AFML-TR-77-50

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SR-52
MAGNETIC CARD CALCULATOR SOLUTIONS TO COMPOSITE MATERIALS WORKBOOK

MECHANICS AND SURFACE INTERACTIONS BRANCH
NONMETALLIC MATERIALS DIVISION

JANUARY 1979

TECHNICAL REPORT AFML-TR-77-50

AIR FORCE MATERIALS LABORATORY
AIR FORCE WRIGHT AERONAUTICAL LABORATORIES
AIR FORCE SYSTEMS COMMAND
WRIGHT-PATTERSON AIR FORCE BASE, OHIO 45433

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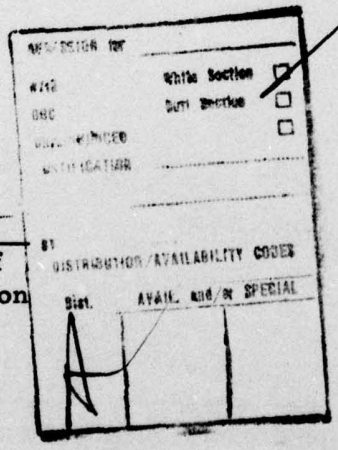
This technical report has been reviewed and is approved for publication.

STEPHEN W. TSAI
Project Scientist

FOR THE COMMANDER

JEROME M. KELBLE, Chief
Nonmetallic Materials Division



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AIR FORCE/56780/6 April 1979 — 400

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REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

(14) 1. REPORT NUMBER: AFML-TR-77-50

2. GOVT ACCESSION NO.

3. RECIPIENT'S CATALOG NUMBER

(6) 4. TITLE (and Subtitle): SR-52 MAGNETIC CARD CALCULATOR SOLUTIONS TO COMPOSITE MATERIALS WORKBOOK.

5. TYPE OF REPORT & PERIOD COVERED: Interim Report 1/1/77 - 3/1/78

6. PERFORMING ORG. REPORT NUMBER

(10) 7. AUTHOR(s): Stephen W. Tsai, H. Thomas Hahn, Frank Huber

8. CONTRACT OR GRANT NUMBER(s)

9. PERFORMING ORGANIZATION NAME AND ADDRESS: Air Force Materials Laboratory, AFML/MBM, Wright-Patterson AFB, OH 45433

10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS: 03P1 (16) 24190310 (17) P3, P1

11. CONTROLLING OFFICE NAME AND ADDRESS: Air Force Materials Laboratory, AFML/MB, Wright-Patterson AFB, OH 45433

12. REPORT DATE: March 1978 (11) Jan 79

13. NUMBER OF PAGES: (12) 185 p.

14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office): Same

15. SECURITY CLASS. (of this report): Unclassified

15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

(9) Interim rept. 1 Jan 77-1 Mar 78,

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

Same

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Composite materials; cards for material properties; preprogrammed magnetic cards for: stress, strain, stress/strain relations, micromechanics, laminated plate theory, curing and swelling stresses, fatigue, fracture, diffusion, and statistical methodology.

AD58 534

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

This is a companion volume to AFML-TR-78-33, "Composite Materials Workbook." Both workbook and its solutions are intended to present to the users of composite materials a set of tools to solve most commonly encountered problems in design and testing. Attempts are made to simplify both the operational and conceptual aspects. Subjects are selected from the standpoint of practical application rather than elegance.

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Unclassified 012 320

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20. ABSTRACT (continued)

Programmable pocket calculators, preferably with magnetic card capability, are found to be most suitable to perform the calculations in the workbook. In this report, the description, operating instructions, program listing, and sample problems for these calculations have been compiled for Texas Instruments SR-52.

It is believed that composite materials are simple in concept and easy to describe quantitatively with the aid of card programmable calculators. The performance characteristics of composite materials can now be fully appreciated and utilized.

FOREWORD

This report was prepared in the Mechanics and Surface Interactions Branch (AFML/MBM), Nonmetallic Materials Division, Air Force Materials Laboratory, Wright-Patterson AFB, Ohio. The work was performed under the joint support of Project No. 2419 "Nonmetallic Structural Materials," Task No. 241903 "Composite Materials and Mechanics Technology," and Project No. 2307 "Aerospace Sciences," Task No. 2307P1 "Life Analysis and Failure Mechanics in Engine and Airframe Structural Metals and Composites." The time period covered by the effort was 1 January 1976 to 1 March 1977. Stephen W. Tsai (AFML/MBM) was the laboratory project engineer.

Same numbers for equations and figures are used in this report as those in the AFML-TR-78-33, "Composite Materials Workbook," in order to facilitate cross referencing. Program card numbers are assigned the page number on which the instructions for the program card appear.

The technical assistance from Texas Instruments Incorporated during the preparation of this report is gratefully acknowledged. Standard production cards by Texas Instruments relevant to this report and the Workbook are listed in the Appendix.

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TITLE: SI/ENGINEERING CONVERSION				NO: 001
A' $Pa \sim kgf/m^2$	B' J~ft lbf	C' J~cal	D' $kgm^{-3} \sim lb/yd^3$	E' $ms^{-1} \sim ft/sec$
A $MPa \sim ksi$	B $MNm^{-1} \sim kip/in$	C N ~ lbf	D kW ~ Hp	E $MPam^{\frac{1}{2}} \sim ksi\sqrt{in}$

INSTRUCTIONS

OPERATION	STEP	READ	ENTER	PRESS	DISPLAY
SI ⇨ Eng	1	001 A&B			
	2	Q_{ij} σ_i	kPa	A	psi
			MPa	A	ksi
Set angular mode switch to D (degree)			GPa	A	10^6 psi
			Pa	A'	kgf/m^2
			MPa	A'	kgf/mm^2
		A_{ij}, N_i	MNm^{-1}	B	kip/in
			Nm^{-1}	A'	kgf/m
		B_{ij}, M_i	N	C	lbf
			kN	C	kip
			N	A'	kgf
		D_{ij}, J	Nm	B'	ft lbf
			Nm	A'	kgf/m
SI ⇦ Eng	3	Q_{ij}, σ_i	psi	A	kPa
Set angular mode switch to R (radian)			ksi	A	MPa
			10^6 psi	A	GPa
			kgf/m^2	A'	Pa
			kgf/mm^2	A'	MPa
		A_{ij}, N_i	kip/in	B	MNm^{-1}
			kgf/m	A'	Nm^{-1}
		B_{ij}, M_i	lbf	C	N
			kip	C	kN
			kgf	A'	N
		D_{ij}, J	ft lbf	B'	Nm
			kgf m	A'	Nm

PROGRAM TITLE: SI/ENGINEERING CONVERSION NO: 001

PROGRAM LISTING:

```
000 46  045 51  090 03  135 95
001 11  046 89  091 07  136 81
002 42  047 01  092 06  137 46
003 00  048 93  093 95  138 89
004 00  049 03  094 81  139 59
005 51  050 04  095 46  140 32
006 89  051 01  096 18  141 90
007 93  052 95  097 42  142 87
008 01  053 81  098 00  143 43
009 04  054 46  099 00  144 00
010 05  055 15  100 51  145 00
011 95  056 42  101 89  146 65
012 81  057 00  102 93  147 56
013 46  058 00  103 02  148 46
014 12  059 51  104 03  149 87
015 42  060 89  105 08  150 43
016 00  061 93  106 09  151 00
017 00  062 09  107 95  152 00
018 51  063 00  108 81  153 55
019 89  064 09  109 46  154 56
020 05  065 08  110 19
021 93  066 95  111 42
022 07  067 81  112 00
023 01  068 46  113 00
024 95  069 16  114 51
025 81  070 42  115 89
026 46  071 00  116 01
027 13  072 00  117 93
028 42  073 51  118 06
029 00  074 89  119 08
030 00  075 93  120 06
031 51  076 01  121 95
032 89  077 00  122 81
033 93  078 02  123 46
034 02  079 95  124 10
035 02  080 81  125 42
036 04  081 46  126 00
037 08  082 17  127 00
038 95  083 42  128 51
039 81  084 00  129 89
040 46  085 00  130 03
041 14  086 51  131 93
042 42  087 89  132 02
043 00  088 93  133 08
044 00  089 07  134 01
045 51  090 03  135 95
```

TITLE: VOID CONTENT				NO: 003
A' v_v	B' v_f'	C' v_f	D'	E'
A ρ_f	B ρ_m	C m_f	D m_m	E ρ

00					
01 ρ_f	02 ρ_m	03 m_f	04 m_m	05 ρ	06 v_v
07 v_f'	08	09	10		
11	12	13	14	15	16
17	18	19			
98	99				

REGISTER

INSTRUCTIONS

OPERATION	STEP	READ	ENTER	PRESS	DISPLAY
Input Data	1	003 A&B	ρ_f	A	ρ_f
			ρ_m	B	ρ_m
			m_f	C	m_f
			m_m	D	m_m
			ρ	E	ρ
Compute	2			A'	v_v
				B'	v_f'
				C'	v_f

PROGRAM TITLE: VOID CONTENT NO: 003

FORMULAS:

$m_{f,m}$: mass fraction of fiber, matrix

$\rho_{f,m}$: density of fiber, matrix

a. Void content

$$v_v = 1 - \rho (m_f/\rho_f + m_m/\rho_m)$$

b. Apparent fiber volume fraction ($v_v = 0$)

$$v_f' = \frac{1}{1 + \frac{m_m}{m_f} \frac{\rho_f}{\rho_m}}$$

c. True fiber volume fraction

$$v_f = (1 - v_v)v_f'$$

PROGRAM TITLE: VOID CONTENT **NO:** 003

PROGRAM LISTING:

```
000 46   045 01   090 53
001 11   046 85   091 01
002 42   047 43   092 75
003 00   048 00   093 43
004 01   049 04   094 00
005 81   050 55   095 06
006 46   051 43   096 54
007 12   052 00   097 65
008 42   053 02   098 43
009 00   054 54   099 00
010 02   055 95   100 07
011 81   056 42   101 95
012 46   057 00   102 81
013 13   058 06
014 42   059 81
015 00   060 46
016 03   061 17
017 81   062 01
018 46   063 55
019 14   064 53
020 42   065 01
021 00   066 85
022 04   067 43
023 81   068 00
024 46   069 04
025 15   070 65
026 42   071 43
027 00   072 00
028 05   073 01
029 81   074 55
030 46   075 43
031 16   076 00
032 01   077 03
033 75   078 55
034 43   079 43
035 00   080 00
036 05   081 02
037 65   082 54
038 53   083 95
039 43   084 42
040 00   085 00
041 03   086 07
042 55   087 81
043 43   088 46
044 00   089 18
```

SAMPLE PROBLEM: 003 VOID CONTENT NO: 003

1) Input: $\rho_f = 2600\ Kg\text{-}m^{-3}$

$\rho_m = 1200\ Kg\text{-}m^{-3}$

$m_f = .65$

$m_m = .35$

$\rho = 1800\ Kg\text{-}m^{-3}$

Output: $V_v = .025$

$V_f' = .462$

$V_f = .450$

2) Input: $\rho_f = 1700\ Kg\text{-}m^{-3}$

$\rho_m = 1200\ Kg\text{-}m^{-3}$

$m_f = .70$

$m_m = .30$

$\rho = 1500\ Kg\text{-}m^{-3}$

Output: $V_v = .007$

$V_f' = .622$

$V_f = .618$

TITLE: STRESS COMPONENTS				NO: 007
A' σ_i	B'	C'	D'	E'
A Initialize	B σ_1'	C σ_2'	D σ_6'	E δ

00 $2(\theta-\delta)$	REGISTER				
01	02	03	04	05	06
07 σ_1', σ_1	08 σ_2', σ_2	09 σ_6', σ_6	10 $(\sigma_1-\sigma_2)/2$		
11	12	13	14	15 I	16 R
17	18	19			
98	99 δ				

INSTRUCTIONS

OPERATION	STEP	READ	ENTER	PRESS	DISPLAY
Input σ_i	1	007 A&B	σ_1' σ_2 σ_6	A' RUN RUN	σ_1' σ_2 σ_6
Compute Invariants	2			A	δ*
Input θ	3		θ(deg)	B	σ_1'
				C	σ_2'
				D	σ_6'
				E	δ
Principal Stresses	4		δ(deg)	B	σ_I
				C	σ_{II}
				D	0
Max Shear	5		δ + 45°	B	σ_1' = I
				C	σ_2' = I
				D	σ_6' = R

*Note: Display will flash "0" when $\sigma_1 = \sigma_2$ (See Eq. 30). Stabilize display with CE and proceed, computations are not effected.

PROGRAM TITLE: STRESS COMPONENTS NO: 007

FORMULAS:

TABLE 3 STRESS TRANSFORMATION RELATIONS

	I	R
σ'_x	1	$\cos 2(\theta-\delta)$
σ'_y	1	$-\cos 2(\theta-\delta)$
σ'_{xy}	0	$-\sin 2(\theta-\delta)$

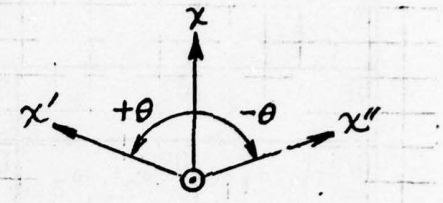


Figure 12 New coordinates x'- y' in terms of old coordinates x-y, or given x-y and θ, find x'-y'. Arrow of rotation is pointing up. If θ is negative, new coordinates are x''-y''.

$$I = \frac{\sigma_x + \sigma_y}{2} \qquad (28)$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \sigma_{xy}^2} \qquad (29)$$

$$\tan 2\delta_1 = \frac{2\sigma_{xy}}{\sigma_x - \sigma_y} \qquad (30)$$

$\delta = \delta_1 + 90$ if $\sigma_x < \sigma_y$; $\delta = \delta_1$ if $\sigma_x > \sigma_y$

Both I and R are invariants, as shown in the Mohr's Circle.

Special orientations:

(1) When $\theta = \delta$, i.e., Principal Directions

σ'_x, σ'_y = maximum or minimum

σ'_{xy} = 0

(2) When $\theta - \delta = \pm \pi/4$, i.e., max. shear orientation

$\sigma'_x = \sigma'_y = I$

σ'_{xy} = max. = R

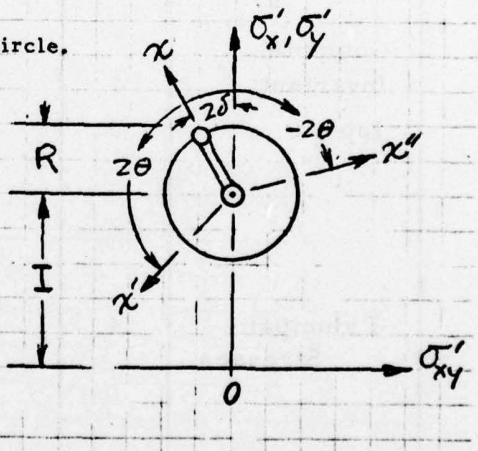


Figure 13 Mohr's Circle is defined by invariants I and R. Phase angle δ is defined by a specific combination of stress components σ_x, σ_y and σ_{xy}. As reference coordinates change by +θ, the rotation in Mohr's Circle is 2θ.

PROGRAM TITLE: STRESS COMPONENTS NO: 007

PROGRAM LISTING:

```
000 46  045 85  090 09  135 01
001 16  046 43  091 09  136 06
002 42  047 00  092 81  137 95
003 00  048 09  093 46  138 42
004 07  049 40  094 12  139 00
005 81  050 95  095 75  140 08
006 42  051 30  096 43  141 81
007 00  052 42  097 09  142 46
008 08  053 01  098 09  143 14
009 81  054 06  099 95  144 43
010 42  055 43  100 65  145 00
011 00  056 00  101 02  146 00
012 09  057 09  102 95  147 32
013 81  058 55  103 42  148 94
014 46  059 43  104 00  149 65
015 11  060 01  105 00  150 43
016 43  061 00  106 43  151 01
017 00  062 95  107 01  152 06
018 07  063 22  108 05  153 95
019 85  064 34  109 85  154 42
020 43  065 55  110 43  155 00
021 00  066 02  111 00  156 09
022 08  067 95  112 00  157 81
023 95  068 42  113 33  158 46
024 55  069 09  114 65  159 15
025 02  070 09  115 43  160 43
026 95  071 43  116 01  161 09
027 42  072 01  117 06  162 09
028 01  073 00  118 95  163 81
029 05  074 80  119 42
030 43  075 87  120 00
031 00  076 43  121 07
032 07  077 09  122 81
033 75  078 09  123 46
034 43  079 85  124 13
035 00  080 09  125 43
036 08  081 00  126 01
037 95  082 95  127 05
038 55  083 42  128 75
039 02  084 09  129 43
040 95  085 09  130 00
041 42  086 81  131 00
042 01  087 46  132 33
043 00  088 87  133 65
044 40  089 43  134 43
```

SAMPLE PROBLEM: 007 STRESS COMPONENTS NO: 007

1. Given $\sigma_i = \begin{Bmatrix} 1 \\ 2 \\ 3 \end{Bmatrix}$ (MPa)

I = 1.5 R = -3.041 δ = 49.731

	δ-45	δ-30	δ-15	δ	δ+15	δ+30	δ+45	δ-δ=0
σ'_x	1.5	3.021	4.134	4.541	4.134	3.021	1.5	1
σ'_y	1.5	-.021	-1.134	-1.541	-1.134	-.021	1.5	2
σ'_{xy}	3.041	2.634	1.521	0	-1.521	-2.634	-3.041	3

2. Given $\sigma_i = \begin{Bmatrix} -1 \\ -2 \\ -3 \end{Bmatrix}$ (MPa)

I = -1.5 R = -3.041 δ = -40.269

	δ-45	δ-30	δ-15	δ	δ+15	δ+30	δ+45	δ-δ=0
σ'_x	-1.5	.021	1.134	1.541	1.134	.021	-1.5	-1
σ'_y	-1.5	-3.021	-4.134	-4.541	-4.134	-3.021	-1.5	-2
σ'_{xy}	3.041	2.634	1.521	.0	-1.521	-2.634	-3.041	-3

TITLE: STRAIN COMPONENTS				NO: 011
A' e_i	B'	C'	D'	E'
A Initialize	B e_1'	C e_2'	D e_6'	E δ

00 $2(\theta-\delta)$		REGISTER			
01	02	03	04	05 I	06 R
07	08	09	10 $(e_1-e_2)/2$		
11	12	13	14	15	16
17 e_1', e_1	18 e_2', e_2	19 e_6', e_6			
98 δ	99				

INSTRUCTIONS

OPERATION	STEP	READ	ENTER	PRESS	DISPLAY
Input e_i	1	01' A&B	e_1 e_2 e_6	A' RUN RUN	e_1 e_2 e_6
Compute I's Invariants	2			A	δ*
Input θ	3		θ (deg)	B	e_1'
Recall	3'			C	e_2'
				D	e_6'
				E	δ
Principal Strains	4		δ (deg)	B	e_I
				C	e_{II}
				D	0
Max Shear	5		$\delta + 45°$	B	$e_1' = I$
				C	$e_2' = I$
				D	$e_6' = 2R$

*Note: Display will flash "0" when $e_x = e_y$ (See Eq. 44). Stabilize display with CE and proceed, computations are not effected.

PROGRAM TITLE: STRAIN COMPONENTS NO: 011

FORMULAS:

Only the shear component is the difference between the strain and stress transformation relations, i.e.,

TABLE 7 STRAIN TRANSFORMATION RELATIONS

	I	R
e'_x	1	$\cos 2(\theta - \delta)$
e'_y	1	$-\cos 2(\theta - \delta)$
e'_{xy}	0	$-2\sin 2(\theta - \delta)$

$$I = \frac{e_x + e_y}{2} \qquad (42)$$

$$R = \sqrt{\left(\frac{e_x - e_y}{2}\right)^2 + \left(\frac{e_{xy}}{2}\right)^2} \qquad (43)$$

$$\tan 2\delta = \frac{e_{xy}}{e_x - e_y} \qquad (44)$$

$\delta = \delta_1$ if $e_x > e_y$

$\delta = \delta_1 + 90$ if $e_x < e_y$

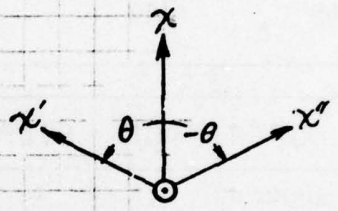


Figure 21 Positive and negative rotations for transformation.

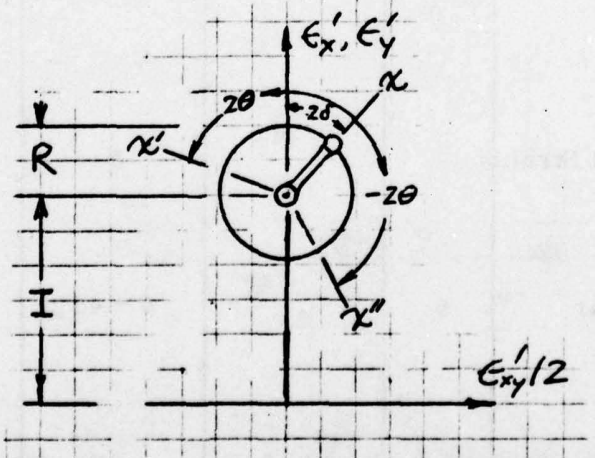


Figure 22 Mohr's Circle representation of strain transformation. Only the one-half factor for the shear strain is the difference between the strain representation and that for stress.

PROGRAM TITLE: STRAIN COMPONENTS **NO:** 011

PROGRAM LISTING:

```
000 46  045 08  090 00  135 00
001 16  046 95  091 95  136 05
002 42  047 55  092 42  137 75
003 01  048 02  093 09  138 43
004 07  049 95  094 08  139 00
005 81  050 42  095 81  140 00
006 42  051 01  096 46  141 33
007 01  052 00  097 87  142 65
008 08  053 40  098 43  143 43
009 81  054 85  099 09  144 00
010 42  055 43  100 08  145 06
011 01  056 01  101 81  146 95
012 09  057 09  102 46  147 42
013 81  058 40  103 12  148 01
014 46  059 95  104 75  149 08
015 11  060 30  105 43  150 81
016 43  061 42  106 09  151 46
017 01  062 00  107 08  152 14
018 08  063 06  108 95  153 43
019 85  064 43  109 65  154 00
020 43  065 01  110 02  155 00
021 01  066 09  111 95  156 32
022 07  067 55  112 42  157 94
023 95  068 43  113 00  158 65
024 55  069 01  114 00  159 02
025 02  070 00  115 43  160 65
026 95  071 95  116 00  161 43
027 42  072 22  117 05  162 00
028 00  073 34  118 85  163 06
029 05  074 55  119 43  164 95
030 43  075 02  120 00  165 42
031 01  076 95  121 00  166 01
032 09  077 42  122 33  167 09
033 55  078 09  123 65  168 81
034 02  079 08  124 43  169 46
035 95  080 43  125 00  170 15
036 42  081 01  126 06  171 43
037 01  082 00  127 95  172 09
038 09  083 80  128 42  173 08
039 43  084 87  129 01  174 81
040 01  085 43  130 07
041 07  086 09  131 81
042 75  087 08  132 46
043 43  088 85  133 13
044 01  089 09  134 43
```

SAMPLE PROBLEM: 011 STAIN COMPONENTS NO: 011

1. Given $e_i = \begin{Bmatrix} 1 \\ 2 \\ 3 \end{Bmatrix}$ (mm/m or $10^3 \mu m/m$)

$I = 1.5$ $R = -1.581$ $\delta = 54.217$

	δ-45	δ-30	δ-15	δ	δ+15	δ+30	δ+45	δ-δ=0
e'_x	1.5	2.291	2.869	3.081	2.869	2.291	1.5	1
e'_y	1.5	.709	.131	-.081	.131	.709	1.5	2
e'_{xy}	3.162	2.739	1.581	0	-1.581	-2.739	-3.162	3

2. Given $e_i = \begin{Bmatrix} -1 \\ -2 \\ -3 \end{Bmatrix}$ (mm/m or $10^3 \mu m/m$)

$I = -1.5$ $R = -1.581$ $\delta = -35.783$

	δ-45	δ-30	δ-15	δ	δ+15	δ+30	δ+45	δ-δ=0
e'_x	-1.50	-.709	-.131	.081	-.131	-.709	-1.50	-1
e'_y	-1.50	-2.291	-2.869	-3.081	-2.869	-2.291	-1.50	-2
e'_{xy}	3.162	2.739	1.581	0	-1.581	-2.739	-3.162	-3

TITLE: STRAIN ROSETTES				NO: 015
A'	B'	C'	D'	E'
A θ_i	B e_i	C Initialize	D	E Solve 3x3

00 $\cos^2 A$					
01 $\sin^2 A$	02 sinAcosA	03 e_A	04 z	05 $\cos^2 B$	06 $\sin^2 B$
07 sinBcosB	08 e_B	09 y	10 $\cos^2 C$		
11 $\sin^2 C$	12 sinCcosC	13 e_C	14 θ_A	15 θ_B	16 θ_C
17	18	19			
98	99				

REGISTER

INSTRUCTIONS

OPERATION	STEP	READ	ENTER	PRESS	DISPLAY
Compute Coeff	1	015 A&B		C	
Input Angles	2		θ_A(deg)	A	θ_A
			θ_B(deg)	RUN	θ_B
			θ_C(deg)	RUN	θ_C
Input Strains	3		e_A	B	e_A
			e_B	RUN	e_B
			e_C	RUN	e_C
Solve 3 x 3	4	MA1-05		E	e_x
				RUN	e_y
				RUN	e_{xy}

PROGRAM TITLE: STRAIN ROSETTES NO: 015

FORMULAS:

a. Three-Element Rosettes

Since there are 3 strain components at each point, 3-element rosettes are in general needed to solve for 3 unknowns. Assuming 3 elements are mounted at 3 different angles from some reference coordinates, each rosette must satisfy the first equation in Eq. 45, i.e.

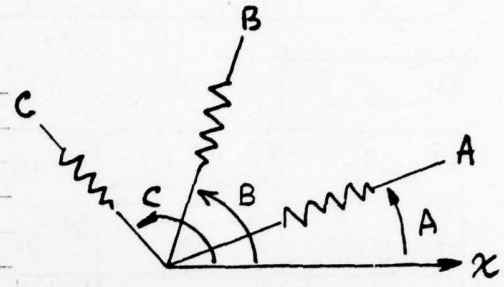


Figure 25 Strain rosette orientation with positive angles. If an angle is negative, it shall be so entered into Equation and

$$e_A = (\cos^2 A)e_x + (\sin^2 A)e_y + (\sin A \cos A)e_{xy}$$

$$e_B = (\cos^2 B)e_x + (\sin^2 A)e_y + (\sin B \cos B)e_{xy} \quad (45)$$

$$e_C = (\cos^2 C)e_x + (\sin^2 C)e_y + (\sin C \cos C)e_{xy}$$

In matrix form

$$\begin{bmatrix} \cos^2 A & \sin^2 A & \sin A \cos A \\ \cos^2 B & \sin^2 B & \sin B \cos B \\ \cos^2 C & \sin^2 C & \sin C \cos C \end{bmatrix} \begin{Bmatrix} e_x \\ e_y \\ e_{xy} \end{Bmatrix} = \begin{Bmatrix} e_A \\ e_B \\ e_C \end{Bmatrix} \quad (46)$$

PROGRAM TITLE: STRAIN ROSETTES NO: 015

PROGRAM LISTING:

```
000 46  045 01  090 06
001 11  046 43  091 33
002 42  047 01  092 40
003 01  048 04  093 42
004 04  049 33  094 01
005 81  050 65  095 00
006 42  051 43  096 43
007 01  052 01  097 01
008 05  053 04  098 06
009 81  054 32  099 32
010 42  055 95  100 40
011 01  056 42  101 42
012 06  057 00  102 01
013 81  058 02  103 01
014 46  059 43  104 43
015 12  060 01  105 01
016 42  061 05  106 06
017 00  062 33  107 33
018 03  063 40  108 65
019 81  064 42  109 43
020 42  065 00  110 01
021 00  066 05  111 06
022 08  067 43  112 32
023 81  068 01  113 95
024 42  069 05  114 42
025 01  070 32  115 01
026 03  071 40  116 02
027 81  072 42  117 81
028 46  073 00
029 13  074 06
030 43  075 43
031 01  076 01
032 04  077 05
033 33  078 33
034 40  079 65
035 42  080 43
036 00  081 01
037 00  082 05
038 43  083 32
039 01  084 95
040 04  085 42
041 32  086 00
042 40  087 07
043 42  088 43
044 00  089 01
```

SAMPLE PROBLEM: 015 STRAIN ROSETTES NO: 015

1) Given A=0, B=60°, C=-60° and ϵ_A, ϵ_B, ϵ_C = 10, -3, -3

Find Poisson's ratio along a-axis

ϵ_x = 10

ϵ_y = -7.333

ϵ_{xy} = $-4.618 \times 10^{-12} \simeq 0$

ν = .733

TITLE: STRESS-STRAIN RELATION				NO: 019
A' S_{ij}	B'	C'	D'	E'
A σ_i	B e_1	C e_2	D e_6	E

00					
01 S_{11}	02 S_{22}	03 S_{12}	04 S_{66}	05 S_{16}	06 S_{26}
07 σ_1	08 σ_2	09 σ_6	10		
11	12	13	14	15	16
17 e_1	18 e_2	19 e_6			
98	99				

REGISTER

INSTRUCTIONS

OPERATION	STEP	READ	ENTER	PRESS	DISPLAY
Input S_{ij}	1	019 A&B	S_{11}	A'	S_{11}
			S_{22}	RUN	S_{22}
			S_{12}	RUN	S_{12}
			S_{66}	RUN	S_{66}
			S_{16}	RUN	S_{16}
			S_{26}	RUN	S_{26}
Input σ_i	2		σ_1	A	σ_1
			σ_2	RUN	σ_2
			σ_6	RUN	σ_6
Strains	3			B	e_1
				C	e_2
				D	e_6
Transf. Strains	4	011 A&B		A	δ^*
			θ (deg)	B	e_1'
				C	e_2'
				D	e_6'

PROGRAM TITLE: STRESS-STRAIN RELATION NO: 019

FORMULAS:

$$\begin{aligned} e_1 &= S_{1j}\sigma_j = S_{11}\sigma_1 + S_{12}\sigma_2 + S_{16}\sigma_6 \\ e_2 &= S_{2j}\sigma_j = S_{21}\sigma_1 + S_{22}\sigma_2 + S_{26}\sigma_6 \\ &\cdot \\ e_6 &= S_{6j}\sigma_j = S_{61}\sigma_1 + S_{62}\sigma_2 + S_{66}\sigma_6 \end{aligned} \tag{49}$$

$$\begin{Bmatrix} e_1 \\ e_2 \\ e_6 \end{Bmatrix} = \begin{bmatrix} S_{11} & S_{12} & S_{16} \\ S_{21} & S_{22} & S_{26} \\ S_{61} & S_{62} & S_{66} \end{bmatrix} \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_6 \end{Bmatrix} \tag{52}$$

S_{ij} is symmetric, i.e., $S_{ij} = S_{ji}$, or $S_{12} = S_{21}$, $S_{16} = S_{61}$, $S_{26} = S_{62}$

PROGRAM TITLE: STRESS-STRAIN RELATION NO: 019

PROGRAM LISTING:

```
000 46  045 65  090 06
001 16  046 43  091 65
002 42  047 00  092 43
003 00  048 07  093 00
004 01  049 85  094 09
005 81  050 43  095 95
006 42  051 00  096 42
007 00  052 03  097 01
008 02  053 65  098 08
009 81  054 43  099 81
010 42  055 00  100 46
011 00  056 08  101 14
012 03  057 85  102 43
013 81  058 43  103 00
014 42  059 00  104 05
015 00  060 05  105 65
016 04  061 65  106 43
017 81  062 43  107 00
018 42  063 00  108 07
019 00  064 09  109 85
020 05  065 95  110 43
021 81  066 42  111 00
022 42  067 01  112 06
023 00  068 07  113 65
024 06  069 81  114 43
025 81  070 46  115 00
026 46  071 13  116 08
027 11  072 43  117 85
028 42  073 00  118 43
029 00  074 03  119 00
030 07  075 65  120 04
031 81  076 43  121 65
032 42  077 00  122 43
033 00  078 07  123 00
034 08  079 85  124 09
035 81  080 43  125 95
036 42  081 00  126 42
037 00  082 02  127 01
038 09  083 65  128 09
039 81  084 43  129 81
040 46  085 00
041 12  086 08
042 43  087 85
043 00  088 43
044 01  089 00
```

1. Given:

$$\sigma_i = \begin{Bmatrix} 400 \\ 60 \\ 15 \end{Bmatrix} (\text{MPa}) = \begin{Bmatrix} .40 \\ .06 \\ .015 \end{Bmatrix} (\text{TPa})$$

$$S_{ij} = \begin{bmatrix} 5.5249 & -1.5470 & 0 \\ -1.5470 & 97.0874 & 0 \\ 0 & 0 & 139.4700 \end{bmatrix} (\text{TPa})^{-1}$$

(for T-300/5208)

$$\epsilon_i = \begin{Bmatrix} 2.1171 \\ 5.2064 \\ 2.0921 \end{Bmatrix} (\text{mm/m})$$

2. Given:

$$\sigma_i = \begin{Bmatrix} .01 \\ .02 \\ .03 \end{Bmatrix} (\text{TPa})$$

S_{ij} $(\text{TPa})^{-1}$ for T-300/5208

$$\epsilon_i = \begin{Bmatrix} .0243 \\ 1.9263 \\ 4.1841 \end{Bmatrix} (\text{mm/m})$$

TITLE: STRAIN-STRESS RELATION				NO: 023
A' Q_{ij}	B'	C'	D'	E'
A e_i	B σ_1	C σ_2	D σ_6	E

00					
REGISTER					
01	02	03	04	05	06
07 σ_1	08 σ_2	09 σ_6	10		
11 Q_{11}	12 Q_{22}	13 Q_{12}	14 Q_{66}	15 Q_{16}	16 Q_{26}
17 e_1	18 e_2	19 e_6			
98	99				

INSTRUCTIONS

OPERATION	STEP	READ	ENTER	PRESS	DISPLAY
Input Q_{ij}	1	023 A&B	Q_{11}	A'	Q_{11}
			Q_{22}	RUN	Q_{22}
			Q_{12}	RUN	Q_{12}
			Q_{66}	RUN	Q_{66}
			Q_{16}	RUN	Q_{16}
			Q_{26}	RUN	Q_{26}
Input e_i	2		e_1	A	e_1
			e_2	RUN	e_2
			e_6	RUN	e_6
Stresses	3			B	σ_1
				C	σ_2
				D	σ_6
Transf. Stresses	4	011 A&B		A	δ^*
			θ (deg)	B	σ_1'
				C	σ_2'
				D	σ_6'

PROGRAM TITLE: STRAIN-STRESS RELATION NO: 023

FORMULAS:

$$\sigma_i = Q_{ij} e_j \tag{50}$$

$$\sigma_1 = Q_{1i} e_i = Q_{11} e_1 + Q_{12} e_2 + Q_{16} e_6$$

$$\sigma_2 = Q_{2i} e_i = Q_{21} e_1 + Q_{22} e_2 + Q_{26} e_6 \tag{51}$$

$$\sigma_6 = Q_{6i} e_i = Q_{61} e_1 + Q_{62} e_2 + Q_{66} e_6$$

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_6 \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & Q_{16} \\ Q_{21} & Q_{22} & Q_{26} \\ Q_{61} & Q_{62} & Q_{66} \end{bmatrix} \begin{Bmatrix} e_1 \\ e_2 \\ e_6 \end{Bmatrix} \tag{53}$$

Q_{ij} is also symmetric, i.e., $Q_{ij} = Q_{ji}$, or $Q_{12} = Q_{21}$, $Q_{16} = Q_{61}$, $Q_{26} = Q_{62}$

PROGRAM TITLE: STRAIN-STRESS RELATION **NO:** 023

PROGRAM LISTING:

```
000 46   045 65   090 06
001 16   046 43   091 65
002 42   047 01   092 43
003 01   048 07   093 01
004 01   049 85   094 09
005 81   050 43   095 95
006 42   051 01   096 42
007 01   052 03   097 00
008 02   053 65   098 08
009 81   054 43   099 81
010 42   055 01   100 46
011 01   056 08   101 14
012 03   057 85   102 43
013 81   058 43   103 01
014 42   059 01   104 05
015 01   060 05   105 65
016 04   061 65   106 43
017 81   062 43   107 01
018 42   063 01   108 07
019 01   064 09   109 85
020 05   065 95   110 43
021 81   066 42   111 01
022 42   067 00   112 06
023 01   068 07   113 65
024 06   069 81   114 43
025 81   070 46   115 01
026 46   071 13   116 08
027 11   072 43   117 85
028 42   073 01   118 43
029 01   074 03   119 01
030 07   075 65   120 04
031 81   076 43   121 65
032 42   077 01   122 43
033 01   078 07   123 01
034 08   079 85   124 09
035 81   080 43   125 95
036 42   081 01   126 42
037 01   082 02   127 00
038 09   083 65   128 09
039 81   084 43   129 81
040 46   085 01
041 12   086 08
042 43   087 85
043 01   088 43
044 01   089 01
```

SAMPLE PROBLEM: 023 STRAIN-STRESS RELATIONS NO: 023

1. Given

$$e_i = \begin{Bmatrix} 2.1171 \\ 5.2064 \\ 2.0921 \end{Bmatrix} \quad mm/m$$

$$Q_{ij} = \begin{bmatrix} 181.811 & 2.897 & 0 \\ 2.897 & 10.346 & 0 \\ 0 & 0 & 7.170 \end{bmatrix} \quad (GPa)$$

(For T-300/5208)

$$\sigma_i = \begin{Bmatrix} 400 \\ 60 \\ 15 \end{Bmatrix} \quad (MPa)$$

2. Given

$$e_i = \begin{Bmatrix} .0243 \\ 1.9263 \\ 4.1841 \end{Bmatrix} \quad mm/m$$

A_{ij} (GPa) (for T-300/5208)

$$\sigma_i = \begin{Bmatrix} 10 \\ 20 \\ 30 \end{Bmatrix} \cdot MPa$$

TITLE: MODULUS TO COMPLIANCE				NO: 027
A' Q_{ij}	B'	C'	D'	E'
A $\|Q\|$	B	C	D	E

00					
01 S_{11}	02 S_{22}	03 S_{12}	04 S_{66}	05 S_{16}	06 S_{26}
07	08	09	10 $\|Q\|$		
11 Q_{11}	12 Q_{22}	13 Q_{12}	14 Q_{66}	15 Q_{16}	16 Q_{26}
17	18	19			
98	99				

REGISTER

INSTRUCTIONS

OPERATION	STEP	READ	ENTER	PRESS	DISPLAY
Input Q_{ij}	1	027 A&B	Q_{11}	A'	Q_{11}
			Q_{22}	RUN	Q_{22}
			Q_{12}	RUN	Q_{12}
			Q_{66}	RUN	Q_{66}
			Q_{16}	RUN	Q_{16}
			Q_{26}	RUN	Q_{26}
Compute S_{ij}	2			A	S_{26}
				RCL 01	S_{11}
				RCL 02	S_{22}
				RCL 03	S_{12}
				RCL 04	S_{66}
				RCL 05	S_{16}
				RCL 06	S_{26}

PROGRAM TITLE: MODULUS TO COMPLIANCE NO: 027

FORMULAS:

In-Plane Modulus and Compliance

$$\begin{Bmatrix} N_1 \\ N_2 \\ N_6 \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{21} & A_{22} & A_{26} \\ A_{61} & A_{62} & A_{66} \end{bmatrix} \begin{Bmatrix} e_1^o \\ e_2^o \\ e_6^o \end{Bmatrix} \quad \text{or} \quad N_i = A_{ij}\, e_j^o \tag{203}$$

$$\text{Let Compliance} = {}_{ij} = [A_{ij}]^{-1} \quad \text{or} \quad {}_{ij}A_{jk} = \delta_{ik} \tag{204}$$

$$\begin{Bmatrix} e_1^o \\ e_2^o \\ e_6^o \end{Bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{16} \\ a_{21} & a_{22} & a_{26} \\ a_{61} & a_{62} & a_{66} \end{bmatrix} \begin{Bmatrix} N_1 \\ N_2 \\ N_6 \end{Bmatrix} \quad \text{or} \quad e_i^o = a_{ij}N_j \tag{205}$$

where

$$\begin{aligned} a_{11} &= \frac{1}{\Delta}(A_{22}A_{66}-A_{26}^2), & a_{22} &= \frac{1}{\Delta}(A_{11}A_{66}-A_{16}^2) \\ a_{12} &= \frac{1}{\Delta}(A_{16}A_{26}-A_{12}A_{66}), & a_{66} &= \frac{1}{\Delta}(A_{11}A_{22}-A_{12}^2) \\ a_{16} &= \frac{1}{\Delta}(A_{12}A_{26}-A_{22}A_{16}), & a_{26} &= \frac{1}{\Delta}(A_{12}A_{16}-A_{11}A_{26}) \end{aligned} \tag{206}$$

$$\Delta = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{21} & A_{22} & A_{26} \\ A_{61} & A_{62} & A_{66} \end{bmatrix}$$

In-Plane Engineering Constants:

$$E_{11}^o = \frac{1}{ha_{11}}, \quad E_{22}^o = \frac{1}{ha_{22}}, \quad \nu_{12}^o = -\frac{a_{12}}{a_{11}}, \quad \nu_{21}^o = -\frac{a_{12}}{a_{22}}, \quad G_{12}^o = \frac{1}{ha_{66}} \tag{207}$$

PROGRAM TITLE: MODULUS TO COMPLIANCE NO: 027

PROGRAM LISTING:

```
000 46   045 01   090 04   135 01   180 01
001 16   046 06   091 75   136 03   181 05
002 42   047 65   092 43   137 40   182 65
003 01   048 43   093 01   138 95   183 43
004 01   049 01   094 06   139 55   184 01
005 81   050 05   095 40   140 43   185 02
006 42   051 65   096 95   141 01   186 95
007 01   052 02   097 55   142 00   187 55
008 02   053 75   098 43   143 95   188 43
009 81   054 43   099 01   144 42   189 01
010 42   055 01   100 00   145 00   190 00
011 01   056 05   101 95   146 04   191 95
012 03   057 40   102 42   147 43   192 42
013 81   058 65   103 00   148 01   193 00
014 42   059 43   104 01   149 05   194 05
015 01   060 01   105 43   150 65   195 43
016 04   061 02   106 01   151 43   196 01
017 81   062 75   107 01   152 01   197 03
018 42   063 43   108 65   153 06   198 65
019 01   064 01   109 43   154 75   199 43
020 05   065 03   110 01   155 43   200 01
021 81   066 40   111 04   156 01   201 05
022 42   067 65   112 75   157 03   202 75
023 01   068 43   113 43   158 65   203 43
024 06   069 01   114 01   159 43   204 01
025 81   070 04   115 05   160 01   205 01
026 46   071 75   116 40   161 04   206 65
027 11   072 43   117 95   162 95   207 43
028 43   073 01   118 55   163 55   208 01
029 01   074 06   119 43   164 43   209 06
030 01   075 40   120 01   165 01   210 95
031 65   076 65   121 00   166 00   211 55
032 43   077 43   122 95   167 95   212 43
033 01   078 01   123 42   168 42   213 01
034 02   079 01   124 00   169 00   214 00
035 65   080 95   125 02   170 03   215 95
036 43   081 42   126 43   171 43   216 42
037 01   082 01   127 01   172 01   217 00
038 04   083 00   128 01   173 03   218 06
039 85   084 43   129 65   174 65   219 81
040 43   085 01   130 43   175 43
041 01   086 02   131 01   176 01
042 03   087 65   132 02   177 06
043 65   088 43   133 75   178 75
044 43   089 01   134 43   179 43
```

SAMPLE PROBLEM: 027 MODULUS TO COMPLIANCE NO: 027

$$Q_{ij} = \begin{bmatrix} 181.8111 & 2.8969 & 0 \\ 2.8969 & 10.3462 & 0 \\ 0 & 0 & 7.1700 \end{bmatrix} \quad \text{(MPa)}$$

$$S_{ij} = \begin{bmatrix} 5.5249 & -1.5469 & 0 \\ -1.5469 & 97.0870 & 0 \\ 0 & 0 & 139.4700 \end{bmatrix} \quad (\text{TPa})^{-1}$$

TITLE: COMPLIANCE TO MODULUS				NO: 031
A' S_{ij}	B'	C'	D'	E'
A $\|S\|$	B	C	D	E

00					
	REGISTER				
01 S_{11}	02 S_{22}	03 S_{12}	04 S_{66}	05 S_{16}	06 S_{26}
07	08	09	10 $\|S\|$		
11 Q_{11}	12 Q_{22}	13 Q_{12}	14 Q_{66}	15 Q_{16}	16 Q_{26}
17	18	19			
98	99				

INSTRUCTIONS

OPERATION	STEP	READ	ENTER	PRESS	DISPLAY
Input S_{ij}	1	031 A&B	S_{11}	A'	S_{11}
			S_{22}	RUN	S_{12}
			S_{12}	RUN	S_{12}
			S_{66}	RUN	S_{66}
			S_{16}	RUN	S_{16}
			S_{26}	RUN	S_{26}
Compute Q_{ij}	2			A	Q_{11}
				RCL 11	Q_{11}
				RCL 12	Q_{22}
				RCL 13	Q_{12}
				RCL 14	Q_{66}
				RCL 15	Q_{16}
				RCL 16	Q_{26}

PROGRAM TITLE: COMPLIANCE TO MODULUS NO: 031

PROGRAM LISTING:

```
000 46   045 00   090 02   135 00   180 03
001 16   046 06   091 75   136 03   181 75
002 42   047 65   092 43   137 65   182 43
003 00   048 43   093 00   138 43   183 00
004 01   049 00   094 03   139 00   184 01
005 81   050 05   095 40   140 04   185 65
006 42   051 65   096 95   141 95   186 43
007 00   052 02   097 55   142 55   187 00
008 02   053 75   098 43   143 43   188 06
009 81   054 43   099 01   144 01   189 95
010 42   055 00   100 00   145 00   190 55
011 00   056 05   101 95   146 95   191 43
012 03   057 40   102 42   147 42   192 01
013 81   058 65   103 01   148 01   193 00
014 42   059 43   104 04   149 03   194 95
015 00   060 00   105 43   150 43   195 42
016 04   061 02   106 00   151 00   196 01
017 81   062 75   107 01   152 03   197 06
018 42   063 43   108 65   153 65   198 43
019 00   064 00   109 43   154 43   199 00
020 05   065 03   110 00   155 00   200 02
021 81   066 40   111 04   156 06   201 65
022 42   067 65   112 75   157 75   202 43
023 00   068 43   113 43   158 43   203 00
024 06   069 00   114 00   159 00   204 04
025 81   070 04   115 05   160 05   205 75
026 46   071 75   116 40   161 65   206 43
027 11   072 43   117 95   162 43   207 00
028 43   073 00   118 55   163 00   208 06
029 00   074 06   119 43   164 02   209 40
030 01   075 40   120 01   165 95   210 95
031 65   076 65   121 00   166 55   211 55
032 43   077 43   122 95   167 43   212 43
033 00   078 00   123 42   168 01   213 01
034 02   079 01   124 01   169 00   214 00
035 65   080 95   125 02   170 95   215 95
036 43   081 42   126 43   171 42   216 42
037 00   082 01   127 00   172 01   217 01
038 04   083 00   128 05   173 05   218 01
039 85   084 43   129 65   174 43   219 81
040 43   085 00   130 43   175 00
041 00   086 01   131 00   176 05
042 03   087 65   132 06   177 65
043 65   088 43   133 75   178 43
044 43   089 00   134 43   179 00
```

TITLE: INVARIANTS OF ELASTICITY				NO: 033
A'	B'	C'	D'	E'
A Initialize	B	C	D	E

REGISTER

00 $\frac{1}{m}$					
01 I_{1Q}	02 I_{2Q}	03 R_{1Q}	04 R_{2Q}	05 Q_{11}	06 Q_{22}
07	08	09 ν_{LT}	10 G_{LT}		
11 I_{1S}	12 I_{2S}	13 R_{1S}	14 R_{2S}	15 Q_{12}	16 S_{12}
17	18	19			
98 E_L	99 E_T				

INSTRUCTIONS

OPERATION	STEP	READ	ENTER	PRESS	DISPLAY
Input Engineering Constants	1		E_L	STO 98	E_L
			E_T	STO 99	E_T
			ν_{LT}	STO 09	ν_{LT}
			G_{LT}	STO 10	G_{LT}
Compute invariants	2	033 A&B		A	R_{2S}
				RCL 11	I_{1S}
				RCL 12	I_{2S}
				RCL 13	R_{1S}
				RCL 14	R_{2S}
				RCL 01	I_{1Q}
				RCL 02	I_{2Q}
				RCL 03	R_{1Q}
				RCL 04	R_{2Q}

PROGRAM TITLE: INVARIANTS OF ELASTICITY NO: 033

FORMULAS:

TABLE 10 COMPLIANCE AND MODULUS IN TERMS OF ELASTIC SYMMETRIES

Symmetry (No Indep. Const)	Compliance Matrix S_{ij} $(TPa)^{-1}$	Modulus Matrix Q_{ij} (MPa)
Anisotropic (6) or Generally Orthotropic (4)	$\begin{bmatrix} \frac{1}{E_{11}} & -\frac{\nu_{12}}{E_{11}} & S_{16} \\ -\frac{\nu_{21}}{E_{22}} & \frac{1}{E_{22}} & S_{26} \\ S_{61} & S_{62} & \frac{1}{G_{12}} \end{bmatrix}$	$\begin{bmatrix} Q_{11} & Q_{12} & Q_{16} \\ Q_{21} & Q_{22} & Q_{26} \\ Q_{61} & Q_{62} & Q_{66} \end{bmatrix}$
Specially Orthotropic (4)	$\begin{bmatrix} \frac{1}{E_L} & -\frac{\nu_{LT}}{E_L} & 0 \\ -\frac{\nu_{TL}}{E_T} & \frac{1}{E_T} & 0 \\ 0 & 0 & \frac{1}{G_{LT}} \end{bmatrix}$	$\begin{bmatrix} mE_L & \nu_{TL}mE_L & 0 \\ \nu_{LT}mE_T & mE_T & 0 \\ 0 & 0 & G_{LT} \end{bmatrix}$

$$I_1 = \frac{1}{4}(Q_{11} + Q_{22} + 2Q_{12}) \quad (89)$$

$$I_2 = \frac{1}{8}(Q_{11} + Q_{22} - 2Q_{12} + 4Q_{66}) \quad (90)$$

$$R_1 = \frac{1}{2}\sqrt{(-Q_{11} + Q_{22})^2} \quad (91)$$

$$R_2 = \frac{1}{8}\sqrt{(Q_{11} + Q_{22} - 2Q_{12} - 4Q_{66})^2} \quad (92)$$

$$I_1 = \frac{1}{4}(S_{11} + S_{22} + 2S_{12}) \quad (81)$$

$$I_2 = \frac{1}{8}(S_{11} + S_{22} - 2S_{12} + S_{66}) \quad (82)$$

$$R_1 = \frac{1}{2}\sqrt{(-S_{11} + S_{22})^2} \quad (83)$$

$$R_2 = \frac{1}{8}\sqrt{(S_{11} + S_{22} - 2S_{12} - S_{66})^2} \quad (84)$$

$$m = \frac{1}{1 - \nu_{LT}\nu_{TL}} = \frac{1}{1 - \nu_{LT}^2 \frac{E_T}{E_L}}$$

PROGRAM TITLE: INVARIANTS OF ELASTICITY NO: 033

PROGRAM LISTING:

```
000 46   045 42   090 02   135 54   180 09
001 11   046 01   091 43   136 42   181 09
002 01   047 05   092 00   137 01   182 20
003 75   048 65   093 05   138 06   183 75
004 43   049 02   094 75   139 65   184 43
005 00   050 85   095 43   140 02   185 09
006 09   051 43   096 00   141 95   186 08
007 40   052 00   097 06   142 55   187 20
008 65   053 05   098 95   143 04   188 95
009 43   054 85   099 55   144 95   189 55
010 09   055 43   100 02   145 51   190 02
011 09   056 00   101 95   146 87   191 95
012 55   057 06   102 42   147 42   192 51
013 43   058 95   103 00   148 01   193 87
014 09   059 55   104 03   149 01   194 42
015 08   060 04   105 43   150 43   195 01
016 95   061 95   106 00   151 09   196 03
017 42   062 42   107 02   152 08   197 43
018 00   063 00   108 75   153 20   198 01
019 00   064 01   109 43   154 85   199 00
020 55   065 43   110 01   155 43   200 20
021 43   066 00   111 00   156 09   201 51
022 09   067 05   112 95   157 09   202 87
023 08   068 85   113 42   158 20   203 55
024 95   069 43   114 00   159 75   204 04
025 20   070 00   115 04   160 02   205 75
026 42   071 06   116 43   161 65   206 43
027 00   072 75   117 09   162 43   207 01
028 05   073 02   118 08   163 01   208 02
029 43   074 65   119 20   164 06   209 95
030 09   075 43   120 85   165 85   210 42
031 09   076 01   121 43   166 43   211 01
032 55   077 05   122 09   167 01   212 04
033 43   078 85   123 09   168 00   213 81
034 00   079 04   124 20   169 20   214 46
035 00   080 65   125 85   170 95   215 87
036 95   081 43   126 53   171 55   216 65
037 42   082 01   127 43   172 08   217 01
038 00   083 00   128 00   173 95   218 00
039 06   084 95   129 09   174 51   219 00
040 65   085 55   130 55   175 87   220 00
041 43   086 08   131 43   176 42   221 95
042 00   087 95   132 09   177 01   222 56
043 09   088 42   133 08   178 02
044 95   089 00   134 94   179 43
```

SAMPLE PROBLEM: 033 INVARIANTS OF ELASTICITY NO: 033

Given:

Material	Type	E_L (GPa)	E_T (GPa)	ν_{LT}	G_{LT} (GPa)	Fiber Vol. v_f	Specific Gravity
B/Ep	B(4) 5505	204	18.5	.23	5.79	.5	2.0
Gr/Ep	T-300 5208	181	10.3	.28	7.17	.70	1.60
Gl/Ep	Scotchply 1002	38.6	8.27	.26	4.14	.45	1.8

INVARIANTS FOR COMPLIANCE MATRIX

Material	I_{1S}	I_{2S}	R_{1S}	R_{2S}	$\delta_1 = \delta_2$
B/Ep	14.18	29.24	24.58	13.94	0
Gr/Ep	24.88	30.65	45.78	4.22	0
Gl/Ep	33.3	50.2	47.5	10.2	0

INVARIANTS FOR MODULUS MATRIX

Material	I_{1Q}	I_{2Q}	R_{1Q}	R_{2Q}	$\delta_1 = \delta_2$
B/Ep	58.03	29.77	93.20	23.98	0
Gr/Ep	49.49	26.88	85.73	19.71	0
Gl/Ep	13.0	7.47	15.4	3.33	0

TITLE: COMPLIANCE COMPONENTS				NO: 037
A'	B' S_{16}	C' S_{26}	D'	E'
A Invariants	B S_{11}	C S_{22}	D S_{12}	E S_{66}

REGISTER

00 θ					
01 S_{11}	02 S_{22}	03 S_{12}	04 S_{66}	05 S_{16}	06 S_{26}
07	08	09	10		
11 I_{1S}	12 I_{2S}	13 R_{1S}	14 R_{2S}	15	16
17 δ_{1S}	18 δ_{2S}	19 $2(\theta-\delta_1)$			
98 $4(\theta-\delta_2)$	99				

INSTRUCTIONS

OPERATION	STEP	READ	ENTER	PRESS	DISPLAY
Input Invariants	1	037 A&B	I_{1S}	A	I_{1S}
			I_{2S}	RUN	I_{2S}
			R_{1S}	RUN	R_{1S}
			R_{2S}	RUN	R_{2S}
			δ_{1S}	RUN	δ_{1S}
			δ_{2S}	RUN	δ_{2S}
Input θ & Compute	2		θ (deg)	B	S_{11}
				C	S_{22}
				D	S_{12}
				E	S_{66}
				B'	S_{16}
				C'	S_{26}

PROGRAM TITLE: COMPLIANCE COMPONENTS NO: 037

FORMULAS:

TABLE 17 TRANSFORMATION OF COMPLIANCE

	I_1	I_2	R_1	R_2
S'_{11}	1	1	$-\cos 2(\theta - \delta_1)$	$-\cos 4(\theta - \delta_2)$
S'_{22}	1	1	$\cos 2(\theta - \delta_1)$	$-\cos 4(\theta - \delta_2)$
S'_{12}	1	-1	0	$\cos 4(\theta - \delta_2)$
S'_{66}	0	4	0	$4 \cos 4(\theta - \delta_2)$
S'_{16}	0	0	$\sin 2(\theta - \delta_1)$	$2 \sin 4(\theta - \delta_2)$
S'_{26}	0	0	$\sin 2(\theta - \delta_1)$	$-2 \sin 4(\theta - \delta_2)$

$$I_1 = \frac{1}{4}(S_{11} + S_{22} + 2S_{12}) \tag{82}$$

$$I_2 = \frac{1}{8}(S_{11} + S_{22} - 2S_{12} + S_{66}) \tag{83}$$

$$R_1 = \frac{1}{2}\sqrt{(-S_{11} + S_{22})^2 + (S_{16} + S_{26})^2} \tag{84}$$

$$R_2 = \frac{1}{8}\sqrt{(S_{11} + S_{22} - 2S_{12} - S_{66})^2 + 4(S_{26} - S_{16})^2} \tag{85}$$

$$\tan 2\delta_1 = \frac{S_{16} + S_{26}}{S_{11} - S_{22}} \tag{86}$$

$$\tan 4\delta_2 = \frac{2(S_{16} - S_{26})}{S_{11} + S_{22} - 2S_{12} - S_{66}} \tag{87}$$

For orthotropic material

$$\delta_1 = \delta_2 \tag{88}$$

For anisotropic material

$$\delta_1 \neq \delta_2 \tag{89}$$

PROGRAM TITLE: COMPLIANCE COMPONENTS NO: 037

PROGRAM LISTING:

```
000 46   045 01   090 06   135 85   180 14
001 11   046 03   091 43   136 43   181 43
002 42   047 85   092 09   137 01   182 00
003 01   048 53   093 08   138 09   183 03
004 01   049 53   094 33   139 33   184 81
005 81   050 43   095 65   140 65   185 46
006 42   051 00   096 43   141 43   186 15
007 01   052 00   097 01   142 01   187 43
008 02   053 75   098 04   143 03   188 00
009 81   054 43   099 85   144 75   189 04
010 42   055 01   100 43   145 43   190 81
011 01   056 08   101 01   146 09   191 46
012 03   057 54   102 01   147 08   192 17
013 81   058 65   103 75   148 33   193 43
014 42   059 04   104 43   149 65   194 00
015 01   060 54   105 01   150 43   195 05
016 04   061 42   106 02   151 01   196 81
017 81   062 09   107 95   152 04   197 46
018 42   063 08   108 42   153 95   198 18
019 01   064 32   109 00   154 42   199 43
020 07   065 65   110 03   155 00   200 00
021 81   066 02   111 75   156 02   201 06
022 42   067 65   112 43   157 75   202 81
023 01   068 43   113 01   158 43
024 08   069 01   114 01   159 01
025 81   070 04   115 85   160 09
026 46   071 95   116 02   161 33
027 12   072 42   117 65   162 65
028 42   073 00   118 43   163 43
029 00   074 05   119 01   164 01
030 00   075 94   120 02   165 03
031 75   076 85   121 95   166 65
032 43   077 43   122 65   167 02
033 01   078 01   123 04   168 95
034 07   079 09   124 95   169 42
035 95   080 32   125 42   170 00
036 65   081 65   126 00   171 01
037 02   082 43   127 04   172 81
038 95   083 01   128 43   173 46
039 42   084 03   129 01   174 13
040 01   085 65   130 01   175 43
041 09   086 02   131 85   176 00
042 32   087 95   132 43   177 02
043 65   088 42   133 01   178 81
044 43   089 00   134 02   179 46
```

SAMPLE PROBLEM: 037 COMPLIANCE COMPONENTS NO: 037

Given:

Material	Type	I_{1S}	I_{2S}	R_{1S}	R_{2S}	$\delta_1=\delta_2$
B/Ep	B(4) 5505	14.18	29.24	24.58	13.94	0
Gr/Ep	T-300 5208	24.88	30.65	45.78	4.22	0
Gl/Ep	Scotchply 1002	33.3	50.2	47.5	10.2	0

	T-300 / 5208					
	S_{ij} $(TPa)^{-1}$			Engr. Const. (GPa)		
	5.52	-1.55	0	E_{11}'	=	181
				E_{22}'	=	10.3
0^o	-1.55	97.09	0	ν_{12}'	=	.28
				ν_{21}'	=	.016
	0	0	139.47	G_{12}'	=	7.2
	34.75	-7.88	46.96	E_{11}'	=	28.78
				E_{22}'	=	12.42
30^o	-7.88	80.53	32.34	ν_{12}'	=	.23
				ν_{21}'	=	.10
	46.96	32.34	114.15	G_{12}'	=	8.76
	59.75	-9.99	45.78	E_{11}'	=	16.74
				E_{22}'	=	16.74
45^o	-9.99	59.75	45.78	ν_{12}'	=	.17
				ν_{21}'	=	.17
	45.78	45.78	105.71	G_{12}'	=	9.46

TITLE: MODULUS COMPONENTS				NO: 041
A'	B' Q_{16}	C' Q_{26}	D'	E'
A Invariants	B Q_{11}	C Q_{22}	D Q_{12}	E Q_{66}

00 θ					
	REGISTER				
01 I_{1Q}	02 I_{2Q}	03 R_{1Q}	04 R_{2Q}	05	06
07 δ_{1Q}	08 δ_{2Q}	09 $2(\theta-\delta_1)$	10		
11 Q_{11}	12 Q_{22}	13 Q_{12}	14 Q_{66}	15 Q_{16}	16 Q_{26}
17	18	19			
98	99 $4(\theta-\delta_2)$				

INSTRUCTIONS

OPERATION	STEP	READ	ENTER	PRESS	DISPLAY
Input Invariants	1	041 A&B	I_{1Q}	A	I_{1Q}
			I_{2Q}	RUN	I_{2Q}
			R_{1Q}	RUN	R_{1Q}
			R_{2Q}	RUN	R_{2Q}
			δ_{1Q}	RUN	δ_{1Q}
			δ_{2Q}	RUN	δ_{2Q}
Input θ & Compute	2		θ(deg)	B	Q_{11}
				C	Q_{22}
				D	Q_{12}
				E	Q_{66}
				B'	Q_{16}
				C'	Q_{26}

PROGRAM TITLE: MODULUS COMPONENTS NO: 041

FORMULAS:

TABLE 20 TRANSFORMATION OF MODULUS

	I_1	I_2	R_1	R_2
Q'_{11}	1	1	$\cos 2(\theta - \delta_1)$	$\cos 4(\theta - \delta_2)$
Q'_{22}	1	1	$-\cos 2(\theta - \delta_1)$	$\cos 4(\theta - \delta_2)$
Q'_{12}	1	-1	0	$-\cos 4(\theta - \delta_2)$
Q'_{66}	0	1	0	$-\cos 4(\theta - \delta_2)$
Q'_{16}	0	0	$-\frac{1}{2}\sin 2(\theta - \delta_1)$	$-\sin 4(\theta - \delta_2)$
Q_{26}	0	0	$-\frac{1}{2}\sin 2(\theta - \delta_1)$	$\sin 4(\theta - \delta_2)$

$$I_1 = \frac{1}{4}(Q_{11} + Q_{22} + 2Q_{12}) \tag{90}$$

$$I_2 = \frac{1}{8}(Q_{11} + Q_{22} - 2Q_{12} + 4Q_{66}) \tag{91}$$

$$R_1 = \frac{1}{2}\sqrt{(-Q_{11} + Q_{22})^2 + 4(Q_{16} + Q_{26})^2} \tag{92}$$

$$R_2 = \frac{1}{8}\sqrt{(Q_{11} + Q_{22} - 2Q_{12} - 4Q_{66})^2 + 16(Q_{16} - Q_{26})^2} \tag{93}$$

$$\tan 2\delta_1 = \pm \frac{2(Q_{16} + Q_{26})}{Q_{11} - Q_{22}} \tag{94}$$

$$\tan 4\delta_2 = \pm \frac{4(Q_{16} - Q_{26})}{Q_{11} + Q_{22} - 2Q_{12} - 4Q_{66}} \tag{95}$$

$\delta_1 = \delta_2$ for orthotropic materials

$\delta_1 \neq \delta_2$ for anisotropic materials

PROGRAM TITLE: MODULUS COMPONENTS **NO:** 041

PROGRAM LISTING:

```
000 46   045 94   090 06   135 65   180 15
001 11   046 65   091 43   136 43   181 43
002 42   047 43   092 09   137 00   182 01
003 00   048 00   093 09   138 03   183 04
004 01   049 03   094 33   139 85   184 81
005 81   050 75   095 94   140 43   185 46
006 42   051 53   096 65   141 00   186 17
007 00   052 53   097 43   142 01   187 43
008 02   053 43   098 00   143 85   188 01
009 81   054 00   099 04   144 43   189 05
010 42   055 00   100 85   145 00   190 81
011 00   056 75   101 43   146 02   191 46
012 03   057 43   102 00   147 95   192 18
013 81   058 00   103 02   148 42   193 43
014 42   059 08   104 95   149 01   194 01
015 00   060 54   105 42   150 02   195 06
016 04   061 65   106 01   151 85   196 81
017 81   062 04   107 04   152 43
018 42   063 54   108 85   153 00
019 00   064 42   109 43   154 09
020 07   065 09   110 00   155 33
021 81   066 09   111 01   156 65
022 42   067 32   112 75   157 43
023 00   068 65   113 02   158 00
024 08   069 43   114 65   159 03
025 81   070 00   115 43   160 65
026 46   071 04   116 00   161 02
027 12   072 95   117 02   162 95
028 42   073 42   118 95   163 42
029 00   074 01   119 42   164 01
030 00   075 05   120 01   165 01
031 75   076 85   121 03   166 81
032 43   077 43   122 43   167 46
033 00   078 09   123 09   168 13
034 07   079 09   124 09   169 43
035 95   080 32   125 33   170 01
036 65   081 65   126 65   171 02
037 02   082 43   127 43   172 81
038 95   083 00   128 00   173 46
039 42   084 04   129 04   174 14
040 00   085 65   130 75   175 43
041 09   086 02   131 43   176 01
042 32   087 95   132 00   177 03
043 55   088 42   133 09   178 81
044 02   089 01   134 33   179 46
```

SAMPLE PROBLEM: 041 MODULUS COMPONENTS NO: 041

Given:

Material	Type	I_{1Q}	I_{2Q}	R_{1Q}	R_{2Q}	$\delta_1=\delta_2$
B/Ep	B(4) / 5505	58.03	29.77	93.20	23.98	0
Gr/Ep	T-300 5208	49.49	26.88	85.73	19.71	0
Gl/Ep	Scotchply 1002	13.0	7.47	15.4	3.33	0

	T-300 / 5208			B(4) / 5504		
	Q'_{ij}			Q'_{ij}		
0^o	181.81	42.90	0	204.98	4.28	0
	42.90	10.35	0	4.28	18.59	0
	0	0	7.17	0	0	5.79
30^o	109.38	32.46	-54.19	122.41	40.25	-61.13
	32.46	23.65	-20.05	40.25	29.21	-19.59
	-54.19	-20.05	36.74	-61.13	-19.59	41.76
45^o	56.66	42.32	-42.89			
	42.32	56.66	-42.87			
	-42.87	-42.87	46.59			

TITLE: PLY DATA: T-300/5208 Gr/Ep				NO: 045
A' X, e_L	B' X', e_L'	C' Y, e_T	D' Y', e_T'	E' S, e_S
A E_L	B E_T	C ν_{LT}	D G_{LT}	E'

REGISTER

00 S					
01 I_{1Q}	02 I_{2Q}	03 R_{1Q}	04 R_{2Q}	05 E_L	06 E_T
07 0	08 0	09 ν_{LT}	10 G_{LT}		
11 I_{1S}	12 I_{2S}	13 R_{1S}	14 R_{2S}	15 X	16 X'
17 0	18 0	19 0			
98 Y	99 Y'				

INSTRUCTIONS

OPERATION	STEP	READ	ENTER	PRESS	DISPLAY
Initialize	1	045 A&B		A	E_L
				B	E_T
				C	ν_{LT}
				D	G_{LT}
				A'	X
				RUN	e_L
				B'	X'
				RUN	e_L'
				C'	Y
				RUN	e_T
				D'	Y'
				RUN	e'_T
				E'	S
				RUN	e_S

PROGRAM TITLE: PLY DATA: T300/5208 Gr/Ep NO: 045

PROGRAM LISTING:

```
000 46   045 02   090 03   135 08   180 43
001 11   046 08   091 04   136 42   181 00
002 01   047 05   092 93   137 00   182 05
003 08   048 93   093 02   138 07   183 95
004 01   049 07   094 02   139 43   184 81
005 42   050 03   095 00   140 00   185 46
006 00   051 02   096 42   141 05   186 18
007 05   052 42   097 01   142 81   187 43
008 01   053 00   098 04   143 46   188 09
009 00   054 03   099 01   144 12   189 08
010 93   055 01   100 05   145 43   190 81
011 03   056 09   101 00   146 00   191 55
012 42   057 93   102 00   147 06   192 43
013 00   058 07   103 42   148 81   193 00
014 06   059 01   104 01   149 46   194 06
015 93   060 00   105 05   150 13   195 95
016 02   061 42   106 01   151 43   196 81
017 08   062 00   107 05   152 00   197 46
018 42   063 04   108 00   153 09   198 19
019 00   064 02   109 00   154 81   199 43
020 09   065 04   110 42   155 46   200 09
021 07   066 93   111 01   156 14   201 09
022 93   067 09   112 06   157 43   202 81
023 01   068 08   113 04   158 01   203 55
024 07   069 00   114 00   159 00   204 43
025 42   070 42   115 42   160 81   205 00
026 01   071 01   116 09   161 46   206 06
027 00   072 01   117 08   162 16   207 95
028 04   073 03   118 02   163 43   208 81
029 09   074 00   119 04   164 01   209 46
030 93   075 93   120 06   165 05   210 10
031 04   076 06   121 42   166 81   211 43
032 08   077 04   122 09   167 55   212 00
033 08   078 07   123 09   168 43   213 00
034 42   079 42   124 06   169 00   214 81
035 00   080 01   125 08   170 05   215 55
036 01   081 02   126 42   171 95   216 43
037 02   082 04   127 00   172 81   217 01
038 06   083 05   128 00   173 46   218 00
039 93   084 93   129 00   174 17   219 95
040 08   085 07   130 42   175 43   220 81
041 08   086 08   131 00   176 01
042 00   087 01   132 07   177 06
043 42   088 42   133 42   178 81
044 00   089 01   134 00   179 55
```

TITLE: PLY DATA: B/5505 B/Ep				NO: 047
A' X, e_L	B' X', e_L'	C' Y, e_T	D' Y', e_T'	E' S, e_S
A E_L	B E_T	C ν_{LT}	D G_{LT}	E

00 S					
REGISTER					
01 I_{1Q}	02 I_{2Q}	03 R_{1Q}	04 R_{2Q}	05 E_L	06 E_T
07 0	08 0	09 ν_{LT}	10 G_{LT}		
11 I_{1S}	12 I_{2S}	13 R_{1S}	14 R_{2S}	15 X	16 X'
17 0	18 0	19 0			
98 Y	99 Y'				

INSTRUCTIONS

OPERATION	STEP	READ	ENTER	PRESS	DISPLAY
Initialize	1	047 A&B		A	E_L
				B	E_T
				C	ν_{LT}
				D	G_{LT}
				A'	X
				RUN	e_L
				B'	X'
				RUN	e_L'
				C'	Y
				RUN	e_T
				D'	Y'
				RUN	e_T
				E'	S
				RUN	e_S

PROGRAM TITLE: PLY DATA: B/5505 B/Ep NO: 047

PROGRAM LISTING:

```
000 46   045 00   090 00   135 81   180 43
001 11   046 03   091 42   136 46   181 09
002 02   047 02   092 06   137 14   182 09
003 00   048 04   093 09   138 43   183 81
004 04   049 42   094 06   139 01   184 55
005 42   050 00   095 01   140 00   185 43
006 00   051 04   096 42   141 81   186 00
007 05   052 01   097 09   142 46   187 06
008 01   053 04   098 08   143 16   188 95
009 08   054 93   099 02   144 43   189 81
010 93   055 02   100 00   145 01   190 46
011 05   056 42   101 02   146 05   191 10
012 42   057 01   102 42   147 81   192 43
013 00   058 01   103 09   148 55   193 00
014 06   059 02   104 09   149 43   194 00
015 93   060 09   105 06   150 00   195 81
016 02   061 93   106 07   151 05   196 55
017 03   062 02   107 42   152 95   197 43
018 42   063 42   108 00   153 81   198 01
019 01   064 01   109 00   154 46   199 00
020 06   065 02   110 00   155 17   200 95
021 05   066 02   111 42   156 43   201 81
022 93   067 04   112 00   157 06
023 07   068 93   113 07   158 09
024 09   069 06   114 42   159 81
025 42   070 42   115 00   160 55
026 01   071 01   116 08   161 43
027 00   072 03   117 42   162 00
028 05   073 01   118 00   163 05
029 08   074 03   119 09   164 95
030 42   075 93   120 43   165 81
031 00   076 09   121 00   166 46
032 01   077 42   122 05   167 18
033 02   078 01   123 81   168 43
034 09   079 04   124 46   169 09
035 93   080 01   125 12   170 08
036 08   081 02   126 43   171 81
037 42   082 06   127 00   172 55
038 00   083 00   128 06   173 43
039 02   084 42   129 81   174 00
040 09   085 01   130 46   175 06
041 03   086 05   131 13   176 95
042 93   087 02   132 43   177 81
043 02   088 05   133 01   178 46
044 42   089 00   134 06   179 19
```

TITLE: PLY DATA: SCOTCHPLY 1002				NO: 049
A' X, e_L	B' X', e_L'	C' Y, e_T	D' Y', e_T'	E' S, e_S
A E_L	B E_T	C ν_{LT}	D G_{LT}	E

00 S					
01 I_{1Q}	02 I_{2Q}	03 R_{1Q}	04 R_{2Q}	05 E_L	06 E_T
07 0	08 0	09 ν_{LT}	10 G_{LT}		
11 I_{1S}	12 I_{2S}	13 R_{1S}	14 R_{2S}	15 X	16 X'
17 0	18 0	19 0			
98 Y	99 Y'				

REGISTER

INSTRUCTIONS

OPERATION	STEP	READ	ENTER	PRESS	DISPLAY
Initialize	1	049 A&B		A	E_L
				B	E_T
				C	ν_{LT}
				D	G_{LT}
				A'	X
				RUN	e_L
				B'	X'
				RUN	e_L'
				C'	Y
				RUN	e_T
				D'	Y'
				RUN	e_T
				E'	S
				RUN	e_S

PROGRAM TITLE: PLY DATA: SCOTCHPLY 1002 Gl/Ep NO: 049

PROGRAM LISTING:

```
000 46  045 42  090 05  135 06  180 19
001 11  046 00  091 02  136 81  181 43
002 03  047 03  092 42  137 46  182 09
003 08  048 03  093 06  138 14  183 09
004 93  049 93  094 09  139 43  184 81
005 06  050 03  095 02  140 01  185 55
006 42  051 03  096 00  141 00  186 43
007 00  052 42  097 42  142 81  187 00
008 05  053 00  098 09  143 46  188 06
009 08  054 04  099 08  144 16  189 95
010 93  055 03  100 01  145 43  190 81
011 02  056 03  101 03  146 01  191 46
012 07  057 93  102 08  147 05  192 10
013 42  058 03  103 42  148 81  193 43
014 00  059 42  104 09  149 55  194 00
015 06  060 01  105 09  150 43  195 00
016 93  061 01  106 02  151 00  196 81
017 02  062 05  107 09  152 05  197 55
018 06  063 00  108 42  153 95  198 43
019 42  064 93  109 00  154 81  199 01
020 01  065 02  110 00  155 46  200 00
021 06  066 42  111 00  156 17  201 95
022 04  067 01  112 42  157 43  202 81
023 93  068 02  113 00  158 06
024 01  069 04  114 07  159 09
025 04  070 07  115 42  160 81
026 42  071 93  116 00  161 55
027 01  072 05  117 08  162 43
028 00  073 42  118 42  163 00
029 01  074 01  119 00  164 05
030 03  075 03  120 09  165 95
031 42  076 01  121 43  166 81
032 00  077 00  122 00  167 46
033 01  078 93  123 05  168 18
034 07  079 03  124 81  169 43
035 93  080 42  125 46  170 09
036 04  081 01  126 12  171 08
037 07  082 04  127 43  172 81
038 42  083 07  128 00  173 55
039 00  084 05  129 06  174 43
040 02  085 08  130 81  175 00
041 01  086 42  131 46  176 06
042 05  087 01  132 13  177 95
043 93  088 05  133 43  178 81
044 04  089 05  134 01  179 46
```

TITLE: PLY DATA:				NO: 051
A' X, e_L	B' X', e_L'	C' Y, e_T	D' Y', e_T'	E' S, e_S
A E_L	B E_T	C ν_{LT}	D G_{LT}	E

REGISTER

00 S					
01 I_{1Q}	02 I_{2Q}	03 R_{1Q}	04 R_{2Q}	05 E_L	06 E_T
07 0	08 0	09 ν_{LT}	10 G_{LT}		
11 I_{1S}	12 I_{2S}	13 R_{1S}	14 R_{2S}	15 X	16 X'
17 0	18 0	19 0			
98 Y	99 Y'				

INSTRUCTIONS

OPERATION	STEP	READ	ENTER	PRESS	DISPLAY
Initialize	1	051 A&B		A	E_L
				B	E_T
				C	ν_{LT}
				D	G_{LT}
				A'	X
				RUN	e_L
				B'	X'
				RUN	e_L'
				C'	Y
				RUN	e_T
				D'	Y'
				RUN	e_T
				E'	S
				RUN	e_S

PROGRAM TITLE: PLY DATA NO: 051

PROGRAM LISTING:

TITLE: PLY DATA:				NO: 053
A' X, e_L	B' X', e_L'	C' Y, e_T	D' Y', e_T'	E' S, e_S
A E_L	B E_T	C ν_{LT}	D G_{LT}	E

REGISTER

00 S					
01 I_{1Q}	02 I_{2Q}	03 R_{1Q}	04 R_{2Q}	05 E_L	06 E_T
07 0	08 0	09 ν_{LT}	10 G_{LT}		
11 I_{1S}	12 I_{2S}	13 R_{1S}	14 R_{2S}	15 X	16 X'
17 0	18 0	19 0			
98 Y	99 Y'				

INSTRUCTIONS

OPERATION	STEP	READ	ENTER	PRESS	DISPLAY
Initialize	1	053 A&B		A	E_L
				B	E_T
				C	ν_{LT}
				D	G_{LT}
				A'	X
				RUN	e_L
				B'	X'
				RUN	e_L'
				C'	Y
				RUN	e_T
				D'	Y'
				RUN	e_T
				E'	S
				RUN	e_S

PROGRAM TITLE: PLY DATA NO: 053

PROGRAM LISTING:

TITLE: PLY DATA:				NO: 055
A' X, e_L	B' X', e_L'	C' Y, e_T	D' Y', e_T'	E' S, e_S
A E_L	B E_T	C ν_{LT}	D G_{LT}	E

REGISTER

00 S					
01 I_{1Q}	02 I_{2Q}	03 R_{1Q}	04 R_{2Q}	05 E_L	06 E_T
07 0	08 0	09 ν_{LT}	10 G_{LT}		
11 I_{1S}	12 I_{2S}	13 R_{1S}	14 R_{2S}	15 X	16 X'
17 0	18 0	19 0			
98 Y	99 Y'				

INSTRUCTIONS

OPERATION	STEP	READ	ENTER	PRESS	DISPLAY
Initialize	1	055 A&B		A	E_L
				B	E_T
				C	ν_{LT}
				D	G_{LT}
				A'	X
				RUN	e_L
				B'	X'
				RUN	e_L'
				C'	Y
				RUN	e_T
				D'	Y'
				RUN	e_T
				E'	S
				RUN	e_S

PROGRAM TITLE: PLY DATA NO: 055

PROGRAM LISTING:

TITLE: PLY DATA:				NO: 057
A' X, e_L	B' X', e_L'	C' Y, e_T	D' Y', e_T'	E' S, e_S
A E_L	B E_T	C ν_{LT}	D G_{LT}	E

REGISTER

00 S					
01 I_{1Q}	02 I_{2Q}	03 R_{1Q}	04 R_{2Q}	05 E_L	06 E_T
07 0	08 0	09 ν_{LT}	10 G_{LT}		
11 I_{1S}	12 I_{2S}	13 R_{1S}	14 R_{2S}	15 X	16 X'
17 0	18 0	19 0			
98 Y	99 Y'				

INSTRUCTIONS

OPERATION	STEP	READ	ENTER	PRESS	DISPLAY
Initialize	1	057 A&B		A	E_L
				B	E_T
				C	ν_{LT}
				D	G_{LT}
				A'	X
				RUN	e_L
				B'	X'
				RUN	e_L'
				C'	Y
				RUN	e_T
				D'	Y'
				RUN	e_T
				E'	S
				RUN	e_S

PROGRAM TITLE: PLY DATA NO: 057

PROGRAM LISTING:

TITLE: STORAGE OF CONSTITUENT PROPERTIES				NO: 059
A' ν_m	B' η_1	C' η_2	D' η_6 or η	E' v_f
A E_f, α_f	B G_f, α_m	C ν_f	D E_m	E G_m

00					
		REGISTER			
01 E_f	02 G_f	03 ν_f	04 E_m	05 G_m	06 ν_m
07 η_1	08 η_2	09 η_6 or η	10 v_f		
11 v_m	12 α_f	13 α_m	14	15	16
17	18	19			
98	99				

INSTRUCTIONS

OPERATION	STEP	READ	ENTER	PRESS	DISPLAY
Initialize	1	059 A&B		INV	
				st fg.	
				0	
Input data	2		E_f	A	E_f
			G_f	B	G_f
			ν_f	C	ν_f
			E_m	D	E_m
			G_m	E	G_m
			ν_m	A'	ν_m
			η_1	B'	η_1
			η_2	C'	η_2
			η_6 or η	D'	η_6 or η
			v_f	E'	v_m
				st flg	
				0	
			α_f	A	
			α_m	B	

PROGRAM TITLE: STORAGE OF CONSTITUENT PROPERTIES NO: 059

PROGRAM LISTING:

```
000 46   045 46   090 46
001 11   046 14   091 17
002 60   047 60   092 42
003 00   048 00   093 00
004 00   049 00   094 07
005 01   050 05   095 81
006 01   051 06   096 46
007 42   052 42   097 18
008 00   053 00   098 42
009 01   054 04   099 00
010 81   055 81   100 08
011 42   056 42   101 81
012 01   057 01   102 46
013 02   058 05   103 19
014 81   059 81   104 42
015 46   060 46   105 00
016 12   061 15   106 09
017 60   062 60   107 81
018 00   063 00   108 46
019 00   064 00   109 10
020 02   065 07   110 42
021 06   066 01   111 01
022 42   067 42   112 00
023 00   068 00   113 01
024 02   069 05   114 75
025 81   070 81   115 43
026 42   071 42   116 01
027 01   072 01   117 00
028 03   073 06   118 95
029 81   074 81   119 42
030 46   075 46   120 01
031 13   076 16   121 01
032 60   077 60   122 81
033 00   078 00
034 00   079 00
035 04   080 08
036 01   081 06
037 42   082 42
038 00   083 00
039 03   084 06
040 81   085 81
041 42   086 42
042 01   087 01
043 04   088 07
044 81   089 81
```

TITLE: COMPOSITE MODULI I				NO: 061
A' E	B' ν	C'	D'	E'
A E_L	B G_{LT}	C ν_{LT}	D G_{TT}	E

00					
01 E_f	02 G_f	03 ν_f	04 E_m	05 G_m	06 ν_m
07 η_1	08 η_2	09 η_6 or η	10 v_f		
11 v_m	12 α_f	13 α_m	14	15	16
17 used	18 used	19			
98	99				

REGISTER

INSTRUCTIONS

OPERATION	STEP	READ	ENTER	PRESS	DISPLAY
Input data	1	059 A&B			
Compute moduli	2	061 A&B		A B C D A' B'	E_L G_{LT} ν_{LT} G_{TT} E ν

PROGRAM TITLE: COMPOSITE MODULI I NO: 061

FORMULAS:

$$E_1^* = \frac{1}{\eta_1 v_f + v_m} (\eta_1 v_f E_f + v_m E_m) \quad (128)$$

$$\nu_{12}^* = \frac{1}{\eta_1 v_f + v_m} (\eta_1 v_f \nu_f + v_m \nu_m) \quad (129)$$

$$\frac{1}{G_{cc}^*} = \left(\frac{v_f}{G_f} + \frac{\eta_c v_m}{G_m} \right) \frac{1}{v_f + \eta_c v_m} \quad (132)$$

$$\frac{1}{E^*} = \frac{v_f}{E_f} + \frac{\eta v_m}{E_m} \frac{1}{v_f + \eta v_m} \quad (146)$$

$$\nu^* = \frac{v_f \nu_f E_m + \eta v_m \nu_m E_f}{v_f E_m + \eta v_m E_f} \quad (147)$$

PROGRAM TITLE: COMPOSITE MODULI I NO: 061

PROGRAM LISTING:

```
000 46   045 43   090 43   135 00   180 65
001 11   046 01   091 01   136 05   181 43
002 43   047 01   092 08   137 09   182 01
003 00   048 54   093 85   138 46   183 01
004 01   049 95   094 43   139 16   184 65
005 42   050 81   095 01   140 43   185 43
006 01   051 46   096 09   141 00   186 00
007 08   052 12   097 65   142 01   187 06
008 43   053 43   098 43   143 42   188 65
009 00   054 00   099 01   144 01   189 43
010 04   055 09   100 01   145 08   190 00
011 42   056 42   101 55   146 43   191 01
012 01   057 01   102 43   147 00   192 54
013 09   058 09   103 01   148 04   193 55
014 53   059 43   104 07   149 42   194 53
015 43   060 00   105 54   150 01   195 43
016 00   061 02   106 95   151 07   196 01
017 07   062 42   107 81   152 43   197 00
018 65   063 01   108 46   153 00   198 65
019 43   064 08   109 13   154 09   199 43
020 01   065 43   110 43   155 42   200 00
021 00   066 00   111 00   156 01   201 04
022 65   067 05   112 03   157 09   202 85
023 43   068 42   113 42   158 41   203 43
024 01   069 01   114 01   159 00   204 00
025 08   070 07   115 08   160 07   205 09
026 85   071 53   116 43   161 01   206 65
027 43   072 43   117 00   162 46   207 43
028 01   073 01   118 06   163 17   208 01
029 01   074 00   119 42   164 53   209 01
030 65   075 85   120 01   165 43   210 65
031 43   076 43   121 09   166 01   211 43
032 01   077 01   122 41   167 00   212 00
033 09   078 09   123 00   168 65   213 01
034 54   079 65   124 01   169 43   214 54
035 55   080 43   125 04   170 00   215 95
036 53   081 01   126 46   171 03   216 81
037 43   082 01   127 14   172 65
038 00   083 54   128 43   173 43
039 07   084 55   129 00   174 00
040 65   085 53   130 08   175 04
041 43   086 43   131 42   176 85
042 01   087 01   132 01   177 43
043 00   088 00   133 09   178 00
044 85   089 55   134 41   179 09
```

SAMPLE PROBLEM: 061 COMPOSITE MODULI I NO: 061

Input data with Program 059

E_f	=	10.5×10^6	E_m	=	$.5 \times 10^6$
G_f	=	4.38×10^6	G_m	=	$.185 \times 10^6$
ν_f	=	.2	ν_m	=	.35

n_1	=	1
n_2	=	.5
n_6 or n	=	.4
v_f	=	.6

E_L	=	6.5×10^6
G_{LT}	=	758.6×10^3
ν_{LT}	=	.25
G_{TT}	=	656.8×10^6
E	=	2.0152×10^6
ν	=	.3273

TITLE: COMPOSITE MODULI II				NO: 065
A'	B'	C'	D'	E'
A E_T	B ν_{TT}	C	D	E

00					
01 E_f	02 G_f	03 ν_f	04 E_m	05 G_m	06 ν_m
07 η_1	08 η_2	09 η_6 or η	10 v_f		
11 v_m	12 α_f	13 α_m	14	15	16
17	18	19			
98	99				

REGISTER

INSTRUCTIONS

OPERATION	STEP	READ	ENTER	PRESS	DISPLAY
Store data with Program 059	1				
Compute moduli	2	065 A&B	G_{TT}	A RUN	E_T ν_{TT}

PROGRAM TITLE: COMPOSITE MODULI II NO: 065

FORMULAS:

$$\frac{1}{E_a^*} = \left(\frac{v_f}{E_f} + \frac{\eta_a v_m}{E_m}\right) \frac{1}{v_f + \eta_a v_m} - v_f v_m \frac{(\eta_1 \eta_a E_f \nu_m - E_m \nu_f)(\nu_m/E_m - \nu_f/E_f)}{(\eta_1 v_f E_f + v_m E_m)(v_f + \eta_a v_m)} \qquad (130)$$

$$\nu_{TT} = \frac{E_T}{2G_{TT}} - 1$$

PROGRAM TITLE: COMPOSITE MODULI II **NO:** 065

PROGRAM LISTING:

```
000 46   045 65   090 53   135 55
001 11   046 53   091 53   136 43
002 53   047 43   092 43   137 01
003 43   048 00   093 00   138 09
004 01   049 07   094 07   139 54
005 00   050 65   095 65   140 20
006 55   051 43   096 43   141 75
007 43   052 00   097 01   142 01
008 00   053 08   098 00   143 95
009 01   054 65   099 65   144 81
010 85   055 43   100 43
011 43   056 00   101 00
012 00   057 01   102 01
013 08   058 65   103 85
014 65   059 43   104 43
015 43   060 00   105 01
016 01   061 06   106 01
017 01   062 75   107 65
018 55   063 43   108 43
019 43   064 00   109 00
020 00   065 04   110 04
021 04   066 65   111 54
022 54   067 43   112 65
023 55   068 00   113 53
024 53   069 03   114 43
025 43   070 54   115 01
026 01   071 65   116 00
027 00   072 53   117 85
028 85   073 43   118 43
029 43   074 00   119 00
030 00   075 06   120 08
031 08   076 55   121 65
032 65   077 43   122 43
033 43   078 00   123 01
034 01   079 04   124 01
035 01   080 75   125 54
036 54   081 43   126 54
037 75   082 00   127 95
038 43   083 03   128 20
039 01   084 55   129 42
040 00   085 43   130 01
041 65   086 00   131 09
042 43   087 01   132 81
043 01   088 54   133 65
044 01   089 55   134 02
```

SAMPLE PROBLEM: 065 COMPOSITE MODULI NO: 065

Input data with Program 059

E_f	= 10.5×10^6	E_m	= $.5 \times 10^6$
G_f	= 4.38×10^6	G_m	= $.185 \times 10^6$
ν_f	= .2	ν_m	= .35

η_1	= 1		
η_2	= .5	G_{TT}	= 656.8×10^3
η_6 or η	= .4		
v_f	= .6		

E_T	= 1.9349×10^6
ν	= .473

TITLE: THERMAL EXPANSION COEFFICIENTS				NO: 069
A'	B'	C'	D'	E'
A α_L	B α_T	C	D	E

00					
01 E_f	02 G_f	03 ν_f	04 E_m	05 G_m	06 ν_m
07 η_1	08 η_2	09 η_6 or η	10 v_f		
11 v_m	12 α_f	13 α_m	14	15	16
17	18 used	19 used			
98	99				

REGISTER

INSTRUCTIONS

OPERATION	STEP	READ	ENTER	PRESS	DISPLAY
Store data with Program 059	1				
Compute expansion	2	069 A&B		A B	α_L α_T

PROGRAM TITLE: THERMAL EXPANSION COEFFICIENTS NO: 069

FORMULAS:

$$\alpha_L = \alpha_1^* = (v_f E_f \alpha_f + v_m E_m \alpha_m)/(\eta_1 v_f E_f + v_m E_m) \quad (162)$$

$$\alpha_T = \alpha_2^* = v_f \alpha_f + v_m \alpha_m + (v_f \alpha_f \nu_f + v_m \alpha_m \nu_m) - (\eta_1 v_f \nu_f + v_m \nu_m)\alpha_1^* \quad (163)$$

PROGRAM LISTING:

P

```
000 46   030 00   060 00   090 00
001 11   031 07   061 03   091 07
002 53   032 65   062 54   092 65
003 43   033 43   063 65   093 43
004 01   034 01   064 43   094 00
005 00   035 00   065 01   095 03
006 65   036 65   066 00   096 85
007 43   037 43   067 65   097 43
008 00   038 00   068 43   098 00
009 01   039 01   069 01   099 06
010 65   040 85   070 02   100 54
011 43   041 43   071 85   101 65
012 01   042 01   072 53   102 43
013 02   043 01   073 01   103 01
014 85   044 65   074 85   104 01
015 43   045 43   075 43   105 65
016 01   046 00   076 00   106 43
017 01   047 04   077 06   107 01
018 65   048 54   078 54   108 09
019 43   049 95   079 85   109 95
020 00   050 42   080 43   110 81
021 04   051 01   081 01
022 65   052 09   082 01
023 43   053 81   083 65
024 01   054 46   084 43
025 03   055 12   085 01
026 54   056 53   086 03
027 55   057 01   087 75
028 53   058 85   088 53
029 43   059 43   089 43
```

TITLE: LONGITUDINAL STRENGTH				NO: 071
A' X (brittle)	B' X (ductile)	C' δ	D' X (Statist)	E'
A X_f	B X_m	C δ_f	D X_{fo}	E α

REGISTER

00					
01 E_f	02 G_f	03 ν_f	04 E_m	05 G_m	06 ν_m
07	08	09	10 v_f		
11 v_m	12	13	14 X_f	15 X_m	16 δ_f
17 X_{fo}	18 α	19			
98	99				

INSTRUCTIONS

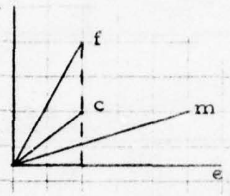
OPERATION	STEP	READ	ENTER	PRESS	DISPLAY
Store data with Program 059	1				
Input data	2	071 A&B	X_f	A	X_f
			X_m	B	X_m
			δ_f	C	δ_f
			X_{fo}	D	X_{fo}
			α	E	α
Compute strength	3			A'	X(Brittle)
				B'	X(Ductile)
				C'	δ
				RUN	X(Statistical)

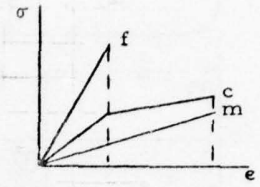
PROGRAM TITLE: LONGITUDINAL STRENGTH NO: 071

FORMULAS:

Brittle matrix

$$X = [v_f + (1-v_f)E_m/E_f]X_f \qquad (164)$$





Ductile matrix (tough)

$$X = v_f X_f + v_m X_m \qquad (165)$$

$$\bar{\delta} = \frac{\delta}{d_f} = \left[\frac{1}{2}\frac{E_f}{G_m}\frac{1-\sqrt{v_f}}{\sqrt{v_f}}\right]^{1/2} \qquad (188)$$

X = v_f (strength of bundle of length δ)

$$= v_f X_B (L/\delta)^{-1/\alpha} \qquad (182)$$

$$= v_f X_{fo} (\delta \alpha e)^{-1/\alpha} \qquad (183)$$

PROGRAM TITLE: LONGITUDINAL STRENGTH NO: 071

PROGRAM LISTING:

```
000 46   045 43   090 43
001 11   046 00   091 00
002 42   047 01   092 01
003 01   048 54   093 55
004 04   049 65   094 43
005 81   050 43   095 00
006 46   051 01   096 05
007 12   052 04   097 55
008 42   053 95   098 02
009 01   054 81   099 95
010 05   055 46   100 30
011 81   056 17   101 65
012 46   057 43   102 43
013 13   058 01   103 01
014 42   059 00   104 06
015 01   060 65   105 95
016 06   061 43   106 81
017 81   062 01   107 65
018 46   063 04   108 43
019 14   064 85   109 01
020 42   065 43   110 08
021 01   066 01   111 65
022 07   067 01   112 01
023 81   068 65   113 22
024 46   069 43   114 23
025 15   070 01   115 95
026 42   071 05   116 45
027 01   072 95   117 43
028 08   073 81   118 01
029 81   074 46   119 08
030 46   075 18   120 20
031 16   076 53   121 94
032 53   077 01   122 95
033 43   078 75   123 65
034 01   079 43   124 43
035 00   080 01   125 01
036 85   081 00   126 07
037 43   082 30   127 65
038 01   083 54   128 43
039 01   084 55   129 01
040 65   085 43   130 00
041 43   086 01   131 95
042 00   087 00   132 81
043 04   088 30
044 55   089 65
```

TITLE: IN-PLANE INVARIANTS FOR $[0_p/90_q/45_r/-45_s]$	NO: 075			
A'	B' δ_{1A}	C' δ_{2A}	D'	E'
A Initialize	B I_{1A}	C I_{2A}	D R_{1A}	E R_{2A}

00					
		REGISTER			
01 $I_1 = I_{1A}$	02 $I_2 = I_{2A}$	03 R_1, R_{1A}	04 R_2, R_{2A}	05	06
07 δ_1	08 δ_2	09	10		
11	12	13	14	15	16
17 n_0	18 n_{90}	19 n			
98 n_{45}	99 n_{-45}				

INSTRUCTIONS

OPERATION	STEP	READ	ENTER	PRESS	DISPLAY
Input ply data and initialize	1	045 A&B		A	E_L
Input n_θ and	2	075 A&B	N	A	N
Compute invariants of A_{ij}			n_θ	RUN	n_0/N
			n_{90}	RUN	n_{90}/N
			n_{45}	RUN	n_{45}/N
			n_{-45}	RUN	n_{-45}/N
				B	I_{1A}
				C	I_{2A}
				D	R_{1A}
				E	R_{2A}
				B'	δ_{1A}
				C'	δ_{2A}

PROGRAM TITLE: IN-PLANE INVARIANTS FOR $[0_p/90_q/45_r/-45_s]$ NO: 075

FORMULAS:

TABLE 29 FORMULAS FOR IN-PLANE MODULUS FOR $[0_p/90_q/45_r/-45_s]$

	I_1	I_2	$\sqrt{V_1^2 + V_3^2}\ R_1$	$\sqrt{V_2^2 + V_4^2}\ R_2$
A'_{11}/h	1	1	$\cos 2(\theta - \delta_1)$	$\cos 4\theta$
A'_{22}/h	1	1	$-\cos 2(\theta - \delta_1)$	$\cos 4\theta$
A'_{12}/h	1	-1	0	$-\cos 4\theta$
A'_{66}/h	0	1	0	$-\cos 4\theta$
A'_{16}/h	0	0	$-\frac{1}{2}\sin 2(\theta-\delta_1)$	$-\sin 4\theta$
A'_{26}/h	0 0	0	$-\frac{1}{2}\sin 2(\theta-\delta_1)$	$\sin 4\theta$

$$\sqrt{V_1^2 + V_3^2} = \frac{1}{N}\sqrt{(n_o - n_{90})^2 + (n_{45} - n_{-45})^2} = R_{1A} / R_1 \qquad (229)$$

$$\sqrt{V_2^2 + V_4^2} = \frac{1}{N}\left|(n_o + n_{90} - n_{45} - n_{-45})\right| = R_{2A} / R_2 \qquad (230)$$

$$\tan 2\delta_1 = -\frac{n_{45} - n_{-45}}{n_o - n_{90}} \qquad (231)$$

Laminate is orthotropic if $n_{45} = n_{-45}$.

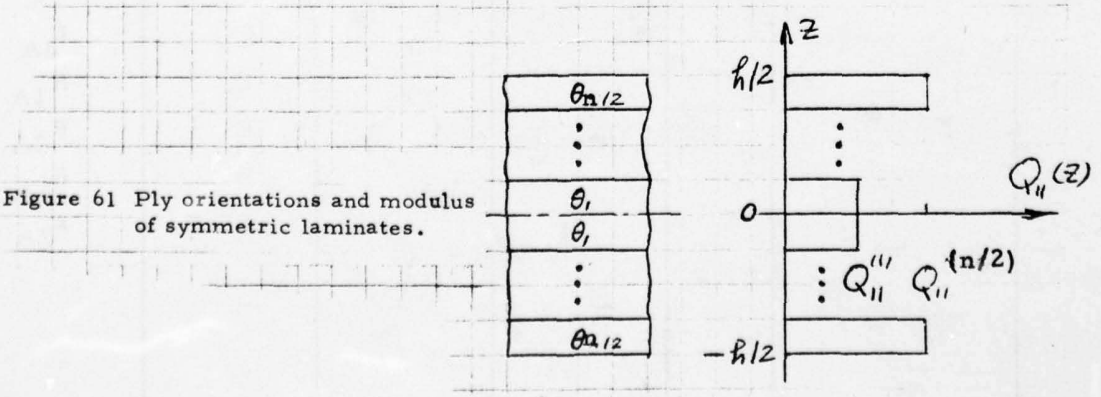


Figure 61 Ply orientations and modulus of symmetric laminates.

PROGRAM TITLE: IN-PLANE INVARIANTS FOR $[0_p/90_q/45_r/-45_s]$ NO: 075

PROGRAM LISTING:

```
000 46   045 43   090 49   135 81   180 07
001 11   046 09   091 00   136 46   181 81
002 42   047 08   092 04   137 87   182 46
003 01   048 95   093 43   138 43   183 18
004 09   049 40   094 09   139 01   184 43
005 81   050 85   095 08   140 06   185 00
006 55   051 53   096 75   141 90   186 08
007 43   052 43   097 43   142 88   187 81
008 01   053 01   098 09   143 04
009 09   054 07   099 09   144 05
010 95   055 75   100 95   145 42
011 42   056 43   101 42   146 00
012 01   057 01   102 01   147 07
013 07   058 08   103 06   148 41
014 81   059 54   104 55   149 89
015 55   060 40   105 53   150 46
016 43   061 95   106 43   151 88
017 01   062 30   107 01   152 00
018 09   063 42   108 07   153 42
019 95   064 00   109 75   154 00
020 42   065 05   110 43   155 07
021 01   066 49   111 01   156 41
022 08   067 00   112 08   157 89
023 81   068 03   113 54   158 46
024 55   069 43   114 90   159 13
025 43   070 01   115 87   160 43
026 01   071 07   116 95   161 00
027 09   072 85   117 94   162 02
028 95   073 43   118 22   163 81
029 42   074 01   119 34   164 46
030 09   075 08   120 55   165 14
031 08   076 75   121 02   166 43
032 81   077 43   122 95   167 00
033 55   078 09   123 42   168 03
034 43   079 08   124 00   169 81
035 01   080 75   125 07   170 46
036 09   081 43   126 46   171 15
037 95   082 09   127 89   172 43
038 42   083 09   128 00   173 00
039 09   084 95   129 42   174 04
040 09   085 40   130 00   175 81
041 81   086 30   131 08   176 46
042 46   087 42   132 43   177 17
043 12   088 00   133 00   178 43
044 75   089 06   134 01   179 00
```

NO: 075

SAMPLE PROBLEM: 075 IN-PLANE INVARIANTS FOR $[0_p/90_q/45_r/45_s]$

1. T-300/5208

$I_1 = 49.49$, $I_2 = 26.88$, $R_1 = 85.732$, $R_2 = 19.717$

a) [0/90] $I_{1A} = 49.49$ $R_{1A} = 0$ $\delta_{1A} = 0 = \delta_2$
$I_{2A} = 26.88$ $R_{2A} = 19.72$

b) [45/-45] $I_{1A} = 49.49$ $R_{1A} = 0$ $\delta_{1A} = 0 = \delta_2$
$I_{2A} = 26.88$ $R_{2A} = 19.72$

c) [0/90/45/-45] $I_{1A} = 49.49$ $R_{1A} = 0$ $\delta_{1A} = 0 = \delta_2$
$I_{2A} = 26.88$ $R_{2A} = 0$

d) [0/45] $I_{1A} = 49.49$ $R_{1A} = 60.62$ $\delta_{1A} = -22.50$
$I_{2A} = 26.88$ $R_{2A} = 0$ $\delta_{2A} = 0$

2. Same as 1 but for B(4)/5505

a) [0/90] $I_{1A} = 58.0$ $R_{1A} = 0$ $\delta_1 = 0 = \delta_2$
$I_{2A} = 29.80$ $R_{2A} = 24.0$

TITLE: IN-PLANE MODULUS AND COMPLIANCE				NO: 079
A'	B' $\frac{A}{h}16$, ha_{16}	C' $\frac{A}{h}26$, ha_{26}	D'	E'
A $\frac{1}{h}\|A_{ij}\|$	B $\frac{A}{h}11$, ha_{11}	C $\frac{A}{h}22$, ha_{22}	D $\frac{A}{h}12$, ha_{12}	E $\frac{A}{h}66$, ha_{66}

00 θ	REGISTER				
01 I_{1A}, ha_{11}	02 I_{2A}, ha_{22}	03 R_{1A}, ha_{12}	04 R_{2A}, ha_{66}	05 ha_{16}	06 ha_{26}
07 δ_{1A}	08 δ_{2A}	09	10		
11 $\frac{A}{h}11$	12 $\frac{A}{h}22$	13 $\frac{A}{h}12$	14 $\frac{A}{h}66$	15 $\frac{A}{h}16$	16 $\frac{A}{h}26$
17	18	19			
98	99				

INSTRUCTIONS

OPERATION	STEP	READ	ENTER	PRESS	DISPLAY
Continue from 075					
Compute A_{ij}	3	041 A&B	θ (deg)	B	A_{11}/h
				C	A_{22}/h
				D	A_{12}/h
				E	A_{66}/h
				B'	A_{16}/h
				C'	A_{26}/h
Invert A_{ij}	4	027 A&B		A	ha_{26}
				RCL 01	ha_{11}
				RCL 02	ha_{22}
				RCL 03	ha_{12}
				RCL 04	ha_{66}
				RCL 05	ha_{16}
				RCL 06	ha_{26}

PROGRAM TITLE: IN-PLANE MODULUS AND COMPLIANCE NO: 079

FORMULAS:

TABLE 29 FORMULAS FOR IN-PLANE MODULUS FOR $[0_p/90_q/45_r/-45_s]$

	I_1	I_2	R_{1A}	R_{2A}
A'_{11}/h	1	1	$\cos 2(\theta-\delta_1)$	$\cos 4\theta$
A'_{22}/h	1	1	$-\cos 2(\theta-\delta_1)$	$\cos 4\theta$
A'_{12}/h	1	-1	0	$-\cos 4\theta$
A'_{66}/h	0	1	0	$-\cos 4\theta$
A'_{16}/h	0	0	$-\frac{1}{2}\sin 2(\theta-\delta_1)$	$-\sin 4\theta$
A'_{26}/h	0	0	$-\frac{1}{2}\sin 2(\theta-\delta_1)$	$\sin 4\theta$

Let Compliance $= a_{ij} = [A_{ij}]^{-1}$ or $a_{ij}A_{jk} = \delta_{ik}$ (204)

where

$$a_{11} = \frac{1}{\Delta}(A_{22}A_{66}-A_{26}^2), \quad a_{22} = \frac{1}{\Delta}(A_{11}A_{66}-A_{16}^2)$$

$$a_{12} = \frac{1}{\Delta}(A_{16}A_{26}-A_{12}A_{66}), \quad a_{66} = \frac{1}{\Delta}(A_{11}A_{22}-A_{12}^2) \qquad (206)$$

$$a_{16} = \frac{1}{\Delta}(A_{12}A_{26}-A_{22}A_{16}), \quad a_{26} = \frac{1}{\Delta}(A_{12}A_{16}-A_{11}A_{26})$$

$$\Delta = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{21} & A_{22} & A_{26} \\ A_{61} & A_{62} & A_{66} \end{bmatrix}$$

In-Plane Engineering Constants:

$$E_{11}^o = \frac{1}{ha_{11}}, \quad E_{22}^o = \frac{1}{ha_{22}}, \quad \nu_{12}^o = -\frac{a_{12}}{a_{11}}, \quad \nu_{21}^o = -\frac{a_{12}}{a_{22}}, \quad G_{12}^o = \frac{1}{ha_{66}}$$

TITLE: IN-PLANE STRAIN				NO: 081
A'	B'	C'	D'	E'
A N_i	B e_1^o	C e_2^o	D e_6^o	E

00					
01 a_{11}	02 a_{22}	03 a_{12}	04 a_{66}	05 a_{16}	06 a_{26}
07 N_1	08 N_2	09 N_6	10		
11	12	13	14	15	16
17 e_1^o	18 e_2^o	19 e_6^o			
98	99				

REGISTER

INSTRUCTIONS

OPERATION	STEP	READ	ENTER	PRESS	DISPLAY
Continue from 079					
Input N_i	5	019 A&B	N_1	A	N_1
			N_2	RUN	N_2
			N_6	RUN	N_6
Compute e_i^o				B	e_1^o
				C	e_2^o
				D	e_6^o

PROGRAM TITLE: IN-PLANE STRAIN NO: 081

FORMULAS:

$$\text{Let Compliance} = a_{ij} = [A_{ij}]^{-1} \quad \text{or} \quad a_{ij}A_{jk} = \delta_{ik} \tag{204}$$

$$\begin{Bmatrix} \epsilon_1^o \\ \epsilon_2^o \\ \epsilon_6^o \end{Bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{16} \\ a_{21} & a_{22} & a_{26} \\ a_{61} & a_{62} & a_{66} \end{bmatrix} \begin{Bmatrix} N_1 \\ N_2 \\ N_6 \end{Bmatrix} \quad \text{or} \quad \epsilon_i^o = a_{ij}N_j \tag{205}$$

TITLE: PLY STRESS FOR $[0_p/90_q/45_r/-45_s]$				NO: 083
A'	B' Q_{16}	C' Q_{26}	D'	E'
A	B Q_{11}, σ_1	C Q_{22}, σ_2	D Q_{12}, σ_6	E Q_{66}

00 α_i					
01 I_1	02 I_2	03 R_1	04 R_2	05	06
07 σ_1	08 σ_2	09 σ_6	10		
11 Q_{11}	12 Q_{22}	13 Q_{12}	14 Q_{66}	15 Q_{16}	16 Q_{26}
17 ϵ_1^o	18 ϵ_2^o	19 ϵ_6^o			
98	99				

REGISTER

INSTRUCTIONS

OPERATION	STEP	READ	ENTER	PRESS	DISPLAY
Continue from 081					
Input ply data and initialize	6	045 A&B		A	E_L
Compute $Q_{ij}(\alpha)$	7	041 A&B	α_1(deg)	B	Q_{11}
Compute $\sigma_i^{(\alpha)}$	8	023 A&B		B	σ_1
				C	σ_2
				D	σ_6
Compute $Q_{ij}(\beta)$	Rpt 7	041 A&B	α_i (deg) $(i=2, 3, \cdots n)$	B	Q_{11}
Compute $\sigma_i^{(\beta)}$	Rpt 8	023 A&B		B	σ_1
				C	σ_2
				D	σ_6
Repeat steps	7 and 8	for each successive ply.			

PROGRAM TITLE: PLY STRESS FOR $[0_p/90_q/45_r/-45_s]$ NO: 083

FORMULAS:

a. Calculation of In-Plane Strains

$$\begin{aligned} \epsilon_1^o &= a_{11}N_1 + a_{12}N_2 + a_{16}N_6 \\ \epsilon_2^o &= a_{21}N_1 + a_{22}N_2 + a_{26}N_6 \\ \epsilon_6^o &= a_{61}N_1 + a_{62}N_2 + a_{66}N_6 \end{aligned} \tag{233}$$

or

$$\begin{Bmatrix} \epsilon_1^o \\ \epsilon_2^o \\ \epsilon_6^o \end{Bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{16} \\ a_{21} & a_{22} & a_{26} \\ a_{61} & a_{62} & a_{66} \end{bmatrix} \begin{Bmatrix} N_1 \\ N_2 \\ N_6 \end{Bmatrix} \quad \text{or} \quad \epsilon_i^o = a_{ij}N_j \tag{234}$$

b. Calculation of Stresses of the t-th ply.

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_6 \end{Bmatrix}^{(t)} = \begin{bmatrix} Q_{11} & Q_{12} & Q_{16} \\ Q_{21} & Q_{22} & Q_{26} \\ Q_{61} & Q_{62} & Q_{66} \end{bmatrix}^{(t)} \begin{Bmatrix} \epsilon_1^o \\ \epsilon_2^o \\ \epsilon_6^o \end{Bmatrix} \tag{235}$$

For the a oriented ply, replace index "t" by "a" in the equation above

c. TABLE 31 SAMPLE CALCULATIONS OF PLY STRESSES
(For T-300/5208)

Laminate	Imposed N_i	Resulting ϵ_i^o	$\sigma_i^{(o)}$	$\sigma_i^{(90)}$	$\sigma_i^{(45)}$	$\sigma_i^{(-45)}$
	1	.0104	1.893	.1088		
[0,90]	0	-.0003	.0269	.0269		
	0	0	0	.0282		
	1	.0101	1.8662	-.2279		
[0/90]	1	.0101	.1338	1.4495		
	1	.1395	1.0	.9432		
	1	.0104			.5769	.5591
[45/-45]	0	-.003			.4231	.4224
	0	0			-.4331	.4268
	1	.0144	2.5971	.1412	.6399	.6060
[0/90/45/-45]	0	-.0042	-.0024	-.7294	.3679	.3584
	0	0	0	-.0393	-.4144	.4236

TITLE: INVARIANTS OF FLEXURAL RIGIDITY				NO: 085
A'	B'	C'	D'	E'
A Initialize	B	C	D	E

00 $2\alpha_t$					
01 I_1 , I_{1D}	02 I_2 , I_{2D}	03 R_1, R_{1D}	04 R_2, R_{2D}	05 t	06 $3t^2-3t+1$
07 δ_{1D}	08 δ_{2D}	09	10		
11 Fc2D	12 Fs2D	13 Fc4D	14 Fs4D	15 $2z_o/h$	16
17	18	19 $4\alpha_t$			
98	99 n/2				

REGISTER

INSTRUCTIONS

OPERATION	STEP	READ	ENTER	PRESS	DISPLAY
Input ply data	1	045 A&B		A	E_L
Input data	2.		N/2	STO 99	N/2
			$2z_o/h$	STO 15	$2z_o/h$
Initialize	3	085 A&B		A	$F_t(1)$
Compute invariants of D_{ij}			α_1 (deg)	B	$F_t(2)$
			α_2 (deg)	RUN	$F_t(3)$
			.	.	.
			.	.	.
			.	.	.
			$\alpha_{n/2}$	RUN	$\frac{R_{1D}}{R_1}$
Recall				RCL 01	I_{1D}
				RCL 02	I_{2D}
				RCL 03	R_{1D}
				RCL 04	R_{2D}
				RCL 07	δ_1
				RCL 08	δ_2

PROGRAM TITLE: INVARIANTS OF FLEXURAL RIGIDITY NO: 085

FORMULAS:

TABLE 33 FORMULAS FOR FLEXURAL RIGIDITY WITH ISOTROPIC SUBSTRUCTURE

	$\left[1-\left(\frac{2Z_o}{h}\right)^3\right]I_1$	$\left[1-\left(\frac{2Z_o}{h}\right)^3\right]I_2$	$\sqrt{V_1^2+V_3^2}\ R$	$\sqrt{V_2^2+V_4^2}\ R_2$
$\frac{12}{h^3} D'_{11}-\left(\frac{2z_o}{h}\right)^3 Q^o_{11}$	1	1	$\cos 2(\theta-\delta_1)$	$\cos 4(\theta-\delta_2)$
$\frac{12}{h^3} D'_{22}-\left(\frac{2z_o}{h}\right)^3 Q^o_{11}$	1	1	$-\cos 2(\theta-\delta_1)$	$\cos 4(\theta-\delta_2)$
$\frac{12}{h^3} D'_{12}-\left(\frac{2z_o}{h}\right)^3 Q^o_{12}$	1	-1	0	$-\cos 4(\theta-\delta_2)$
$\frac{12}{h^3} D'_{66}-\frac{1}{2}\left(\frac{2z_o}{h}\right)^3 (Q^o_{11}-Q^o_{12})$	0	1	0	$-\cos 4(\theta-\delta_2)$
$\frac{12}{h^3} D'_{16}$	0	0	$-\frac{1}{2}\sin 2(\theta-\delta_1)$	$-\sin 4(\theta-\delta_2)$
$\frac{12}{h^3} D'_{26}$	0	0	$-\frac{1}{2}\sin 2(\theta-\delta_1)$	$\sin 4(\theta-\delta_2)$

$$\sqrt{V_1^2+V_3^2} = \left(\frac{2}{N}\right)^3 \sqrt{\left(\sum F_t \cos 2\alpha_t\right)^2+\left(\sum F_t \sin 2\alpha_t\right)^2} = R_{1D}/R_1 \qquad (259)$$

$$\sqrt{V_2^2+V_4^2} = \left(\frac{2}{N}\right)^3 \sqrt{\left(\sum F_t \cos 4\alpha_t\right)^2+\left(\sum F_t \sin 4\alpha_t\right)^2} = R_{2D}/R_2 \qquad (260)$$

$$\tan 2\delta_2 = \frac{2(D_{16}+D_{26})}{D_{11}-D_{22}} = -\frac{V_3}{V_1} \qquad (261)$$

$$\tan 4\delta_2 = \frac{4(D_{16}-D_{26})}{D_{11}+D_{22}-2D_{12}-4D_{66}} = -\frac{V_4}{V_2} \qquad (262)$$

$$F_t = 3\left(t+\frac{z_o}{h_o}\right)^2 - 3\left(t+\frac{z_o}{h_o}\right)+1 \qquad (263)$$

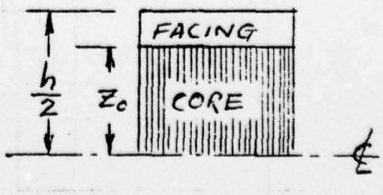


Figure 70 Sandwich Plate

PROGRAM TITLE: INVARIANTS OF FLEXURAL RIGIDITY NO: 085

PROGRAM LISTING:

```
000 46  045 65  090 00  135 05  180 40
001 11  046 03  091 06  136 85  181 85
002 01  047 75  092 95  137 01  182 43
003 75  048 43  093 44  138 95  183 01
004 43  049 00  094 01  139 41  184 04
005 01  050 05  095 03  140 88  185 40
006 05  051 65  096 43  141 46  186 95
007 45  052 03  097 00  142 87  187 30
008 03  053 85  098 00  143 43  188 55
009 95  054 01  099 32  144 01  189 43
010 49  055 95  100 65  145 02  190 09
011 00  056 42  101 43  146 55  191 09
012 01  057 00  102 00  147 43  192 45
013 49  058 06  103 06  148 01  193 03
014 00  059 81  104 95  149 01  194 95
015 02  060 46  105 44  150 94  195 49
016 00  061 12  106 01  151 95  196 00
017 42  062 65  107 02  152 22  197 04
018 01  063 04  108 43  153 34  198 43
019 01  064 94  109 01  154 55  199 01
020 42  065 95  110 09  155 02  200 01
021 01  066 42  111 32  156 95  201 40
022 02  067 01  112 65  157 42  202 85
023 42  068 09  113 43  158 00  203 43
024 01  069 55  114 00  159 07  204 01
025 03  070 02  115 06  160 43  205 02
026 42  071 95  116 95  161 01  206 40
027 01  072 42  117 44  162 04  207 95
028 04  073 00  118 01  163 55  208 30
029 01  074 00  119 04  164 43  209 55
030 85  075 33  120 43  165 01  210 43
031 43  076 65  121 09  166 03  211 09
032 01  077 43  122 09  167 94  212 09
033 05  078 00  123 75  168 95  213 45
034 65  079 06  124 43  169 22  214 03
035 43  080 95  125 00  170 34  215 95
036 09  081 44  126 05  171 55  216 49
037 09  082 01  127 95  172 04  217 00
038 95  083 01  128 90  173 95  218 03
039 46  084 43  129 87  174 42  219 81
040 88  085 01  130 22  175 00
041 42  086 09  131 80  176 08
042 00  087 33  132 87  177 43
043 05  088 65  133 43  178 01
044 40  089 43  134 00  179 03
```

TITLE: FLEXURAL RIGIDITY AND COMPLIANCE				NO: 088
A'	B' D_{16}, d_{16}	C' D_{26}, d_{26}	D'	E'
A $\lvert D_{ij} \rvert$	B D_{11}, d_{11}	C D_{22}, d_{22}	D D_{12}, d_{12}	E D_{66}, d_{66}

00 θ					
REGISTER					
01 I_1, d_{11}	02 I_2, d_{22}	03 R_1, d_{12}	04 R_2, d_{66}	05 d_{16}	06 d_{26}
07 δ_1	08 δ_2	09	10		
11 D_{11}	12 D_{22}	13 D_{12}	14 D_{66}	15 D_{16}	16 D_{26}
17	18	19			
98	99				

INSTRUCTIONS

OPERATION	STEP	READ	ENTER	PRESS	DISPLAY
Continue from 085					
Compute D_{ij}	4	041, A&B	θ(deg)	B	D_{11}
				C	D_{22}
				D	D_{12}
				E	D_{66}
				B'	D_{16}
				C'	D_{26}
Invert D_{ij}	5	027 A&B		A	d_{26}
				RCL 01	d_{11}
				RCL 02	d_{22}
				RCL 03	d_{12}
				RCL 04	d_{66}
				RCL 05	d_{16}
				RCL 06	d_{26}

PROGRAM TITLE: FLEXURAL RIGIDITY AND COMPLIANCE NO: 088

FORMULAS:

TABLE 33 FORMULAS FOR FLEXURAL RIGIDITY WITH ISOTROPIC SUBSTRUCTURE

	$\left[1-\left(\frac{2Z_o}{h}\right)^3\right]I_1$	$\left[1-\left(\frac{2Z_o}{h}\right)^3\right]I_2$	R_{1D}	R_{2D}
$\frac{12}{h^3} D'_{11}-\left(\frac{2z_o}{h}\right)^3 Q^o_{11}$	1	1	$\cos 2(\theta-\delta_1)$	$\cos 4(\theta-\delta_2)$
$\frac{12}{h^3} D'_{22}-\left(\frac{2z_o}{h}\right)^3 Q^o_{11}$	1	1	$-\cos 2(\theta-\delta_1)$	$\cos 4(\theta-\delta_2)$
$\frac{12}{h^3} D'_{12}-\left(\frac{2z_o}{h}\right)^3 Q^o_{12}$	1	-1	0	$-\cos 4(\theta-\delta_2)$
$\frac{12}{h^3} D'_{66}-\frac{1}{2}\left(\frac{2z_o}{h}\right)^3 (Q^o_{11}-Q^o_{12})$	0	1	0	$-\cos 4(\theta-\delta_2)$
$\frac{12}{h^3} D'_{16}$	0	0	$-\frac{1}{2}\sin 2(\theta-\delta_1)$	$-\sin 4(\theta-\delta_2)$
$\frac{12}{h^3} D'_{26}$	0	0	$-\frac{1}{2}\sin 2(\theta-\delta_1)$	$\sin 4(\theta-\delta_2)$

AD-A067 987 AIR FORCE MATERIALS LAB WRIGHT-PATTERSON AFB OH F/G 9/2
SR-52 MAGNETIC CARD CALCULATOR SOLUTIONS TO COMPOSITE MATERIALS--ETC(U)
JAN 79 S W TSAI, H T HAHN, F HUBER

UNCLASSIFIED AFML-TR-77-50 NL

2 OF 2
AD
A067987

END
DATE
FILMED
6-79
DDC

TITLE: CURVATURE DUE TO BENDING				NO: 090
A'	B'	C'	D'	E'
A M_i	B k_1	C k_2	D k_6	E

00	REGISTER				
01 d_{11}	02 d_{22}	03 d_{12}	04 d_{66}	05 d_{16}	06 d_{26}
07 M_1	08 M_2	09 M_6	10		
11	12	13	14	15	16
17 k_1	18 k_2	19 k_6			
98	99				

INSTRUCTIONS

OPERATION	STEP	READ	ENTER	PRESS	DISPLAY
Continue from VI-1b					
Input M_i	6	019 A&B	M_1	A	M_1
			M_2	RUN	M_2
			M_6	RUN	M_6
Compute k_i				B	k_1
				C	k_2
				D	k_6

PROGRAM TITLE: CURVATURE DUE TO BENDING NO: 090

FORMULAS:

TABLE 50 FLEXURE-CURVATURE RELATIONS

	k_1	k_2	k_6
M_1	D_{11}	D_{12}	D_{16}
M_2	D_{21}	D_{22}	D_{26}
M_6	D_{61}	D_{62}	D_{66}

	M_1	M_2	M_6
k_1	d_{11}	d_{12}	d_{16}
k_2	d_{21}	d_{22}	d_{26}
k_6	d_{61}	d_{62}	d_{66}

Since $e_1 = zk_1$, $e_2 = zk_2$, $e_6 = zk_6$

Finally, from stress-strain relation of the t-th layer.

$$\left.\begin{aligned} \sigma_1 &= Q_{11}e_1 + Q_{12}e_2 + Q_{16}e_6 \\ \sigma_2 &= Q_{21}e_1 + Q_{22}e_2 + Q_{26}e_6 \\ \sigma_6 &= Q_{61}e_1 + Q_{62}e_2 + Q_{66}e_6 \end{aligned}\right\} \qquad (281)$$

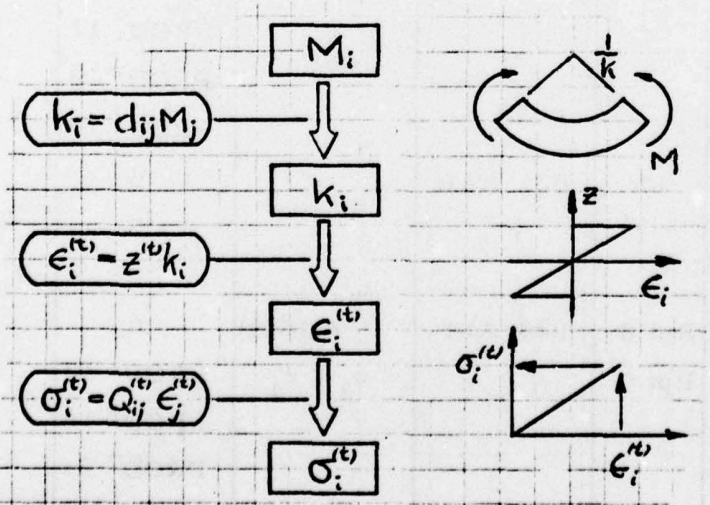


Figure 73 Flow diagram for ply stress calculation.

TITLE: PLY STRESS DUE TO BENDING				NO: 092
A'	B' Q_{16}	C' Q_{26}	D'	E'
A	B Q_{11}, σ_1	C Q_{22}, σ_2	D Q_{12}, σ_6	E Q_{66}

00					
REGISTER					
01 I_1	02 I_2	03 R_1	04 R_2	05	06
07 σ_1	08 σ_2	09 σ_6	10		
11 Q_{11}	12 Q_{22}	13 Q_{12}	14 Q_{66}	15 Q_{16}	16 Q_{26}
17 ϵ_1	18 ϵ_2	19 ϵ_6			
98	99				

INSTRUCTIONS

OPERATION	STEP	READ	ENTER	PRESS	DISPLAY
Continue from (90)					
Input ply data & initialize	7	045 A&B		A	E_L
Compute $Q_{ij}(\alpha_1)$	8	041 A&B	α_1(deg)	B	Q_{11}
Compute $\epsilon_i(z_1)$	9		z_1	PROD* 17	
				PROD* 18	
				PROD* 19	
Compute $\sigma_i(z_1)$	10	023 A&B		B	σ_1
				C	σ_2
				D	σ_6
Compute $Q_{ij}(\alpha_2)$	Rpt 8	023 A&B	α_2(deg)	B	Q_{11}
Compute $\epsilon_i(z_2)$	Rpt 9		z_2 / z_1	PROD* 17	
				PROD* 18	
				PROD* 19	
Compute $\sigma_i(z_2)$	Rpt 10	023 A&B		B, C, D	σ_i

TITLE: INVARIANTS OF COUPLING MODULUS				NO: 093
A' σ_i	B'	C'	D'	E'
A Initialize	B a_t, R_{1B}	C δ_{1B}	D R_{2B}	E δ_{2B}

00 $-2a_t$					
01	02	03 R_1, R_{1B}	04 R_2, R_{2B}	05 t	06 2t - 1
07 δ_{1B}	08 δ_{2B}	09	10		
11 Fc2B	12 Fs2B	13 Fc4B	14 Fs4B	15	16
17	18	19 $-4a_t$			
98	99				

REGISTER

INSTRUCTIONS

OPERATION	STEP	READ	ENTER	PRESS	DISPLAY
Input ply data	1	045 A&B		A	E_L
Input data	2	093 A&B	n	A	n
Compute invariants	3		a_1(deg)	B	a_1
			a_2(deg)	RUN	a_2
			.	.	.
			.	.	.
			.	.	.
			a_N	RUN	R_{1B}
Recall	4			C	δ_{1B}
				D	R_{2B}
				E	δ_{2B}

PROGRAM TITLE: INVARIANTS OF COUPLING MODULUS NO: 093

FORMULAS:

TABLE 56 FORMULA FOR TRANSFORMED COUPLING MODULUS

	$\sqrt{V_1^2+V_3^2}\ R_1$	$\sqrt{V_2^2+V_4^2}\ R_2$
$\frac{h^2}{2} B'_{11}$	$\cos 2(\theta-\delta_1)$	$\cos 4(\theta-\delta_2)$
$\frac{h^2}{2} B'_{22}$	$-\cos 2(\theta-\delta_1)$	$\cos 4(\theta-\delta_2)$
$\frac{h^2}{2} B'_{12}$	0	$-\cos 4(\theta-\delta_2)$
$\frac{h^2}{2} B'_{66}$	0	$-\cos 4(\theta-\delta_2)$
$\frac{h^2}{2} B'_{16}$	$-\frac{1}{2}\sin 2(\theta-\delta_1)$	$-\sin 4(\theta-\delta_2)$
$\frac{h^2}{2} B'_{26}$	$-\frac{1}{2}\sin 2(\theta-\delta_1)$	$\sin 4(\theta-\delta_2)$

$$V_1 = \frac{1}{N^2}\sum F_t \cos 2\alpha, \qquad V_2 = \frac{1}{N^2}\sum F_t \cos 4\alpha \tag{322}$$

$$V_3 = -\frac{1}{N^2}\sum F_t \sin 2\alpha \qquad V_4 = -\frac{1}{N^2}\sum F_t \sin 4\alpha \tag{323}$$

$$\tan 2\delta_1 = -\frac{V_3}{V_1} \tag{320}$$

$$\tan 4\delta_2 = -\frac{V_4}{V_2} \tag{321}$$

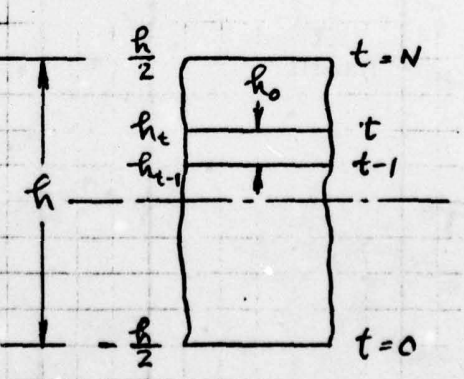


Figure 78 Integration of unsymmetric laminates must be performed for the entire thickness from -h/2 to h/2. (For symmetric laminates, integration can be limited to from 0 to h/2).

PROGRAM TITLE: INVARIANTS OF COUPLING MODULUS NO: 093

PROGRAM LISTING:

```
000 46
001 11
002 42
003 09
004 09
005 00
006 42
007 01
008 01
009 42
010 01
011 02
012 42
013 01
014 03
015 42
016 01
017 04
018 42
019 00
020 05
021 46
022 88
023 42
024 00
025 05
026 65
027 02
028 75
029 01
030 95
031 42
032 00
033 06
034 81
035 46
036 12
037 65
038 02
039 94
040 95
041 42
042 00
043 00
044 65
045 02
046 95
047 42
048 01
049 09
050 43
051 00
052 00
053 33
054 65
055 43
056 00
057 06
058 95
059 44
060 01
061 01
062 43
063 01
064 09
065 33
066 65
067 43
068 00
069 06
070 95
071 44
072 01
073 03
074 43
075 00
076 00
077 32
078 65
079 43
080 00
081 06
082 95
083 44
084 01
085 02
086 43
087 01
088 09
089 32
090 65
091 43
092 00
093 06
094 95
095 44
096 01
097 04
098 43
099 09
100 09
101 75
102 43
103 00
104 05
105 95
106 90
107 87
108 43
109 00
110 05
111 85
112 01
113 95
114 41
115 88
116 46
117 87
118 43
119 01
120 02
121 55
122 43
123 01
124 01
125 94
126 95
127 22
128 34
129 55
130 02
131 95
132 42
133 00
134 07
135 43
136 01
137 04
138 55
139 43
140 01
141 03
142 94
143 95
144 22
145 34
146 55
147 04
148 95
149 42
150 00
151 08
152 43
153 01
154 03
155 40
156 85
157 43
158 01
159 04
160 40
161 95
162 30
163 55
164 53
165 43
166 09
167 09
168 65
169 02
170 54
171 40
172 95
173 49
174 00
175 04
176 43
177 01
178 01
179 40
180 85
181 43
182 01
183 02
184 40
185 95
186 30
187 55
188 53
189 43
190 09
191 09
192 65
193 02
194 54
195 40
196 95
197 49
198 00
199 03
200 43
201 00
202 03
203 81
204 46
205 13
206 43
207 00
208 07
209 81
210 46
211 14
212 43
213 00
214 04
215 81
216 46
217 15
218 43
219 00
220 08
221 81
```

NO: 093

TITLE: COUPLING MODULUS				NO: 097
A'	B' B_{16}	C' B_{26}	D'	E'
A	B B_{11}	C B_{22}	D B_{12}	E B_{66}

00					
01	02	03 R_{1B}	04 R_{2B}	05	06
07 δ_{1B}	08 δ_{2B}	09	10		
11	12	13	14	15	16
17	18	19			
98	99				

REGISTER

INSTRUCTIONS

OPERATION	STEP	READ	ENTER	PRESS	DISPLAY
Continue from 093					
Compute B_{ij}	3	041 A&B	θ (deg)	B	B_{11}
Recall	3'			C	B_{22}
				D	B_{12}
				E	B_{66}
				B'	B_{16}
				C'	B_{26}

PROGRAM TITLE: COUPLING MODULUS NO: 097

FORMULAS:

TABLE 56 FORMULA FOR TRANSFORMED COUPLING MODULUS

	$\sqrt{V_1^2+V_3^2}\ R_1$	$\sqrt{V_2^2+V_4^2}\ R_2$
$\frac{h^2}{2} B'_{11}$	$\cos 2(\theta-\delta_1)$	$\cos 4(\theta-\delta_2)$
$\frac{h^2}{2} B'_{22}$	$-\cos 2(\theta-\delta_1)$	$\cos 4(\theta-\delta_2)$
$\frac{h^2}{2} B'_{12}$	0	$-\cos 4(\theta-\delta_2)$
$\frac{h^2}{2} B'_{66}$	0	$-\cos 4(\theta-\delta_2)$
$\frac{h^2}{2} B'_{16}$	$-\frac{1}{2}\sin 2(\theta-\delta_1)$	$-\sin 4(\theta-\delta_2)$
$\frac{h^2}{2} B'_{26}$	$-\frac{1}{2}\sin 2(\theta-\delta_1)$	$\sin 4(\theta-\delta_2)$

$$V_1 = \frac{1}{N^2}\sum F_t \cos 2\alpha, \qquad V_2 = \frac{1}{N^2}\sum F_t \cos 4\alpha \tag{322}$$

$$V_3 = -\frac{1}{N^2}\sum F_t \sin 2\alpha \qquad V_4 = -\frac{1}{N^2}\sum F_t \sin 4\alpha \tag{323}$$

$$\tan 2\delta_1 = -\frac{V_3}{V_1} \tag{320}$$

$$\tan 4\delta_2 = = -\frac{V_4}{V_2} \tag{321}$$

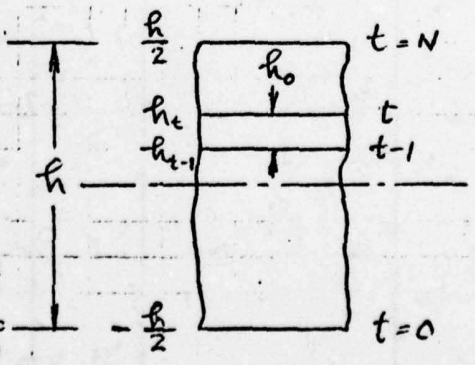


Figure 78 Integration of unsymmetric laminates must be performed for the entire thickness from -h/2 to h/2. (For symmetric laminates, integration can be limited to from 0 to h/2).

TITLE: MOISTURE CONTENT AND SWELLING STRAIN				NO: 099
A' e_T^H	B'	C'	D'	E'
A s	B v_v	C c	D α_T^H	E c_o

00					
01 s	02 v_v	03 c	04	05	06
07	08	09	10		
11	12	13	14	15	16
17	18	19			
98	99				

REGISTER

INSTRUCTIONS

OPERATION	STEP	READ	ENTER	PRESS	DISPLAY
Input data	1	099 A	s	A	s
			v_v	B	v_v
Compute c	2		c_m	C	c_m
			v_m	RUN	$c_m c_v$
			s_m	RUN	c
Compute α_T^H	3		ν_m	D	α_T^H
Compute c_o	4			E	c_o
Compute e_T^H	5			A'	
			ρ	RUN	
			ρ'	RUN	e_T^H

PROGRAM TITLE: MOISTURE CONTENT AND SWELLING STRAIN NO: 099

FORMULAS:

$$c = \frac{M' - M}{M} = c_m m_m + c_f m_f + M_{vw} / M$$

$$= (c_m v_m \rho_m + c_f v_f \rho_f + v_v \rho_w) / \rho$$

$$= (c_m v_m s_m + c_f v_f s_f + v_v) / s \qquad (374)$$

$$\alpha_T^H = \frac{1 + \nu_m}{3} s$$

$$c_o = v_v / s$$

$$e_T^H = \frac{1}{2} \left[(1 + c) \frac{\rho}{\rho'} - 1 \right] \qquad \text{cf. (402)}$$

PROGRAM LISTING:

```
000 46  030 00  060 01
001 11  031 03  061 85
002 42  032 81  062 43
003 00  033 46  063 00
004 01  034 14  064 03
005 81  035 85  065 54
006 46  036 01  066 65
007 12  037 95  067 81
008 42  038 55  068 55
009 00  039 03  069 81
010 02  040 65  070 75
011 81  041 43  071 01
012 46  042 00  072 95
013 13  043 01  073 55
014 81  044 95  074 02
015 65  045 81  075 95
016 81  046 46  076 81
017 65  047 15
018 81  048 43
019 85  049 00
020 43  050 02
021 00  051 55
022 02  052 43
023 95  053 00
024 55  054 01
025 43  055 95
026 00  056 81
027 01  057 46
100 028 95  058 16
029 42  059 53
```

TITLE: RESIDUAL STRESSES (MICRO)				NO: 101
A' Input Data	B'	C'	D'	E'
A e_f	B e_m	C $\bar{\sigma}^R_{mL}, \bar{\sigma}^R_{fL}$	D	E

00					
01 E_f	02	03	04 E_m	05	06
07 η_1	08	09	10 v_f		
11 v_m	12 e_f	13 e_m	14	15	16
17	18	19			
98	99				

REGISTER

INSTRUCTIONS

OPERATION	STEP	READ	ENTER	PRESS	DISPLAY
Enter data	1	101 A	E_f	A'	
			E_m	RUN	
			η_1	RUN	
			v_f	RUN	
			e_f	RUN	
Compute stress	2		e_m	RUN	
				A	$\bar{\sigma}^R_{mL}$
				RUN	$\bar{\sigma}^R_{fL}$

PROGRAM TITLE: RESIDUAL STRESSES (MICRO) NO: 101

FORMULAS:

Matrix

$$\bar{\sigma}^{R}_{mL} = \frac{v_f E_f E_m (e_f - \eta_1 e_m)}{\eta_1 v_f E_f + v_m E_m} \quad (387)$$

$$\bar{\sigma}^{R}_{mT} = 0 \quad (388)$$

Fiber

$$\bar{\sigma}^{R}_{fL} = -\frac{v_m}{v_f} \bar{\sigma}^{R}_{mL} = \frac{v_m E_m E_f (\eta_1 e_m - e_f)}{\eta_1 v_f E_f + v_m E_m} \quad (389)$$

$$\bar{\sigma}^{R}_{fT} = 0 \quad (390)$$

PROGRAM LISTING:

```
000 46  030 01  060 55  090 01
001 16  031 03  061 53  091 00
002 42  032 81  062 43  092 94
003 00  033 46  063 00  093 95
004 01  034 11  064 07  094 81
005 81  035 43  065 65
006 42  036 01  066 43
007 00  037 00  067 01
008 04  038 65  068 00
009 81  039 43  069 65
010 42  040 00  070 43
011 00  041 01  071 00
012 07  042 65  072 01
013 81  043 43  073 85
014 42  044 00  074 43
015 01  045 04  075 01
016 00  046 65  076 01
017 94  047 53  077 65
018 85  048 43  078 43
019 01  049 01  079 00
020 95  050 02  080 04
021 42  051 75  081 54
022 01  052 43  082 95
023 01  053 00  083 81
024 81  054 07  084 65
025 42  055 65  085 43
026 01  056 43  086 01
027 02  057 01  087 01
028 81  058 03  088 55
029 42  059 54  089 43
```

TITLE: STRESS ANALYSIS PARAMETERS				NO: 103
A' Eng'g Const.	B'	C'	D'	E'
A Parallel	B Perpendicular	C k, n, et al	D	E

00	REGISTER				
01 E_L	02 E_T	03 G_{LT}	04 ν_{LT}	05 S_{11}	06 S_{22}
07 S_{66}	08 S_{12}	09 k	10 n		
11 $\sqrt{n^2-4k}$	12 μ_1	13 μ_2	14	15	16
17	18	19			
98	99				

INSTRUCTIONS

OPERATION	STEP	READ	ENTER	PRESS	DISPLAY
Enter data	1	103 A&B	E_L	A'	
			E_T	RUN	
			G_{LT}	RUN	
			ν_{LT}	RUN	S_{12}
Compute parameters	2				
Crack parallel to fibers				A	k
				RUN	n
				RUN	μ_1
				RUN	μ_2
Crack perpendicular to fibers				B	k
				RUN	n
				RUN	μ_1
				RUN	μ_2

PROGRAM TITLE: STRESS ANALYSIS PARAMETERS NO: 103

FORMULAS:

$$\mu_1 = \frac{1}{2}\left[n+(n^2-4k)^{1/2}\right] \quad , \quad \mu_2 = \frac{1}{2}\left[n-(n^2-4k)^{1/2}\right] \tag{511}$$

$$n = \left[2\left(\sqrt{\frac{S_{22}}{S_{11}}} + \frac{S_{12}}{S_{11}}\right) + \frac{S_{66}}{S_{11}}\right]^{1/2} \quad , \quad k = \left(\frac{S_{22}}{S_{11}}\right)^{1/2} \tag{512}$$

PROGRAM TITLE: STRESS ANALYSIS PARAMETERS **NO:** 103

PROGRAM LISTING:

```
000 46  045 95  090 81  135 01
001 16  046 94  091 40  136 20
002 42  047 42  092 75  137 42
003 00  048 00  093 04  138 00
004 01  049 08  094 65  139 06
005 81  050 81  095 43  140 43
006 42  051 46  096 00  141 00
007 00  052 11  097 09  142 02
008 02  053 43  098 95  143 20
009 81  054 00  099 30  144 42
010 42  055 06  100 42  145 00
011 00  056 55  101 01  146 05
012 03  057 43  102 01  147 43
013 81  058 00  103 85  148 00
014 42  059 05  104 43  149 03
015 00  060 95  105 01  150 20
016 04  061 30  106 00  151 42
017 43  062 42  107 95  152 00
018 00  063 00  108 95  153 07
019 01  064 09  109 55  154 41
020 20  065 81  110 02  155 11
021 42  066 85  111 95  156 81
022 00  067 43  112 42
023 05  068 00  113 01
024 43  069 08  114 02
025 00  070 55  115 81
026 02  071 43  116 43
027 20  072 00  117 01
028 42  073 05  118 00
029 00  074 95  119 75
030 06  075 65  120 43
031 43  076 02  121 01
032 00  077 85  122 01
033 03  078 43  123 95
034 20  079 00  124 55
035 42  080 07  125 02
036 00  081 55  126 95
037 07  082 43  127 42
038 43  083 00  128 01
039 00  084 05  129 03
040 04  085 95  130 81
041 55  086 30  131 46
042 43  087 42  132 12.
043 00  088 01  133 43
044 01  089 00  134 00
```

SAMPLE PROBLEM: 103 STRESS ANALYSIS PARAMETERS NO: 103

$E_L = 181 \times 10^9$

$E_T = 10.3 \times 10^9$

$G_{LT} = 7.17 \times 10^9$

$\nu_{LT} = .28$

Parallel to fibers

$k = 4.192$

$n = 5.750$

$\mu_1 = 4.894$

$\mu_2 = 856.6 \times 10^{-3}$

Perpendicular to fibers

$k = 238.5 \times 10^{-3}$

$n = 1.372$

$\mu_1 = 1.167$

$\mu_2 = 204.3 \times 10^{-3}$

TITLE: CRACK TIP STRESSES				NO: 107
A'	B'	C'	D'	E'
A σ_x	B σ_y	C τ_{xy}	D	E

00					
01 E_L	02 E_T	03 G_{LT}	04 ν_{LT}	05 S_{11}	06 S_{12}
07 S_{66}	08 S_{12}	09 k	10 n		
11 $\sqrt{n^2-4k}$	12 μ_1	13 μ_2	14	15	16
17	18	19			
98	99				

REGISTER

INSTRUCTIONS

OPERATION	STEP	READ	ENTER	PRESS	DISPLAY
Compute parameters with 103	1				
Compute crack tip stresses	2	107 A&B		A	$\sigma_x\sqrt{2\pi r}/K_I$
				B	$\sigma_y\sqrt{2\pi r}/K_I$
				C	$\tau_{xy}\sqrt{2\pi r}/K_I$

PROGRAM TITLE: CRACK TIP STRESSES NO: 107

FORMULAS:

$$\theta = 90^\circ$$

$$\sigma_x = \frac{K_I}{\sqrt{2\pi r}} \frac{k}{\sqrt{2}\ \bar{\mu}} \left(\sqrt{\mu_1} - \sqrt{\mu_2}\right), \quad \sigma_y = \frac{K_I}{\sqrt{2\pi r}} \frac{1}{\sqrt{2}\ \bar{\mu}} \left(\frac{\mu_1}{\sqrt{\mu_2}} - \frac{\mu_2}{\sqrt{\mu_1}}\right) \tag{518}$$

$$\tau_{xy} = \frac{K_I}{\sqrt{2\pi r}} \frac{k}{\sqrt{2}\ \bar{\mu}} \left(\frac{1}{\sqrt{\mu_1}} - \frac{1}{\sqrt{\mu_2}}\right), \quad \bar{\mu} = \mu_1 - \mu_2$$

PROGRAM LISTING:

```
000 46   030 75   060 54
001 11   031 43   061 95
002 43   032 01   062 46
003 00   033 03   063 87
004 09   034 55   064 55
005 65   035 43   065 02
006 53   036 01   066 30
007 43   037 02   067 55
008 01   038 30   068 53
009 02   039 51   069 43
010 30   040 87   070 01
011 75   041 81   071 02
012 43   042 46   072 75
013 01   043 13   073 43
014 03   044 43   074 01
015 30   045 00   075 03
016 54   046 09   076 54
017 51   047 65   077 95
018 87   048 53   078 56
019 81   049 43
020 46   050 01
021 12   051 02
022 43   052 30
023 01   053 20
024 02   054 75
025 55   055 43
026 43   056 01
027 01   057 03
028 03   058 30
029 30   059 20
```

TITLE: Y FOR CENTER-CRACKED PLATE AND COD				NO: 109
A' Constants	B' E_L, γ	C' τ_y	D'	E'
A a	B w	C Y	D COD	E

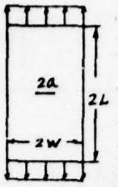
00					
		REGISTER			
01 a	02 w	03 a/w	04 1.5254	05 -0.2881	06 0.1282
07 E_L	08 $\gamma = 2n$	09 τ_y	10 Y		
11 σ	12	13	14	15	16
17	18	19			
98	99				

INSTRUCTIONS

OPERATION	STEP	READ	ENTER	PRESS	DISPLAY
Enter constants	1	109 A&B	0.1282	A'	.1282
			-0.2881	RUN	-0.2881
			1.5254	RUN	1.5254
Enter data	2		a	A	a
			w	B	w
Compute Y	3			C	Y
Enter data	4		E_L	B'	E_L
			γ	RUN	γ
			τ_y	C'	τ_y
Compute COD	5		σ	D	COD

PROGRAM TITLE: Y FOR CENTER-CRACKED PLATE AND COD NO: 109

FORMULAS:



Isotropic plates

$$Y = 1 + 0.1282\,(a/w) - 0.2881\,(a/w)^2 + 1.5254\,(a/w)^3 \qquad (516)$$

Total COD

COD = elastic COD + plastic CTOD

$$= \frac{Y a \sigma}{E_L}\left(\gamma + \frac{\pi}{2}\frac{Y}{\tau_y}\sigma\right) \qquad (526)$$

PROGRAM TITLE: Y FOR CENTER-CRACKED PLATE AND COD **NO:** 109

PROGRAM LISTING:

```
000 46  045 06  090 09
001 11  046 85  091 54
002 42  047 01  092 95
003 00  048 95  093 81
004 01  049 42  094 46
005 81  050 01  095 16
006 46  051 00  096 42
007 12  052 81  097 00
008 42  053 46  098 06
009 00  054 14  099 81
010 02  055 42  100 42
011 81  056 01  101 00
012 46  057 01  102 05
013 13  058 65  103 81
014 55  059 43  104 42
015 43  060 01  105 00
016 00  061 00  106 04
017 01  062 65  107 81
018 95  063 43  108 46
019 20  064 00  109 17
020 42  065 01  110 42
021 00  066 55  111 00
022 03  067 43  112 07
023 45  068 00  113 81
024 03  069 07  114 42
025 65  070 65  115 00
026 43  071 53  116 08
027 00  072 43  117 81
028 04  073 00  118 46
029 85  074 08  119 18
030 43  075 85  120 42
031 00  076 59  121 00
032 03  077 55  122 09
033 40  078 02  123 81
034 65  079 65
035 43  080 43
036 00  081 01
037 05  082 00
038 85  083 65
039 43  084 43
040 00  085 01
041 03  086 01
042 65  087 55
043 43  088 43
044 00  089 00
```

TITLE: ECDZ AND NOTCHED STRENGTH				NO: .113
A'	B'	C'	D'	E'
A Y, c_o	B Constants	C X	D YX_n/X	E

00					
01 a	02 w	03 a/w	04 1.5254	05 -0.2881	06 0.1282
07 X	08	09	10 $(\frac{x}{YX_n})^2 - 1$		
11 used	12 used	13 c_o	14 a	15	16
17	18	19			
98	99				

REGISTER

INSTRUCTIONS

OPERATION	STEP	READ	ENTER	PRESS	DISPLAY
Enter constants	1	113 A&B	0.1282	A	0.1282
			-0.2881	RUN	-0.2881
			1.5254	RUN	1.5254
Enter unnotched strength	2		X	B	X
Enter data to determine c_o Repeat for all data	3			C	
			a	RUN	a
			w	RUN	Y
			X_n	RUN	c_o
				st flg	
				6	
				RUN	c_o (best fit)
Compute YX_n/X	4			D	
			c_o	RUN	c_o
			a_o	RUN	a_o
			θ(deg)	RUN	YX_n/X

PROGRAM TITLE: ECDZ AND NOTCHED STRENGTH NO: 113

FORMULAS:

$$c_o = \frac{a}{\left(\frac{X}{YX_n}\right)^2 - 1} \qquad \text{cf. Eq. (540)}$$

$$c_o \text{ (best fit)} = \frac{\Sigma a\left[\left(\frac{X}{YX_n}\right)^2 - 1\right]}{\Sigma\left[\left(\frac{X}{YX_n}\right)^2 - 1\right]^2}$$

$$\frac{YX_n}{X} = \left(\frac{c_o}{a\sin\theta + c_o}\right)^{1/2} \qquad (546)$$

PROGRAM TITLE: ECDZ AND NOTCHED STRENGTH **NO:** 113

PROGRAM LISTING:

```
000 46   045 00   090 01   135 01
001 13   046 03   091 00   136 02
002 22   047 65   092 95   137 81
003 50   048 43   093 81   138 50
004 00   049 00   094 60   139 00
005 42   050 06   095 00   140 81
006 01   051 85   096 01   141 46
007 01   052 01   097 00   142 14
008 42   053 95   098 06   143 81
009 01   054 65   099 41   144 42
010 02   055 81   100 00   145 01
011 81   056 95   101 01   146 03
012 42   057 20   102 02   147 81
013 00   058 65   103 43   148 42
014 01   059 43   104 01   149 01
015 81   060 00   105 02   150 04
016 42   061 07   106 55   151 81
017 00   062 95   107 43   152 32
018 02   063 40   108 01   153 65
019 55   064 75   109 01   154 43
020 43   065 01   110 95   155 01
021 00   066 95   111 81   156 04
022 01   067 42   112 46   157 85
023 95   068 01   113 11   158 43
024 20   069 00   114 42   159 01
025 42   070 40   115 00   160 03
026 00   071 44   116 06   161 95
027 03   072 01   117 81   162 20
028 45   073 01   118 42   163 65
029 03   074 43   119 00   164 43
030 65   075 00   120 05   165 01
031 43   076 01   121 81   166 03
032 00   077 65   122 42   167 95
033 04   078 43   123 00   168 30
034 85   079 01   124 04   169 81
035 43   080 00   125 81   170 00
036 00   081 95   126 46
037 03   082 44   127 12
038 40   083 01   128 42
039 65   084 02   129 00
040 43   085 43   130 07
041 00   086 00   131 81
042 05   087 01   132 01
043 85   088 55   133 01
044 43   089 43   134 42
```

TITLE: OFF-AXIS FATIGUE STRENGTH				NO: 117
A'	B'	C'	D'	E'
A Constants	B a_θ, b_θ, etal	C S_θ (S)	D S_θ (T)	E

00					
	REGISTER				
01 a_S	02 b_S	03 a_T	04 b_T	05 θ	06 1/mn
07 a_θ(S)	08 b_θ(S)	09 m^2	10 a_θ(T)		
11 b_θ(T)	12	13	14	15	16
17	18	19			
98	99				

INSTRUCTIONS

OPERATION	STEP	READ	ENTER	PRESS	DISPLAY
Enter data	1	117 A&B	a_S	A	a_S
			b_S	RUN	b_S
			a_T	RUN	a_T
			b_T	RUN	b_T
Compute a_θ, b_θ	2		θ (deg)	B	a_θ(S)
				RUN	b_θ(S)
				RUN	a_θ(T)
				RUN	b_θ(T)
Compute S_θ at N	3		N	C	S_θ(S)
			N	D	S_θ(T)

PROGRAM TITLE: OFF-AXIS FATIGUE STRENGTH NO: 117

FORMULAS:

$$a_\theta = \frac{a_s}{mn} \quad , \quad b_\theta = \frac{b_s}{mn}$$

$$a_\theta = \frac{a_s}{n^2} \quad , \quad b_\theta = \frac{b_s}{n^2}$$

PROGRAM LISTING:

```
000 46  030 20  060 42  090 65
001 11  031 42  061 00  091 43
002 42  032 00  062 09  092 00
003 00  033 06  063 43  093 07
004 01  034 65  064 00  094 95
005 81  035 43  065 03  095 94
006 42  036 00  066 55  096 85
007 00  037 01  067 43  097 43
008 02  038 95  068 00  098 00
009 81  039 42  069 09  099 08
010 42  040 00  070 95  100 95
011 00  041 07  071 42  101 81
012 03  042 81  072 01  102 46
013 81  043 43  073 00  103 14
014 42  044 00  074 81  104 28
015 00  045 06  075 43  105 65
016 04  046 65  076 00  106 43
017 81  047 43  077 04  107 01
018 46  048 00  078 55  108 00
019 12  049 02  079 43  109 95
020 42  050 95  080 00  110 94
021 00  051 42  081 09  111 85
022 05  052 00  082 95  112 43
023 33  053 08  083 42  113 01
024 65  054 81  084 01  114 01
025 43  055 43  085 01  115 95
026 00  056 00  086 81  116 81
027 05  057 05  087 46
028 32  058 32  088 13
029 95  059 40  089 28
```

TITLE: FATIGUE STRENGTH OF ANGLE-PLY LAMINATE NO: 119				
A'	B'	C'	D'	E'
A Data	B u_1, u_2	C k_S, k_T	D	E

REGISTER

00					
01 E_L	02 E_T	03 G_{LT}	04 ν_{LT}	05 a_S	06 b_S
07 a_T	08 b_T	09 2θ	10 u_1		
11 u_2	12 k_S	13 k_T	14	15 used	16
17	18	19			
98	99				

INSTRUCTIONS

OPERATION	STEP	READ	ENTER	PRESS	DISPLAY
Enter data	1	119 A&B	E_L	A	E_L
			E_T	RUN	E_T
			G_{LT}	RUN	G_{LT}
			ν_{LT}	RUN	ν_{LT}
			a_S	RUN	a_S
			b_S	RUN	b_S
			a_T	RUN	a_T
			b_T	RUN	b_T
Compute parameters	2			B	u_1
				RUN	u_2
			θ(deg)	C	k_S
				RUN	k_T

PROGRAM TITLE: FATIGUE STRENGTH OF ANGLE-PLY LAMINATE NO: 119

FORMULAS:

$$k_T = \frac{1}{2}\left[1 - \sec 2\theta + \frac{(u_1 + \sec 2\theta)\tan^2 2\theta}{u_2 + \tan^2 2\theta}\right] \quad (555)$$

$$k_S = -\frac{1}{2}\,\frac{(u_1 + \sec 2\theta)\tan 2\theta}{u_2 + \tan^2 2\theta} \quad (556)$$

$$u_1 = \frac{1 - E_L/E_T}{1 + 2\nu_{LT} + E_L/E_T} \quad (557)$$

$$u_2 = \frac{E_L/G_{LT}}{1 + 2\nu_{LT} + E_L/E_T} \quad (558)$$

PROGRAM TITLE: FATIGUE STRENGTH OF ANGLE-PLY LAMINATE **NO:** 119

PROGRAM LISTING:

```
000 46  045 95  090 13  135 95  180 09
001 11  046 55  091 65  136 94  181 32
002 42  047 53  092 02  137 55  182 40
003 00  048 01  093 95  138 02  183 55
004 01  049 85  094 42  139 95  184 43
005 81  050 02  095 00  140 42  185 01
006 42  051 65  096 09  141 01  186 06
007 00  052 43  097 53  142 03  187 75
008 02  053 00  098 43  143 81  188 01
009 81  054 04  099 01  144 01  189 54
010 42  055 85  100 00  145 85  190 65
011 00  056 43  101 65  146 43  191 43
012 03  057 00  102 43  147 01  192 00
013 81  058 01  103 00  148 00  193 09
014 42  059 55  104 09  149 65  194 33
015 00  060 43  105 33  150 43  195 20
016 04  061 00  106 65  151 00  196 95
017 81  062 02  107 43  152 09  197 55
018 42  063 54  108 00  153 32  198 02
019 00  064 42  109 09  154 40  199 95
020 05  065 01  110 32  155 55  200 42
021 81  066 05  111 85  156 53  201 01
022 42  067 95  112 43  157 43  202 04
023 00  068 42  113 00  158 01  203 81
024 06  069 01  114 09  159 01
025 81  070 00  115 32  160 65
026 42  071 81  116 54  161 43
027 00  072 43  117 55  162 00
028 07  073 00  118 53  163 09
029 81  074 01  119 43  164 33
030 42  075 55  120 01  165 40
031 00  076 43  121 01  166 85
032 08  077 00  122 65  167 43
033 81  078 03  123 43  168 00
034 46  079 95  124 00  169 09
035 12  080 55  125 09  170 32
036 01  081 43  126 33  171 40
037 75  082 01  127 40  172 54
038 43  083 05  128 85  173 42
039 00  084 95  129 43  174 01
040 01  085 42  130 00  175 06
041 55  086 01  131 09  176 85
042 43  087 01  132 32  177 53
043 00  088 81  133 40  178 43
044 02  089 46  134 54  179 00
```

TITLE: STRENGTH PREDICTION (FAILURE POTENTIAL) NO: 123				
A'	B'	C'	D'	E'
A Data	B x_o, etc.	C	D	E

00					
		REGISTER			
01 $\hat{t}$	02 L	03 C_1	04 β	05 α	06 C_2
07 C_3	08 x_o	09 $\Gamma(\frac{1}{\alpha(\beta+1)}+1)$	10 $\Gamma(\frac{2}{\alpha(\beta+1)}+1)$		
11 γ	12	13	14	15	16
17	18	19			
98	99				

INSTRUCTIONS

OPERATION	STEP	READ	ENTER	PRESS	DISPLAY
Enter data	1	123 A&B	$\hat{t}$	A	$\hat{t}$
			β	RUN	β
			C_1	RUN	C_1
			α	RUN	α
			C_3	RUN	C_3
			C_2	RUN	C_2
			L	RUN	L
			$\gamma(0.5772)$	RUN	γ
Compute	2			B	x_o
				RUN	$1+1/\alpha(\beta+1)$
			$\Gamma(1+1/\alpha(\beta+1))$	RUN	$1+2/\alpha(\beta+1)$
			$\Gamma(1+2/\alpha(\beta+1))$	RUN	$\bar{x}$ (power)
				RUN	c.v. (power)
				C	$\bar{x}$ (exp.)

PROGRAM TITLE: STRENGTH PREDICTION (FAILURE POTENTIAL) NO: 123

FORMULAS:

$$\bar{x}\,(\text{power}) = x_o\ \Gamma\left[1 + \frac{1}{\alpha(\beta+1)}\right]$$

$$x_o = Lt_o = \left[\hat{t}\, L\, C_1^{\beta}(\beta+1)\right]^{1/(\beta+1)} \qquad (580)$$

$$\text{C. V.} = \left[\frac{\Gamma\left[1 + 2/\alpha(\beta+1)\right]}{\Gamma^2\left[1 + 1/\alpha(\beta+1)\right]} - 1\right]^{1/2}$$

$$\bar{x} = \frac{C_3}{\alpha}\left[\alpha\, Ln(\hat{t}LC_2/C_3) - \gamma\right] \qquad \gamma\text{: Euler constant (=0.5772)} \qquad (585)$$

PROGRAM TITLE: STRENGTH PREDICTION (FAILURE POTENTIAL) **NO:** 123

PROGRAM LISTING:

```
000 46  045 43  090 43  135 09  180 54
001 11  046 00  091 00  136 40  181 95
002 42  047 04  092 04  137 75  182 81
003 00  048 85  093 85  138 01  183 00
004 01  049 01  094 01  139 95  184 00
005 81  050 54  095 54  140 30  185 00
006 42  051 95  096 95  141 95
007 00  052 45  097 20  142 81
008 04  053 53  098 42  143 46
009 81  054 43  099 01  144 13
010 42  055 00  100 02  145 43
011 00  056 04  101 85  146 00
012 03  057 85  102 01  147 07
013 81  058 01  103 95  148 55
014 42  059 54  104 81  149 43
015 00  060 20  105 42  150 00
016 05  061 95  106 00  151 05
017 81  062 65  107 09  152 65
018 42  063 43  108 43  153 53
019 00  064 00  109 01  154 43
020 07  065 03  110 02  155 00
021 81  066 45  111 65  156 05
022 42  067 53  112 02  157 65
023 00  068 43  113 85  158 53
024 06  069 00  114 01  159 43
025 81  070 04  115 95  160 00
026 42  071 55  116 81  161 01
027 00  072 53  117 42  162 65
028 02  073 43  118 01  163 43
029 81  074 00  119 00  164 00
030 42  075 04  120 43  165 02
031 01  076 85  121 00  166 65
032 01  077 01  122 08  167 43
033 81  078 54  123 65  168 00
034 46  079 54  124 43  169 06
035 12  080 95  125 00  170 55
036 43  081 42  126 09  171 43
037 00  082 00  127 95  172 00
038 01  083 08  128 81  173 07
039 65  084 81  129 43  174 54
040 43  085 43  130 01  175 23
041 00  086 00  131 00  176 75
042 02  087 05  132 55  177 43
043 65  088 65  133 43  178 01
044 53  089 53  134 00  179 01
```

TITLE: RESIDUAL STRENGTH (a)				NO: 127
A' Parameters	B'	C'	D'	E'
A N_o	B R_ℓ	C R_r	D	E

00					
	REGISTER				
01 α_s	02 x_o	03 α_r	04 α_ℓ	05 β	06 C_1
07 n	08 S_{max}	09 $\hat{x}$	10 N_o		
11	12	13	14	15	16
17	18	19			
98	99				

INSTRUCTIONS

OPERATION	STEP	READ	ENTER	PRESS	DISPLAY
Enter parameters	1	127 A&B	α_S	A'	α_S
			x_o	RUN	x_o
			α_r	RUN	α_r
			α_ℓ	RUN	α_ℓ
			β	RUN	β
			C_1	RUN	C_1
			$\hat{x}$	RUN	$\hat{x}$
Compute N_o	2		S_{max}	A	N_o
Compute R_ℓ	3		n	B	R_ℓ
Compute R_r	4		n	C	R_r

PROGRAM TITLE: RESIDUAL STRENGTH (a) NO: 127

FORMULAS:

$$R_r(\bar{x}_r) = \exp\left[-\frac{\bar{x}_r^{\alpha_s} + (\alpha_s/\alpha_\ell)(S_{max}/C_1)^{\beta_n \alpha_\ell}}{\bar{x}_o^{\alpha_s}} + \left(\frac{\bar{S}_{max}}{\bar{x}_o}\right)^{\alpha_s}\right]$$

$$= \exp\left[-\left(\frac{\bar{x}_r}{\bar{x}_o}\right)^{\alpha_s} - \left(\frac{n}{N_o}\right)^{\alpha_\ell} + \left(\frac{S_{max}}{x_o}\right)^{\alpha_s}\right] \quad (622)$$

$$R_\ell(N) = \exp\left[-\left(\frac{N}{N_o}\right)^{\alpha_\ell}\right] \quad (623)$$

$$N_o(S_{max}/C_1)^{\beta/\alpha_\ell} = (x_o^{\alpha_s}\, \alpha_\ell/\alpha_s)^{1/\alpha_\ell} \quad (624)$$

PROGRAM TITLE: RESIDUAL STRENGTH (a) **NO:** 127

PROGRAM LISTING:

```
000 46  045 43  090 45  135 43
001 16  046 00  091 43  136 00
002 42  047 01  092 00  137 02
003 00  048 95  093 04  138 54
004 01  049 65  094 95  139 45
005 81  050 43  095 94  140 43
006 42  051 00  096 22  141 00
007 00  052 04  097 23  142 01
008 02  053 55  098 81  143 95
009 81  054 43  099 46  144 22
010 42  055 00  100 13  145 23
011 00  056 01  101 42  146 81
012 03  057 95  102 00
013 81  058 55  103 07
014 42  059 53  104 55
015 00  060 43  105 43
016 04  061 00  106 00
017 81  062 08  107 02
018 42  063 55  108 95
019 00  064 43  109 45
020 05  065 00  110 43
021 81  066 06  111 00
022 42  067 54  112 01
023 00  068 45  113 95
024 06  069 43  114 94
025 81  070 00  115 75
026 42  071 05  116 53
027 00  072 95  117 43
028 09  073 45  118 00
029 81  074 43  119 07
030 46  075 00  120 55
031 11  076 04  121 43
032 42  077 20  122 01
033 00  078 95  123 00
034 08  079 42  124 54
035 53  080 01  125 45
036 43  081 00  126 43
037 00  082 81  127 00
038 02  083 46  128 04
039 55  084 12  129 85
040 43  085 55  130 53
041 00  086 43  131 43
042 09  087 01  132 00
043 54  088 00  133 08
044 45  089 95  134 55
```

TITLE: RESIDUAL STRENGTH (b)				NO: 131
A' Parameters	B'	C'	D'	E'
A N_o	B R_ℓ	C R_r	D	E

00					
	REGISTER				
01 α_s	02 x_o	03 α_r	04 α_ℓ	05 β	06 C_1
07 n	08 S_{max}	09 $\hat{x}$	10 N_o		
11	12	13	14	15	16
17	18	19			
98	99				

INSTRUCTIONS

OPERATION	STEP	READ	ENTER	PRESS	DISPLAY
Enter parameters	1	131 A&B	$\dot{\alpha}_s$	A'	
			x_o	RUN	
			α_r	RUN	
			α_ℓ	RUN	
			β	RUN	
			C_1	RUN	
			$\hat{x}$	RUN	
Compute N_o	2		S_{max}	A	N_o
Compute R_ℓ	3		n	B	R_ℓ
Compute R_r	4		n	C	R_r

PROGRAM TITLE: RESIDUAL STRENGTH (b) NO: 131

FORMULAS:

$$R_r(\bar{x}_r) = \exp\left[- \frac{\left(\frac{\bar{x}_r^{\alpha_r}}{} + \alpha_r (S_{max}/C_1)^{\beta_n} \right)^{\alpha_s/\alpha_r}}{\bar{x}_o^{\alpha_s}} + \left(\frac{S_{max}}{x_o} \right)^{\alpha_s} \right]$$

$$= \exp\left\{ - \left[\left(\frac{\bar{x}_r}{\bar{x}_o} \right)^{\alpha_r} + \left(\frac{n}{N_o} \right) \right]^{\alpha_s/\alpha_r} + \left(\frac{S_{max}}{x_o} \right)^{\alpha_s} \right\} \quad (627)$$

$$R_\ell(N) = \exp\left[- \left(\frac{N}{N_o} \right)^{\alpha_s/\alpha_r} \right] \quad (628)$$

$$N_o (S_{max}/C_1)^{\beta} = \bar{x}_o^{\alpha_r}/\alpha_r \quad (629)$$

PROGRAM TITLE: RESIDUAL STRENGTH (b) **NO:** 131

PROGRAM LISTING:

```
000 46   045 20   090 23   135 02
001 16   046 65   091 81   136 54
002 42   047 53   092 46   137 45
003 00   048 43   093 13   138 43
004 01   049 00   094 42   139 00
005 81   050 02   095 00   140 01
006 42   051 55   096 07   141 95
007 00   052 43   097 55   142 22
008 02   053 00   098 43   143 23
009 81   054 09   099 00   144 81
010 42   055 54   100 02
011 00   056 45   101 95
012 03   057 43   102 45
013 81   058 00   103 43
014 42   059 03   104 00
015 00   060 95   105 03
016 04   061 55   106 85
017 81   062 43   107 43
018 42   063 00   108 00
019 00   064 03   109 07
020 05   065 95   110 55
021 81   066 42   111 43
022 42   067 01   112 01
023 00   068 00   113 00
024 06   069 81   114 95
025 81   070 46   115 45
026 42   071 12   116 53
027 00   072 55   117 43
028 09   073 43   118 00
029 81   074 01   119 01
030 46   075 00   120 55
031 11   076 95   121 43
032 42   077 45   122 00
033 00   078 53   123 03
034 08   079 43   124 54
035 55   080 00   125 95
036 43   081 01   126 94
037 00   082 55   127 85
038 06   083 43   128 53
039 95   084 00   129 43
040 45   085 03   130 00
041 43   086 54   131 08
042 00   087 95   132 55
043 05   088 94   133 43
044 95   089 22   134 00
```

TITLE: CUMULATIVE DAMAGE				NO: 135
A' Data	B'	C'	D'	E'
A $R_{\ell n_1}$	B $R_{\ell n_1}$	C	D	E

00					
01 N_{o1}	02 N_{o2}	03 n_1	04 α_ℓ	05 α_s	06 α_r
07	08	09	10		
11	12	13	14	15	16
17	18	19			
98	99				

REGISTER

INSTRUCTIONS

OPERATION	STEP	READ	ENTER	PRESS	DISPLAY
Enter data	1	135 A	N_{o1}	A'	N_{o1}
			N_{o2}	RUN	N_{o2}
			n_1	RUN	n_1
			α_ℓ	RUN	α_ℓ
			α_s	RUN	α_s
			α_r	RUN	α_r
Compute $R_{\ell n_1}$ model (a)	2		N_2	A	$R_{\ell n_1}$
model (b)			N_2	B	$R_{\ell n_1}$

PROGRAM TITLE: CUMULATIVE DAMAGE NO: 135

FORMULAS:

(a) $\alpha_r = \alpha_s$

$$R_{\ell n1}(N_2) = \exp\left[-\left(\frac{n_1}{N_{o1}}\right)^{\alpha_\ell} - \left(\frac{N_2}{N_{o2}}\right)^{\alpha_\ell}\right] \quad (634)$$

(b) $\alpha_\ell = 1$

$$R_{\ell n1}(N_2) = \exp\left[-\left(\frac{n_1}{N_{o1}} + \frac{N_2}{N_{o2}}\right)^{\alpha_s/\alpha_r}\right] \quad (636)$$

PROGRAM TITLE: CUMULATIVE DAMAGE NO: 135

PROGRAM LISTING:

```
000 46   045 00
001 16   046 01
002 42   047 54
003 00   048 45
004 01   049 43
005 81   050 00
006 42   051 04
007 00   052 95
008 02   053 94
009 81   054 22
010 42   055 23
011 00   056 95
012 03   057 81
013 81   058 46
014 42   059 12
015 00   060 55
016 04   061 43
017 81   062 00
018 42   063 02
019 00   064 85
020 05   065 43
021 81   066 00
022 42   067 03
023 00   068 55
024 06   069 43
025 81   070 00
026 46   071 01
027 11   072 95
028 55   073 45
029 43   074 53
030 00   075 43
031 02   076 00
032 95   077 05
033 45   078 55
034 43   079 43
035 00   080 00
036 04   081 06
037 95   082 54
038 85   083 95
039 53   084 94
040 43   085 22
041 00   086 23
042 03   087 95
043 55   088 81
044 43
```

TITLE: COMPOSITE CONDUCTIVITIES				NO: 139
A'	B'	C'	D'	E'
A K_{11}	B K_{22}	C K_x	D	E

00 α					
01 K_m	02 K_f	03 v_f	04 B_k	05 $B_k^2 v_f/\pi$	06 K_{22}
07 K_{11}	08 K_x	09	10		
11	12	13	14	15	16
17	18	19			
98	99				

REGISTER

INSTRUCTIONS

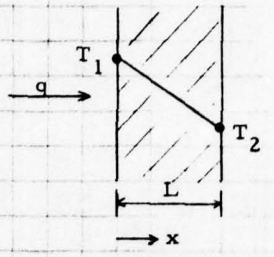
OPERATION	STEP	READ	ENTER	PRESS	DISPLAY
Input data	1	139 A&B	K_m	A	K_m
			K_f	RUN	K_f
			v_f	RUN	K_{11}
				B	K_{22}
			α	C	K_x

PROGRAM TITLE: COMPOSITE CONDUCTIVITIES NO: 139

FORMULAS:

Heat Transfer (temperature effects)

$$q = -K\nabla T \qquad (638)$$

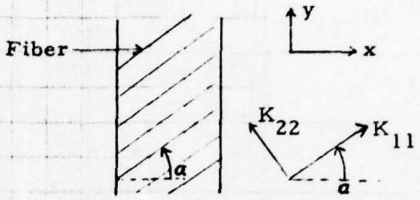


One dimensional steady

$$q_x = +K_x \frac{T_1 - T_2}{L} \qquad (639)$$

Composite Materials

$$K_x = K_{11}\cos^2\alpha + K_{22}\sin^2\alpha \qquad (640)$$



Approximations ($v_f < 0.785$)

$$K_{11} = (1-v_f)K_m + v_f K_f \qquad (641)$$

$$K_{22} = K_m\left\{(1 - 2\sqrt{v_f/\pi}) + \frac{1}{B_K}\left[\pi - \frac{4}{\sqrt{1 - B_K^2 v_f/\pi}}\tan^{-1}\frac{\sqrt{1 - (B_K^2 v_f/\pi)}}{1 + B_K(v_f/\pi)^{1/2}}\right]\right\} \qquad (642)$$

$$B_K \equiv 2\left(\frac{K_m}{K_f} - 1\right) \qquad (643)$$

PROGRAM TITLE: COMPOSITE CONDUCTIVITIES NO: 139

PROGRAM LISTING:

```
000 46  045 02  090 20  135 81
001 11  046 75  091 22  136 46
002 42  047 01  092 34  137 13
003 00  048 95  093 65  138 42
004 01  049 65  094 04  139 00
005 81  050 02  095 55  140 00
006 42  051 95  096 53  141 33
007 00  052 42  097 01  142 40
008 02  053 00  098 75  143 65
009 81  054 04  099 43  144 43
010 42  055 40  100 00  145 00
011 00  056 65  101 05  146 07
012 03  057 43  102 54  147 85
013 43  058 00  103 30  148 43
014 00  059 03  104 95  149 00
015 01  060 55  105 75  150 06
016 65  061 59  106 59  151 65
017 53  062 95  107 95  152 43
018 01  063 42  108 94  153 00
019 75  064 00  109 55  154 00
020 43  065 05  110 43  155 32
021 00  066 43  111 00  156 40
022 03  067 00  112 04  157 95
023 54  068 03  113 85  158 42
024 85  069 55  114 01  159 00
025 43  070 59  115 75  160 08
026 00  071 54  116 02  161 81
027 03  072 30  117 65
028 65  073 65  118 53
029 43  074 43  119 43
030 00  075 00  120 00
031 02  076 04  121 03
032 95  077 85  122 55
033 42  078 01  123 59
034 00  079 54  124 54
035 07  080 55  125 30
036 81  081 53  126 95
037 46  082 01  127 65
038 12  083 75  128 43
039 43  084 43  129 00
040 00  085 00  130 01
041 01  086 05  131 95
042 55  087 54  132 42
043 43  088 30  133 00
044 00  089 95  134 06
```

TITLE: MOISTURE GAIN AND DISTRIBUTION				NO: 143
A'	B'	C' t	D'	E'
A D_x, S, h, x*	B	C t^*	D C^*	E G

00					
01 D_x	02 D_x/S^2	03 h	04 $x^*=x/h$	05 $t^*=D_x t/S^2$	06 (2j+1)
07 SUM G	08 SUM C^*	09 exp[ ]	10 G		
11 C^*	12	13	14	15	16
17	18	19			
98	99				

REGISTER

INSTRUCTIONS

OPERATION	STEP	READ	ENTER	PRESS	DISPLAY
Input Data	1	143 A&B	D_x	A	D_x
			S	RUN	D_x/S^2
			h	RUN	h
			x	RUN	$x^*=x/h$
			t	RUN	$t^*=D_x t/S^2$
Compute C* and G	2		j=0	B	
			1	B	
			2	B	
			3	B	
			.	.	
			.	.	
			.	.	G
				C	C*
				D	t

PROGRAM TITLE: MOISTURE GAIN AND DISTRIBUTION NO: 143

FORMULAS:

Moisture Effects

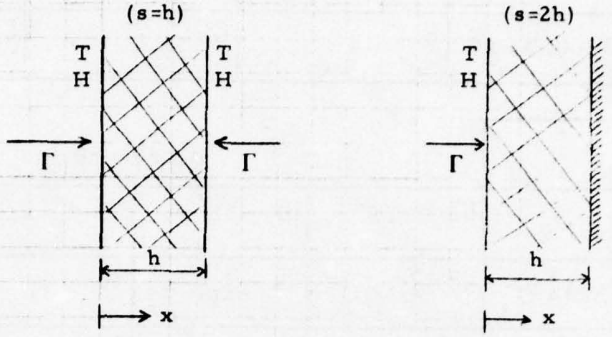
$$\Gamma = -D\nabla C \qquad (644)$$

Steady Bound. Cond.

One Dimensional

Temp Constant

Uniform Initial Distributions



Moisture Distribution:

$$C^* = \frac{C-C_i}{C_m-C_i} = 1 - \frac{4}{\pi}\sum_{j=0}^{\infty}\frac{1}{(2j+1)}\sin\frac{(2j+1)\pi x}{h}\exp\left\{-\left[(2j+1)^2\pi^2\right]\frac{D_x t}{s^2}\right\} \qquad (645)$$

Moisture Content (weight gain):

$$M = \frac{W-W_D}{W_D} = G(M_m-M_i) + M_i \qquad (646)$$

$$G \equiv 1 - \frac{8}{\pi^2}\sum_{j=0}^{\infty}\frac{\exp\left[-(2j+1)^2\pi^2\left(\frac{D_x t}{s^2}\right)\right]}{(2j+1)^2} \qquad (647)$$

Time to reach 99.9% saturation:

$$t_m = \frac{0.67\, s^2}{D_x} \qquad (648)$$

PROGRAM TITLE: MOISTURE GAIN AND DISTRIBUTION NO: 143

PROGRAM LISTING:

```
045 07  000 46  090 59  135 54
046 42  001 11  091 40  136 10
047 00  002 42  092 75  137 30
048 08  003 00  093 01  138 14
049 81  004 01  094 95  139 00
050 46  005 81  095 94  140 10
051 12  006 40  096 42  141 68
052 65  007 55  097 01  142 34
053 02  008 43  098 00  143 41
054 85  009 00  099 43  144 14
055 01  010 01  100 00  145 10
056 95  011 95  101 06  146 10
057 42  012 20  102 65  147 68
058 00  013 42  103 43  148 44
059 06  014 00  104 00  149 31
060 40  015 02  105 04  150 04
061 65  016 81  106 65  151 10
062 59  017 42  107 59  152 50
063 40  018 00  108 95  153 35
064 65  019 03  109 22  154 04
065 43  020 81  110 38  155 20
066 00  021 55  111 32  156 50
067 05  022 43  112 36  157 19
068 95  023 00  113 04  158 08
069 94  024 03  114 92  159 30
070 22  025 95  115 38
071 23  026 42  116 34
072 42  027 00  117 52
073 00  028 04  118 00
074 09  029 00  119 14
075 55  030 42  120 49
076 43  031 00  121 04
077 00  032 08  122 80
078 06  033 81  123 30
079 40  034 65  124 04
080 95  035 43  125 80
081 44  036 00  126 50
082 00  037 02  127 56
083 07  038 95  128 50
084 43  039 42  129 95
085 00  040 00  130 55
086 07  041 05  131 17
087 65  042 00  132 50
088 08  043 42  133 49
089 55  044 00  134 29
```

TITLE: BEST FIT FOR WEIBULL DISTRIBUTION				NO: 147
A'	B'	C'	D'	E'
A x	B j	C r^2(C.D.)	D α	E x_o

00					
01	02	03	04	05	06
07	08	09	10		
11	12	13	14	15	16
17	18	19			
98	99				

REGISTER

INSTRUCTIONS

OPERATION	STEP	READ	ENTER	PRESS	DISPLAY
Initialize	1	147 A&B		CM_s	
			N	STO	
				1	
				4	N
Enter data	2		x	A	lnx
Repeat for all data			j	B	j
Compute	3			C	α
				RUN	x_o
				E	r^2(C.D.)

PROGRAM TITLE: BEST FIT FOR WEIBULL DISTRIBUTION NO: 147

FORMULAS:

$$F = 1 - \exp\left[-(x/x_o)^{\alpha}\right] \qquad (702)$$

$$F = \frac{j - 0.3}{N + 0.4}$$

PROGRAM TITLE: BEST FIT FOR WEIBULL DISTRIBUTION **NO:** 147

PROGRAM LISTING:

```
000 46   045 02   090 43   135 00   180 41
001 88   046 44   091 00   136 04   181 87
002 44   047 00   092 04   137 95   182 46
003 00   048 04   093 75   138 55   183 13
004 00   049 40   094 43   139 43   184 14
005 43   050 44   095 00   140 00   185 65
006 00   051 00   096 05   141 00   186 43
007 00   052 07   097 95   142 95   187 01
008 75   053 43   098 94   143 94   188 03
009 01   054 00   099 42   144 55   189 55
010 95   055 02   100 01   145 43   190 53
011 30   056 65   101 03   146 01   191 43
012 42   057 43   102 55   147 01   192 00
013 00   058 00   103 53   148 95   193 04
014 08   059 01   104 43   149 22   194 40
015 25   060 95   105 00   150 23   195 55
016 43   061 44   106 06   151 56   196 43
017 00   062 00   107 75   152 46   197 00
018 00   063 05   108 43   153 16   198 00
019 56   064 43   109 00   154 51   199 94
020 46   065 00   110 03   155 77   200 85
021 12   066 01   111 65   156 94   201 43
022 75   067 44   112 43   157 85   202 00
023 93   068 00   113 01   158 43   203 07
024 03   069 03   114 00   159 01   204 95
025 95   070 40   115 95   160 02   205 56
026 55   071 44   116 42   161 95   206 46
027 53   072 00   117 01   162 55   207 11
028 43   073 06   118 01   163 46   208 23
029 01   074 01   119 56   164 87   209 42
030 04   075 41   120 46   165 43   210 00
031 85   076 88   121 77   166 01   211 01
032 93   077 46   122 42   167 01   212 56
033 04   078 14   123 01   168 95
034 54   079 43   124 02   169 56
035 95   080 00   125 46   170 46
036 94   081 03   126 15   171 17
037 85   082 55   127 14   172 51
038 01   083 43   128 65   173 77
039 95   084 00   129 43   174 85
040 23   085 00   130 00   175 53
041 94   086 65   131 03   176 43
042 23   087 42   132 94   177 01
043 42   088 01   133 85   178 02
044 00   089 00   134 43   179 65
```

TITLE: A AND B ALLOWABLES (WEIBULL)				NO: 151
A' Parameters	B'	C'	D'	E'
A x_A	B x_B	C i + 1	D	E

00					
		REGISTER			
01 α	02 n	03 $X^2_{2n;\gamma}$	04 j	05 0.99	06 0.90
07 $\sum x_i^{\alpha}$	08	09	10		
11	12	13	14	15	16
17	18	19			
98	99				

INSTRUCTIONS

OPERATION	STEP	READ	ENTER	PRESS	DISPLAY
Enter parameters	1	151 A		A'	
			α	RUN	α
			n	RUN	n
			$X^2_{2n;\gamma}$	RUN	$X^2_{2n;\gamma}$
Enter data	2			C	
Repeat for all data			x_i	RUN	i + 1
Compute	3			A	x_A
				B	x_B

PROGRAM TITLE: A AND B ALLOWABLES (WEIBULL) NO: 151

FORMULAS:

For "A" allowable x_A $\quad \hat{R}_\gamma = 0.99, \quad \gamma = 0.95$

For "B" allowable x_B $\quad \hat{R}_\gamma = 0.90, \quad \gamma = 0.95$

$$x_{A,B} = \left[-2n\ell n\hat{R}_\gamma / X^2_{2n;\gamma} \right]^{1/\alpha} \hat{x}_o \tag{728}$$

$$\hat{x}_o = \left[\frac{1}{n} \sum_{i=1}^{n} x_i^\alpha \right]^{1/\alpha} \tag{729}$$

PROGRAM TITLE: A AND B ALLOWABLES (WEIBULL) NO: 151

PROGRAM LISTING:

```
000 46
001 16
002 81
003 42
004 00
005 01
006 81
007 42
008 00
009 02
010 81
011 42
012 00
013 03
014 81
015 46
016 13
017 00
018 42
019 00
020 04
021 42
022 00
023 07
024 01
025 44
026 00
027 04
028 43
029 00
030 04
031 81
032 45
033 43
034 00
035 01
036 95
037 44
038 00
039 07
040 41
041 00
042 02
043 04
044 81
045 46
046 11
047 43
048 00
049 07
050 55
051 43
052 00
053 04
054 65
055 02
056 65
057 43
058 00
059 02
060 65
061 93
062 09
063 09
064 23
065 95
066 55
067 43
068 00
069 03
070 95
071 94
072 45
073 43
074 00
075 01
076 20
077 95
078 81
079 46
080 12
081 43
082 00
083 07
084 55
085 43
086 00
087 04
088 65
089 02
090 65
091 43
092 00
093 02
094 65
095 93
096 09
097 00
098 23
099 95
100 55
101 43
102 00
103 03
104 95
105 94
106 45
107 43
108 00
109 01
110 20
111 95
112 81
113 00
114 03
115 65
116 43
117 01
118 00
119 95
120 42
121 01
122 01
123 56
124 46
125 77
126 42
127 01
128 02
129 46
130 15
131 14
132 65
133 43
134 00
135 03
136 94
137 85
138 43
139 00
140 04
141 95
142 55
143 43
144 00
145 00
146 95
147 94
148 55
149 43
150 01
151 01
152 95
153 22
154 23
155 56
156 46
157 16
158 51
159 77
160 94
161 85
162 43
163 01
164 02
165 95
166 55
167 46
168 87
169 43
170 01
171 01
172 95
173 56
174 46
175 17
176 51
177 77
178 85
179 53
180 43
181 01
182 02
183 65
184 41
185 87
186 46
187 13
188 14
189 65
190 43
191 01
192 03
193 55
194 53
195 43
196 00
197 04
198 40
199 55
200 43
201 00
202 00
203 94
204 85
205 43
206 00
207 07
208 95
209 56
210 46
211 11
212 23
213 42
214 00
215 01
216 56
```

TITLE: SIZE EFFECT (a)				NO: 155
A' Dimensions	B' Dimensions	C'	D'	E'
A V_t, A_t, ℓ_t	B V_f, A_f, ℓ_f	C Volume, 3-pt	D Volume, 4-pt	E

REGISTER

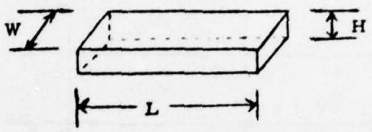
00					
01 W_t	02 L_t	03 H_t	04 W_f	05 L_f	06 H_f
07 V_t	08 A_t	09 ℓ_t	10 V_f		
11 A_f	12 B_f	13 ℓ_f	14 α	15	16
17	18	19			
98	99				

INSTRUCTIONS

OPERATION	STEP	READ	ENTER	PRESS	DISPLAY
Enter dimensions	1	155 A&B		A'	
			W_t	RUN	W_t
			L_t	RUN	L_t
			H_t	RUN	H_t
			W_f	B'	W_f
			L_f	B'	L_f
			H_f	B'	H_f
Compute	2			A	V_t
				RUN	A_t
				RUN	ℓ_t
				B	V_f
				RUN	A_f
				RUN	ℓ_f
Compute (volume model)	3			C	
			α	RUN	X_{of}/X_{ot} (3-pt)
				D	
			α	RUN	X_{of}/X_{ot} (4-pt)

PROGRAM TITLE: SIZE EFFECT (a) NO: 155

FORMULAS:

3-pt. flexure (center point loading)	4-pt. flexure (quarter point loading)
Volume $\frac{(X_o)_f}{(X_o)_t} = \left[2(a+1)^2 \frac{V_t}{V_f}\right]^{\frac{1}{a}}$	$\frac{(X_o)_f}{(X_o)_t} = \left[\frac{4(a+1)^2}{a+2} \frac{V_t}{V_f}\right]^{\frac{1}{a}}$
$V_t = WLH$, $V_f = WLH$ $A_t = 2L(W+H)$, $A_f = WL$, $B_f = LH$ $\ell_t = 4L$, $\ell_f = 2L$	

PROGRAM TITLE: SIZE EFFECT (a) **NO:** 155

PROGRAM LISTING:

```
000 46  045 00  090 06  135 04  180 00
001 16  046 03  091 95  136 85  181 07
002 81  047 54  092 42  137 01  182 55
003 42  048 95  093 01  138 95  183 43
004 00  049 42  094 00  139 40  184 01
005 01  050 00  095 81  140 65  185 00
006 81  051 08  096 43  141 02  186 95
007 42  052 81  097 00  142 65  187 45
008 00  053 04  098 04  143 43  188 43
009 02  054 65  099 65  144 00  189 01
010 81  055 43  100 43  145 07  190 04
011 42  056 00  101 00  146 55  191 20
012 00  057 02  102 05  147 43  192 95
013 03  058 95  103 95  148 01  193 81
014 81  059 42  104 42  149 00  194 53
015 46  060 00  105 01  150 95  195 43
016 11  061 09  106 01  151 45  196 00
017 43  062 81  107 81  152 43  197 04
018 00  063 46  108 43  153 01  198 40
019 01  064 17  109 00  154 04  199 55
020 65  065 81  110 05  155 20  200 43
021 43  066 42  111 65  156 95  201 00
022 00  067 00  112 43  157 81  202 00
023 02  068 04  113 00  158 46  203 94
024 65  069 81  114 06  159 14  204 85
025 43  070 42  115 95  160 81  205 43
026 00  071 00  116 42  161 42  206 00
027 03  072 05  117 01  162 01  207 07
028 95  073 81  118 02  163 04  208 95
029 42  074 42  119 81  164 85  209 56
030 00  075 00  120 43  165 01  210 46
031 07  076 06  121 00  166 95  211 11
032 81  077 81  122 05  167 40  212 23
033 43  078 46  123 65  168 65  213 42
034 00  079 12  124 02  169 04  214 00
035 02  080 43  125 95  170 55  215 01
036 65  081 00  126 42  171 53  216 56
037 02  082 04  127 01  172 43
038 65  083 65  128 03  173 01
039 53  084 43  129 81  174 04
040 43  085 00  130 46  175 85
041 00  086 05  131 13  176 02
042 01  087 65  132 81  177 54
043 85  088 43  133 42  178 65
044 43  089 00  134 01  179 43
```

TITLE: SIZE EFFECT (b)				NO: 159
A'	B'	C'	D'	E'
A X_{of}/X_{ot}	B Surface, 3-pt	C Surface, 4-pt	D Edge, 3-pt	E Edge, 4-pt

00					
01 W_t	02 L_t	03 H_t	04 W_f	05 L_f	06 H_f
07 V_t	08 A_t	09 ℓ_t	10 V_f		
11 A_f	12 B_f	13 ℓ_f	14 α	15	16
17	18	19			
98	99				

REGISTER

INSTRUCTIONS

OPERATION	STEP	READ	ENTER	PRESS	DISPLAY
Compute dimensions with 155	1				
Compute Surface, 3-pt	2	159 A&B		A	
			α	RUN	X_{of}/X_{ot}
Surface, 4-pt				B	
			α	RUN	X_{of}/X_{ot}
Edge, 3-pt				C	
			α	RUN	X_{of}/X_{ot}
Edge, 4-pt				D	
			α	RUN	X_{of}/X_{ot}

PROGRAM TITLE: SIZE EFFECT (b) NO: 159

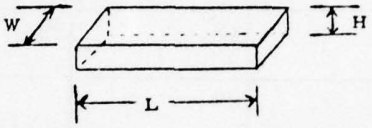
FORMULAS:

	3-pt. flexure (center point loading)	4-pt. flexure (quarter point loading)
Surface	$\frac{(X_o)_f}{(X_o)_t} = \left[\frac{(a+1)\,A_t}{A_f + B_f/(a+1)}\right]^{\frac{1}{a}}$	$\frac{(X_o)_f}{(X_o)_t} = \left[\frac{(a+1)^2\,A_t}{B_f/2 + (a+1)(A_f + B_f)/2 + (a+1)^2 A_f/2}\right]^{\frac{1}{a}}$
Edge	$\frac{(X_o)_f}{(X_o)_t} = \left[(a+1)\frac{\ell_t}{\ell_f}\right]^{\frac{1}{a}}$	$\frac{(X_o)_f}{(X_o)_t} = \left[\frac{a+1}{a+2}\,\frac{\ell_t}{\ell_f/2}\right]^{\frac{1}{a}}$

$V_t = WLH$, $V_f = WLH$

$A_t = 2L(W+H)$, $A_f = WL$, $B_f = LH$

$\ell_t = 4L$, $\ell_f = 2L$



PROGRAM TITLE: SIZE EFFECT (b) **NO:** 159

PROGRAM LISTING:

```
000 46  045 85  090 43  135 85  180 00
001 11  046 01  091 01  136 01  181 07
002 81  047 95  092 01  137 95  182 55
003 42  048 40  093 55  138 55  183 43
004 01  049 65  094 02  139 53  184 01
005 04  050 43  095 54  140 43  185 00
006 85  051 00  096 95  141 01  186 95
007 01  052 08  097 45  142 04  187 45
008 95  053 55  098 43  143 85  188 43
009 65  054 53  099 01  144 02  189 01
010 43  055 43  100 04  145 54  190 04
011 00  056 01  101 20  146 65  191 20
012 08  057 02  102 95  147 43  192 95
013 55  058 55  103 81  148 00  193 81
014 53  059 02  104 46  149 09  194 53
015 43  060 85  105 13  150 55  195 43
016 01  061 53  106 81  151 43  196 00
017 01  062 43  107 42  152 01  197 04
018 85  063 01  108 01  153 03  198 40
019 43  064 04  109 04  154 65  199 55
020 01  065 85  110 85  155 02  200 43
021 02  066 01  111 01  156 95  201 00
022 55  067 54  112 95  157 45  202 00
023 53  068 65  113 65  158 43  203 94
024 43  069 53  114 43  159 01  204 85
025 01  070 43  115 00  160 04  205 43
026 04  071 01  116 09  161 20  206 00
027 85  072 01  117 55  162 95  207 07
028 01  073 85  118 43  163 81  208 95
029 54  074 43  119 01  164 85  209 56
030 54  075 01  120 03  165 01  210 46
031 95  076 02  121 95  166 95  211 11
032 45  077 54  122 45  167 40  212 23
033 43  078 55  123 43  168 65  213 42
034 01  079 02  124 01  169 04  214 00
035 04  080 85  125 04  170 55  215 01
036 20  081 53  126 20  171 53  216 56
037 95  082 43  127 95  172 43
038 81  083 01  128 81  173 01
039 46  084 04  129 46  174 04
040 12  085 85  130 14  175 85
041 81  086 01  131 81  176 02
042 42  087 54  132 42  177 54
043 01  088 40  133 01  178 65
044 04  089 65  134 04  179 43
```

TITLE: DATA AVERAGING (a)				NO: 163
A'	B'	C'	D'	E'
A σ_2	B σ_6	C A_{ij}, B_i	D	E

REGISTER

00					
01 σ_2	02 σ_2^2	03 σ_6	04	05	06
07	08	09	10 $\Sigma\sigma_2^2$		
11 $\Sigma\sigma_2^3$	12 $\Sigma\sigma_2\sigma_6^2$	13 $\Sigma\sigma_2^4$	14 $\Sigma\sigma_2^2\sigma_6^2$	15 $\Sigma\sigma_6^4$	16 $\Sigma\sigma_2$
17 $\Sigma\sigma_2^2$	18 $\Sigma\sigma_6^2$	19			
98	99				

INSTRUCTIONS

OPERATION	STEP	READ	ENTER	PRESS	DISPLAY
Enter data	1	163 A&B	σ_2	A	$\Sigma\sigma_2$
Repeat for all data			σ_6	B	$\Sigma\sigma_6^2$
Display results	2			C	A_{11}
				RUN	A_{12}
				RUN	A_{13}
				RUN	A_{22}
				RUN	A_{23}
				RUN	A_{33}
				RUN	B_1
				RUN	B_2
				RUN	B_3

PROGRAM TITLE: DATA AVERAGING (a) NO: 163

FORMULAS:

$$[A] = [\sigma]^T [\sigma]$$

cf. Eq. (769)

$$\{B\} = [\sigma]^T \{1\}$$

PROGRAM LISTING:

```
000 46
001 11
002 42
003 00
004 01
005 40
006 42
007 00
008 02
009 44
010 01
011 00
012 65
013 43
014 00
015 01
016 95
017 44
018 01
019 01
020 43
021 00
022 02
023 40
024 44
025 01
026 03
027 81
028 46
029 12
030 42
031 00
032 03
033 40
034 65
035 43
036 00
037 01
038 95
039 44
040 01
041 02
042 43
043 00
044 02
045 65
046 43
047 00
048 03
049 40
050 95
051 44
052 01
053 04
054 43
055 00
056 03
057 40
058 40
059 44
060 01
061 05
062 43
063 00
064 01
065 44
066 01
067 06
068 43
069 00
070 02
071 44
072 01
073 07
074 43
075 00
076 03
077 40
078 44
079 01
080 08
081 81
082 46
083 13
084 43
085 01
086 00
087 81
088 43
089 01
090 01
091 81
092 43
093 01
094 02
095 81
096 43
097 01
098 03
099 81
100 43
101 01
102 04
103 81
104 43
105 01
106 05
107 81
108 43
109 01
110 06
111 81
112 43
113 01
114 07
115 81
116 43
117 01
118 08
119 81
```

TITLE: DATA AVERAGING (b)				NO: 165
A'	B'	C'	D'	E'
A F_{ij}	B f_m	C	D	E

00					
01 F_2	02 F_{22}	03 F_{66}	04 σ_2	05	06
07	08	09	10		
11	12	13	14	15	16
17	18	19			
98	99				

REGISTER

INSTRUCTIONS

OPERATION	STEP	READ	ENTER	PRESS	DISPLAY
Enter data	1	165 A		A	
			F_2	RUN	F_2
			F_{22}	RUN	F_{22}
			F_{66}	RUN	F_{66}
Compute	2		σ_2	B	$F_2\sigma_2+F_{22}\sigma_2^2$
			σ_6	RUN	f_n

PROGRAM TITLE: DATA AVERAGING (b) NO: 165

FORMULAS:

$$f_m(\sigma_2, \sigma_6) = F_2\sigma_2 + F_{22}\sigma_2^2 + F_{66}\sigma_6^2 . \qquad (775)$$

PROGRAM LISTING:

```
000 46   020 65   040 95   060 01   080 08
001 11   021 43   041 81   061 05   081 81
002 81   022 00   042 43   062 43
003 42   023 01   043 00   063 00
004 00   024 85   044 02   064 01
005 01   025 43   045 65   065 44
006 81   026 00   046 43   066 01
007 42   027 02   047 00   067 06
008 00   028 65   048 03   068 43
009 02   029 43   049 40   069 00
010 81   030 00   050 95   070 02
011 42   031 04   051 44   071 44
012 00   032 40   052 01   072 01
013 03   033 85   053 04   073 07
014 81   034 81   054 43   074 43
015 46   035 40   055 00   075 00
016 12   036 65   056 03   076 03
017 42   037 43   057 40   077 40
018 00   038 00   058 40   078 44
019 04   039 03   059 44   079 01
```

APPENDIX A

TEXAS INSTRUMENTS PRERECORDED CARDS

SIMULTANEOUS EQUATIONS IN 2 OR 3 UNKNOWNS

Given the coefficients of two simultaneous equations with two unknowns each, the solutions are effected as follows.

$$\begin{aligned} a_0x + a_1y &= a_2 \\ b_0x + b_1y &= b_2 \end{aligned}$$

$$x = \frac{\begin{vmatrix} a_2 & a_1 \\ b_2 & b_1 \end{vmatrix}}{\begin{vmatrix} a_0 & a_1 \\ b_0 & b_1 \end{vmatrix}} = \frac{a_2b_1 - a_1b_2}{a_0b_1 - a_1b_0}$$

$$y = \frac{\begin{vmatrix} a_0 & a_2 \\ b_0 & b_2 \end{vmatrix}}{\begin{vmatrix} a_0 & a_1 \\ b_0 & b_1 \end{vmatrix}} = \frac{a_0b_2 - a_2b_0}{a_0b_1 - a_1b_0}, \text{ where } a_0b_1 - a_1b_0 \neq 0.$$

For three unknowns:

$$\begin{aligned} a_0x + a_1y + a_2z &= a_3 \\ b_0x + b_1y + b_2z &= b_3 \\ c_0x + c_1y + c_2z &= c_3 \end{aligned}$$

The program solves for x in the first equation and substitutes the result into the second and third equations. Now only two, two-unknown equations exist and are solved as above.

NOTE: a_0, b_0, and $c_0 \neq 0$.

Example: Solve for x and y in the following two equations:

$5.2x + 3y = 2.56$

$10.9x - 4.3y = -34.87$

Enter	Press	Display	Comments
	2nd CMs	0.	Clear data memories
5.2	2nd A	5.2	a_0
3	RUN	3.	a_1
2.56	RUN	2.56	a_2
10.9	2nd B	10.9	b_0
4.3	+/- RUN	−4.3	b_1
34.87	+/- RUN	−34.87	b_2
	D	−1.7	x
	RUN	3.8	y

Example: Solve for x, y, and z.

$.25x - 3.1y - .3z = .675$

$1.25x + .18y + .73z = .901$

$-.8x + 1.1y - 1.6z = -2.074$

$x = .18,\ y = -.3,\ z = 1.0$

TEXAS INSTRUMENTS ©1976

MA1-05 SIMULTANEOUS EQUATIONS ◆B◆

◆A◆ SIMULTANEOUS EQUATIONS MA1-05

a0,a1,a2,	b0,b1,b2,			
a0,a1,a2,a3	b0,b1,b2,b3	c0,c1,c2,c3	x,y	x,y,z

USER INSTRUCTIONS

STEP	PROCEDURE	ENTER	PRESS	DISPLAY
1	Enter Program (A and B)			
2	Clear Data Memories		2nd CMs	
	Perform Steps 3-4 for			
	2 unknowns OR			
	Perform Steps 5-6 for			
	3 unknowns			
3*	Enter coefficients for	a_0	2nd A	a_0
	2 unknowns	a_1	RUN	a_1
		a_2	RUN	a_2
		b_1	2nd B	b_1
		b_2	RUN	b_2
		b_3	RUN	b_3
4	Calculate x and y		D	x
			RUN	y
5*	Enter coefficients for	a_0	A	a_0
	3 unknowns	a_1	RUN	a_1
		a_2	RUN	a_2
		a_3	RUN	a_3
		b_0	B	b_0
		b_1	RUN	b_1
		b_2	RUN	b_2
		b_3	RUN	b_3
		c_0	C	c_0
		c_1	RUN	c_1
		c_2	RUN	c_2
		c_3	RUN	c_3
6	Calculate x, y, and z		E	x
			RUN	y
			RUN	z

*NOTE: To correct input error return to last user defined key and reenter associated values

PROGRAM LISTING

DISPLAY	KEY	DISPLAY	KEY	DISPLAY	KEY	DISPLAY	KEY
000 46	*LBL	028 06	6	056 03	3	084 65	×
001 11	A	029 56	*rtn	057 56	*rtn	085 43	RCL
002 42	STO	030 42	STO	058 46	*LBL	086 01	1
003 00	0	031 00	0	059 14	D	087 01	1
004 00	0	032 07	7	060 10	*E'	088 95	=
005 56	*rtn	033 56	*rtn	061 46	*LBL	089 55	÷
006 42	STO	034 42	STO	062 85	+	090 53	(
007 00	0	035 00	0	063 43	RCL	091 43	RCL
008 01	1	036 08	8	064 00	0	092 00	0
009 56	*rtn	037 56	*rtn	065 09	9	093 06	6
010 42	STO	038 46	*LBL	066 56	*rtn	094 65	×
011 00	0	039 13	C	067 43	RCL	095 43	RCL
012 02	2	040 42	STO	068 00	0	096 01	1
013 56	*rtn	041 01	1	069 04	4	097 02	2
014 42	STO	042 00	0	070 56	*rtn	098 75	−
015 00	0	043 56	*rtn	071 46	*LBL	099 43	RCL
016 03	3	044 46	*LBL	072 10	*E'	100 00	0
017 56	*rtn	045 17	*B'	073 43	RCL	101 07	7
018 46	*LBL	046 42	STO	074 00	0	102 65	×
019 12	B	047 01	1	075 06	6	103 43	RCL
020 42	STO	048 01	1	076 65	×	104 01	1
021 00	0	049 56	*rtn	077 43	RCL	105 01	1
022 05	5	050 42	STO	078 01	1	106 95	=
023 56	*rtn	051 01	1	079 03	3	107 42	STO
024 46	*LBL	052 02	2	080 75	−	108 00	0
025 16	*A'	053 56	*rtn	081 43	RCL	109 04	4
026 42	STO	054 42	STO	082 00	0	110 65	×
027 00	0	055 01	1	083 08	8	111 43	RCL

*Denotes 2nd function key

REGISTERS

00 a_0	05 $-b_0$	10 $-c_0$	15
01 a_1	06 b_1, a_0	11 c_1, b_0	16
02 a_2	07 b_2, a_1	12 c_2, b_1	17
03 a_3	08 b_3, a_2	13 c_3, b_2	18
04 z,y	09 y,x	14	19

FLAGS

0	1	2	3	4

PROGRAM LISTING

DISPLAY	KEY	DISPLAY	KEY	DISPLAY	KEY	DISPLAY	KEY
112 00	0	140 02	2	168 42	STO	196 95	=
113 07	7	141 49	*PROD	169 00	0	197 56	*rtn
114 94	+/−	142 00	0	170 09	9	198 41	GTO
115 85	+	143 03	3	171 02	2	199 85	+
116 43	RCL	144 43	RCL	172 44	SUM	200 46	*LBL
117 00	0	145 00	0	173 01	1	201 19	*D'
118 08	8	146 05	5	174 04	4	202 43	RCL
119 95	=	147 42	STO	175 19	*D'	203 00	0
120 55	÷	148 00	0	176 19	*D'	204 04	4
121 43	RCL	149 04	4	177 19	*D'	205 65	×
122 00	0	150 01	1	178 10	*E'	206 36	*IND
123 06	6	151 42	STO	179 65	×	207 43	RCL
124 95	=	152 00	0	180 43	RCL	208 00	0
125 42	STO	153 09	9	181 00	0	209 09	9
126 00	0	154 06	6	182 01	1	210 95	=
127 09	9	155 42	STO	183 94	+/−	211 94	+/−
128 56	*rtn	156 01	1	184 75	−	212 36	*IND
129 46	*LBL	157 04	4	185 43	RCL	213 44	SUM
130 15	E	158 19	*D'	186 00	0	214 01	1
131 43	RCL	159 19	*D'	187 04	4	215 04	4
132 00	0	160 19	*D'	188 65	×	216 01	1
133 00	0	161 43	RCL	189 43	RCL	217 44	SUM
134 20	*1/x	162 01	1	190 00	0	218 00	0
135 49	*PROD	163 00	0	191 02	2	219 09	9
136 00	0	164 42	STO	192 85	+	220 44	SUM
137 01	1	165 00	0	193 43	RCL	221 01	1
138 49	*PROD	166 04	4	194 00	0	222 04	4
139 00	0	167 01	1	195 03	3	223 56	*rtn

*Denotes 2nd function key

MATRIX INVERSION AND DETERMINANT (3 x 3)

This program will evaluate and take the inverse of a 3 x 3 matrix.

$$\text{Matrix A} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

The determinant of matrix A is:

$$|A| = a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} - a_{13}a_{22}a_{31} - a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33}$$

The inverse of matrix A is $B = A^{-1}$, where $A \times A^{-1} = 1$.

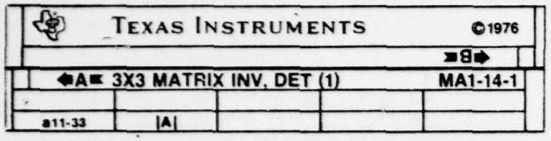
The elements of inverted matrix B are:

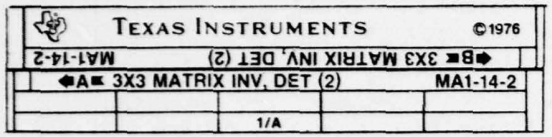
$$b_{11} = (a_{22}a_{33} - a_{32}a_{23}) / |A|$$
$$b_{21} = (a_{32}a_{13} - a_{12}a_{33}) / |A|$$
$$b_{31} = (a_{12}a_{23} - a_{22}a_{13}) / |A|$$
$$b_{12} = (a_{31}a_{23} - a_{21}a_{33}) / |A|$$
$$b_{22} = (a_{11}a_{33} - a_{31}a_{13}) / |A|$$
$$b_{32} = (a_{21}a_{13} - a_{11}a_{23}) / |A|$$
$$b_{13} = (a_{21}a_{32} - a_{31}a_{22}) / |A|$$
$$b_{23} = (a_{31}a_{12} - a_{11}a_{32}) / |A|$$
$$b_{33} = (a_{11}a_{22} - a_{21}a_{12}) / |A|$$

NOTES: 1. if $|A| = 0$, the display will flash 0 when [C] is pressed.
2. The determinant must be calculated before its inverse.

Example: Let $A = \begin{pmatrix} 2 & 4 & 9 \\ 1 & 3 & 5 \\ 7 & 8 & 6 \end{pmatrix}$. Evaluate $|A|$ and find A^{-1}

Enter	Press	Display	Comments
2	[A]	2.	a_{11}
4	[RUN]	4.	a_{12}
9	[RUN]	9.	a_{13}
1	[RUN]	1.	a_{21}
3	[RUN]	3.	a_{22}
5	[RUN]	5.	a_{23}
7	[RUN]	7.	a_{31}
8	[RUN]	8.	a_{32}
6	[RUN]	6.	a_{33}
	[B]	−45.	$\|A\|$
Enter card 2			
	[C]	.4888888889	b_{11}
	[RUN]	−1.066666667	b_{21}
	[RUN]	.1555555556	b_{31}
	[RUN]	−.6444444444	b_{12}
	[RUN]	1.133333333	b_{22}
	[RUN]	.0222222222	b_{32}
	[RUN]	.2888888889	b_{13}
	[RUN]	−.2666666667	b_{23}
	[RUN]	−.0444444444	b_{33}





USER INSTRUCTIONS

STEP	PROCEDURE	ENTER	PRESS	DISPLAY
1	Enter Card 1 (Side A)			
2	Enter Matrix A	a_{11}	A	a_{11}
		a_{12}	RUN	a_{12}
		a_{13}	RUN	a_{13}
		a_{21}	RUN	a_{21}
		a_{22}	RUN	a_{22}
		a_{23}	RUN	a_{23}
		a_{31}	RUN	a_{31}
		a_{32}	RUN	a_{32}
		a_{33}	RUN	a_{33}
3	Calculate determinant		B	A
4	Enter Card 2 (A and B)			
5	Calculate inverse matrix		C	b_{11}
			RUN	b_{21}
			RUN	b_{31}
			RUN	b_{12}
			RUN	b_{22}
			RUN	b_{32}
			RUN	b_{13}
			RUN	b_{23}
			RUN	b_{33}

PROGRAM LISTING

DISPLAY	KEY	DISPLAY	KEY	DISPLAY	KEY	DISPLAY	KEY
000 46	*LBL	028 12	B	056 03	3	084 08	8
001 11	A	029 43	RCL	057 65	×	085 65	×
002 48	*EXC	030 00	0	058 43	RCL	086 43	RCL
003 00	0	031 09	9	059 00	0	087 00	0
004 00	0	032 65	×	060 04	4	088 06	6
005 01	1	033 43	RCL	061 65	×	089 75	−
006 48	*EXC	034 00	0	062 43	RCL	090 43	RCL
007 00	0	035 01	1	063 00	0	091 00	0
008 00	0	036 65	×	064 08	8	092 04	4
009 46	*LBL	037 43	RCL	065 75	−	093 65	×
010 68	*8	038 00	0	066 43	RCL	094 43	RCL
011 36	*IND	039 05	5	067 00	0	095 00	0
012 42	STO	040 85	+	068 07	7	096 02	2
013 00	0	041 53	(	069 65	×	097 65	×
014 00	0	042 43	RCL	070 43	RCL	098 43	RCL
015 56	*rtn	043 00	0	071 00	0	099 00	0
016 48	*EXC	044 02	2	072 05	5	100 09	9
017 00	0	045 65	×	073 65	×	101 95	=
018 00	0	046 43	RCL	074 43	RCL	102 42	STO
019 85	+	047 00	0	075 00	0	103 01	1
020 01	1	048 06	6	076 03	3	104 09	9
021 95	=	049 65	×	077 75	−	105 56	*rtn
022 48	*EXC	050 43	RCL	078 43	RCL	106 00	
023 00	0	051 00	0	079 00	0	107 00	
024 00	0	052 07	7	080 01	1	108 00	
025 41	GTO	053 85	+	081 65	×	109 00	
026 68	*8	054 43	RCL	082 43	RCL	110 00	
027 46	*LBL	055 00	0	083 00	0	111 00	

*Denotes 2nd function key

REGISTERS

00	05 a_{22}	10	15
01 a_{11}	06 a_{23}	11	16
02 a_{12}	07 a_{31}	12	17
03 a_{13}	08 a_{32}	13	18
04 a_{21}	09 a_{33}	14	19 Det.

FLAGS

0	1	2	3	4

PROGRAM LISTING

DISPLAY	KEY	DISPLAY	KEY	DISPLAY	KEY	DISPLAY	KEY
000 46	*LBL	028 75	–	056 42	STO	084 02	2
001 13	C	029 43	RCL	057 01	1	085 75	–
002 43	RCL	030 00	0	058 01	1	086 43	RCL
003 00	0	031 07	7	059 43	RCL	087 00	0
004 07	7	032 65	×	060 00	0	088 01	1
005 65	×	033 43	RCL	061 01	1	089 65	×
006 43	RCL	034 00	0	062 65	×	090 43	RCL
007 00	0	035 05	5	063 43	RCL	091 00	0
008 06	6	036 10	*E'	064 00	0	092 08	8
009 75	–	037 43	STO	065 09	9	093 10	*E'
010 43	RCL	038 01	1	066 75	–	094 42	STO
011 00	0	039 06	6	067 43	RCL	095 01	1
012 04	4	040 43	RCL	068 00	0	096 07	7
013 65	×	041 00	0	069 07	7	097 43	RCL
014 43	RCL	042 08	8	070 65	×	098 00	0
015 00	0	043 65	×	071 43	RCL	099 02	2
016 09	9	044 43	RCL	072 00	0	100 65	×
017 10	*E'	045 00	0	073 03	3	101 43	RCL
018 42	STO	046 03	3	074 10	*E'	102 00	0
019 01	1	047 75	–	075 42	STO	103 06	6
020 03	3	048 43	RCL	076 01	1	104 75	–
021 43	RCL	049 00	0	077 04	4	105 43	RCL
022 00	0	050 02	2	078 43	RCL	106 00	0
023 04	4	051 65	×	079 00	0	107 05	5
024 65	×	052 43	RCL	080 07	7	108 65	×
025 43	RCL	053 00	0	081 65	×	109 43	RCL
026 00	0	054 09	9	082 43	RCL	110 00	0
027 08	8	055 10	*E'	083 00	0	111 03	3

*Denotes 2nd function key

REGISTERS

00	05 a_{22}	10 b_{11}	15 b_{23}
01 a_{11}	06 a_{23}	11 b_{12}	16 b_{31}
02 a_{12}	07 a_{31}	12 b_{13}	17 b_{32}
03 a_{13}	08 a_{32}	13 b_{21}	18 b_{33}
04 a_{21}	09 a_{33}	14 b_{22}	19 Det.

FLAGS

0	1	2	3	4

PROGRAM LISTING

DISPLAY	KEY	DISPLAY	KEY	DISPLAY	KEY	DISPLAY	KEY
112 10	*E'	140 00	0	168 06	6	196 43	RCL
113 42	STO	141 05	5	169 10	*E'	197 01	1
114 01	1	142 75	–	170 42	STO	198 09	9
115 02	2	143 43	RCL	171 01	1	199 95	=
116 43	RCL	144 00	0	172 00	0	200 70	*if err
117 00	0	145 04	4	173 56	*rtn	201 67	*7'
118 04	4	146 65	×	174 01	1	202 56	*rtn
119 65	×	147 43	RCL	175 01	1	203 46	*LBL
120 43	RCL	148 00	0	176 42	STO	204 67	*7'
121 00	0	149 02	2	177 00	0	205 00	0
122 03	3	150 10	*E'	178 00	0	206 20	*1/x
123 75	–	151 42	STO	179 46	*LBL	207 00	0
124 43	RCL	152 01	1	180 89	*3'	208 56	*rtn
125 00	0	153 08	8	181 36	*IND	209 00	
126 01	1	154 43	RCL	182 43	RCL	210 00	
127 65	×	155 00	0	183 00	0	211 00	
128 43	RCL	156 05	5	184 00	0	212 00	
129 00	0	157 65	×	185 56	*rtn	213 00	
130 06	6	158 43	RCL	186 01	1	214 00	
131 10	*E'	159 00	0	187 44	SUM	215 00	
132 42	STO	160 09	9	188 00	0	216 00	
133 01	1	161 75	–	189 00	0	217 00	
134 05	5	162 43	RCL	190 41	GTO	218 00	
135 43	RCL	163 00	0	191 89	*3'	219 00	
136 00	0	164 08	8	192 46	*LBL	220 00	
137 01	1	165 65	×	193 10	*E'	221 00	
138 65	×	166 43	RCL	194 95	=	222 00	
139 43	RCL	167 00	0	195 55	÷	223 00	

*Denotes 2nd function key

MATRIX ARITHMETIC (2)

The product AB in that order of the m x p matrix $A = [a_{ij}]$ and the p x n matrix $B = [b_{ij}]$ is the m x n matrix $C = [c_{ij}]$ where $0 \le m \le 3$, $0 \le n \le 3$, $0 \le p \le 3$.

$$AB = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{32} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix}$$

$$= \begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} + a_{13}b_{31} & a_{11}b_{12} + a_{12}b_{22} + a_{13}b_{32} & a_{11}b_{13} + a_{12}b_{23} + a_{13}b_{33} \\ a_{21}b_{11} + a_{22}b_{21} + a_{23}b_{31} & a_{21}b_{12} + a_{22}b_{22} + a_{23}b_{32} & a_{21}b_{13} + a_{22}b_{23} + a_{23}b_{33} \\ a_{31}b_{11} + a_{32}b_{21} + a_{33}b_{31} & a_{31}b_{12} + a_{32}b_{22} + a_{33}b_{32} & a_{31}b_{13} + a_{32}b_{23} + a_{33}b_{33} \end{bmatrix}$$

$$= \begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{bmatrix} = C$$

NOTE: if $m < 3$, $n < 3$, $p < 3$, it is not necessary to enter zeros for the remainder of the matrix elements. Matrix 'A' is not destroyed, thus you may do constant multiplication by matrix 'A'.

Reference: *Computer Methods for Science and Engineering*, Robert L. LaFara, Hayden Book Company, 1973.

Example: Evaluate $\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

Enter	Press	Display	Comments
	E	0.	Initialize
2	A	1.	a_{11}
3	RUN	1.	a_{12}
1	B	1.	a_{21}
2	RUN	1.	a_{22}
	D	1.	Multiply
1	A	0.	$a_{31} \times b_{11}$
2	RUN	0.	$a_{31} \times b_{21}$
3	B	0.	$a_{32} \times b_{12}$
4	RUN	0.	$a_{32} \times b_{22}$
	2nd A	11.	c_{11}
	RUN	16.	c_{12}
	2nd B	7.	c_{21}
	RUN	10.	c_{22}

Example: $\begin{bmatrix} 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} = \begin{bmatrix} 32 \end{bmatrix}$

Example: $\begin{bmatrix} 1 & 4 & -2 \\ 6 & 3 & 0 \\ 6 & -1 & -2 \end{bmatrix} \begin{bmatrix} 0 & 4 & 2 \\ -1 & 2 & 6 \\ 4 & 3 & 2 \end{bmatrix} = \begin{bmatrix} -12 & 6 & 22 \\ -3 & 30 & 30 \\ -7 & 16 & 2 \end{bmatrix}$

TEXAS INSTRUMENTS ©1976

MA1-16 MATRIX ARITHMETIC (2) ◆B■

◆A■ MATRIX ARITHMETIC (2) MA1-16

c1j	c2j	c3j		
a1j(b1j)	a2j(b2j)	a3j(b3j)	MULT	INIT

USER INSTRUCTIONS

STEP	PROCEDURE	ENTER	PRESS	DISPLAY
1	*Enter Program (A and B)*			
2	Initialize		E	
3	Enter matrix	a_{11} (b_{11})	A	$a_{31} \times b_{11}$
4		a_{12} (b_{12})	RUN	$a_{31} \times b_{12}$
5		a_{13} (b_{13})	RUN	$a_{31} \times b_{13}$
6		a_{21} (b_{21})	B	$a_{32} \times b_{21}$
7		a_{22} (b_{22})	RUN	$a_{32} \times b_{22}$
8		a_{23} (b_{23})	RUN	$a_{32} \times b_{23}$
9		a_{31} (b_{31})	C	$a_{33} \times b_{31}$
10		a_{32} (b_{32})	RUN	$a_{33} \times b_{32}$
11		a_{33} (b_{33})	RUN	$a_{33} \times b_{33}$
	(If Matrix B has been			
	entered go to 13)			
12	Setup for multiply		D	
	(Repeat 3-11 for Matrix B)			
13	Recall Matrix C		2nd A	c_{11}
			RUN	c_{12}
			RUN	c_{13}
			2nd B	c_{21}
			RUN	c_{22}
			RUN	c_{23}
			2nd C	c_{31}
			RUN	c_{32}
			RUN	c_{33}
14	To use Matrix A again, ——	go to Step 12		

PROGRAM LISTING

DISPLAY	KEY	DISPLAY	KEY	DISPLAY	KEY	DISPLAY	KEY
000 25	CLR	028 52	EE	056 56	*rtn	084 00	0
001 56	*rtn	029 19	*D'	057 46	*LBL	085 00	0
002 46	*LBL	030 44	SUM	058 42	STO	086 07	7
003 15	E	031 00	0	059 19	*D'	087 51	SBR
004 47	*CMs	032 01	1	060 44	SUM	088 24	CE
005 86	*rset	033 10	*E'	061 00	0	089 56	*rtn
006 46	*LBL	034 44	SUM	062 03	3	090 42	STO
007 19	*D'	035 00	0	063 10	*E'	091 00	0
008 42	STO	036 04	4	064 44	SUM	092 00	0
009 01	1	037 10	*E'	065 00	0	093 07	7
010 09	9	038 44	SUM	066 06	6	094 51	SBR
011 46	*LBL	039 00	0	067 10	*E'	095 42	STO
012 10	*E'	040 07	7	068 44	SUM	096 56	*rtn
013 03	3	041 56	*rtn	069 00	0	097 46	*LBL
014 44	SUM	042 46	*LBL	070 09	9	098 88	*2'
015 01	1	043 24	CE	071 56	*rtn	099 08	8
016 09	9	044 19	*D'	072 46	*LBL	100 51	SBR
017 43	RCL	045 44	SUM	073 14	D	101 52	EE
018 00	0	046 00	0	074 50	*st flg	102 56	*rtn
019 00	0	047 02	2	075 00	0	103 42	STO
020 65	×	048 10	*E'	076 56	*rtn	104 00	0
021 36	*IND	049 44	SUM	077 46	*LBL	105 00	0
022 43	RCL	050 00	0	078 87	*1'	106 08	8
023 01	1	051 05	5	079 07	7	107 51	SBR
024 09	9	052 10	*E'	080 51	SBR	108 24	CE
025 95	=	053 44	SUM	081 52	EE	109 56	*rtn
026 56	*rtn	054 00	0	082 56	*rtn	110 42	STO
027 46	*LBL	055 08	8	083 42	STO	111 00	0

*Denotes 2nd function key

REGISTERS

00 Used	05 c_{22}	10 a_{11}	15 a_{22}
01 c_{11}	06 c_{23}	11 a_{12}	16 a_{31}
02 c_{12}	07 c_{31}	12 a_{13}	17 a_{32}
03 c_{13}	08 c_{32}	13 a_{21}	18 a_{33}
04 c_{21}	09 c_{33}	14 a_{22}	19 Used

FLAGS

0 Used	1	2	3	4

PROGRAM LISTING

DISPLAY	KEY	DISPLAY	KEY	DISPLAY	KEY	DISPLAY	KEY
112 00	0	140 00	0	168 00	0	196 00	0
113 08	8	141 00	0	169 00	0	197 46	*LBL
114 51	SBR	142 60	*if flg	170 60	*if flg	198 65	×
115 42	STO	143 00	0	171 00	0	199 01	1
116 56	*rtn	144 87	*1'	172 88	*2'	200 44	SUM
117 46	*LBL	145 01	1	173 01	1	201 00	0
118 89	*3'	146 00	0	174 03	3	202 00	0
119 09	9	147 46	*LBL	175 41	GTO	203 25	CLR
120 51	SBR	148 32	sin	176 32	sin	204 36	*IND
121 52	EE	149 48	*EXC	177 46	*LBL	205 43	RCL
122 56	*rtn	150 00	0	178 13	C	206 00	0
123 42	STO	151 00	0	179 42	STO	207 00	0
124 00	0	152 46	*LBL	180 00	0	208 56	*rtn
125 00	0	153 33	cos	181 00	0	209 41	GTO
126 09	9	154 36	*IND	182 60	*if flg	210 65	×
127 51	SBR	155 42	STO	183 00	0	211 46	*LBL
128 24	CE	156 00	0	184 89	*3'	212 17	*B'
129 56	*rtn	157 00	0	185 01	1	213 03	3
130 42	STO	158 01	1	186 06	6	214 41	GTO
131 00	0	159 44	SUM	187 41	GTO	215 85	+
132 00	0	160 00	0	188 32	sin	216 46	*LBL
133 09	9	161 00	0	189 46	*LBL	217 18	*C'
134 51	SBR	162 56	*rtn	190 16	*A'	218 06	6
135 42	STO	163 41	GTO	191 00	0	219 41	GTO
136 56	*rtn	164 33	cos	192 46	*LBL	220 85	+
137 46	*LBL	165 46	*LBL	193 85	+	221 00	
138 11	A	166 12	B	194 42	STO	222 00	
139 42	STO	167 42	STO	195 00	0	223 00	

*Denotes 2nd function key

BASIC STATISTICS FOR ONE OR TWO VARIABLES

This program calculates means, standard deviations, and standard errors of the mean for one or two variables, and covariance and correlation coefficient for two variables.

The input is a set of data points:
$\{(x_1,y_1),(x_2,y_2), \ldots (x_i,y_i), \ldots (x_n,y_n)\}$

in which the y_i's may be omitted if only one variable is being used.

The statistics computed are:

$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$ = mean of x

$\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i$ = mean of y

$S_x = \sqrt{\frac{\sum_{i=1}^{n} x_i^2 - n\bar{x}^2}{n-1}}$ = std. dev. of x by n−1 method

$S'_x = \sqrt{\frac{\sum_{i=1}^{n} x_i^2 - n\bar{x}^2}{n}}$ = std. dev. of x by n method

$S_y = \sqrt{\frac{\sum_{i=1}^{n} y_i^2 - n\bar{y}^2}{n-1}}$ = std. dev. of y by n−1 method

$S'_y = \sqrt{\frac{\sum_{i=1}^{n} y_i^2 - n\bar{y}^2}{n}}$ = std. dev. of y by n method

$S_{\bar{x}} = \frac{S_x}{\sqrt{n}}$ = std. error of the mean of x by n−1 method

$S'_{\bar{x}} = \frac{S'_x}{\sqrt{n}}$ = std. error of the mean of x by n method

$S_{\bar{y}} = \frac{S_y}{\sqrt{n}}$ = std. error of the mean of y by n−1 method

$S'_{\bar{y}} = \frac{S'_y}{\sqrt{n}}$ = std. error of the mean of y by n method

$S_{xy} = \frac{1}{n-1}\left[\sum_{i=1}^{n} x_i y_i - \frac{1}{n}\sum_{i=1}^{n} x_i \sum_{i=1}^{n} y_i\right]$ = covariance of x and y by n−1 method

$S'_{xy} = \frac{1}{n}\left[\sum_{i=1}^{n} x_i y_i - \frac{1}{n}\sum_{i=1}^{n} x_i \sum_{i=1}^{n} y_i\right]$ = covariance of x and y by n method

$r_{xy} = \frac{S_{xy}}{S_x S_y} = \frac{S'_{xy}}{S'_x S'_y}$ = correlation coefficient of x and y

NOTES: 1. The number of data points must be > 1.
2. This program may be used with the Linear Regression program (ST1-08) to determine additional statistics. Data points entered on either program may be used for the other. Do not initialize if data from the other program is being used.

Example: Calculate $\bar{x}$, $\bar{y}$, S_x, S'_x, S_y, S'_y, $S_{\bar{x}}$, $S'_{\bar{x}}$, $S_{\bar{y}}$, $S'_{\bar{y}}$, S_{xy}, S'_{xy}, r_{xy} for the data in the following table, then change the x and y values for i = 5 from (23, 17) to (5, 19) and find the new r_{xy} and S'_{xy}.

i	x_i	y_i
1	31	6
2	20.5	19
3	14	14
4	9	12.5
5	23	17
6	37	22
7	38	19.5
8	18	21
9	29	16

Enter	Press	Display	Comment
	2nd CMs		Initialize
31	A	1.	x_1
6	RUN	1.	y_1
20.5	A	2.	x_2
19	RUN	2.	y_2
14	A	3.	x_3
14	RUN	3.	y_3
etc.			
	B	24.38888889	compute $\bar{x}$
	2nd B'	16.33333333	compute $\bar{y}$
	C	10.07403152	compute S_x
	RUN	9.497888004	compute S'_x
	2nd C'	4.981214711	compute S_y
	RUN	4.696334268	compute S'_y
	D	3.358010507	compute $S_{\bar{x}}$
	RUN	3.165962668	compute $S'_{\bar{x}}$
	2nd D'	1.660404904	compute $S_{\bar{y}}$
	RUN	1.565444756	compute $S'_{\bar{y}}$
	E	10.85416667	compute S_{xy}
	RUN	9.648148148	compute S'_{xy}
	2nd E'	.2163006968	compute r_{xy}
23	2nd A'	8.	delete 23 (23, 17)
17	RUN	8.	delete 17 (23, 17)
5	A	9.	insert 5 (5, 19)
19	RUN	9.	insert 19 (5, 19)
	2nd E'	0825567005	compute new r_{xy}
	E	5.006944445	compute new S_{xy}
	RUN	4.450617284	compute new S'_{xy}

TEXAS INSTRUMENTS

ST1-01 ◆B■ STATS FOR 1 OR 2 VARIABLES

◆A■ STATS FOR 1 OR 2 VARIABLES ST1-01

delete	$\bar{y}$, n	Sy, Sy′	$S\bar{y}$, $S\bar{y}'$	rxy
x, y	$\bar{x}$, n	Sx, Sx′	$S\bar{x}$, $S\bar{x}'$	Sxy, Sxy′

USER INSTRUCTIONS

STEP	PROCEDURE	ENTER	PRESS	DISPLAY
1	Enter Program (A and B)			
2	Initialize		2nd CMs	
3	Enter Data	x_1	A	1
	If only one	y_1	RUN	1
	variable is used	x_2	A	2
	omit the y steps	y_2	RUN	2
4	To delete (x_i, y_i)	x_i	2nd A'	
		y_i	RUN	
5	Compute outputs		B	$\bar{x}$
			RUN	n
	All outputs except		2nd B'	$\bar{y}$
	those obtained by		RUN	n
	pressing run, may		C	S_x
	be obtained in		RUN	S'_x
	any order. Outputs		2nd C'	S_y
	obtained by pressing		RUN	S'_y
	run must be		D	$S_{\bar{x}}$
	obtained immediately		RUN	$S'_{\bar{x}}$
	following the		2nd D'	$S_{\bar{y}}$
	corresponding		RUN	$S'_{\bar{y}}$
	keypush shown		E	S_{xy}
			RUN	S'_{xy}
			2nd E'	r_{xy}

PROGRAM LISTING

DISPLAY	KEY	DISPLAY	KEY	DISPLAY	KEY	DISPLAY	KEY
000	46 *LBL	028	00 0	056	56 *rtn	084	44 SUM
001	11 A	029	56 *rtn	057	46 *LBL	085	00 0
002	42 STO	030	42 STO	058	16 *A'	086	04 4
003	00 0	031	00 0	059	95 =	087	40 *x^2
004	01 1	032	02 2	060	94 +/−	088	94 +/−
005	44 SUM	033	44 SUM	061	42 STO	089	41 GTO
006	00 0	034	00 0	062	00 0	090	32 sin
007	03 3	035	04 4	063	01 1	091	46 *LBL
008	40 *x^2	036	40 *x^2	064	44 SUM	092	12 B
009	44 SUM	037	46 *LBL	065	00 0	093	43 RCL
010	00 0	038	32 sin	066	03 3	094	00 0
011	06 6	039	44 SUM	067	40 *x^2	095	03 3
012	46 *LBL	040	00 0	068	94 +/−	096	46 *LBL
013	34 tan	041	07 7	069	44 SUM	097	33 cos
014	43 RCL	042	43 RCL	070	00 0	098	65 X
015	00 0	043	00 0	071	06 6	099	43 RCL
016	00 0	044	01 1	072	02 2	100	00 0
017	30 *$\sqrt{\ }\bar{x}$	045	65 X	073	94 +/−	101	00 0
018	42 STO	046	43 RCL	074	44 SUM	102	20 *1/x
019	00 0	047	00 0	075	00 0	103	54)
020	08 8	048	02 2	076	00 0	104	56 *rtn
021	01 1	049	95 =	077	51 SBR	105	43 RCL
022	44 SUM	050	44 SUM	078	34 tan	106	00 0
023	00 0	051	00 0	079	56 *rtn	107	00 0
024	00 0	052	05 5	080	42 STO	108	56 *rtn
025	25 CLR	053	43 RCL	081	00 0	109	46 *LBL
026	43 RCL	054	00 0	082	02 2	110	17 *B'
027	00 0	055	00 0	083	94 +/−	111	43 RCL

*Denotes 2nd function key

REGISTERS

00 n	05 $\Sigma x_i y_i$	10	15
01 x_i	06 Σx_i^2	11	16
02 y_i	07 Σy_i^2	12	17
03 Σx_i	08 $\sqrt{n-1}$	13	18
04 Σy_i	09	14	19

FLAGS

0	1	2	3	4

PROGRAM LISTING

DISPLAY	KEY	DISPLAY	KEY	DISPLAY	KEY	DISPLAY	KEY
112	00 0	140	43 RCL	168	13 C	196	00 0
113	04 4	141	65 X	169	41 GTO	197	08 8
114	41 GTO	142	43 RCL	170	42 STO	198	40 *x^2
115	33 cos	143	00 0	171	46 *LBL	199	95 =
116	46 *LBL	144	08 8	172	19 *D'	200	56 *rtn
117	13 C	145	46 *LBL	173	18 *C'	201	65 X
118	43 RCL	146	42 STO	174	41 GTO	202	43 RCL
119	00 0	147	55 ÷	175	42 STO	203	00 0
120	06 6	148	43 RCL	176	46 *LBL	204	08 8
121	75 −	149	00 0	177	15 E	205	40 *x^2
122	53 (	150	00 0	178	43 RCL	206	55 ÷
123	12 B	151	30 *$\sqrt{\ }\bar{x}$	179	00 0	207	43 RCL
124	46 *LBL	152	95 =	180	05 5	208	00 0
125	24 CE	153	56 *rtn	181	75 −	209	00 0
126	40 *x^2	154	41 GTO	182	43 RCL	210	95 =
127	65 X	155	43 RCL	183	00 0	211	56 *rtn
128	43 RCL	156	46 *LBL	184	03 3	212	46 *LBL
129	00 0	157	18 *C'	185	65 X	213	10 *E'
130	00 0	158	43 RCL	186	43 RCL	214	15 E
131	54)	159	00 0	187	00 0	215	55 ÷
132	30 *$\sqrt{\ }\bar{x}$	160	07 7	188	04 4	216	53 (
133	55 ÷	161	75 −	189	55 ÷	217	53 (
134	43 RCL	162	53 (	190	43 RCL	218	13 C
135	00 0	163	17 *B'	191	00 0	219	55 ÷
136	08 8	164	41 GTO	192	00 0	220	53 (
137	95 =	165	24 CE	193	95 =	221	53 (
138	56 *rtn	166	46 *LBL	194	55 ÷	222	18 *C'
139	46 *LBL	167	14 D	195	43 RCL	223	56 *rtn

*Denotes 2nd function key

MEANS AND MOMENTS

For a given set of input data, $\{x_1, x_2, \ldots x_n\}$, with associated frequencies, $\{f_1, f_2, \ldots f_n\}$ (for grouped data), the following means, moments, and skewness and kurtosis of distribution are calculated. If $f_i = 1$ for all i, the calculations are for ungrouped data, otherwise for grouped data.

Means

Arithmetic $= A = \frac{1}{N}\sum_{i=1}^{n} f_i x_i$ Harmonic $= H = N \Big/ \sum_{i=1}^{n} \frac{f_i}{x_i}$

Geometric $= G = \left(\prod_{i=1}^{n} x_i^{f_i}\right)^{1/N}$ Generalized $= M(t) = \left(\frac{1}{N}\sum_{i=1}^{n} f_i (x_i)^t\right)^{1/t}$

Moments

$m_1 = A = \frac{1}{N}\sum_{i=1}^{n} f_i x_i$ $m_4 = \frac{1}{N}\sum_{i=1}^{n} f_i (x_i - A)^4$

$m_2 = \frac{1}{N}\sum_{i=1}^{n} f_i (x_i - A)^2$ Kurtosis of distribution $= \frac{m_4}{(m_2)^2}$

$m_3 = \frac{1}{N}\sum_{i=1}^{n} f_i (x_i - A)^3$ Skewness of distribution $= \frac{m_3}{(m_2)^{3/2}}$

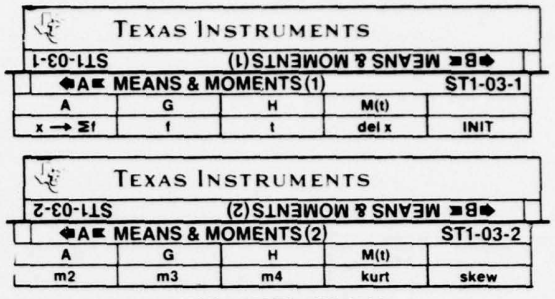
$N = \sum_{i=1}^{n} f_i$ n = Number of different groups

NOTES: 1. The initialization routine assumes a frequency of 1 and t = 1.
2. The value of t may be changed before any x_i's are entered, if changed after any x_i, M(t) will be meaningless.
3. For grouped data if $f_i = f_{i-1}$, it need not be entered.
4. A negative frequency will cause the next x entered to be deleted |f| times.
5. A negative x will cause the geometric and generalized means to be invalid.

Error Indications:
A frequency or t of 0 will cause the display to flash 9.999999999 99 and the old value to be retained.

Example: The ungrouped ($f_i = 1$) data $\{x_i = 1.1, 2.3, 4.7, 3.5, 1.8, 2.9\}$ yields the following: A = 2.716666667, G = 2.451843924, H = 2.187595899, m(1) = 2.716666667, m_2 = 1.368055556, m_3 = .5367592591, m_4 = 3.903019215; kurtosis of distribution = 2.085418497, skewness of distribution = .3354470243

References: *Handbook of Mathematical Tables and Formulas*, R. S. Burington, McGraw-Hill, Fourth Edition, 1965, p. 164. *International Dictionary of Applied Mathematics*, W. F. Frieberger, D. Van Nostrand, 1960



USER INSTRUCTIONS

STEP	PROCEDURE	ENTER	PRESS	DISPLAY
1	Enter Program ST1-03-1			
2	Initialize		E	1.
3	Enter t (for gen. mean)	$t \neq 0$	C	t
4	Enter data			
	a. for grouped data,			
	ent. f_i if $f_i \neq f_{i-1}$	$f_i \neq 0$	B	f_i
	b. enter x_i	x_i	A	Σf
	repeat a and b for each i			
5	Delete data			
	a. for grouped data,			
	enter f_i if $f_i \neq f_{i-1}$	$f_i \neq 0$	B	f_i
	b. enter x_i to be deleted	x_i	D	corrected Σf
6	Calculate			
	a. arithmetic mean		2nd A	A
	b. geometric mean		2nd B	G
	c. harmonic mean		2nd C	H
	d. generalized mean		2nd D	M(t)
.7	Enter Program ST1-03-2			
8	Calculate			
	a. second moment		A	m_2
	b. third moment		B	m_3
	c. fourth moment		C	m_4
	d. kurtosis		D	kurtosis
	e. skewness		E	skewness
	f. means, see step 6			

PROGRAM LISTING

DISPLAY	KEY	DISPLAY	KEY	DISPLAY	KEY	DISPLAY	KEY
000 46	*LBL	028 45	y^x	056 43	RCL	084 00	0
001 11	A	029 43	RCL	057 00	0	085 05	5
002 42	STO	030 00	0	058 02	2	086 55	÷
003 00	0	031 02	2	059 65	X	087 43	RCL
004 04	4	032 95	=	060 44	SUM	088 00	0
005 65	X	033 49	*PROD	061 00	0	089 01	1
006 43	RCL	034 00	0	062 09	9	090 95	=
007 00	0	035 06	6	063 43	RCL	091 42	STO
008 02	2	036 43	RCL	064 00	0	092 01	1
009 95	=	037 00	0	065 04	4	093 02	2
010 44	SUM	038 04	4	066 65	X	094 25	CLR
011 00	0	039 45	y^x	067 44	SUM	095 43	RCL
012 05	5	040 43	RCL	068 01	1	096 00	0
013 43	RCL	041 00	0	069 00	0	097 01	1
014 00	0	042 03	3	070 43	RCL	098 56	*rtn
015 04	4	043 65	X	071 00	0	099 46	*LBL
016 20	*1/x	044 43	RCL	072 04	4	100 12	B
017 65	X	045 00	0	073 95	=	101 90	*if zro
018 43	RCL	046 02	2	074 44	SUM	102 87	*1'
019 00	0	047 95	=	075 01	1	103 42	STO
020 02	2	048 44	SUM	076 01	1	104 00	0
021 95	=	049 00	0	077 43	RCL	105 02	2
022 44	SUM	050 08	8	078 00	0	106 56	*rtn
023 00	0	051 43	RCL	079 02	2	107 46	*LBL
024 07	7	052 00	0	080 44	SUM	108 15	E
025 43	RCL	053 04	4	081 00	0	109 25	CLR
026 00	0	054 40	*x^2	082 01	1	110 47	*CMs
027 04	4	055 65	X	083 43	RCL	111 01	1

*Denotes 2nd function key

REGISTERS

00	05 Σf	10 Σfx^3	15
01 n	06 Πx	11 Σfx^4	16
02 f	07 $\Sigma f/x$	12 $\Sigma fx/n$	17
03 t	08 Σfx	13	18
04 last x	09 Σfx^2	14	19

FLAGS

0	1	2	3	4

PROGRAM LISTING

DISPLAY	KEY	DISPLAY	KEY	DISPLAY	KEY	DISPLAY	KEY
112 42	STO	140 11	A	168 46	*LBL	196 47	*1'
113 00	0	141 01	1	169 18	*C'	197 20	*1/x
114 02	2	142 94	+/-	170 43	RCL	198 56	*rtn
115 42	STO	143 22	INV	171 00	0	199 00	
116 00	0	144 49	*PROD	172 01	1	200 00	
117 06	6	145 00	0	173 55	÷	201 00	
118 46	*LBL	146 02	2	174 43	RCL	202 00	
119 13	C	147 43	RCL	175 00	0	203 00	
120 90	*if zro	148 00	0	176 07	7	204 00	
121 87	*1'	149 01	1	177 95	=	205 00	
122 42	STO	150 56	*rtn	178 56	*rtn	206 00	
123 00	0	151 46	*LBL	179 46	*LBL	207 00	
124 03	3	152 16	*A'	180 19	*D'	208 00	
125 56	*rtn	153 43	RCL	181 43	RCL	209 00	
126 46	*LBL	154 01	1	182 00	0	210 00	
127 14	D	155 02	2	183 08	8	211 00	
128 42	STO	156 56	*rtn	184 55	÷	212 00	
129 00	0	157 46	*LBL	185 43	RCL	213 00	
130 04	4	158 17	*B'	186 00	0	214 00	
131 01	1	159 43	RCL	187 01	1	215 00	
132 94	+/-	160 00	0	188 95	=	216 00	
133 22	INV	161 06	6	189 35	$\sqrt{y}$	217 00	
134 49	*PROD	162 35	$\sqrt{y}$	190 43	RCL	218 00	
135 00	0	163 43	RCL	191 00	0	219 00	
136 02	2	164 00	0	192 03	3	220 00	
137 43	RCL	165 01	1	193 95	=	221 00	
138 00	0	166 95	=	194 56	*rtn	222 00	
139 04	4	167 56	*rtn	195 46	*LBL	223 00	

*Denotes 2nd function key

PROGRAM LISTING

DISPLAY	KEY	DISPLAY	KEY	DISPLAY	KEY	DISPLAY	KEY
000 46	*LBL	028 03	3	056 55	÷	084 09	9
001 11	A	029 65	X	057 43	RCL	085 55	÷
002 43	RCL	030 43	RCL	058 00	0	086 43	RCL
003 01	1	031 01	1	059 01	1	087 00	0
004 02	2	032 02	2	060 75	−	088 01	1
005 40	*x^2	033 65	X	061 04	4	089 75	−
006 94	+/−	034 43	RCL	062 65	X	090 03	3
007 85	+	035 00	0	063 43	RCL	091 65	X
008 53	(	036 09	9	064 01	1	092 43	RCL
009 43	RCL	037 55	÷	065 02	2	093 01	1
010 00	0	038 43	RCL	066 65	X	094 02	2
011 09	9	039 00	0	067 43	RCL	095 40	*x^2
012 55	÷	040 01	1	068 01	1	096 40	*x^2
013 43	RCL	041 85	+	069 00	0	097 95	=
014 00	0	042 43	RCL	070 55	÷	098 56	*rtn
015 01	1	043 01	1	071 43	RCL	099 46	*LBL
016 95	=	044 02	2	072 00	0	100 14	D
017 56	*rtn	045 65	X	073 01	1	101 11	A
018 46	*LBL	046 40	*x^2	074 85	+	102 40	*x^2
019 12	B	047 65	X	075 06	6	103 42	STO
020 43	RCL	048 02	2	076 65	X	104 01	1
021 01	1	049 95	=	077 43	RCL	105 03	3
022 00	0	050 56	*rtn	078 01	1	106 13	C
023 55	÷	051 46	*LBL	079 02	2	107 46	*LBL
024 43	RCL	052 13	C	080 40	*x^2	108 87	*1'
025 00	0	053 43	RCL	081 65	X	109 55	÷
026 01	1	054 01	1	082 43	RCL	110 43	RCL
027 75	−	055 01	1	083 00	0	111 01	1

*Denotes 2nd function key

REGISTERS

00	05 Σf_i	10 Σfx^3	15
01 n	06 $\Pi x'$	11 Σfx^4	16
02 f	07 $\Sigma f/x$	12 $\Sigma fx/n$	17
03 t	08 $\Sigma fx'$	13 Used	18
04 last x	09 Σfx^2	14	19

FLAGS

0	1	2	3	4

PROGRAM LISTING

DISPLAY	KEY	DISPLAY	KEY	DISPLAY	KEY	DISPLAY	KEY
112 03	3	140 35	$\sqrt[x]{y}$	168 43	RCL	196 00	
113 95	=	141 43	RCL	169 00	0	197 00	
114 56	*rtn	142 00	0	170 03	3	198 00	
115 46	*LBL	143 01	1	171 95	=	199 00	
116 15	E	144 95	=	172 56	*rtn	200 00	
117 11	A	145 56	*rtn	173 00		201 00	
118 65	X	146 46	*LBL	174 00		202 00	
119 40	*x^2	147 18	*C'	175 00		203 00	
120 95	=	148 43	RCL	176 00		204 00	
121 30	*$\sqrt{x}$	149 00	0	177 00		205 00	
122 95	=	150 01	1	178 00		206 00	
123 42	STO	151 55	÷	179 00		207 00	
124 01	1	152 43	RCL	180 00		208 00	
125 03	3	153 00	0	181 00		209 00	
126 12	B	154 07	7	182 00		210 00	
127 41	GTO	155 95	=	183 00		211 00	
128 87	*1'	156 56	*rtn	184 00		212 00	
129 46	*LBL	157 46	*LBL	185 00		213 00	
130 16	*A'	158 19	*D'	186 00		214 00	
131 43	RCL	159 43	RCL	187 00		215 00	
132 01	1	160 00	0	188 00		216 00	
133 02	2	161 08	8	189 00		217 00	
134 56	*rtn	162 55	÷	190 00		218 00	
135 46	*LBL	163 43	RCL	191 00		219 00	
136 17	*B'	164 00	0	192 00		220 00	
137 43	RCL	165 01	1	193 00		221 00	
138 00	0	166 95	=	194 00		222 00	
139 06	6	167 35	$\sqrt[x]{y}$	195 00		223 00	

*Denotes 2nd function key

LINEAR REGRESSION

The linear least-squares fit of input data points (x,y) is calculated using the following:

$$\text{Slope} = m = \frac{\frac{\Sigma x \Sigma y}{n} - \Sigma xy}{\frac{(\Sigma x)^2}{n} - \Sigma x^2}$$

$$\text{Intercept} = b = \frac{\Sigma y - m\Sigma x}{n}$$

The y' for entered x and x' for entered y are calculated as follows:

$$y' = mx + b$$
$$x' = (y - b)/m$$

The coefficient of determination is calculated as:

$$r^2 = m\left(\frac{\Sigma x \Sigma y}{n} - \Sigma xy\right) \Big/ \left(\Sigma y^2 - \frac{(\Sigma y)^2}{n}\right)$$

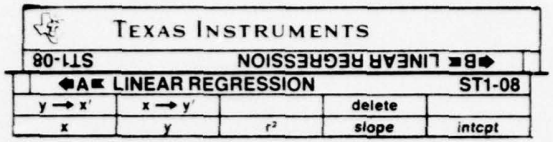
NOTES: 1. The calculator must be initialized by pressing 2nd CMs before each new set of data is entered.
2. The number of data points entered must be greater than 1 before any of the calculations are attempted.
3. This program may be used with the Basic Statistics program (ST1-01) to determine additional statistics. Data points entered on either program may be used for the other. Do not initialize if data from the other program is being used.

Example:

x	20.4	19.7	21.8	20.1	20.7
y	9.2	8.9	11.4	9.4	10.3

$r^2 = .9256678121$
$m = 1.22906793$
$b = -15.40505529$

For $y = 12$, $x' = 22.29742931$
For $x = 22.3$, $y' = 12.00315956$



USER INSTRUCTIONS

STEP	PROCEDURE	ENTER	PRESS	DISPLAY
1	Enter Program (A and B)			
2	Initialize		2nd CMs	
3	Enter data	x	A	x
	Repeat 3 for each x,y	y	B	n
4	Delete data			
	a. Enter x (not necessary	x	A	x
	if deleting last pair			
	entered)			
	b. Enter y	y	2nd B'	n − 1
5	Calculate			
	a. r^2		C	r^2
	b. slope		D	m
	c. intercept		E	b
	d. x'	y	2nd A'	x
	e. y'	x	2nd B'	y

PROGRAM LISTING

DISPLAY	KEY	DISPLAY	KEY	DISPLAY	KEY	DISPLAY	KEY
000 46	*LBL	028 01	1	056 43	RCL	084 00	0
001 19	*D'	029 94	+/−	057 00	0	085 01	1
002 95	=	030 44	SUM	058 00	0	086 44	SUM
003 94	+/−	031 00	0	059 56	*rtn	087 00	0
004 42	STO	032 03	3	060 46	*LBL	088 03	3
005 00	0	033 40	*x^2	061 12	B	089 40	*x^2
006 02	2	034 94	+/−	062 42	STO	090 44	SUM
007 44	SUM	035 44	SUM	063 00	0	091 00	0
008 00	0	036 00	0	064 02	2	092 06	6
009 04	4	037 06	6	065 44	SUM	093 01	1
010 40	*x^2	038 01	1	066 00	0	094 41	GTO
011 94	+/−	039 94	+/−	067 04	4	095 88	*2'
012 44	SUM	040 46	*LBL	068 40	*x^2	096 46	*LBL
013 00	0	041 88	*2'	069 44	SUM	097 14	D
014 07	7	042 44	SUM	070 00	0	098 43	RCL
015 43	RCL	043 00	0	071 07	7	099 00	0
016 00	0	044 00	0	072 43	RCL	100 03	3
017 02	2	045 43	RCL	073 00	0	101 55	÷
018 65	X	046 00	0	074 02	2	102 43	RCL
019 43	RCL	047 00	0	075 65	X	103 00	0
020 00	0	048 75	−	076 43	RCL	104 00	0
021 01	1	049 01	1	077 00	0	105 65	X
022 95	=	050 95	=	078 01	1	106 42	STO
023 44	SUM	051 30	*\ $\bar{x}$	079 95	=	107 01	1
024 00	0	052 42	STO	080 44	SUM	108 00	0
025 05	5	053 00	0	081 00	0	109 43	RCL
026 43	RCL	054 08	8	082 05	5	110 00	0
027 00	0	055 25	CLR	083 43	RCL	111 04	4

*Denotes 2nd function key

REGISTERS

00 n	04 Σxy	08 $\bar{x}$	12
01 x	05 Σx^2	09 m	13
02 y	06 Σy^2	10 Used	14
03 Σx	07 $\sqrt{n-1}$	11 Used	15
04 Σy	08	12	16

FLAGS

0		2		4

PROGRAM LISTING

DISPLAY	KEY	DISPLAY	KEY	DISPLAY	KEY	DISPLAY	KEY
112 75	−	140 77	*4'	168 85	+	196 65	X
113 43	RCL	141 42	STO	169 43	RCL	197 43	RCL
114 00	0	142 01	1	170 01	1	198 01	1
115 05	5	143 02	2	171 02	2	199 03	3
116 95	=	144 46	*LBL	172 95	=	200 55	÷
117 94	+/−	145 15	E	173 55	÷	201 53	(
118 42	STO	146 14	D	174 46	*LBL	202 43	RCL
119 01	1	147 65	X	175 87	*1'	203 00	0
120 03	3	148 43	RCL	176 43	RCL	204 04	4
121 55	÷	149 00	0	177 01	1	205 40	*x^2
122 53	(	150 03	3	178 01	1	206 55	÷
123 43	RCL	151 94	+/−	179 95	=	207 43	RCL
124 00	0	152 85	+	180 56	*rtn	208 00	0
125 06	6	153 43	RCL	181 46	*LBL	209 00	0
126 75	−	154 00	0	182 17	*B'	210 94	+/−
127 43	RCL	155 04	4	183 51	SBR	211 85	+
128 00	0	156 95	=	184 77	*4'	212 43	RCL
129 03	3	157 55	÷	185 85	+	213 00	0
130 65	X	158 43	RCL	186 53	(	214 07	7
131 43	RCL	159 00	0	187 43	RCL	215 95	=
132 01	1	160 00	0	188 01	1	216 56	*rtn
133 00	0	161 95	=	189 02	2	217 46	*LBL
134 95	=	162 56	*rtn	190 65	X	218 11	A
135 42	STO	163 46	*LBL	191 41	GTO	219 42	STO
136 01	1	164 16	*A'	192 87	*1'	220 00	0
137 01	1	165 51	SBR	193 46	*LBL	221 01	1
138 56	*rtn	166 77	*4'	194 13	C	222 56	*rtn
139 46	*LBL	167 94	+/−	195 14	D	223 00	

*Denotes 2nd function key

POWER CURVE FIT

Given a set of data points:

$$\{x_i, y_i),\ i = 1, 2, \ldots, n\}$$

Where: $x_i > 0$ and $y_i > 0$, this program will fit a power curve:

$$y = ax^b \qquad (a > 0)$$

The problem may be interpreted as a linear regression problem by using the equation in the form:

$$\ln y = b \ln x + \ln a$$

The computed statistics are:

1. Regression coefficients

$$a = \exp\left[\frac{\Sigma \ln y_i}{n} - \frac{b \Sigma \ln x_i}{n}\right]$$

$$b = \frac{\Sigma(\ln x_i)(\ln y_i) - \frac{(\Sigma \ln x_i)(\Sigma \ln y_i)}{n}}{\Sigma(\ln x_i)^2 - \frac{(\Sigma \ln x_i)^2}{n}}$$

NOTE: n is a positive integer ≠ 1.

2. Correlation coefficient

$$r^2 = \frac{\left[\Sigma (\ln x_i)(\ln y_i) - \frac{(\Sigma \ln x_i)(\Sigma \ln y_i)}{n}\right]^2}{\left[\Sigma (\ln x_i)^2 - \frac{(\Sigma \ln x_i)^2}{n}\right]\left[\Sigma (\ln y_i)^2 - \frac{(\Sigma \ln y_i)^2}{n}\right]}$$

3. Estimated value x′ for given y

$$x' = \sqrt[b]{y/a}$$

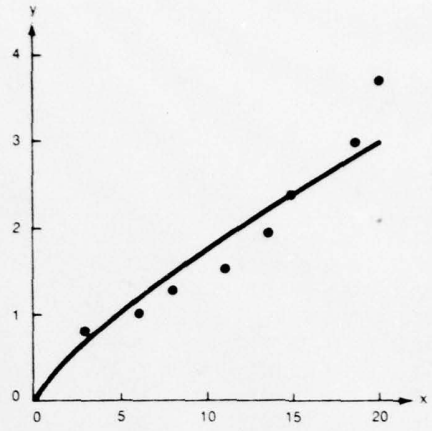
4. Estimated value y′ for given x

$$y' = ax^b$$

Example:

x_i	3	6	8	11	13.5	15	18.5	20
y_i	.8	1.12	1.3	1.53	1.95	2.4	3.0	3.7

a = .2901902009
b = .7759169351
r^2 = 0.921727263
for x = 9 , y′ = 1.596230726
for y = 2.8, x′ = 18.56904361



TEXAS INSTRUMENTS

ST1-09 ◀B▶ POWER CURVE FIT

◀A▶ POWER CURVE FIT ST1-09

delete	y → x′	x → y′	b	
INIT	x_i	y_i	a	r^2

USER INSTRUCTIONS

STEP	PROCEDURE	ENTER	PRESS	DISPLAY
1	Enter Program (A and B)			
2	*Initialize*		A	0
3	Enter data	x_i	B	x_i
	(repeat for all i)	y_i	C	n
4	Delete data			
	a. Enter x	x	B	x
	b. Enter y	y	2nd B′	n − 1
5	Calculate regression		D	a
	coefficients		2nd D′	b
6	Calculate correlation		E	r^2
	coefficient			
7	Compute estimated			
	value for			
	x′ given y	y	2nd E′	x
	y′ given x	x	2nd C′	y

PROGRAM LISTING

DISPLAY	KEY	DISPLAY	KEY	DISPLAY	KEY	DISPLAY	KEY
000 46	*LBL	028 00	0	056 46	*LBL	084 00	0
001 11	A	029 06	6	057 16	*A'	085 02	2
002 47	*CMs	030 44	SUM	058 23	lnx	086 43	RCL
003 25	CLR	031 00	0	059 42	STO	087 00	0
004 56	*rtn	032 01	1	060 00	0	088 07	7
005 46	*LBL	033 49	*PROD	061 07	7	089 22	INV
006 12	B	034 00	0	062 22	INV	090 44	SUM
007 23	lnx	035 07	7	063 44	SUM	091 00	0
008 42	STO	036 40	*x^2	064 00	0	092 04	4
009 00	0	037 44	SUM	065 03	3	093 01	1
010 06	6	038 00	0	066 40	*x^2	094 94	+/−
011 22	INV	039 02	2	067 22	INV	095 41	GTO
012 23	lnx	040 43	RCL	068 44	SUM	096 87	*1'
013 56	*rtn	041 00	0	069 00	0	097 46	*LBL
014 46	*LBL	042 07	7	070 05	5	098 14	D
015 13	C	043 44	SUM	071 43	RCL	099 43	RCL
016 23	lnx	044 00	0	072 00	0	100 00	0
017 42	STO	045 04	4	073 06	6	101 02	2
018 00	0	046 01	1	074 22	INV	102 75	−
019 07	7	047 46	*LBL	075 44	SUM	103 43	RCL
020 44	SUM	048 87	*1'	076 00	0	104 00	0
021 00	0	049 44	SUM	077 01	1	105 01	1
022 03	3	050 00	0	078 49	*PROD	106 40	*x^2
023 40	*x^2	051 00	0	079 00	0	107 55	÷
024 44	SUM	052 43	RCL	080 07	7	108 43	RCL
025 00	0	053 00	0	081 40	*x^2	109 00	0
026 05	5	054 00	0	082 22	INV	110 00	0
027 43	RCL	055 56	*rtn	083 44	SUM	111 95	=

*Denotes 2nd function key

REGISTERS

00 n	05 $\Sigma(lny)^2$	10 Used	15
01 Σlnx	06 lnx	11 Used	16
02 $\Sigma(lnx)^2$	07 lny	12 r^2	17
03 Σlny	08 b	13	18
04 Σ(lnx)(lny)	09 a	14	19

FLAGS

	1	2	3	4

PROGRAM LISTING

DISPLAY	KEY	DISPLAY	KEY	DISPLAY	KEY	DISPLAY	KEY
112 20	*1/x	140 08	8	168 46	*LBL	196 01	1
113 42	STO	141 94	+/−	169 17	*B'	197 00	0
114 01	1	142 65	X	170 55	÷	198 65	X
115 00	0	143 43	RCL	171 43	RCL	199 53	(
116 65	X	144 00	0	172 00	0	200 43	RCL
117 53	(	145 01	1	173 09	9	201 00	0
118 43	RCL	146 85	+	174 95	=	202 05	5
119 00	0	147 43	RCL	175 35	$\sqrt{\bar{y}}$	203 75	−
120 04	4	148 00	0	176 43	RCL	204 43	RCL
121 75	−	149 03	3	177 00	0	205 00	0
122 43	RCL	150 95	=	178 08	8	206 03	3
123 00	0	151 55	÷	179 95	=	207 40	*x^2
124 01	1	152 43	RCL	180 56	*rtn	208 55	÷
125 65	X	153 00	0	181 46	*LBL	209 43	RCL
126 43	RCL	154 00	0	182 18	*C'	210 00	0
127 00	0	155 95	=	183 45	y^x	211 00	0
128 03	3	156 22	INV	184 43	RCL	212 54	)
129 55	÷	157 23	lnx	185 00	0	213 20	*1/x
130 43	RCL	158 42	STO	186 08	8	214 65	X
131 00	0	159 00	0	187 65	X	215 43	RCL
132 00	0	160 09	9	188 43	RCL	216 01	1
133 54	)	161 56	*rtn	189 00	0	217 01	1
134 42	STO	162 46	*LBL	190 09	9	218 40	*x^2
135 01	1	163 19	*D'	191 95	=	219 95	=
136 01	1	164 43	RCL	192 56	*rtn	220 42	STO
137 95	=	165 00	0	193 46	*LBL	221 01	1
138 42	STO	166 08	8	194 15	E	222 02	2
139 00	0	167 56	*rtn	195 43	RCL	223 56	*rtn

*Denotes 2nd function key

EXPONENTIAL CURVE FIT

Given a set of data points:

$$\{(x_i, y_i), i = 1, 2, \ldots, n\} \qquad y_i > 0$$

This program finds the least-squares fit for an exponential function of the form:

$$y = ae^{bx} \quad (a > 0)$$

The problem is computed according to the linear equation

$$\ln y = \ln a + bx$$

Statistical outputs:

1. Coefficients a, b

$$a = \exp\left[\frac{\Sigma \ln y_i}{n} - b\frac{\Sigma x_i}{n}\right]$$

$$b = \frac{\Sigma x_i \ln y_i - \frac{\Sigma x_i \Sigma \ln y_i}{n}}{\Sigma x_i^2 - \frac{(\Sigma x_i)^2}{n}}$$

Note: n is positive integer ≥ 1.

2. Correlation coefficient

$$r^2 = \frac{\left[\Sigma x_i \ln y_i - \frac{\Sigma x_i \Sigma \ln y_i}{n}\right]^2}{\left[\Sigma x_i^2 - \frac{(\Sigma x_i)^2}{n}\right]\left[\Sigma (\ln y_i)^2 - \frac{(\Sigma \ln y_i)^2}{n}\right]},$$

3. Estimated value for x′ given y

$$x' = \frac{1}{b}\ln\left(\frac{y}{a}\right)$$

4. Estimated value for y′ given x

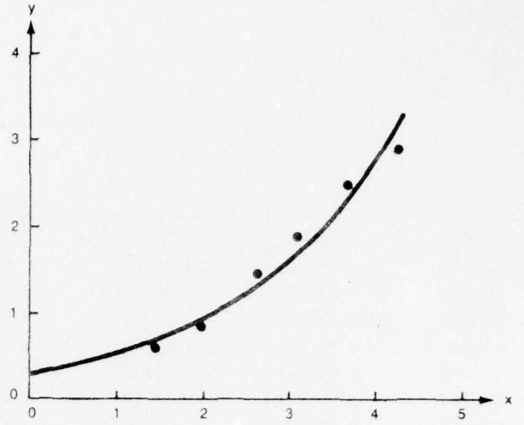
$$y' = ae^{bx}$$

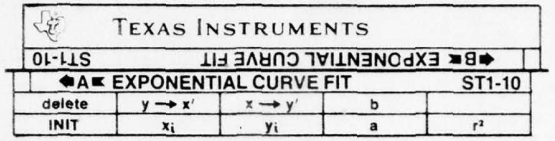
Example:

x_i	1.42	1.94	2.63	3.11	3.7	4.25
y_i	.63	.9	1.42	1.9	2.43	2.89

$a = .3131935086$
$b = .5477927735$
$r^2 = .9810626986$

For $x = 1.6$, $y' = .7524162355$
For $y = 2.2$, $x' = 3.558629277$





USER INSTRUCTIONS

STEP	PROCEDURE	ENTER	PRESS	DISPLAY
1	Enter Program (A and B)			
2	Initialize		A	0.
3	Enter data	x_i	B	x_i
	(repeat for all i)	y_i	C	n
4	Delete data			
	a. Enter x	x	B	x
	b. Enter y	y	2nd A′	n − 1
5	Calculate coefficients		D	a
			2nd D′	b
6	Calculate correlation			
	coefficient		E	r^2
7	Compute estimated			
	value for			
	x′ given y	y	2nd B′	x′
	y′ given x	x	2nd C′	y′

PROGRAM LISTING

DISPLAY	KEY	DISPLAY	KEY	DISPLAY	KEY	DISPLAY	KEY
000	46	*LBL	028	00	0	056	56	*rtn	084	44	SUM
001	10	*E'	029	05	5	057	46	*LBL	085	00	0
002	43	RCL	030	43	RCL	058	16	*A'	086	02	2
003	00	0	031	00	0	059	23	lnx	087	43	RCL
004	00	0	032	06	6	060	42	STO	088	00	0
005	56	*rtn	033	44	SUM	061	00	0	089	07	7
006	46	*LBL	034	00	0	062	07	7	090	22	INV
007	11	A	035	01	1	063	22	INV	091	44	SUM
008	47	*CMs	036	49	*PROD	064	44	SUM	092	00	0
009	25	CLR	037	00	0	065	00	0	093	04	4
010	56	*rtn	038	07	7	066	03	3	094	01	1
011	46	*LBL	039	40	*x^2	067	40	*x^2	095	94	+/−
012	12	B	040	44	SUM	068	22	INV	096	41	GTO
013	42	STO	041	00	0	069	44	SUM	097	87	*1'
014	00	0	042	02	2	070	00	0	098	46	*LBL
015	06	6	043	43	RCL	071	05	5	099	14	D
016	56	*rtn	044	00	0	072	43	RCL	100	43	RCL
017	46	*LBL	045	07	7	073	00	0	101	00	0
018	13	C	046	44	SUM	074	06	6	102	02	2
019	23	lnx	047	00	0	075	22	INV	103	75	−
020	42	STO	048	04	4	076	44	SUM	104	43	RCL
021	00	0	049	01	1	077	00	0	105	00	0
022	07	7	050	46	*LBL	078	01	1	106	01	1
023	44	SUM	051	87	*1'	079	49	*PROD	107	40	*x^2
024	00	0	052	44	SUM	080	00	0	108	55	÷
025	03	3	053	00	0	081	07	7	109	10	*E'
026	40	*x^2	054	00	0	082	40	*x^2	110	95	=
027	44	SUM	055	10	*E'	083	22	INV	111	20	*1/x

*Denotes 2nd function key

REGISTERS

00 n	05 $\Sigma(\ln y)^2$	10 Used	15
01 Σx	06 x	11 Used	16
02 Σx^2	07 x lny	12 r^2	17
03 $\Sigma \ln y$	08 b	13	18
04 $\Sigma x \ln y$	09 a	14	19

FLAGS

0	1	2	3	4

PROGRAM LISTING

DISPLAY	KEY	DISPLAY	KEY	DISPLAY	KEY	DISPLAY	KEY
112	42	STO	140	43	RCL	168	09	9	196	00	0
113	01	1	141	00	0	169	95	=	197	65	X
114	00	0	142	01	1	170	23	lnx	198	53	(
115	65	X	143	85	+	171	55	÷	199	43	RCL
116	53	(	144	43	RCL	172	43	RCL	200	00	0
117	43	RCL	145	00	0	173	00	0	201	05	5
118	00	0	146	03	3	174	08	8	202	75	−
119	04	4	147	95	=	175	95	=	203	43	RCL
120	75	−	148	55	÷	176	56	*rtn	204	00	0
121	43	RCL	149	10	*E'	177	46	*LBL	205	03	3
122	00	0	150	95	=	178	18	*C'	206	40	*x^2
123	01	1	151	22	INV	179	65	X	207	55	÷
124	65	X	152	23	lnx	180	43	RCL	208	10	*E'
125	43	RCL	153	42	STO	181	00	0	209	54	)
126	00	0	154	00	0	182	08	8	210	20	*1/x
127	03	3	155	09	9	183	95	=	211	65	X
128	55	÷	156	56	*rtn	184	22	INV	212	43	RCL
129	10	*E'	157	46	*LBL	185	23	lnx	213	01	1
130	54	)	158	19	*D'	186	65	X	214	01	1
131	42	STO	159	43	RCL	187	43	RCL	215	40	*x^2
132	01	1	160	00	0	188	00	0	216	95	=
133	01	1	161	08	8	189	09	9	217	42	STO
134	95	=	162	56	*rtn	190	95	=	218	01	1
135	42	STO	163	46	*LBL	191	56	*rtn	219	02	2
136	00	0	164	17	*B'	192	46	*LBL	220	56	*rtn
137	08	8	165	55	÷	193	15	E	221	00	
138	94	+/−	166	43	RCL	194	43	RCL	222	00	
139	65	X	167	00	0	195	01	1	223	00	

*Denotes 2nd function key

LOGARITHMIC CURVE FIT

Given a set of data points:

$$\{(x_i, y_i),\ i = 1, 2, \ldots, n\}$$

Where: $x_i > 0$, this program fits a logarithmic curve:

$$y = a + b\ln x$$

Statistical outputs:

1. Regression coefficients

$$a = \frac{\Sigma y_i - b\Sigma \ln x_i}{n}$$

$$b = \frac{\Sigma y_i \ln x_i - \frac{\Sigma \ln x_i \Sigma y_i}{n}}{\Sigma (\ln x_i)^2 - \frac{\Sigma (\ln x_i)^2}{n}}$$

2. Correlation coefficient

$$r^2 = \frac{\left[\Sigma y_i \ln x_i - \frac{\Sigma \ln x_i \Sigma y_i}{n}\right]^2}{\left[\Sigma (\ln x_i)^2 - \frac{\Sigma (\ln x_i)^2}{n}\right]\left[\Sigma y_i^2 - \frac{(\Sigma y_i)^2}{n}\right]}$$

3. Estimated value for x′ given y

$$x' = \exp\left[\frac{y - a}{b}\right]$$

4. Estimated value for y′ given x

$$y' = a + b\ln x$$

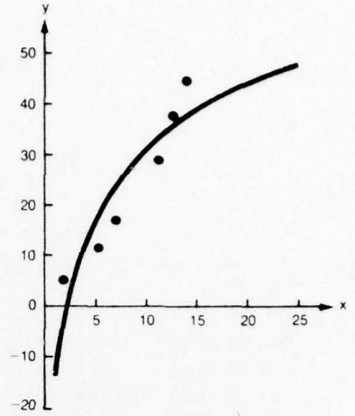
NOTE: n is positive integer ≥ 1.

Example:

x_i	2	5	7	11	13	14
y_i	5.3	10.8	17.5	28.2	36.9	44.4

$a = -13.49953513$
$b = 18.91052316$
$r^2 = .8418448418$

For x = 21. y′ = 44.07397695
For y = 40. x′ = 16.93001448



TEXAS INSTRUMENTS

ST1-11 ◄B► LOGARITHMIC CURVE FIT

◄A► LOGARITHMIC CURVE FIT ST1-11

delete	y → x′	x → y′	b	
INIT	x_i	y_i	*a*	r^2

USER INSTRUCTIONS

STEP	PROCEDURE	ENTER	PRESS	DISPLAY
1	Enter Program (A and B)			
2	Initialize		A	0
3	Enter data	x_i	B	x_i
	(repeat for all i)	y_i	C	n
4	Delete data			
	a. Enter x	x	B	x
	b. Enter y	y	2nd A′	n − 1
5	Calculate coefficients		D	a
			2nd D′	b
6	Calculate correlation			
	coefficient		E	r^2
7	Compute estimated			
	value for			
	x′ given y	y	2nd E′	x′
	y′ given x	x	2nd C′	y′

PROGRAM LISTING

DISPLAY	KEY	DISPLAY	KEY	DISPLAY	KEY	DISPLAY	KEY
000 46	*LBL	028 00	0	056 00	0	084 00	0
001 11	A	029 01	1	057 07	7	085 07	7
002 47	*CMs	030 49	*PROD	058 22	INV	086 22	INV
003 25	CLR	031 00	0	059 44	SUM	087 44	SUM
004 56	*rtn	032 07	7	060 00	0	088 00	0
005 46	*LBL	033 40	*x^2	061 03	3	089 04	4
006 12	B	034 44	SUM	062 40	*x^2	090 01	1
007 42	STO	035 00	0	063 22	INV	091 94	+/−
008 00	0	036 02	2	064 44	SUM	092 41	GTO
009 06	6	037 43	RCL	065 00	0	093 87	*1'
010 56	*rtn	038 00	0	066 05	5	094 46	*LBL
011 46	*LBL	039 07	7	067 43	RCL	095 14	D
012 13	C	040 44	SUM	068 00	0	096 43	RCL
013 42	STO	041 00	0	069 06	6	097 00	0
014 00	0	042 04	4	070 23	lnx	098 02	2
015 07	7	043 01	1	071 22	INV	099 75	−
016 44	SUM	044 46	*LBL	072 44	SUM	100 43	RCL
017 00	0	045 87	*1'	073 00	0	101 00	0
018 03	3	046 44	SUM	074 01	1	102 01	1
019 40	*x^2	047 00	0	075 49	*PROD	103 40	*x^2
020 44	SUM	048 00	0	076 00	0	104 55	÷
021 00	0	049 43	RCL	077 07	7	105 43	RCL
022 05	5	050 00	0	078 40	*x^2	106 00	0
023 43	RCL	051 00	0	079 22	INV	107 00	0
024 00	0	052 56	*rtn	080 44	SUM	108 95	=
025 06	6	053 46	*LBL	081 00	0	109 20	*1/x
026 23	lnx	054 16	*A'	082 02	2	110 42	STO
027 44	SUM	055 42	STO	083 43	RCL	111 01	1

*Denotes 2nd function key

REGISTERS

00 n	05 Σy^2	10 Used	15
01 Σlnx	06 x	11 Used	16
02 $\Sigma(\ln x)^2$	07 y	12 r^2	17
03 Σy	08 b	13	18
04 $\Sigma y \ln x$	09 a	14	19

FLAGS

0	1	2	3	4

PROGRAM LISTING

DISPLAY	KEY	DISPLAY	KEY	DISPLAY	KEY	DISPLAY	KEY
112 00	0	140 43	RCL	168 65	X	196 75	−
113 65	X	141 00	0	169 53	(	197 43	RCL
114 53	(	142 01	1	170 43	RCL	198 00	0
115 43	RCL	143 85	+	171 00	0	199 09	9
116 00	0	144 43	RCL	172 05	5	200 95	=
117 04	4	145 00	0	173 75	−	201 55	÷
118 75	−	146 03	3	174 43	RCL	202 43	RCL
119 43	RCL	147 95	=	175 00	0	203 00	0
120 00	0	148 55	÷	176 03	3	204 08	8
121 01	1	149 43	RCL	177 40	*x^2	205 95	=
122 65	X	150 00	0	178 55	÷	206 22	INV
123 43	RCL	151 00	0	179 43	RCL	207 23	lnx
124 00	0	152 95	=	180 00	0	208 56	*rtn
125 03	3	153 42	STO	181 00	0	209 46	*LBL
126 55	÷	154 00	0	182 54	)	210 18	*C'
127 43	RCL	155 09	9	183 20	*1/x	211 23	lnx
128 00	0	156 56	*rtn	184 65	X	212 65	X
129 00	0	157 46	*LBL	185 43	RCL	213 43	RCL
130 54	)	158 19	*D'	186 01	1	214 00	0
131 42	STO	159 43	RCL	187 01	1	215 08	8
132 01	1	160 00	0	188 40	*x^2	216 85	+
133 01	1	161 08	8	189 95	=	217 43	RCL
134 95	=	162 56	*rtn	190 42	STO	218 00	0
135 42	STO	163 46	*LBL	191 01	1	219 09	9
136 00	0	164 15	E	192 02	2	220 95	=
137 08	8	165 43	RCL	193 56	*rtn	221 56	*rtn
138 94	+/−	166 01	1	194 46	*LBL	222 00	
139 65	X	167 00	0	195 17	*B'	223 00	

*Denotes 2nd function key

NORMAL DISTRIBUTION

In order to calculate the standard normal distribution, the following probability function is used:

$$Z(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}$$

Then: $Q(x) = Z(x)(b_1t + b_2t^2 + b_3t^3 + b_4t^4 + b_5t^5) + \epsilon(x)$

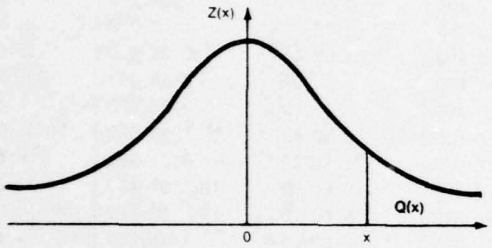
Where: $t = \frac{1}{1 + px}$

$|\epsilon(x)| < 7.5 \times 10^{-8}$
$p = .2316419$
$b_1 = .319381530$
$b_2 = -.356563782$
$b_3 = 1.781477937$
$b_4 = -1.821255978$
$b_5 = 1.330274429$

NOTES: 1. $Z(-x) = Z(x)$, $P(x) = 1 - Q(x)$
2. $|x| < 21.25$
3. $Z(x)$ must be calculated before $Q(x)$

Reference: *Handbook of Mathematical Functions*, National Bureau of Standards, 1964

Example: For $x = 2.02$, $Z(x) = .0518635767$, $Q(x) = .0216916245$



TEXAS INSTRUMENTS

ST1-15 ◆B■ NORMAL DISTRIBUTION

◆A■ NORMAL DISTRIBUTION ST1-15

x ⟶ Z(x)	Q(x)			

USER INSTRUCTIONS

STEP	PROCEDURE	ENTER	PRESS	DISPLAY
1	Enter Program (A and B)			
2	Enter data	x	A	Z(x)
3	Calculate Q(x)		B	Q(x)

PROGRAM LISTING

DISPLAY	KEY	DISPLAY	KEY	DISPLAY	KEY	DISPLAY	KEY
000	46 *LBL	028	30 *√x	056	65 X	084	43 RCL
001	11 A	029	95 =	057	93 •	085	00 0
002	22 INV	030	42 STO	058	03 3	086	02 2
003	50 *st flg	031	00 0	059	01 1	087	45 y`
004	01 1	032	01 1	060	09 9	088	03 3
005	80 *if pos	033	56 *rtn	061	03 3	089	65 X
006	87 *1'	034	46 *LBL	062	08 8	090	01 1
007	94 +/−	035	12 B	063	01 1	091	93 •
008	50 *st flg	036	25 CLR	064	05 5	092	07 7
009	01 1	037	93 •	065	03 3	093	08 8
010	46 *LBL	038	02 2	066	75 −	094	01 1
011	87 *1'	039	03 3	067	43 RCL	095	04 4
012	42 STO	040	01 1	068	00 0	096	07 7
013	00 0	041	06 6	069	02 2	097	07 7
014	00 0	042	04 4	070	45 y`	098	09 9
015	40 *x²	043	01 1	071	02 2	099	03 3
016	55 ÷	044	09 9	072	65 X	100	07 7
017	02 2	045	65 X	073	93 •	101	75 −
018	95 =	046	43 RCL	074	03 3	102	43 RCL
019	94 +/−	047	00 0	075	05 5	103	00 0
020	22 INV	048	00 0	076	06 6	104	02 2
021	23 !nx	049	85 +	077	05 5	105	45 y`
022	55 ÷	050	01 1	078	06 6	106	04 4
023	53 (	051	95 =	079	03 3	107	65 X
024	59 *π	052	20 *1/x	080	07 7	108	01 1
025	65 X	053	42 STO	081	08 8	109	93 •
026	02 2	054	00 0	082	02 2	110	08 8
027	54)	055	02 2	083	85 +	111	02 2

*Denotes 2nd function key

REGISTERS

00 Used	05	10	15
01 Used	06	11	16
02 Used	07	12	17
03	08	13	18
04	09	14	19

FLAGS

0	1 Used	2	3	4

PROGRAM LISTING

DISPLAY	KEY	DISPLAY	KEY	DISPLAY	KEY	DISPLAY	KEY
112	01 1	140	00 0	168	00	196	00
113	02 2	141	01 1	169	00	197	00
114	05 5	142	95 =	170	00	198	00
115	05 5	143	60 *if flg	171	00	199	00
116	09 9	144	01 1	172	00	200	00
117	07 7	145	69 *9'	173	00	201	00
118	08 8	146	56 *rtn	174	00	202	00
119	85 +	147	46 *LBL	175	00	203	00
120	43 RCL	148	69 *9'	176	00	204	00
121	00 0	149	94 +/−	177	00	205	00
122	02 2	150	85 +	178	00	206	00
123	45 y`	151	01 1	179	00	207	00
124	05 5	152	95 =	180	00	208	00
125	65 X	153	56 *rtn	181	00	209	00
126	01 1	154	00	182	00	210	00
127	93 •	155	00	183	00	211	00
128	03 3	156	00	184	00	212	00
129	03 3	157	00	185	00	213	00
130	00 0	158	00	186	00	214	00
131	02 2	159	00	187	00	215	00
132	07 7	160	00	188	00	216	00
132	04 4	161	00	189	00	217	00
134	04 4	162	00	190	00	218	00
135	02 2	163	00	191	00	219	00
136	09 9	164	00	192	00	220	00
137	95 =	165	00	193	00	221	00
138	65 X	166	00	194	00	222	00
139	43 RCL	167	00	195	00	223	00

*Denotes 2nd function key

CHI-SQUARE DISTRIBUTION

The chi-square density function is calculated by

$$p_\nu f(x) = \frac{e^{-\frac{x}{2}} x^{\left(\frac{\nu}{2}-1\right)}}{2^{\frac{\nu}{2}} \Gamma\left(\frac{\nu}{2}\right)}$$

where ν = degrees of freedom, $0 < \nu \leq 141$, $x > 0$ and $\ln x \leq \frac{200 \ln 10}{(\nu - 2)}$

The following series expansion calculates the cumulative distribution:

$$P(x|\nu) = \left(\frac{x}{2}\right)^{\frac{\nu}{2}} \frac{e^{-\frac{\lambda}{2}}}{\Gamma\left(\frac{\nu+2}{2}\right)} \left[1 + \sum_{r=1}^{\infty} \frac{x^r}{(\nu+2)(\nu+4) \ldots (\nu+2r)}\right]$$

Where: $\Gamma(\nu+1) = \nu\Gamma(\nu) = \nu! = \nu(\nu-1)!$

and $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$

NOTES: 1. If the degrees of freedom is entered as a non-integer, it is rounded up to an integer for the calculation.
2. If ν or x is negative or zero, a flashing 0 is returned.
3. A ν must be entered before an x, and both must be entered before P is calculated.
4. A new x may be entered without changing ν.
5. An x must be entered each time P is to be calculated.

Example: For $\nu = 10$, $\Gamma\left(\frac{\nu}{2}\right) = 24$ and if x = 8.5, f(x) = .0969533757, and P = .4198816863

Reference: *An Introduction to Probability and Stochastic Processes*; James L. Melsa, Andrew P. Sage; Prentice-Hall Electrical Eng. Series, 1973

TEXAS INSTRUMENTS

ST1-16 ◀B▶ CHI-SQUARE DISTRIBUTION

◀A▶ CHI-SQUARE DISTRIBUTION ST1-16

$\nu \rightarrow \Gamma(\nu/2)$	x → f(x)	P		

USER INSTRUCTIONS

STEP	PROCEDURE	ENTER	PRESS	DISPLAY
1	Enter Program (A and B)			
2	Enter degrees			
	of freedom	ν	A	$\Gamma(\frac{\nu}{2})$
3	Enter x	x	B	f(x)
4	Calculate P		C	P

PROGRAM LISTING

DISPLAY	KEY	DISPLAY	KEY	DISPLAY	KEY	DISPLAY	KEY
000 46	*LBL	028 75	−	056 04	4	084 52	EE
001 43	RCL	029 48	*EXC	057 49	*PROD	085 57	*fix
002 02	2	030 00	0	058 00	0	086 00	0
003 44	SUM	031 06	6	059 03	3	087 52	EE
004 00	0	032 95	=	060 46	*LBL	088 22	INV
005 08	8	033 22	INV	061 42	STO	089 52	EE
006 43	RCL	034 90	*if zro	062 43	RCL	090 57	*fix
007 00	0	035 43	RCL	063 00	0	091 09	9
008 02	2	036 43	RCL	064 04	4	092 22	INV
009 55	÷	037 00	0	065 75	−	093 80	*if pos
010 43	RCL	038 06	6	066 93	•	094 10	*E'
011 00	0	039 49	*PROD	067 05	5	095 90	*if zro
012 08	8	040 00	0	068 95	=	096 10	*E'
013 65	X	041 05	5	069 22	INV	097 42	STO
014 43	RCL	042 43	RCL	070 90	*if zro	098 00	0
015 00	0	043 00	0	071 44	SUM	099 07	7
016 04	4	044 05	5	072 59	*π	100 55	÷
017 85	+	045 56	*rtn	073 30	*√x	101 01	1
018 42	STO	046 46	*LBL	074 49	*PROD	102 42	STO
019 00	0	047 44	SUM	075 00	0	103 00	0
020 04	4	048 43	RCL	076 03	3	104 03	3
021 43	RCL	049 00	0	077 43	RCL	105 02	2
022 00	0	050 04	4	078 00	0	106 95	=
023 06	6	051 75	−	079 03	3	107 42	STO
024 95	=	052 01	1	080 56	*rtn	108 00	0
025 52	EE	053 95	=	081 46	*LBL	109 01	1
026 22	INV	054 42	STO	082 11	A	110 42	STO
027 52	EE	055 00	0	083 22	INV	111 00	0

*Denotes 2nd function key

REGISTERS

00	Used		
01 Used	06 Used	11	16
02 Used	07 Used	12	17
03 Used	08 Used	13	18
04 Used	09	14	19

FLAGS

0	1	2	3	4

PROGRAM LISTING

DISPLAY	KEY	DISPLAY	KEY	DISPLAY	KEY	DISPLAY	KEY
112 04	4	140 56	*rtn	168 94	+/−	196 43	RCL
113 75	−	141 46	*LBL	169 54	)	197 00	0
114 53	(	142 12	B	170 22	INV	198 01	1
115 57	*fix	143 22	INV	171 23	lnx	199 95	=
116 00	0	144 52	EE	172 55	÷	200 49	*PROD
117 75	−	145 22	INV	173 43	RCL	201 00	0
118 93	•	146 80	*if pos	174 00	0	202 05	5
119 05	5	147 10	*E'	175 03	3	203 43	RCL
120 54	)	148 90	*if zro	176 55	÷	204 00	0
121 52	EE	149 10	*E'	177 02	2	205 07	7
122 22	INV	150 42	STO	178 45	y^	206 42	STO
123 52	EE	151 00	0	179 43	RCL	207 00	0
124 57	*fix	152 02	2	180 00	0	208 08	8
125 09	9	153 45	y^	181 01	1	209 01	1
126 95	=	154 53	(	182 95	=	210 42	STO
127 22	INV	155 43	RCL	183 42	STO	211 00	0
128 90	*if zro	156 00	0	184 00	0	212 06	6
129 42	STO	157 01	1	185 04	4	213 42	STO
130 43	RCL	158 75	−	186 42	STO	214 00	0
131 00	0	159 01	1	187 00	0	215 04	4
132 01	1	160 95	=	188 05	5	216 41	GTO
133 75	−	161 65	X	189 56	*rtn	217 43	RCL
134 01	1	162 53	(	190 46	*LBL	218 46	*LBL
135 95	=	163 43	RCL	191 13	C	219 10	*E'
136 29	*x!	164 00	0	192 43	RCL	220 00	0
137 42	STO	165 02	2	193 00	0	221 20	*1/x
138 00	0	166 55	÷	194 02	2	222 00	0
139 03	3	167 02	2	195 55	÷	223 56	*rtn

*Denotes 2nd function key

*U.S.Government Printing Office: 1979 — 657-002/491