	19	195	01	01
151	15	E	196	22	INV
152	61	GTO	197	59	INT
153	15	E	198	68	NOP
154	42	STO	199	77	GE
155	08	08	200	02	02
156	43	RCL	201	07	07
157	87	87	202	43	RCL
158	75	-	203	09	09
159	01	1	204	74	SM*
160	00	0	205	03	03
161	95	=	206	92	RTN
162	65	×	207	32	X:T
163	43	RCL	208	02	2
164	07	07	209	65	×
165	75	-	210	43	RCL
166	43	RCL	211	78	78
167	87	87	212	65	×
168	42	STO	213	43	RCL
169	04	04	214	09	09
170	49	PRD	215	65	×
171	04	04	216	60	DEG
172	85	+	217	89	π
173	43	RCL	218	95	=
174	08	08	219	42	STO
175	85	+	220	88	88
176	60	DEG	221	93	.
177	09	9	222	03	3
178	85	+	223	67	EQ
179	43	RCL	224	02	02
180	07	07	225	33	33
181	65	×	226	43	RCL
182	01	1	227	88	88
183	68	NOP	228	94	+/-
184	00	0	229	35	1/X
185	95	=	230	42	STO
186	42	STO	231	88	88
187	03	03	232	60	DEG
188	44	SUM	233	43	RCL
189	04	04	234	88	88
190	68	NOP	235	74	SM*
191	93	.	236	04	04
192	02	2	237	92	RTN
193	32	X:T	238	68	NOP
194	73	RC*	239	68	NOP

Program Assumptions

1. Phase output is always between -90° and 270° . (See *Personal Programming*, p. V-31.)
2. No independent current sources exist and all dependent sources are voltage controlled current sources.
3. Node 0 is the reference or ground node, node 1 is the input node, and node 2 is the output node.
4. A one-volt source is connected between node 1 and node 0.
5. The From Node is not zero when entering the node numbers that a component (R, L, C, or g_m) is connected between.
6. Active components (voltage controlled sources) are specified as follows:



The controlling voltage, V , is measured between the nodes V^+ and V^- (may be ground or node 0).

The controlled current $g_m V$ leaves the From node and enters the To node.

7. Networks with inductors are not analyzed at, or very near, zero frequency. (Insure specification of non-zero starting frequency.)
8. DC analysis is realized by redesigning the network, shorting all inductors and specifying a starting frequency of zero.

Outline of Program Usage

1. Review Program Assumptions carefully.
2. Draw circuit to be analyzed.
3. Number the nodes.
4. Enter data as specified. (See User Instructions 1–6.)

EE-19


5. Perform Network Analysis. (See User Instructions 7–9.)
6. Terminate sweep after desired number of intervals. (See User Instruction 10.)

Once a sweep has been completed, the following instructions can be performed to modify existing network. (This assumes that Input Card, side 2 was recorded per User Instruction Note 5.)

1. Perform User Instruction 2a.
2. Perform User Instruction 2b. (Load Input Card side 1. A 1 will appear in the display.)
3. Press [CLR] and repeat User Instruction 2b. (Load Input Card side 2. A 4 will appear in the display.)
4. To effect a change in starting frequency and/or increment, perform User Instructions 3a, 3b, 3c, entering desired frequency changes.

If no changes desired, press [2nd] [B']. Old NAM order, starting frequency, and increment will be printed.

5. To change an existing component value, see User Instructions Note 3.
6. To add component values to existing network, perform User Instructions 5 and 6 as needed.
7. For further network updating, re-record bank 4 data. (See User Instructions Note 5.)
8. Perform Network Analysis using new data. (See User Instructions 7–9.)
9. Terminate sweep after desired number of intervals. (See User Instruction 10.)

 Solid State Software TI © 1979				
NETWORK ANALYSIS				EE-19
g_m				
n	R	L	C	→ Execute

USER INSTRUCTIONS

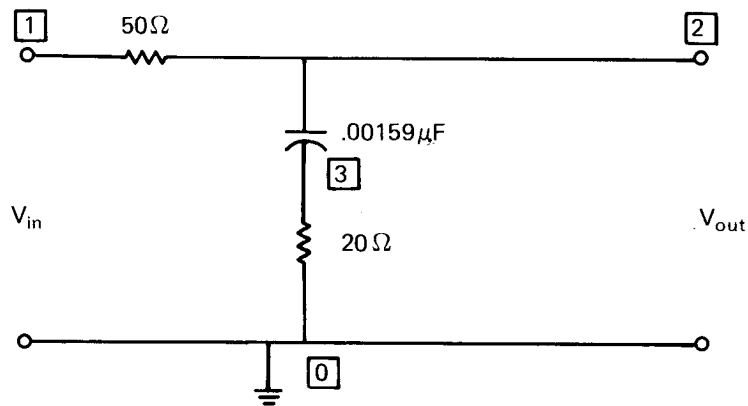
STEP	PROCEDURE	ENTER	PRESS	DISPLAY
1a	Repartition	9	[2nd] [Op] 17	239.89
1b	Initialize		[2nd] [CMs] [RST]	239.89
2a	Prepare to load input card ¹		[2nd] [CP] [CLR]	0
2b	Load input card			1.
3a	Enter order of NAM ²	n	[A]	n^\dagger
3b	Enter starting frequency in hertz	f	[R/S]	f^\dagger
3c	Enter frequency increment in hertz	Δf	[R/S]	Δf^\dagger
4	Initialize for component input		[2nd] [E']	INPUT* VALUE* FM.TO*
	Enter, in any order, values and nodal data for both passive and active components			
	Step 5a, b, c, or d must be followed by Step 6.³			
5a	Enter resistor value in ohms OR	R	[B]	R^\dagger
5b	Enter inductor value in henrys OR	L	[C]	L^\dagger
5c	Enter capacitor value in farads OR	C	[D]	C^\dagger
5d	Enter source value in mhos	g_m	[2nd] [A']	g_m^\dagger
6	Enter nodal data, From node.To node	Fm.To	[R/S]	Fm.To* Component No. [†]
	Repeat Steps 5 and 6 until all components are entered⁵			
7a	Prepare to load Construct NAM card ¹		[2nd] [CP] [CLR]	0
7b	Load Construct NAM card			1.
	If NAM order = 5, go to step 9			
8a	Enter learn mode		[GTO] [E] [LRN]	031 06
8b	If NAM order = 4	5	[LRN]	1.
8c	If NAM order = 3 or 2	3	[LRN]	1.
9	Perform network analysis ⁴ (A linear sweep continues until user termination in step 10)		[E]	Frequency* Magnitude (dB)* Phase (deg)* ⋮
10	Terminate linear sweep and repartition	9	[R/S] [RST] [2nd] [Op] 17	239.89

- NOTES:**
1. Reference: *Personal Programming*, pg. VII-5.
 2. n, order of NAM, corresponds to the total number of nodes (excluding ground node zero) and must meet the constraint ($2 \leq n \leq 5$).
 3. To edit a miskeyed entry in steps 5 and 6, enter component no., press [E] and repeat these steps.
 4. When NAM order = 2, a "?" is printed with frequency during linear sweep. This is a normal occurrence resulting from a two-node network being handled internally in three nodes during matrix reduction. When desired sweep is completed, execute step 10 and press [CE] to stop flashing.
 5. For easy updating, record contents of bank 4 (reference – *Personal Programming*, pg. VII-1) when steps 5 and 6 are completed. Enter 4 in the display, press [2nd] [Write] . Load Input Card, Side 2. A "4" will appear in the display.

† These displayed values are printed.

* These values are printed only.

Example 1: For the passive RC filter shown, determine the frequency response from 1 MHz to 10 MHz at 1 MHz increments. Then continue the sweep from 10 MHz to 100 MHz in increments of 10 MHz.



ENTER	PRESS	DISPLAY	COMMENTS
9	[2nd] [Op] 17	239.89	Repartition
	[2nd] [CMs] [RST]	239.89	Initialize
	[2nd] [CP] [CLR]	0.	Prepare to load Input Card, side 1
		1.	Load card
3	[A]	3.†	Enter order of NAM
1000000	[R/S]	1000000.†	Enter starting f in hertz
1000000	[R/S]	1000000.†	Enter Δf in hertz
	[2nd] [E']	INPUT*	Initialize for input
		VALUE*	
		FM.TO*	
55	[B]	55.†	Enter R in ohms
1.2	[R/S]	1.2*	Enter nodal data
		1.†	
20	[B]	20.†	Enter R in ohms
3.0	[R/S]	3.0*	Enter nodal data
		2.†	
1	[E]	1.†	Change value of component 1
50	[B]	50.†	Enter new value

ENTER	PRESS	DISPLAY	COMMENTS
1.2	[R/S]	1.2*	Enter nodal data
.00159	[EE] 6 [+/-] [D]	1.59-09 [†]	Enter C in farads
1.3	[R/S]	1.3*	Enter nodal data
		3. [†]	
3	[E]	3. [†]	Change nodal data of component 3
.00159	[EE] 6 [+/-] [D]	1.59-09 [†]	Enter C
2.3	[R/S]	2.3*	Enter new nodal data
4	[2nd] [Write]	4.	Record bank 4. Insert Input card, side 2
	[2nd] [CP] [CLR]	0	Prepare to load Construct NAM card
		1.	Load card
	[GTO] [E] [LRN]	031 06	Enter learn mode
3	[LRN]	1.	For NAM order = 3
	[E]	See Tape 1**	Perform analysis
	[R/S] [RST]	-18.44946143	Terminate linear sweep and repartition
9	[2nd] [Op] 17	239.89	
	[2nd] [CP] [CLR]	0.	Prepare to load Input Card, side 1
		1.	Load card
	[CLR]	0	Prepare to load bank 4 data
		4.	Load Input Card, side 2
3	[A]	3. [†]	Enter order of NAM
10000000	[R/S]	10000000. [†]	Enter starting f in hertz
10000000	[R/S]	10000000. [†]	Enter Δf in hertz
4	[2nd] [Write]	4.	Record bank 4. Insert Input Card, side 2
	[2nd] [CP] [CLR]	0	Prepare to load Construct NAM Card
		1.	Load card
	[GTO] [E] [LRN]	031 06	Enter learn mode
3	[LRN]	1.	For NAM order = 3
	[E]	See Tape 2**	Perform analysis
	[R/S] [RST]	-2.045937026	Terminate linear sweep and repartition
9	[2nd] [Op] 17	239.89	

[†]These displayed values are printed

*These values are printed only

**Approximately 19 minutes is required for these calculations.

OUTPUT TAPE FOR EXAMPLE 1

Tape 1

1000000. f(hertz)
 -1.559073999 mag (dB)
 -23.66660142 phase (degrees)

2000000.
 -4.063903011
 -32.6537504

3000000.
 -5.991926308
 -33.57584422

4000000.
 -7.311881441
 -31.69590456

5000000.
 -8.20826577
 -29.06775995

6000000.
 -8.827545839
 -26.42794172

7000000.
 -9.265698392
 -24.01851772

8000000.
 -9.583468724
 -21.89613662

9000000.
 -9.819429644
 -20.05033477

10000000.
 -9.998476016
 -18.44946143

Tape 2

10000000.
 -9.998476016
 -18.44946143

20000000.
 -10.63971792
 -9.959795447

30000000.
 -10.77199574
 -6.742412384

40000000.
 -10.81944351
 -5.084488094

50000000.
 -10.84161403
 -4.077950198

60000000.
 -10.85371347
 -3.40300624

70000000.
 -10.86102831
 -2.91930633

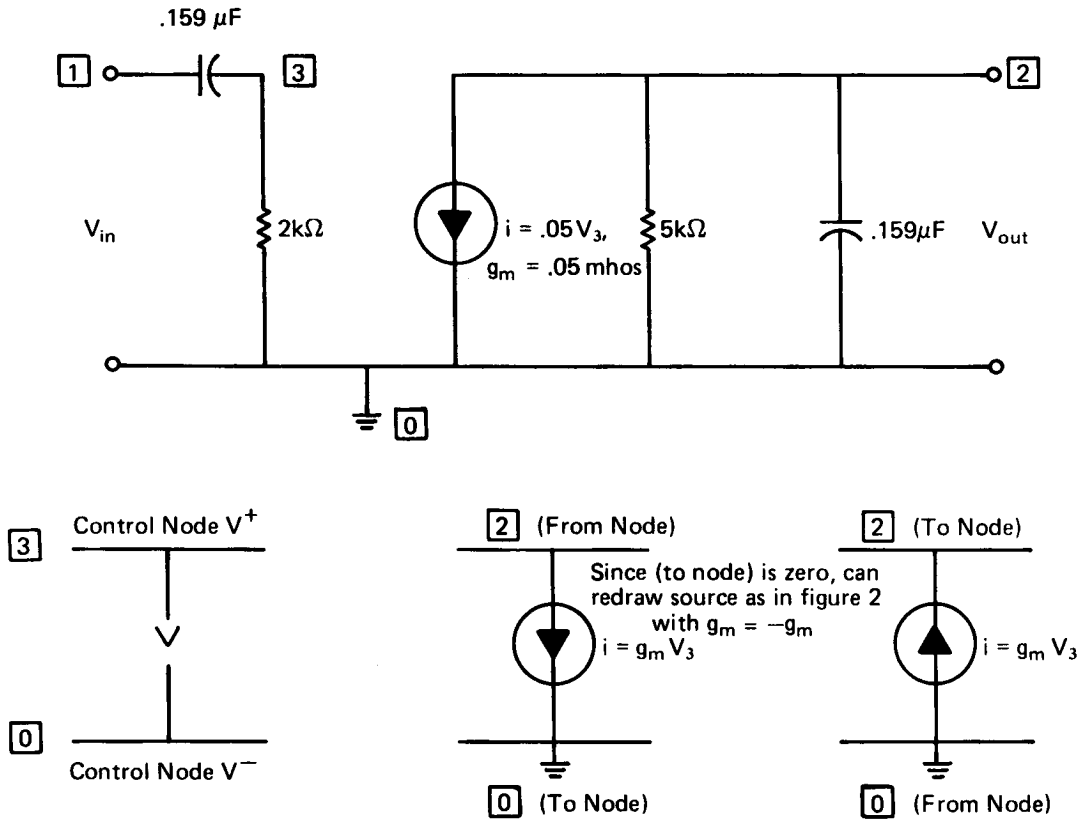
80000000.
 -10.8657837
 -2.555783497

90000000.
 -10.86904753
 -2.272655976

100000000.
 -10.87138391
 -2.045937026

Example 2: Examine the response of a common-emitter amplifier from 1 kHz to 10 kHz at increments of 1 kHz. The equivalent circuit representation is shown below.

COMMON-EMITTER AMPLIFIER (Equivalent Circuit Representation)



ENTER	PRESS	DISPLAY	COMMENTS
9	[2nd] [Op] 17	239.89	Repartition
	[2nd] [CMs] [RST]	239.89	Initialize
	[2nd] [CP] [CLR]	0	Prepare to load Input Card, side 1
		1.	Load card
3	[A]	3.†	Enter order of NAM
1000	[R/S]	1000.†	Enter starting f in hertz
1000	[R/S]	1000.†	Enter Δf in hertz
	[2nd] [E']	INPUT*	Initialize for input
		VALUE*	
		FM.TO*	
.000000159	[D]	.000000159†	Enter C in farads
1.3	[R/S]	1.3*	Enter nodal data
		1.†	
2000	[B]	2000.†	Enter R in ohms
3.0	[R/S]	3.0*	Enter nodal data
		2.†	
.05	[2nd] [A']	.05†	Enter source value in mhos
2.3	[R/S]	2.3*	Enter nodal data
		2.†	
5000	[B]	5000.†	Enter R in ohms
2.0	[R/S]	2.0*	Enter nodal data
		4.†	

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ENTER	PRESS	DISPLAY	COMMENTS
.000000159	[D]	.000000159 [†]	Enter C in farads
2.0	[R/S]	2.0*	Enter nodal data
		5. [†]	
	[2nd] [CP] [CLR]	0	Prepare to load Construct NAM card
		1.	Load card
	[GTO] [E] [LRN]	031 06	Enter learn mode
3	[LRN]	1.	For NAM order = 3
	[E]	See Tape 3**	Perform analysis
	[R/S] [RST]	94.01206902 [†]	Terminate linear sweep and repartition
9	[2nd] [Op] 17	239.89	

[†]These displayed values are printed.

*The values are printed only

**Approximately 17 minutes is required for these calculations.

OUTPUT TAPE FOR EXAMPLE 2

Tape 3

```
1000. f (hertz)
32.84640767 mag (dB)
127.9080497 phase (degrees)
```

```
2000.
27.66017524
109.7655013
```

```
3000.
24.30691736
103.2891567
```

```
4000.
21.86833137
99.99707806
```

```
5000.
19.95820588
98.00896102
```

```
6000.
18.38988951
96.67927389
```

```
7000.
17.06020418
95.72775564
```

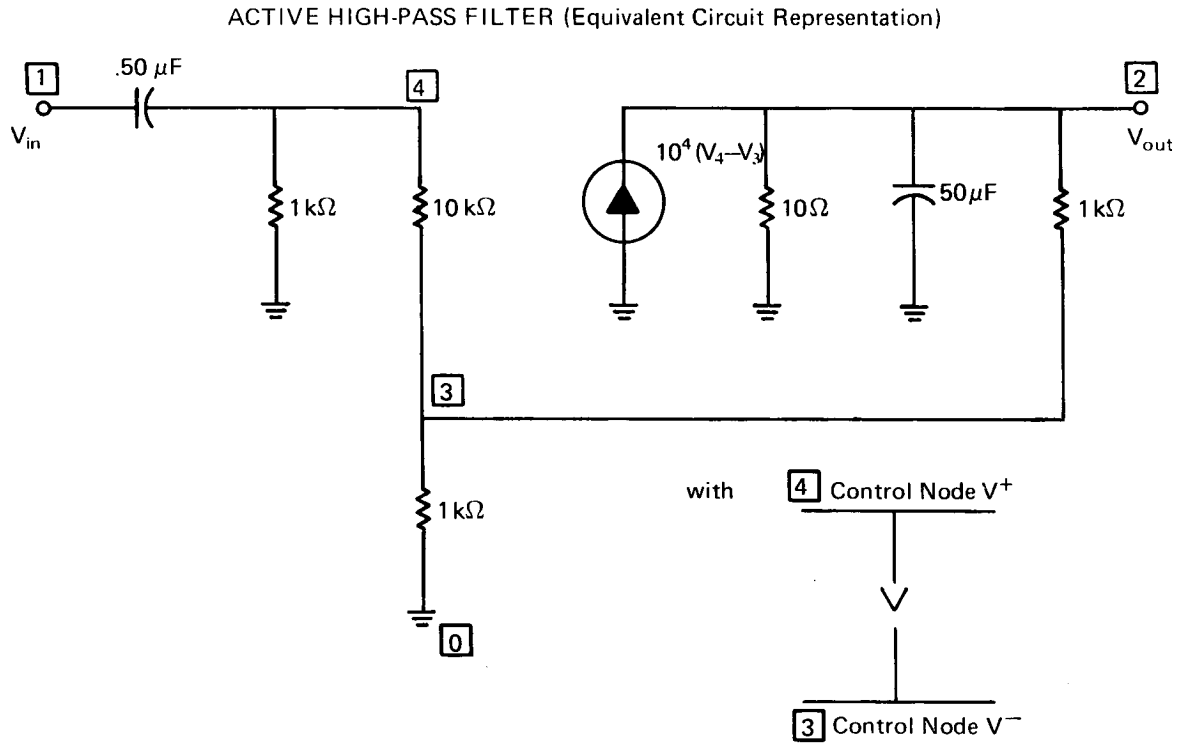
```
8000.
15.90637718
95.01330156
```

```
9000.
14.88745217
94.45719261
```

```
10000.
13.97525509
94.01206902
```

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Example 3: Using the following equivalent circuit, evaluate the gain of an active high-pass filter from 100 kHz to 1 MHz at increments of 100 kHz.



ENTER	PRESS	DISPLAY	COMMENTS
9	[2nd] [Op] 17	239.89	Repartition
	[2nd] [CMs] [RST]	239.89	Initialize
	[2nd] [CP] [CLR]	0	Prepare to load Input Card, side 1
		1.	Load card
4	[A]	4.†	Enter order of NAM
100000	[R/S]	100000.†	Enter starting f in hertz
100000	[R/S]	100000.†	Enter Δf in hertz
	[2nd] [E']	INPUT*	Initialize for input
		VALUE*	
		FM.TO*	
.00005	[D]	.00005†	Enter C in farads
1.4	[R/S]	1.4*	Enter nodal data
		1.†	
1000	[B]	1000.†	Enter R in ohms
4.0	[R/S]	4.0*	Enter nodal data
		2.†	
10000	[B]	10000.†	Enter R in ohms
4.3	[R/S]	4.3*	Enter nodal data
		3.†	
1000	[B]	1000.†	Enter R in ohms
3.0	[R/S]	3.0*	Enter nodal data
		4.†	
10	[B]	10.†	Enter R in ohms
2.0	[R/S]	2.0*	Enter nodal data
		5.†	

ENTER	PRESS	DISPLAY	COMMENTS
1000	[B]	1000. [†]	Enter R in ohms
2.3	[R/S]	2.3*	Enter nodal data
		6. [†]	
10000	[2nd] [A']	10000. [†]	Enter source value in mhos
2.3	[R/S]	2.3*	Enter nodal data
		7. [†]	
10000	[+/-] [2nd] [A']	-10000. [†]	Enter source value in mhos
2.4	[R/S]	2.4*	Enter nodal data
		8. [†]	
.00005	[D]	0.00005 [†]	Enter C in farads
2.0	[R/S]	2.0*	Enter nodal data
		9. [†]	
	[2nd] [CP] [CLR]	0	Prepare to load Construct NAM Card
		1.	Load card
	[GTO] [E] [LRN]	031 06	Enter learn mode
5	[LRN]	1.	For NAM order = 4
	[E]	See Tape 4**	Perform analysis
	[R/S] [RST]	-3.774268231	Terminate linear sweep and repartition
9	[2nd] [Op] 17	239.89	

[†]These displayed values are printed

*These values are printed only

**Approximately 39 minutes is required for these calculations

OUTPUT TAPE FOR EXAMPLE 3

Tape 4

	f (hertz)	mag (dB)	phase (degrees)
100000.			
6.020227407			
-.3761627445			
200000.			
6.019660417			
-.7550282678			
300000.			
6.018715592			
-1.133220083			
400000.			
6.017393182			
-1.511161289			
500000.			
6.015693532			
-1.888910281			
600000.			
6.013617085			
-2.266464726			
700000.			
6.011164382			
-2.643805011			
800000.			
6.008336059			
-3.02090511			
900000.			
6.005132852			
-3.397736221			
1000000.			
6.001555591			
-3.774268231			

Register Contents

R_{00} – R_{09} , R_{80} – R_{86} , R_{90} – R_{99} are work registers having multiple program uses.

R_{78} f

R_{79} Δf

R_{87} n

R_{88} Used

R_{89} Component Counter

NAM Register Assignments

(Admittance y_{ij} is the ij^{th} complex element of the matrix)

NAM order = 5			NAM order = 4			NAM order = 3 or 2		
real y_{ij} register	matrix element i j	imag y_{ij} register	real y_{ij} register	matrix element i j	imag y_{ij} register	real y_{ij} register	matrix element i j	imag y_{ij} register
10	1 1	35	10	1 1	26	10	1 1	19
11	1 2	36	11	1 2	27	11	1 2	20
12	1 3	37	12	1 3	28	12	1 3	21
13	1 4	38	13	1 4	29			
14	1 5	39						
15	2 1	40	14	2 1	30	13	2 1	22
16	2 2	41	15	2 2	31	14	2 2	23
17	2 3	42	16	2 3	32	15	2 3	24
18	2 4	43	17	2 4	33			
19	2 5	44						
20	3 1	45	18	3 1	34	16	3 1	25
21	3 2	46	19	3 2	35	17	3 2	26
22	3 3	47	20	3 3	36	18	3 3	27
23	3 4	48	21	3 4	37			
24	3 5	49						
25	4 1	50	22	4 1	38			
26	4 2	51	23	4 2	39			
27	4 3	52	24	4 3	40			
28	4 4	53	25	4 4	41			
29	4 5	54						
30	5 1	55						
31	5 2	56						
32	5 3	57						
33	5 4	58						
34	5 5	59						
R_{60} – R_{77} used for storage of 9 component values			R_{50} – R_{77} used for storage of 14 component values			R_{30} – R_{77} used for storage of 24 component values		

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Method Used

An electrical network with n nodes (excluding node 0, ground) can be specified for steady-state analysis by its $n \times n$ node admittance matrix (NAM) $[Y]$ which is dependent on the specified frequency, network topology, and component values.

Starting with a null matrix (all elements are zero), the complex admittance matrix $[Y]$ is formed by adding components to the null matrix as follows:

Let y = admittance of a given component

i and j = numbers of the nodes that the component is connected between (i is the from node ($i \neq 0$) and j is the to node)

Let y_{ij} be the element in the i^{th} row and j^{th} column of the NAM $[Y]$ being formed.

Add y to y_{ii} , and if $j \neq 0$, add y to y_{jj} , add $-y$ to y_{ij} and $-y$ to y_{ji} .

Since the matrix element y_{ij} is a complex quantity dependent on frequency, each network component adds to either the real or imaginary part of y_{ij} .

For resistors $y = 1/R$ where R = resistance in ohms

For inductors $y = -(1/\omega L)j$ where L = inductance in henrys
 $\omega = 2\pi f$ where f is frequency in hertz

For capacitors $y = \omega Cj$ where C = capacitance in farads

Once the complex NAM $[Y]$ has been formed, the following matrix equation can be used to represent the relationship between node voltages and currents. This equation describes the steady-state performance of a network at a given frequency.

$$[Y]_{n \times n} [V]_{n \times 1} = [I]_{n \times 1}$$

where

$[V]_{n \times 1}$ is the matrix (column vector) of the node voltages.

$[I]_{n \times 1}$ is the matrix (column vector) of the current sources.

Solving for $[V]_{n \times 1}$:

$$[Y]_{n \times n}^{-1} [Y]_{n \times n} [V]_{n \times 1} = [Y]_{n \times n}^{-1} [I]_{n \times 1}$$

or

$$[V]_{n \times 1} = [Y]_{n \times n}^{-1} [I]_{n \times 1}$$

Since, for this program it is necessary to find only one node voltage, the complex matrix inversion technique required above is dropped in favor of a faster and more accurate matrix reduction method which can be applied to $[Y]$ to find the network transfer function $V_{\text{out}}/V_{\text{in}}$.

At this point, all active elements (i.e., voltage controlled dependent current sources) present in the network should be added to $[Y]$. For each voltage controlled current source, compute the

transconductance g_m . Add $-g_m$ to y_{ij} where i is the number of the node the controlled current is entering and j is the number of the controlling voltage V^+ of the source. If $j \neq 0$, add g_m to y_{ij} where i is the same as before and $j \neq 0$ is the number of the controlling voltage V^- of the source.

(Note that allowances for sources have to be made by the user at input. See User Instruction 5d and program reference 1., pp 9–11, 34, 35 and 2., pp 22, 23, 63–68.)

[Y] is now ready for matrix reduction which is outlined below.

To find the network transfer function $V_{out}/V_{in} = V_2/V_1$

where node 1 is the input node and node 2 is the output node

1. Partition $[Y]_{n \times n}$

$$[Y]_{n \times n} = \begin{bmatrix} [Y_{11}] & [Y_{12}] \\ [Y_{21}] & Y_{nn} \end{bmatrix}$$

where

$[Y_{11}]$ is an $(n-1) \times (n-1)$ square submatrix

$[Y_{12}]$ is a column vector with $(n-1)$ elements

$[Y_{21}]$ is a row vector with $(n-1)$ elements

Y_{nn} is the indicated element of the NAM [Y]

2. Reduce $[Y]_{n \times n}$ to a $(n-1) \times (n-1)$ matrix $[Y^{(1)}]_{(n-1) \times (n-1)}$ by the relation.

$$[Y^{(1)}]_{(n-1) \times (n-1)} = [Y_{11}] - [Y_{12}] [Y_{21}] \left(\frac{1}{Y_{nn}} \right)$$

3. Partition the matrix $[Y^{(1)}]$ in a manner similar to that shown in 1 and repeat the reduction in 2 which produces the matrix $[Y^{(2)}]_{(n-2) \times (n-2)}$. Continue in this manner until a 2×2 matrix

$$[Y^{(n-2)}]_{2 \times 2} = \begin{bmatrix} Y'_{11} & Y'_{12} \\ Y'_{21} & Y'_{22} \end{bmatrix} \quad \text{is derived.}$$

The complex division $\frac{-Y'_{21}}{Y'_{22}} = \frac{V_2}{V_1}$ gives the required transfer function.

$$20 \log \left| \frac{V_2}{V_1} \right| = \text{magnitude in dB and}$$

$$\tan^{-1} \left(\frac{\text{Imag}(V_2/V_1)}{\text{Real}(V_2/V_1)} \right) = \text{phase in degrees where } -90^\circ \leq \text{phase} < 270^\circ$$

(See *Personal Programming*, pg. V-31)

- References:
1. *Active Networks*, edited by Julius J. Hupert, published for Department of Physics, DePaul University by University Microfilms International 1978.
 2. *Theory and Design of Active RC Circuits*, Lawrence P. Huelsman, McGraw-Hill 1968.
 3. *Circuits Matrices and Linear Vector Spaces*, Lawrence P. Huelsman, McGraw-Hill 1963.
 4. *Digital Computations in Basic Circuit Theory*, Lawrence P. Huelsman, McGraw-Hill 1968.

APPENDIX A: PROGRAM REFERENCE DATA

Program Number	Title	No. of Steps	Data Reg. Used	Flags Used	SBR Levels	Paren. Levels	Calls Pgm.	Special Functions Used	x ≥ t	ABS Address	Fix Decimal Format	Angular Mode	Program Number
01	Module Check	32	1-6						CP	X			01
02	Phase-Locked Loop	344	10-20	0,1,2	1	2	11		X	X			02
03	S-Parameter Conversions	273	1-17		2		4	P→R	CP	X		Deg.	03
04	Complex Arithmetic	212	1-4	1	2	1	5	P→R	X	X		Deg.	04
05	Complex Functions	146	1-4		2	2	4	P→R	X	X		Deg.	05
06	Complex Trig Functions	298	0-4	0	2	3	4,5	P→R	X	X		Deg.	06
07	dB, Np, P, V, I, Ratio Conversions	159	10-13	0	1	1				X			07
08	Signal Detection	459	10-18	0,1,2	2	2			X	X			08
09	Roots of a Polynomial	467	0-59	0,1		2			CP	X			09
10	Chained Multiplication of Polynomials	228	0-89	0,1	1	1			X	X			10
11	Reactance Chart	218	10-14	0,1,2		2				X			11
12	Series/Parallel Impedance Conversions	117	10-13	0	1	1	7			X			12
13	Active LP, HP, BP Filters	312	10-18		1	2			X	X			13
14	Passive Lowpass Filters	373	0-13	0	2	6			X	X			14
15	Convolution	178	8-17	0	1	2	0		X	X			15
16	Root Locus Calculations	333	0-59	0	2	1		P→R	X	X	Deg.		16
17	Discrete Fourier Transform	255	0-99	0	1	2		P→R	X	X	Deg.		17
18	Smith Chart Calculations	201	1-10,14-17,20		3	2	4,5,11	P→R	X	X			18
19	Network Analysis	832*	0-99	0	4	3	0,4,19	P→R	X	X	Deg.		19

*Includes program memory

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