AERODYNAMIC CALCULATION METHODS for PROGRAMMABLE CALCULATORS & PERSONAL COMPUTERS

— with programs for the TI-59 —

PAK #1
BASIC AERODYNAMIC RELATIONS

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AEROCAL

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Preface

AEROCAL programs are intended to serve both students and practicing aerodynamicists. For students, they can serve an important role in supplementing theoretical analysis with the actual numerical results so important in developing engineering skills. In aerodynamics, it has been difficult for students to solve meaningful illustrative problems, and this difficulty can now be eliminated by using the new personal computing machines -- either programmable calculators or microcomputers. I have found that the results of numerical calculations inevitably provide a few surprises, which force the analyst to reexamine the theory, leading to a much deeper understanding. AEROCAL programs can thus be used to prevent the calculation from becoming an end in itself. Instead, efforts can be concentrated on the actual aerodynamic problems, with required calculations assuming their proper supporting role. Thus, the availability of the personal computing machines allows the student to gain an appreciation of the role of computational aerodynamic simulations, while developing an engineering attitude.

The second purpose of the work is to provide the practicing aerodynamicist with a readily accessible collection of algorithms designed for use on this class of machine. The availability of such a set of routines will eliminate the most tedious aspects of the software development process so that the code development time can be used to implement the user's unique requirements rather than wasting time creating the basic building blocks.

The material selected for inclusion is, of course, not intended to replace the large computational aerodynamics programs. Instead, it allows students to become familiar with an important part of the set of standard aerodynamic methods representative of those required in aerodynamics. To the experienced user, these methods should be extremely useful, providing results which are more than adequate for a variety of jobs.

The material is organized in workbook fashion, with each program being essentially independent of the others. An example of the style that we intend to follow is found in the IBM SSP or other software package user's manuals. The addition of some examples for each program allows the user to check that the program is properly executing on his own machine.

The choice of the TI59 format for the programs is one of convenience only. Program instructions are similar for other calculators and an Appendix is included to describe the listing nomenclature. Using this information, conversion to other instruction sets should be relatively simple. Microcomputers will typically have more advanced instruction sets, such as BASIC. The information provided in the method description is easily used to write a set of BASIC instructions.

The author acknowledges the contributions of the many aerodynamicists and research scientists who have developed the basic material, which forms the basis for these software paks and with whom he has held discussions on the relative merits of particular methods for performing various aerodynamics calculations.

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September 1981

About the Author

W. H. Mason has spent more than ten years developing and applying computational aerodynamics methodology to transonic and supersonic aircraft design. This work required the use of the full range of computer codes presently used in the industry, so that the author has an unusually broad base of experience to draw upon. He obtained the B.S., M.S. and Ph.D. degrees in Aerospace Engineering at Virginia Polytechnic Institute and is presently employed as a Senior Engineer in the Aerodynamics Section of Grumman Aerospace Corporation. Dr. Mason is a registered Professional Engineer in New York State.

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1.0 INTRODUCTION

There are a number of routine calculations which are a little too difficult to do entirely by hand each time they arise. In this package of basic aerodynamic relations, we provide a summary of the formulas used most often in the general area of compressible flow analysis. This includes isentropic formulas, the methods for normal and oblique shocks, and 1-D gas flows with friction or heat addition. The other method included is the description of the standard atmosphere. When included in the typical library of calculator programs, these routines can eliminate many of the more tedious aspects of aerodynamic flow field calculations.

The methods are presented in a standard format, with the following order:

- o Title
- o Description of what the method does
- o References
- o Detailed outline of the method or listing of equations
- o User Instructions
- o Sample Case
- o Program Description
- o Program Listing

The programs are written in the most direct sequence of instructions possible in order to avoid difficulty in studying the programs and modifying them. The use of TI59 instruction sets is a matter of convenience. These routines will work on a number of other calculators and we can expect that many more powerful hand calculators will appear in the near future.

The routines often make use of the printer. The author has found the printer to be much more valuable in program development than in program execution. Nevertheless, several programs do provide printed results. A description of printed output is included below the user instructions for each program.

1.1 NACA 1135+

This program computes the compressible flow relationships tabulated in NACA Report 1135, "Equations, Tables and Charts for Compressible Flow," by the NACA Ames Center Staff issued in 1953. Until recently, virtually all aerodynamicists had a copy of this report. "1135" contained the isentropic and normal shock relations in tabular form for $\gamma = 1.4$. Unfortunately, 1135 is now out of print. This program also computes several additional useful relations. In addition, the solution can be computed for completely arbitrary γ and Mach number. Together, with Program 1.4, Rayleigh/Fanno Lines, most of the calculations required in 1-D gas dynamics can be carried out quite simply.

No attempt is made to explain the use of these relations. A huge number of texts have been written on basic gas dynamics. The classical book is Shapiro, A. H., The Dynamics and Thermodynamics of Compressible Fluid Flow, Ronald Press, New York, 1953. Two other useful books are: Cambel, A. B. and Jennings, B. H., Gas Dynamics, Dover Edition, 1967 (which should be relatively inexpensive) and Kuethe, A. M. and Chow, C. Y., Foundation of Aerodynamics, John Wiley, New York, 1976 (which is more aerodynamically oriented).

The equations used in the program are contained in the Defining Equations Section. Notable additional parameters computed here, but not normally used in gas dynamics are:

a) Cp_{crit} — The value of the pressure coefficient which corresponds to a local Mach number of unity for a specified reference Mach number.

b) Cp_{vac} — The value of the pressure coefficient which corresponds to a zero (or vacuum) pressure for a specified reference Mach number.

c) δ_{max} — For a specified Mach number, this is the maximum flow deflection angle for which an attached oblique shock can exist (Equation 139a in 1135).

d) $\theta_{\delta_{max}}$ — The shock wave angle corresponding to δ_{max} (Equation 168 in 1135).

The other additional results are simple and require no further elaboration.

DEFINING EQUATIONS

1. Prandtl-Glauert Factor

$$\beta = \sqrt{|M^2 - 1|}$$

2. Stagnation temperature ratio

$$\frac{T}{T_o} = \left[1 + \left(\frac{\gamma-1}{2}\right) M^2\right]^{-1}$$

3. Stagnation pressure ratio

$$\frac{p}{p_o} = \left(\frac{T}{T_o}\right)^{\frac{\gamma}{\gamma-1}}$$

4. Stagnation density ratio

$$\frac{\rho}{\rho_o} = \left(\frac{T}{T_o}\right)^{\left(\frac{1}{\gamma-1}\right)}$$

5. Stagnation speed of sound ratio

$$\frac{a}{a_o} = \left(\frac{T}{T_o}\right)^{\frac{1}{2}}$$

6. Ratio of dynamic pressure, $q = 1/2\rho V^2$, to total pressure

$$\frac{q}{p_o} = \frac{\gamma}{2} M^2 \left(\frac{p}{p_o}\right)$$

7. Cross-sectional area ratio

$$\frac{A^*}{A} = \left(\frac{\gamma+1}{2}\right)^{\frac{1}{2}\frac{(\gamma+1)}{(\gamma-1)}} \cdot M \cdot \left(\frac{T}{T_o}\right)^{\frac{1}{2}\frac{(\gamma+1)}{(\gamma-1)}}$$

8. Ratio of velocity to critical velocity

$$\left(\frac{V}{a^*}\right)^2 = \left(\frac{\gamma+1}{2}\right) M^2 \cdot \left(\frac{T}{T_o}\right)$$

9. Critical pressure coefficient

$$C_{p_{crit}} = -\frac{2}{\gamma M^2}\left[1 - \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}} \frac{1}{p/p_o}\right.$$

10. Vacuum pressure coefficient

$$C_{p_{vac}} = -\frac{2}{\gamma M_\infty^2}$$

11. Prandtl-Meyer angle

$$\nu \sqrt{\frac{\gamma+1}{\gamma-1}} \tan^{-1} \sqrt{\left(\frac{\gamma-1}{\gamma+1}\right)(M^2-1)} - \cos^{-1}\left(\frac{1}{M}\right)$$

12. Mach angle

$$\mu = \sin^{-1}\left(\frac{1}{M}\right)$$

13. Downstream Mach number for normal shock wave

$$M_2^2 = \frac{(\gamma-1)\, M_1^2 + 2}{2\gamma\, M_1^2 - (\gamma-1)}$$

14. Normal shock static pressure ratio

$$\frac{p_2}{p_1} = \frac{2\gamma\, M_1^2 - (\gamma-1)}{(\gamma+1)}$$

15. Normal shock density and velocity rat

$$\frac{\rho_2}{\rho_1} = \frac{u_1}{u_2} = \frac{(\gamma+1)\, M_1^2}{(\gamma-1)\, M_1^2 + 2}$$

16. Normal shock static temperature and speed of sound ratios

$$\frac{T_2}{T_1} = \left(\frac{a_2}{a_1}\right)^2 = \frac{[2\gamma M_1^2-(\gamma-1)][(\gamma-1)M_1^2+2]}{(\gamma+1)^2\, M_1^2}$$

17. Normal shock stagnation pressure ratio

$$\frac{p_{o2}}{p_{o1}} = \frac{\rho_{o2}}{\rho_{o1}} = \left[\frac{(\gamma+1)\, M_1^2}{(\gamma-1)\, M_1^2 + 2}\right]^{\frac{\gamma}{\gamma-1}} \mathrm{X}$$

$$\left[\frac{(\gamma+1)}{2\gamma\, M_1^2 - (\gamma-1)}\right]^{\frac{1}{\gamma-1}}$$

18. Normal shock upstream static to downstream stagnation pressure ratio

$$\frac{p_1}{p_{o2}} = \left(\frac{p_1}{p_{o1}}\right)\left(\frac{p_{o1}}{p_{o2}}\right)$$

Recall that $T_{o2} = T_{o1}$ across shock waves.

19. Oblique shock wave angle for maximum attached shock deflection angle

$$\sin^2 \theta_{\delta_{max}} = \frac{1}{4\gamma M_1^2}\left\{(\gamma+1)\, M_1^2 - 4 + \sqrt{(\gamma+1)\left[(\gamma+1)\, M_1^4 + 8\,(\gamma-1)\, M_1^2 + 16\right]}\right\}$$

20. Maximum flow deflection angle for attached oblique shock

$$\tan \delta_{max} = \frac{2}{\tan \theta_{\delta_{max}}} \quad \mathrm{X}$$

$$\left[\frac{M_1^2 \sin^2 \theta_{\delta_{max}} - 1}{M_1^2\,(\gamma + \cos(2\theta_{\delta_{max}})) + 2}\right]$$

USER INSTRUCTIONS -- PROGRAM 1.1

STEP	ENTER	PRESS
1. Establish γ if $\gamma = 1.4$	-	A
or		
Establish γ if $\gamma \neq 1.4$	γ	B
2. Input Mach number	M	C
(REPEAT STEP 2. AS DESIRED WITHOUT RE-ENTERING γ.)		

OUTPUT: The results are stored in the registers as indicated below and illustrated with two sample cases for $\gamma = 1.4$.

R	VARIABLE	SAMPLE CASES M = .9	SAMPLE CASES M = 2.5
0	M	.9	2.5
1	p/p_o	.5913	.05853
2	ρ/ρ_o	.6870	.1317
3	T/T_o	.8606	.4444
4	β	.4359	2.2910
5	q/p_o	.3352	.2561
6	A/A^*	1.0089	2.6370
7	V/a^*	.91460	1.82574
8	$^*C_{p_{crit}}$	-.1878	1.83456
9	$^*C_{p_{vac}}$	-1.764	- .2286
10	ν		39.124
11	μ		23.58
12	M_2		.5130
13	p_2/p_1		7.125
14	ρ_2/ρ_1		3.333
15	T_2/T_1	SUPERSONIC	2.138
16	p_{o2}/p_{o1}	ONLY	.499
17	p_1/p_{o2}		.1173
18	γ		1.40
19	$^*T_o/T$		2.25
20	$^*a/a_o$		.6666
21	$^*u_2/u_1$	SUPERSONIC	.3
22	$^*\rho_{o2}/\rho_{o1}$	ONLY	.499
23	$^*a_2/a_1$		1.462
24	$^*\delta_{max}$		29.797
25	$^*\theta_{\delta_{max}}$		64.782

* NOT INCLUDED IN NACA 1135.

NOTE: The program also uses R_{27}-R_{29} as work registers.

1.2 PRANDTL-MEYER ANGLE AND INVERSE

This program computes the Prandtl-Meyer (P-M) angle for a specified Mach number, or conversely, for a given P-M angle the corresponding Mach number is found.

A uniform two-dimensional supersonic stream expands isentropically over a convex corner. The P-M angle relates the local Mach number to the angle of expansion from the M = 1 reference condition.

The P-M angle is normally used in the following manner: given an onset Mach number, M_i, and an expansion angle, $\Delta\theta$, find the new Mach number, M_f. As given in Program 1.1, $\nu(M)$ is known, and $M(\nu)$ must be determined through a sequence of guesses.

METHOD OF SOLUTION

The P-M angle is given as:

$$\nu(M) = \sqrt{\frac{\gamma+1}{\gamma-1}} \cdot \tan^{-1} \sqrt{\left(\frac{\gamma-1}{\gamma+1}\right)\left(M^2-1\right)} - \tan^{-1} \sqrt{M^2-1}$$

Defining

$$a = \sqrt{\frac{\gamma+1}{\gamma-1}} \quad \text{and} \quad q = \sqrt{M^2-1}$$

$$\nu(q) = a \tan^{-1} \left\{\frac{q}{a}\right\} - \tan^{-1} q$$

Use Newton's method to find q for a given ν:

$$q^{i+1} = q^i - \frac{f(q^i)}{f'(q^i)}$$

Where

$$f(q) = \nu - a \tan^{-1}\left(\frac{q}{a}\right) + \tan^{-1} q$$

$$f'(q) = -\frac{a^2}{a^2+q^2} + \frac{1}{1+q^2}$$

An initial guess is required to start the iteration and $q^0=2$ has been arbitrarily chosen and found to work satisfactorily. Convergence is usually obtained in 4 or 5 iterations, which takes about 30 seconds on the TI59.

USER INSTRUCTIONS -- PROGRAM 1.2

STEP	ENTER	PRESS	DISPLAY
1. Establish γ if $\gamma = 1.4$	-	A	
or			
Establish γ if $\gamma \neq 1.4$	γ	B	
2. To find ν given M	M	C	ν
3. To find M given ν	ν	D	M

NOTE: ν is given in degrees.

EXAMPLE PROBLEM: An M = 1.60 flow is expanded about a 34.896^0 corner.

A. $\gamma = 1.4$; press A.

B. Enter 1.60 and press C.

C. From Step B, find $\nu = 14.861^0$ in the display.

D. Expanding $34.896^0 \rightarrow \nu = 14.861 + 34.896 = 49.757^0$.
Enter 49.757 and press D.

E. The final Mach number $M_f = 3.00$ is contained in the display.

NOTE: a) $\nu = 130.45^0$ ($\gamma = 1.4$) corresponds to $M_f = \infty$, and hence is the maximum angle.

b) The static pressure ratio can be found from:

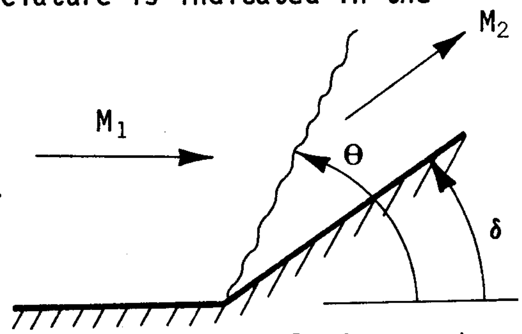
$$\frac{p_i}{p_f} = \left[\frac{1 + \frac{\gamma-1}{2} M_f^2}{1 + \frac{\gamma-1}{2} M_i^2}\right]^{\frac{\gamma}{\gamma-1}} = \frac{p_i/p_o\ (M_i)}{p_f/p_o\ (M_f)}$$

Where p_i/p_o and p_f/p_o for a given M can be found from Program 1.1.

c) ν must be greater than zero. For flow compression, the oblique shock Program 1.3 should be used.

1.3 PROPERTIES OF OBLIQUE SHOCKS

Given the upstream Mach number, the ratio of specific heats, and either the shock wave angle θ, or the flow deflection angle δ, this program determines the downstream flow properties. The nomenclature is indicated in the sketch.



For the basic thermodynamic properties, the Mach number normal to the shock wave ($M_1 \sin\theta$) governs the downstream conditions. [Recall that the flow velocity parallel to the shock is constant across the shock wave.]

If δ is given, a cubic equation must be solved for θ. The solution used in this program was given by V. R. Mascitti in the Journal of Aircraft, Volume 6, No. 1, 1969, page 66.

METHOD OF SOLUTION

The equation for θ takes the form:

$$\sin^6 \theta + b \sin^4 \theta + c \sin^2 \theta + d = 0$$

where

$$b = -\left[\frac{M_1^2 + 2}{M_1^2}\right] - \gamma \sin^2 \delta$$

$$c = \frac{2 M_1^2 + 1}{M_1^4} + \left[\frac{(\gamma+1)^2}{4} + \frac{(\gamma-1)}{M_1^2}\right] \sin^2 \delta$$

$$d = -\frac{\cos^2 \delta}{M_1^4}$$

To find the solution, compute:

$$\cos \phi = \frac{\frac{9}{2} bc - b^3 - \frac{27}{2} d}{(b^2-3c)^{3/2}}$$

And then

$$\sin^2 \theta = A = -\frac{b}{3} + \frac{2}{3} (b^2 - 3c)^{1/2} \cos \left[\frac{\phi + n\pi}{3} \right]$$

where

$n = 0$ for the strong shock solution.

$= 4$ weak shock solution (the weak shock solution is the one employed in the program).

and

$$\theta = \sin^{-1} \sqrt{A}$$

If $A < 0$, then δ has exceeded the angle for the weak shock solution.

If θ is given, δ can be found from:

$$\tan \delta = \frac{2}{\tan \theta} \left[\frac{M_1^2 \sin^2 \theta - 1}{M_1^2 (\gamma + \cos 2\theta) + 2} \right]$$

Once δ and θ are known, the rest of the flowfield is found from the normal shock relations with $M_n = M_1 \sin \theta$. Additional useful relations are:

$$C_p = \frac{4}{(\gamma+1) M_1^2} \left[M_n^2 - 1 \right]$$

$$M_2^2 = \frac{1}{\sin^2 (\theta - \delta)} \left[\frac{1 + \frac{\gamma-1}{2} M_n^2}{\gamma M_n^2 - \frac{\gamma-1}{2}} \right]$$

$$\frac{V_2}{V_1} = \left(\frac{M_2}{M_1} \right) \left(\frac{a_2}{a_1} \right)$$

V_1 and V_2 are the total velocities. a_2/a_1 is found from the normal shock relations using M_n, see Equation 16 on page 1-5.

USER INSTRUCTIONS -- PROGRAM 1.3

STEP	ENTER	PRESS
1. Establish γ if $\gamma = 1.4$	-	A
or		
Establish γ if $\gamma \neq 1.4$	γ	B
2. Input the initial Mach number	M_1	C
3. If δ is given	δ	D
4. If θ is given	θ	E
Steps 3 and 4 can be repeated for a given M_1 without reentering M_1.		

NOTE: θ and δ are given in degrees.

OUTPUT: The results are contained in the registers as indicated below and illustrated with a sample case [$\gamma = 1.4$, $M_1 = 3.0$, $\theta = 41.8103^o$].

R	VARIABLE	SAMPLE CASE
0	N = 4	
1	γ	1.4
2	M_1^2	9
3	δ (DEG)	
4	b	
5	c	
6	d	
7	$\cos \phi$	
8	A	
9	θ (DEG)	41.8103
10	ϕ (DEG)	
11	Cp	.5556
12	M_2	1.8159
13	p_2/p_1	4.5000
14	ρ_2/ρ_1	2.6666
15	T_2/T_1	1.6875
16	a_2/a_1	1.2990
17	V_2/V_1	.7861
18	p_{o2}/p_{o1}	.7209

NOTE: The program also uses R_{20}-R_{22} as work registers.

1.4 RAYLEIGH/FANNO LINE TABLES

This program computes the Rayleigh Line (one-dimensional compressible flow functions for stagnation temperature change in the absence of friction and area change) and the Fanno Line (one-dimensional compressible flow functions for adiabatic flow at constant area with friction) for arbitrary γ and Mach number.

This program completes the information required to perform the calculations required in the classical one-dimensional gas dynamics.

In the tables, the appropriate reference value is the state corresponding to $M = 1$ and this condition is denoted by an asterisk.

A brief description of the use of the tables is included before the equations are listed.

REFERENCES

1. Keenan, J. H. and Kaye, J., Gas Tables, John Wiley and Sons, New York, 1948.

This is the standard table for all of the 1-D compressible flow functions.

2. Cambel, A. B. and Jennings, B. H., Gas Dynamics, Dover Publications, New York, 1967.

This book is very easy to understand and provides a number of detailed numerical examples.

3. Hill, P. G. and Peterson, C. P., Mechanics and Thermodynamics of Propulsion, Addison-Wesley, Reading, 1965.

FRICTIONLESS CONSTANT-AREA FLOW WITH STAGNATION TEMPERATURE CHANGE - RAYLEIGH LINE

Given particular entrance conditions T_{01}, P_{01}, M_1, the exit conditions after a given change in stagnation temperature may be obtained as follows:

i) The value of $T_{01}/T_0{}^*$ (M) is found.

ii) Since T_{01} is known, $T_0{}^*$ can be found.

iii) Add ΔT_0; i.e.,

$$\frac{T_{02}}{T_0{}^*} = \frac{T_{01} + \Delta T_0}{T_0{}^*}$$

iv) Given $\frac{T_{02}}{T_0{}^*}$, find the M_2 that corresponds to this value of $T_0/T_0{}^*$ by some iterative trials using the program.

v) Once M_2 is known, the rest of the properties are read from the registers.

Note that inverses of all the functions could be programmed, but the iterative approach is generally very rapid. There are innumerable combinations of the general problem outlined above, so that it is simplest to keep the standard table format for the calculation.

CONSTANT-AREA ADIABATIC FLOW WITH FRICTION - FANNO LINE

For a duct of specified cross-sectional area and variable length, the inlet mass flow rate and average skin friction are fixed and there is a maximum length of duct which can transmit the flow. At this maximum length, the Mach number is one at the exit plane.

1) At any point 1 in the duct, $\frac{4fL_{MAX}}{D}$ depends only on M_1, γ.

2) At any other point 2 in the duct ($X_2 < L_{MAX}$) (assume f is constant)

$$\left.\frac{4fL_{MAX}}{D}\right|_2 = \left.\frac{4fL_{MAX}}{D}\right|_1 - \frac{4fX_2}{D}$$

3) With the $\left(\frac{4fL_{MAX}}{D}\right)$ at 2 known, determine the Mach number, M_2, corresponding to this condition by iterative application of the p

4) Once M_2 is found, determine the rest of the flow properties by reading the values from the registers.

GOVERNING EQUATIONS

RAYLEIGH LINE

1R. $$\frac{T_o}{T_o^*} = \frac{2\ (\gamma+1)\ M^2\ (1 + \frac{\gamma-1}{2} M^2)}{(1 + \gamma\ M^2)^2}$$

2R. $$\frac{T}{T^*} = \left(\frac{1 + \gamma}{1+\gamma\ M^2}\right)^2 \cdot M^2$$

3R. $$\frac{p}{p^*} = \frac{1 + \gamma}{1 + \gamma\ M^2}$$

4R. $$\frac{p_o}{p_o^*} = \frac{p}{p^*} \cdot \left[\left(\frac{2}{\gamma+1}\right)\left(1 + \frac{\gamma-1}{2}\ M^2\right)\right]^{\frac{\gamma}{\gamma-1}}$$

5R. $$\frac{V}{V^*} = \frac{\rho^*}{\rho} = M^* = \frac{(\gamma+1)\ M^2}{1 + \gamma\ M^2}$$

FANNO LINE

1F. $$\frac{T}{T^*} = \frac{(\gamma + 1)}{2\ (1+\frac{\gamma-1}{2} M^2)}$$

2F. $$\frac{p}{p^*} = \frac{1}{M} \left[\frac{(\gamma + 1)}{2\ (1+\frac{\gamma-1}{2} M^2)}\right]^{1/2}$$

FANNO LINE (Continued)

3F. $$\frac{p_o}{p_o{}^*} = \frac{1}{M}\left[\frac{2}{\gamma+1}\cdot\left(1+\frac{\gamma-1}{2}M^2\right)\right]^{\frac{\gamma+1}{2(\gamma-1)}}$$

4F. $$\frac{V}{V^*} = \frac{\rho}{\rho^*} = M^* = M\sqrt{\frac{(\gamma+1)}{2\left(1+\frac{\gamma-1}{2}M^2\right)}}$$

5F. $$\frac{F}{F^*} = \frac{1+\gamma M^2}{M\sqrt{2(\gamma+1)\left(1+\frac{\gamma-1}{2}M^2\right)}}$$

6F. $$\frac{4f\,L_{max}}{D} = \frac{1-M^2}{\gamma M^2} + \frac{\gamma+1}{2\gamma}\,\ell n\left\{\frac{(\gamma+1)M^2}{2\left(1+\frac{\gamma-1}{2}M^2\right)}\right\}$$

USER INSTRUCTIONS -- PROGRAM 1.4

STEP	ENTER	PRESS
1. Establish γ if $\gamma = 1.4$	-	A
or		
Establish γ if $\gamma \neq 1.4$	γ	B
2. Input the Mach number	M	C
STEP 2 IS REPEATED FOR EACH M WITHOUT RE-ENTERING γ.		

OUTPUT: The results are contained in the registers as indicated below and illustrated with a sample case.

R	VARIABLE		SAMPLE CASE $\gamma = 1.3$, M = 1.75
0	γ		1.30
1	M		1.75
2	M^2		3.0625
3	γM^2		3.98125
4	$1+\gamma M^2$		4.98125
5	$\gamma+1$		2.3
6	$\gamma-1$		0.3
7	$1 + \frac{\gamma-1}{2} M^2$, 0		0
8	T_o/T_o^*	RAYLEIGH LINE	.8286
9	T/T^*	RAYLEIGH LINE	.6529
10	p/p^*	RAYLEIGH LINE	.4617
11	p_o/p_o^*	RAYLEIGH LINE	1.2964
12	V/V^*	RAYLEIGH LINE	1.4141
13	0		0
14	T/T^*	FANNO LINE	.7880
15	p/p^*	FANNO LINE	.5073
16	p_o/p_o^*	FANNO LINE	1.4243
17	V/V^*	FANNO LINE	1.5535
18	F/F^*	FANNO LINE	1.0986
19	$4fL_{max}/D$	FANNO LINE	.2613
20	0		0

NOTE: If the PC-100C printer is used, INV LIST will produce the results, neatly divided between Rayleigh Line and Fanno Line as shown.

1.5 1976 STANDARD ATMOSPHERE

This program provides typical aerodynamic parameters for a standard day temperature for altitudes up to 86 kilometers (282,152 ft.) altitude. The results are presented in either metric or English units depending on the set of constants stored with the program.

The 1976 and 1962 standard atmospheres are identical for the first 51 kilometers above sea level.

METHOD OF COMPUTATION

Given the geometric altitude Z_{in} (in dimensions of either meters or feet), convert to kilometers and find the geopotential altitude H from:

$$H = \frac{Z}{1 + \frac{Z}{r_o}}$$

Where r_o = 6356.766 kilometers (the radius of the earth in kilometers) and $Z = C_1 \cdot Z_{in}$, C_1 = .001 if Z_{in} is in meters, and C_1 = .0003048 if Z_{in} is in feet. The 1962 standard atmosphere used a much more complicated and slightly more accurate relationship.

The inverse relation is given by:

$$Z = \frac{H}{1 - \frac{H}{r_o}}$$

Once the geopotential altitude is found, the temperature is computed. The standard day temperature profile is defined by seven layers, where within each layer the temperature is found by the linear relation (T is given in degrees Kelvin):

$$T = T_{b_i} + L_{m_i} (H - H_{b_i})$$

and T_{b_i}, L_{m_i} and H_{b_i} are the values at the base of the particular layer. The following table defines these constants, as well as the ratio of pressure to sea level pressure, which is also needed.

i	H_{b_i}(Km)	T_{b_i}(°K)	L_{m_i}(°K/Km)	p/p_{SL}	Z(ft.)
1	0	288.15	-6.5	1.0	0
2	11.	216.65	0.0	$2.2336\text{X}10^{-1}$	36,152.
3	20.	216.65	+1.0	$5.4032\text{X}10^{-2}$	65,824.
4	32.	228.65	+2.8	$8.5666\text{X}10^{-3}$	105,518.
5	47.	270.65	0.0	$1.0945\text{X}10^{-3}$	155,348.
6	51.	270.65	-2.8	$6.6063\text{X}10^{-4}$	168,676.
7	71.	214.65	-2.0	$3.9046\text{X}10^{-6}$	235,571.
-	84.852	-	-	-	282,152.

−5 ?← yes

Once the temperature is determined, the pressure is computed using the temperature law, the hydrostatics equation, and the perfect gas law. The resulting formulas are:

$$\frac{p}{p_{SL}} = \frac{p_b}{p_{SL}} \cdot \left[\frac{T_b}{T}\right]^{\frac{K}{L_m}} \qquad L_m \neq 0$$

$$\frac{p}{p_{SL}} = \frac{p_b}{p_{SL}} \cdot e^{\frac{-K(H - H_b)}{T_b}} \qquad L_m = 0$$

where $K = \dfrac{g_o M_o}{R^*}$ = 34.163195 in consistent units.

The remaining fundamental property is the density, which is found using the equation of state to be:

$$\frac{\rho}{\rho_{SL}} = \frac{p/p_{SL}}{T/T_{SL}}$$

Additional parameters of interest in aerodynamics are:

i) The speed of sound $\quad a = a_{SL}\sqrt{\dfrac{T}{T_{SL}}}$

ii) The coefficient of viscosity, found from Sutherland's Law:

$$\mu = \frac{\beta \cdot T^{3/2}}{T + S}$$

where S = 110.4°K and β depends on the system of units and is defined below.

iii) The Reynolds number per unit length and Mach:

$$\frac{R_e}{M \cdot L} = \frac{\rho a}{\mu}$$

iv) The actual temperature, pressure and density:

$$T = T_{SL} \cdot \left(\frac{T}{T_{SL}}\right)$$

$$p = p_{SL} \left(\frac{p}{p_{SL}}\right)$$

$$\rho = \rho_{SL} \left(\frac{\rho}{\rho_{SL}}\right)$$

v) Note that the dynamic pressure normalized by the Mach number squared

$\frac{q}{M^2} = \frac{\gamma p}{2} = .7p$, and hence, does not warrant a separate calculation.

The sea level properties and other required constants are defined in the following table.

	METRIC	ENGLISH
T_{SL}	288.15°K	518.67°R
p_{SL}	1.01325×10^{5} N/m^2	2116.22 lb$_f$/ft^2
ρ_{SL}	1.2250 Kg/m^3	2.3769×10^{-3} slugs/ft^3
a_{SL}	340.294 m/s	1116.45 f/s
μ_{SL}	$1.7894 \times 10^{-5} \frac{kg}{m.sec.}$	$.37373 \times 10^{-6} \frac{slugs}{ft.sec.}$
β	$1.458 \times 10^{-6} \frac{kg}{m.sec.^{1/2}}$	$3.0450963 \times 10^{-8} \frac{slugs}{ft.sec. K^{1/2}}$

The ratio of specific heats, γ, is defined to be 1.40.

Finally, for completeness, we summarize the temperatures at sea level for other days.

	T_{SL}	
	°K	°F
HOT DAY	312.56	103.0
TROPICAL	305.26	89.8
STANDARD	288.16	59.0
POLAR	246.66	- 15.7
COLD	222.06	- 60.0

Aerodynamicists are often required to determine field performance for the standard hot day. For that condition the temperature gradient is approximately -3.91°F per 1000 ft.

The standard reference is U. S. Standard Atmosphere, 1976, NOAA-S/T 76-1562, available from the U. S. Government Printing Office.

USER INSTRUCTIONS -- PROGRAM 1.5

NOTE: Calculator must be partitioned to 50P17 to run program.

STEP	ENTER	PRESS
1. Input Z in ft. or meters	Z	A

The results are found in the following registers:

R	VARIABLE	SAMPLE CASE 10,000 ft.	SAMPLE CASE 10,000 m
12.	H (Km)	3.0464	9.843
16.	T (°K)	268.35	223.25
18.	p/p_{SL}	.6878	.2615
32.	ρ/ρ_{SL}	.7386	.3376
33.	T/T_{SL}	.9313	.7748
44.	p	$1.4556\text{X}10^{3}$ lb/ft^{2}	$2.645\text{X}10^{4}$ N/M^{2}
45.	ρ	$1.7556\text{X}10^{-3}$ slugs/ft^{3}	.4135 Kg/m^{3}
46.	T	483.03°R	223.25°K
47.	a	1077.4 ft/sec	299.5 m/sec
48.	μ	$3.5343\text{X}10^{-7}$ slugs (ft. sec.)	$1.4576\text{X}10^{-5}$ Kg (m. sec.)
49.	$\rho a/\mu$	$5.3517\text{X}10^{6}$ ft^{-1}	$8.4971\text{X}10^{6}$ m^{-1}

NOTE: If 86 Km is exceeded, the display flashes at end of program.
If the PC-100C printer is used, the following output is generated.

REQUIRED CONSTANTS

To run the program, a number of constants are required. These are listed as follows:

INDEPENDENT OF THE SYSTEM OF UNITS

R	Constant		PRINTER LABELS R	
17	K = 34.163195		0	21210.
20	r_0 = 6356.766		1	700363716.
21	288.15		2	4000133730.
22	216.65		3	3236400000.
23	228.65	T_{b_i}	5	37633701.
24	270.65		6	33633301.
25	214.65		7	35633501.
26	.22336		42	30410000.
27	.054032		43	13633100.
28	.0085666	$\frac{p_{b_i}}{p_{SL}}$		
29	.0010945			
30	.00066063			
31	.000039045			
39	110.4			

UNIT DEPENDENT CONSTANTS

VARIABLE	R	METRIC	ENGLISH	PRINTER LABELS R	PRINTER LABELS METRIC	PRINTER LABELS ENGLISH
C_1	19	.001	.0003048	4	304000	4600213740
T_{SL}	34	288.15	518.67	8	13003036	13002136
P_{SL}	35	$1.01325\text{X}10^5$	2116.22	10	33003313	33003321
ρ_{SL}	36	1.2250	.0023769	40	35002630	35003621
a_{SL}	37	340.294	1116.45	41	37006526	37006535
β	38	$1.458\text{X}10^{-6}$	$3.0450963\text{X}10^{-8}$			

NOTE: All other registers are used as work registers during the execution of the program.