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Computer Simulation Applied to the Separation of Porous Leach Residue Solids From Liquor by Horizontal Belt Filtration

By Daniel T. Rogers and Roy T. Sorensen, Jr.

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UNITED STATES DEPARTMENT OF THE INTERIOR

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COMPUTER SIMULATION APPLIED TO THE SEPARATION OF POROUS LEACH RESIDUE SOLIDS FROM LIQUOR BY HORIZONTAL BELT FILTRATION

By Daniel T. Rogers[1] and Roy T. Sorenson, Jr.[2]

ABSTRACT

The Bureau of Mines, in its alumina miniplant project to investigate alumina recovery from domestic, nonbauxitic ores, has conducted research on the use of a hydrochloric acid leaching, gas sparging crystallization technology. An important element of this research is the efficient separation of undissolved, siliceous residue from $AlCl_3$-bearing leach liquors by continuous, horizontal, countercurrent vacuum belt filtration.

In an effort to calculate material balances quickly and to predict material balances based on different belt filtration configurations, the perfect-mixing-cells-in-series model (PMCS) for calculating material balances around belt filters was used. Because of the porous nature of the solids, the model produced erroneous material balances. Therefore a reliable model, the shrinking voids model, was developed postulating the presence of an unwashable voids liquor volume that decreases with decreasing liquor $AlCl_3$ concentration. This volume decrease postulation is equivalent to assuming that dilute liquors flow more freely, causing more voids liquor volume to become washable.

Least-squares based computer programs are provided, which are useful not only in producing material balances from plant data but also in predicting balances for untested configurations using the same feed materials.

INTRODUCTION

As the United States is dependent on foreign sources of bauxite for the alumina required for aluminum production, domestic kaolinitic clay, hereafter referred to as clay, is being explored as an alternate source of aluminum. The Bureau of Mines is presently testing technology in which aqueous hydrochloric acid (HCl) is used to leach aluminum from calcined clay. This leaching produces an aqueous aluminum chloride ($AlCl_3$) solution that must be separated from the siliceous solids not dissolved by HCl. Two possible methods for achieving this separation, vacuum filtration and classifier-thickener systems, were tested.

Because of the time required to calculate material balances from the filtration data obtained in the alumina miniplant operation, and the need to predict material balances for different possible belt filtration configurations, an existing computer model, the perfect-mixing-cells-in-series model (PMCS), was used for these calculations. As this model did not result in accurate calculations because of the porous nature of the solids, a new model, the shrinking voids model, was developed to achieve the following objectives:

1. Calculate material balances rapidly from alumina miniplant data.
2. Predict material balances for belt filtration configurations different from those tested.
3. Predict filtration balances for slurries containing porous solids, when the existing model is not expected to be usable.

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BELT FILTRATION MODELING

BACKGROUND

During investigations in the Bureau's alumina miniplant project for extracting alumina from clay by a hydrochloric acid leaching technology, separation of the undissolved, siliceous residue from the $AlCl_3$-bearing solution was found to be a major problem. The filtration was slow and the washing efficiencies were poor. Two systems for solids-liquid separation, a classifier-thickener system and a belt filtration system, were investigated. The belt filtration system has proven to be a satisfactory approach to the solids-liquid separation problem.

In an effort to calculate accurate material balances quickly around the belt filtration system and to predict results from various belt filtration configurations, it was necessary to devise a new modeling technique.

Filtration and Washing

Process Description

The method of solids-liquid separation covered in this report is horizontal vacuum belt filtration. The alumina miniplant filter consists of a moving belt onto which slurry (with or without the addition of flocculant solution) is deposited under the impetus of a vacuum. This deposition process separates the alumina-bearing liquor from the cake that the belt carries through one or more sprays (washes) for recovery of additional alumina values.

The filtrate at each stage is then transferred countercurrent to the direction of cake movement for use as a wash spray (see fig. 1). Countercurrent filtration and washing is generally considered the most efficient method for maximum product recovery for a given amount of wash water added in the final cake wash.

Definition of Washing Efficiency

Ideally, after the cake is washed in the filtration process, the wash liquor will have displaced all of the more concentrated solute in the liquor with wash liquor to give perfect washing. However, since perfect washing is seldom achieved, a measure of washing efficiency is needed to evaluate the process. The R value (equal to 100 minus the efficiency) measures the percent of solute remaining in the cake after washing, and subtracts any solute added in the wash.[3]

$$R = \frac{C_2 - C_w}{C_1 - C_w}(100), \tag{1}$$

where C_2 = solute concentration (pounds per gallon) in washed cake liquor,
C_1 = solute concentration (pounds per gallon) in feed cake liquor,
and C_w = solute concentration (pounds per gallon) in wash liquor.

This R value will, since it is actually a function of the wash ratio, N, of wash liquor volume to cake liquor volume, change as N changes.

This report will, however, refer to the residuals, R, only in passing because the ultimate goal is to determine solute losses through the final wash cake.

Prior Material Balance Calculations

Heretofore material balances for horizontal belt filtration in the clay-HCl miniplant were made by laborious

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[3] Dahlstrom, D. A., and Silverblatt, C. E. Continuous Vacuum and Pressure Filtration. Ch. in Solid/Liquid Separation Equipment Scaleup, ed. D. B. Purchas. Upland Press Ltd., Croydon, England, 1977, p. 477.

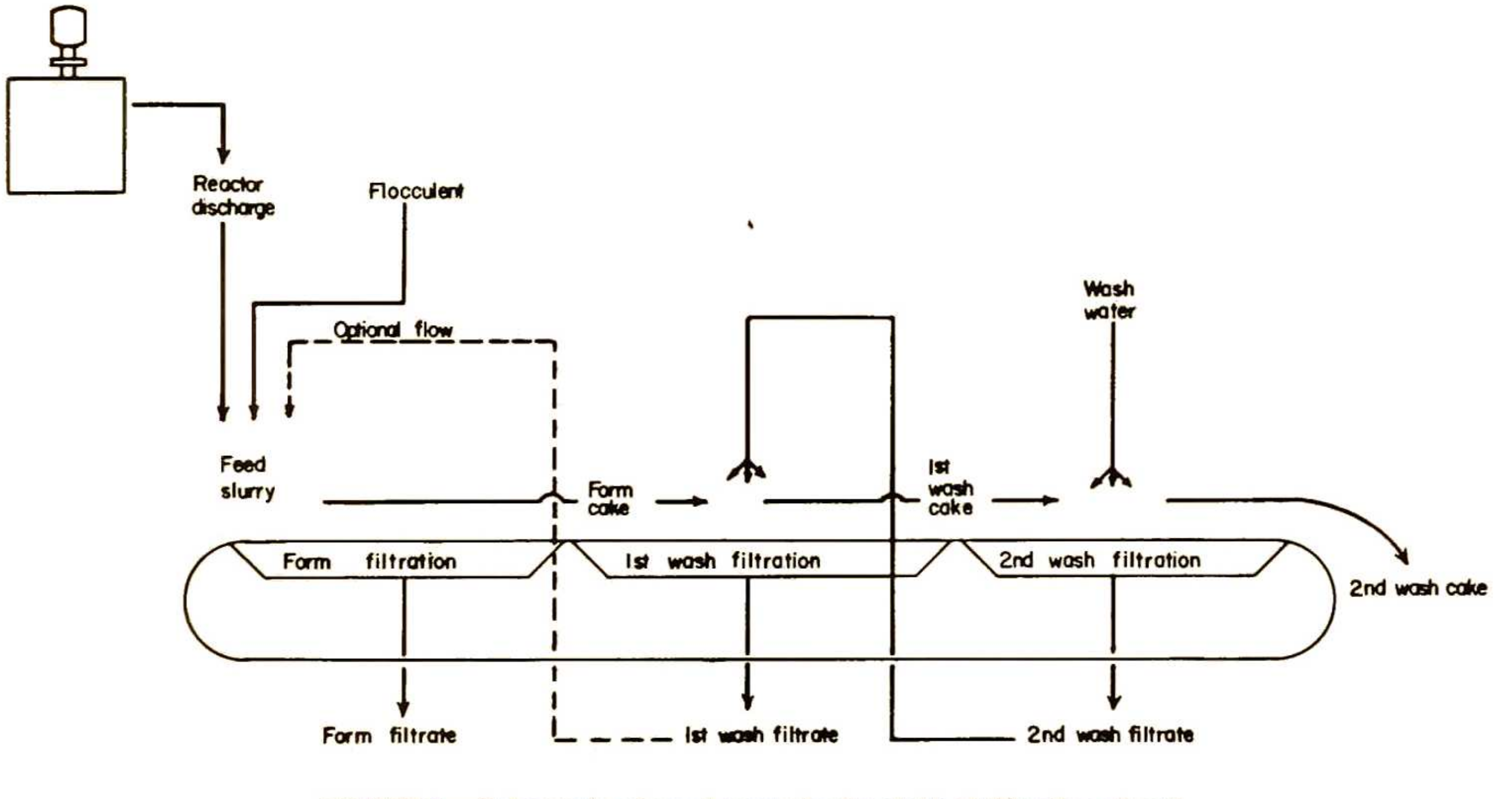

FIGURE 1.—Schematic of continuous horizontal belt filtration circuit.

calculations. A solids-liquid material balance was made from plant input flow rates and cake moisture contents. An alumina balance of the liquor stream was then made by best-fitting or equal-weighting the Al_2O_3 analysis of the liquor in the filtration test samples.

Using these methods, material balances were made for seven horizontal belt filtration tests, and the balances were later used for evaluating the models developed in this report.

Prior Horizontal Belt Filter Modeling

Previously, in order to predict material balances for circuits employing various wash liquor rates or number of washes, the following two assumptions were required:

1. R values for additional stages were equal to the average R value in the first two stages.
2. Cake moisture decreased in a straight line with the number of wash stages.

SELECTION OF A MODEL

Purpose of Model Development

Miniplant filtration data are most easily applied when summarized in a mathematical model. Such models must, of course, predict stream compositions as close as possible to those observed. The models should also predict reliable material balances for continuous filtration system configurations different from those tested in the miniplant. Such models facilitate selection of the most cost-effective filter system configuration. For example, both alumina losses through the final wash cake and the degree of product liquor dilution by wash water for any number of wash stages or any volume of wash water addition are predicted rather than measured.

Ground Rules for Model Development

The models in this study are all guided by theoretically logical constraints on the relationships among filtration flow streams. These constraints are applied in such a manner that the final material balance stream compositions differ minimally from measured compositions—using a least squares error procedure. The following logical constraints common to all these models are referred to as ground rules:

1. No solid or liquid losses occur.
2. The filtrates contain no solids.
3. Liquor volume input equals liquor volume output for each washing-filtration stage.
4. The ratio of solute in the aqueous phase to that in the solid phase remains constant (presumably no solute exists in the solid phase for the study material).
5. Total cake liquor volume is constant from one washing stage to the next.

The Existing Perfect-Mixing-Cells-in-Series (PMCS) Model

Description

The PMCS model[4] uses the previously mentioned ground rules. It assumes that cake washing is described by the complete equilibration of cake liquor with wash liquor as the later moves through each of a number of perfect-mixing cells in the cake. The number of these cells is an intensive property, j, which does not change with cake size.

Discussion

This model specifies that j must be an integer greater than zero. Testing of this model with alumina miniplant data reveals that unless j values are extrapolated to some value less than 1, the model fails completely. It fails because the leached solids are very porous, indicating an imperfect rather than a perfect mixing (PMCS model) condition.

The Diffusion Model

Description

Because the PMCS model inadequately describes filtration of the alumina miniplant solids, it was decided to develop a model that recognized the presence of a large voids volume inside the leached residue particles, which has been determined to be 54.6 pct (see appendix C).

The model assumes the cake liquor to be composed of the following two separate fractions:

Internal—liquor entrained in the pores of the solids.
External—liquor outside the pores and between the solid particles.

During a wash, the external fraction is washed from the cake solids according to the PMCS model (with j cells) and the internal fraction is untouched by the wash liquor. After excess liquor is filtered from the cake, the fractions diffuse into one another for a specified time (see fig. 2).

Discussion

The diffusion model is a considerable improvement over the PMCS model. However, it predicts that alumina residuals, R, are constant from one wash stage to the next. In the miniplant, however, R values actually decrease significantly with decreasing liquor concentrations. Attempts to eliminate this discrepancy lead to the development of a better model.

The Shrinking Voids Model

Description

This model assumes that washing efficiency changes as cake liquor concentration changes. Presumably, liquor viscosity decreases (fluidity increases) as the liquor becomes more dilute, which causes the solid particle voids liquors to be more readily washable (displaceable). This increased washing efficiency is expressed as a decrease in the volume of nondisplaceable voids as the liquor concentration decreases (see fig. 2).

Discussion

This model will be shown to be the best of the three treated in this report. However, it has not been verified using any other configuration than that in the miniplant two-stage washing and filtering system.

It might be objected that a more realistic model would incorporate $AlCl_3$ diffusion during, as well as after, washing. It is, however, felt that diffusion during washing is small, which makes the simplicity of the shrinking voids model very attractive.

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[4] Tomiak, A. Predict Performance of Belt-Filter Washing. Chem. Eng. v. 86, No. 9, Apr. 23, 1979, pp. 143-146.
Tomiak, A. Theoretical Recoveries in Filter Cake Reslurrying and Washing. AIChE J., v. 19, No. 1, January 1973, pp. 76-84.

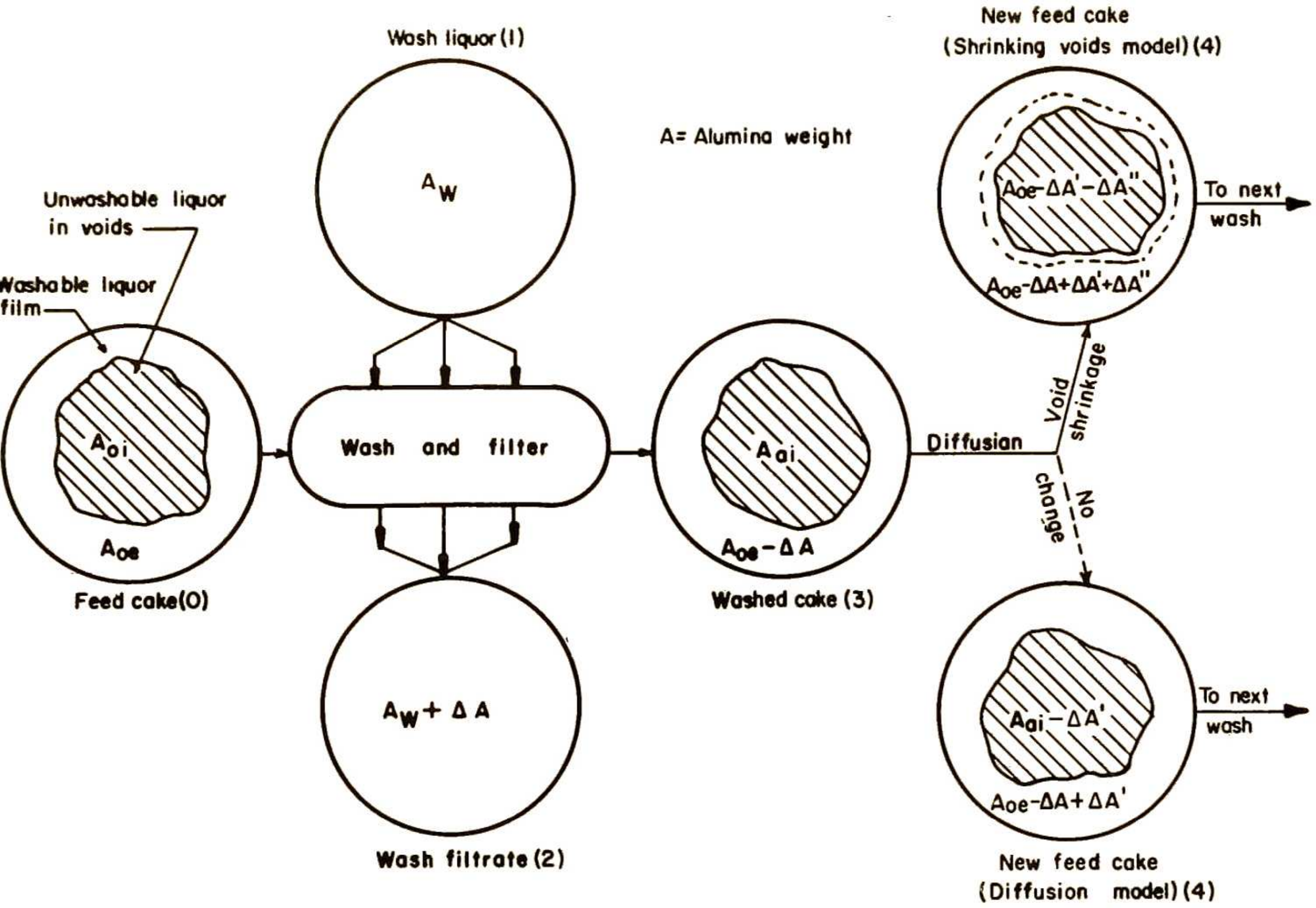

FIGURE 2.—Schematic for two mathematical models (diffusion and shrinking voids) of the displace washing of acid-leached calcined kaolin residues during continuous belt filtration.

DERIVATION OF EQUATIONS FOR A SINGLE WASH-FILTRATION STAGE

Fundamental PMCS Equations

The basic PMCS (with j perfect-mixing cells) equation describing the material balance around a single wash step is

$$\frac{A_1}{V_1} = \left(N\frac{A_2}{V_2} - f\frac{A_o}{V_o}\right) /(N-f), \qquad (2)$$

where N = V_1/V_o, the wash ratio,

f = fraction of salt removed during a wash with salt-free wash liquor,

$$f = 1 - \frac{e^{-jN}}{j} \sum_{k=0}^{j-1} (j-k) \frac{(jN)^k}{k!},$$

and A_n, V_n = alumina weights and liquor volumes for the liquor stream, n, in figure 3 (n = o for feed cake, n = 1 for wash liquor, n = 2 for wash filtrate, n = 3 for initial wash cake, and n = 4 for equilibrated wash cake).

Subscripts e and i attached to cake liquor volumes and alumina weights refer to external and internal fractions respectively.

Development of Diffusion and Shrinking Voids Model Equations

Balance Around the Wash Step

Both the diffusion and shrinking voids models begin with the application of the PMCS equation 2. However, for this application, cake liquor will consist only of the displaceable external fraction. Therefore, the cake liquor alumina weight ($A_o = A_{oi} + A_{oe}$) and liquor volume ($V_o = V_{oi} + V_{oe}$) are temporarily considered equal to A_{oe} and V_{oi} respectively. Thus, equation 2 becomes

$$\frac{A_1}{V_2} = \left(N\frac{A_2}{V_2} - f\frac{A_{oe}}{V_{oe}}\right) /(N-f). \qquad (3)$$

This means the wash ratio is now N = V_1/V_{oe}. Also, since cake volumes do not (ground rule 5) change during washing, V_1 equals V_2. Therefore, the wash liquor alumina weight can be determined from

$$\begin{aligned} A_1 &= V_1 \left(\frac{V_1}{V_{oe}} \bullet \frac{A_2}{V_1} - f\frac{A_{oe}}{V_{oe}}\right) /(N-f), \\ &= \frac{V_1}{V_{oe}} (A_2 - f\,A_{oe})/(N-f), \\ &= N(A_2 - f\,A_{oe})/(N-f). \qquad (4) \end{aligned}$$

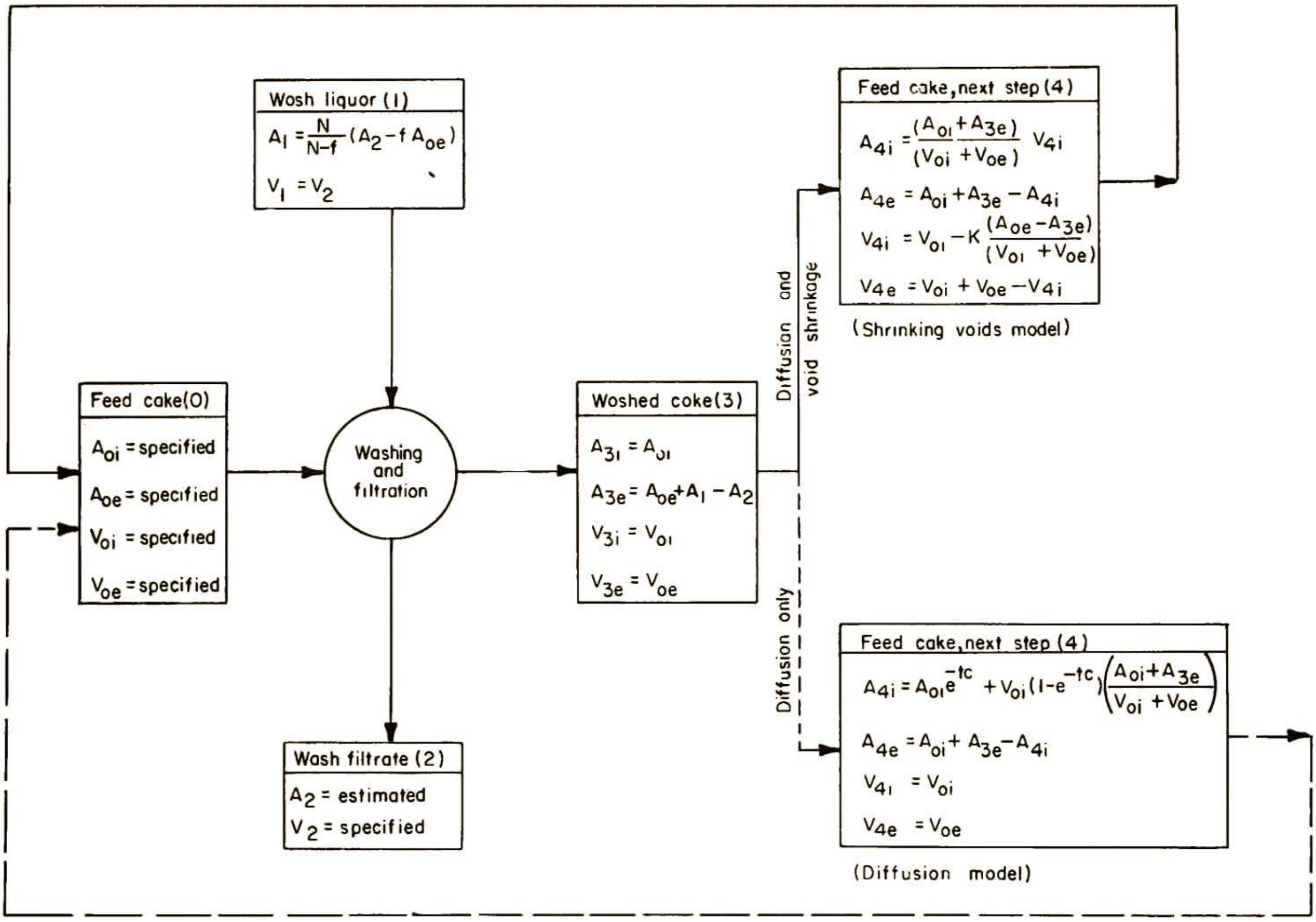

FIGURE 3.—Equations used in calculating a material balance around a single wash step.

Now the internal liquor parameters, A_{oi} and V_{oi}, temporarily set to zero, are again permitted nonzero values. Then the external cake alumina weight follows from material balance $A_{3e} = A_{oi} + A_{oe} + A_1 - A_2 - A_{3i}$, and since $A_{3i} = A_{oi}$, gives $A_{3e} = A_{oe} + A_1 - A_2$.

Diffusion Model: Migration of Al_2O_3 From Cake Voids

At this point, the external cake liquor will be highly deficient in Al_2O_3. It is, therefore, proposed that a specified amount of diffusion will occur to change this situation. The occurrence of this process can be expressed by a first order decay curve illustrated in figure 4. The equation is

$$C_f - C_{eq} = (C_i - C_{eq})\,e^{-tc}, \qquad (5)$$

where tc, a positive time constant, is actually the product of a diffusion time interval, t, and a diffusion rate constant, c (which must be positive). C_i, C_f, and C_{eq} are the initial, final, and equilibrium alumina concentrations in the liquor. The equilibrium concentration is, of course, defined by

$$C_{eq} = \frac{\text{alumina weight}}{\text{liquor volume}} = \frac{A_{3i} + A_{3e}}{V_{3i} + V_{3e}}, \qquad (6)$$

where the subscript 3 refers to the initial product cake liquor. Then for the subscript 4, which refers to the final product cake liquor, the relationships $C_i = A_{3i}/V_{3i}$, $C_f = A_{4i}/V_{4i}$, and C_{eq} of equation 6 can be substituted into equation 5 so that A_{4i} may be determined. This gives

$$\frac{A_{4i}}{V_{4i}} - \frac{A_{3i} + A_{3e}}{V_{3i} + V_{3e}} = \left(\frac{A_{3i}}{V_{3i}} - \frac{A_{3i} + A_{3e}}{V_{3i} + V_{3e}}\right) e^{-tc}. \qquad (7)$$

Now, since cake volume does not change, the relationships $V_{4i} = V_{3i} = V_{oi}$ and $V_{4e} = V_{3e} = V_{oe}$ must hold. Also, $A_{oi} = A_{3i}$ must hold, since no alumina has been removed from the particle interior during washing and before diffusion occurs. Thus, on rearrangement of equation 7, the final internal cake liquor alumina weight must be

$$A_{4i} = A_{oi}e^{-tc} + V_{oi}(1 - e^{-tc}) \left(\frac{A_{oi} + A_{3e}}{V_{oi} + V_{oe}}\right). \qquad (8)$$

This diffusion model is not the best one for giving good predictions. The diffusion model does not, for example, predict the washing efficiency increases (that is, R decreases) observed in the miniplant on passing from one wash stage to a succeeding one. Such increases are contrary to what is typically reported in the literature, where washing efficiency decreases are considered common.[5]

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[5] Dahlstrom, D. A. Predicting Performance of Continuous Filters. Chem. Eng. Prog., v. 74, No. 4, April 1978, pp. 69-74.

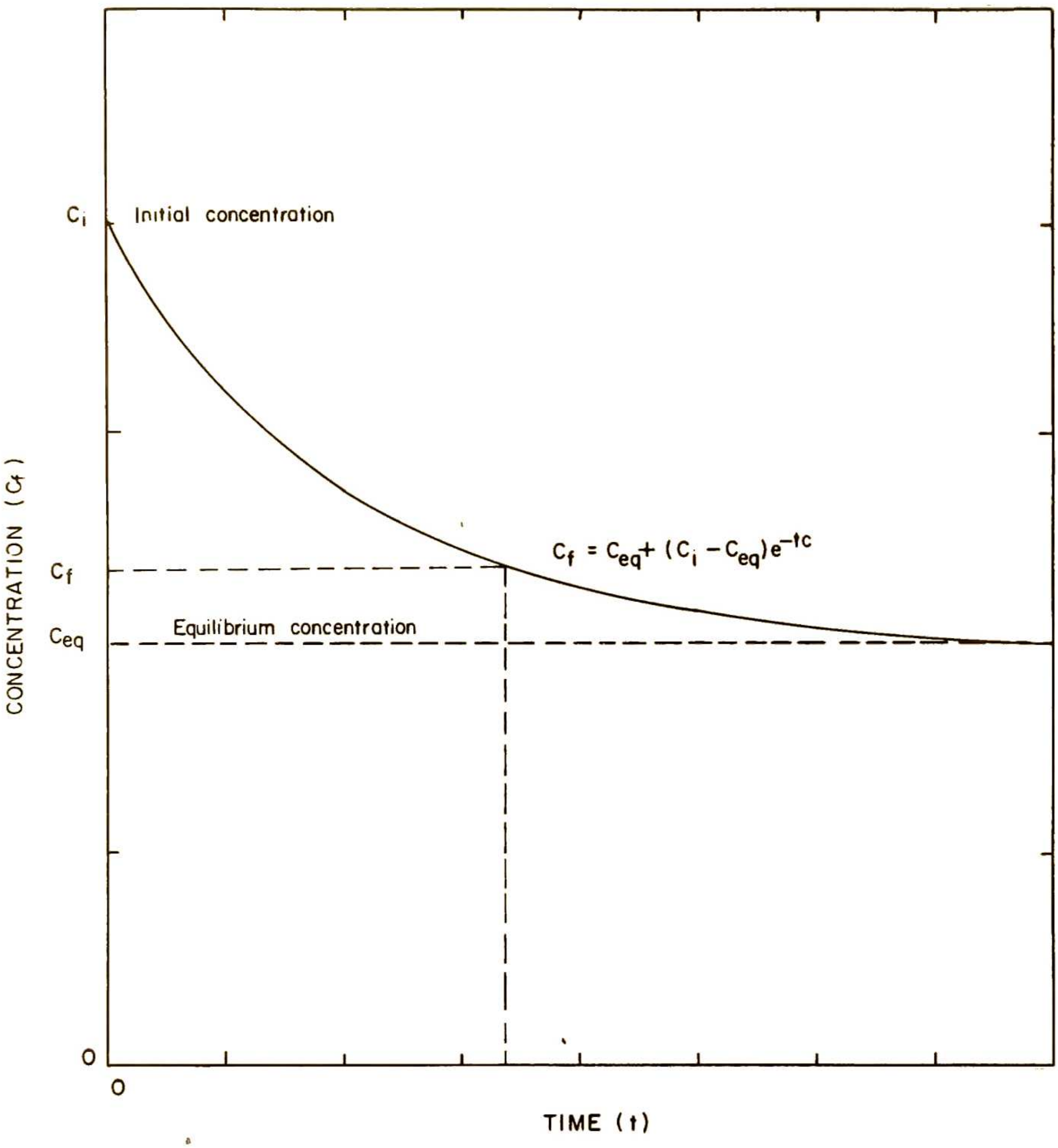

FIGURE 4.—Cake liquor solute concentration, C_f, as a function of diffusion time.

Furthermore, the use of equilibrium equations is rendered nconsequential on discovering that application of the model to miniplant data always requires that the positive ime constant approach infinity.

Shrinking Voids Model: Hypothetical Voids Shrinkage

This model introduces the concept of an unwashable voids fraction that systematically changes (decreases) as he cake passes from one wash to the next. The model thus proposes a voids volume decrease that is directly proportional to the Al_2O_3 concentration decrease, ΔC (concentration decreases being a crude measure of viscosity decreases). Specifically, the new voids volume, V_{4i}, is expressed in terms of the old voids volume, V_{oi}, and a voids shrinkage constant, k

$$V_{4i} = V_{oi} - k\Delta C. \qquad (9)$$

Using the notation of figure 3, ΔC can be expressed as

$$\Delta C = \left(\frac{\Delta A}{V}\right) = \frac{(A_{oe} + A_{oi}) - (A_{3e} + A_{3i})}{V_{oi} + V_{oe}} \qquad (10)$$

Since $A_{oi} = A_{3i}$, this gives

$$\Delta C = \frac{A_{oe} - A_{3e}}{V_{oi} + V_{oe}}, \tag{11}$$

$$\text{giving } V_{4i} = V_{oi} - k\left(\frac{A_{oe} - A_{3e}}{V_{oi} - V_{oe}}\right) \tag{12}$$

Since alumina concentration is uniform (at equilibrium) throughout the cake, the new voids alumina weight must be the voids volume fraction, $V_{4i}/(V_{oi} + V_{oe})$, of the total cake alumina, $A_{3i} + A_{3e}$,

$$A_{4i} = \frac{V_{4i}\,(A_{3i} + A_{3e})}{V_{oi} + V_{oe}}. \tag{13}$$

The quantities V_{4e} and A_{4e} are then determined from material and volume balance as in figure 3.

Completion of Countercurrent Calculations

The essential relationships, summarized in figure 3, are applied to countercurrent washing beginning with a form cake of known composition, an estimated filtrate alumina weight, A_2, and a filtrate volume equal to the wash water volume used in the system (refer to fig. 1). These calculations are repeated for each successive wash and filtration stage. Then, at this point, it is generally found that the alumina weight, A_f, in the final wash liquor does not equal the wash water weight originally specified (A_f is usually zero). Therefore, the first filtrate alumina concentration, A_2, must be reestimated and the whole countercurrent system balance calculation done again. After this second set of calculations is complete, the correct A_2 value may be closely estimated by linear interpolation

$$A_2 = A_2'' - \frac{(A_f'' - A_f)\,(A_2'' - A_2')}{(A_f'' - A_f')}, \tag{14}$$

where the singly primed constants refer to wash water and first wash filtrate aluminas from the first material balance trial and the doubly primed numbers to those from the second trial. This interpolation always gives the correct A_2 value for the diffusion model and generally comes close for the shrinking voids model. In any case, the TI-59[6] calculator and Fortran programs used in doing these calculations do not assume that the equation yields exact answers, but rather use it to produce rapid convergence to the correct A_2 value.

The foregoing calculations have presumed that the parameters, V_i, j, and tc (diffusion model) or V_i and k (shrinking voids model) are known. If these are not known, the values must be varied in the search for another material balance that will cause the balance values to more closely approximate the correct values (measured during a filtration run). The closeness of fit will be optimized using a least squares method to be described at the end of the "Sample Calculations" section.

Equations for Liquor Weight and Volume Balance

Once the optimal set of parameters is established, liquor volumes and weight balances can be made. Fortunately, a simple method exists for converting alumina weight and liquor volume to liquor weight. For pure aqueous $AlCl_3$ liquor, density is reliably calculated from

$$\left(\rho\ \frac{\text{lb}}{\text{gal}}\right) = 8.34 + 2.10\left(\frac{A\ \text{lb}\ Al_2O_3}{V\ \text{gal liquor}}\right) \tag{15}$$

With impurities present, the constant 2.10 may be increased.

With density known, the liquor weight then must be

$$(W\ \text{lb}) = (V\ \text{gal liquor})\left(\rho\ \frac{\text{lb}}{\text{gal}}\right)$$
$$= 8.34\,(V\ \text{gal liquor} + 2.10\,(A\ \text{lb}\ Al_2O_3). \tag{16}$$

SUMMARY OF MATERIAL BALANCE PROCEDURE

To obtain the optimum material balance for the two-wash belt filter shown in figure 1, it is necessary to measure the concentrations and volumes of specified filtration streams. This information is presented in table 1 for miniplant test 1-3. The code 1 in the table specifies that the first wash filtrate is used to dilute the form filtration feed slurry. A code 0 would indicate filtrate was not recycled. For the 123 pounds of dry leach solid passing through the filter each hour, the cake liquor volume can be analytically determined using the methods of appendix D. The parameters V_i and k are merely trial values to use in the first set of shrinking voids model calculations.

TABLE 1.—Miniplant belt filtration data, test 1-3

Number of washes		2
Al_2O_3, weight-percent:		
1st wash filtrate		2.78
Form cake liquor		8.31
2d wash filtrate		1.29
1st wash cake liquor		6.84
Wash water		0
2d wash cake liquor		4.68
Code		1
Reactor discharge:		
Al_2O_3	pounds	80.75
Liquor[1]	gallons	74.40
Wash water	do	28.52
Cake liquor, V_t	do	12.76
Internal cake liquor, V_i	do	e_9
Voids shrinkage constant,k	square gallons per pound	e_5

e Estimated.

[1] Includes 2 gal of flocculant water.

The balance procedure assumes conservation of volume as well as material. Therefore, all stream volumes (fig. 5) are readily specified from the values in table 1, from the assumption that all wash filtrate volumes are the same as those for wash water, and from the assumption that all cake volumes, $V_t = V_i + V_e$, are the same.

The calculation begins with the determination of the form slurry alumina weight:

$$A_{\text{form slurry}} = A_{\text{reactor discharge}} + (A_{\text{1st wash filtrate}})\ (\text{Code}).$$

The alumina weight for the two form filtration liquor fractions is proportional to volume,

$$A_{\text{form filtrate}} = \frac{V_{\text{form filtrate}}}{V_{\text{form slurry}}} \times A_{\text{form slurry}} \text{ and}$$

$$A_{\text{form cake}} = A_{\text{form slurry}} - A_{\text{form filtrate}}.$$

Form cake alumina fractions (internal and external) are then

$$A_{i\text{ form cake}} = \frac{V_i}{V_t} A_{\text{form cake}} \text{ and}$$

$$A_{e\text{ form cake}} = A_{\text{form cake}} - A_{i\text{ form cake}}.$$

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[6] Reference to specific equipment does not imply endorsement by the Bureau of Mines.

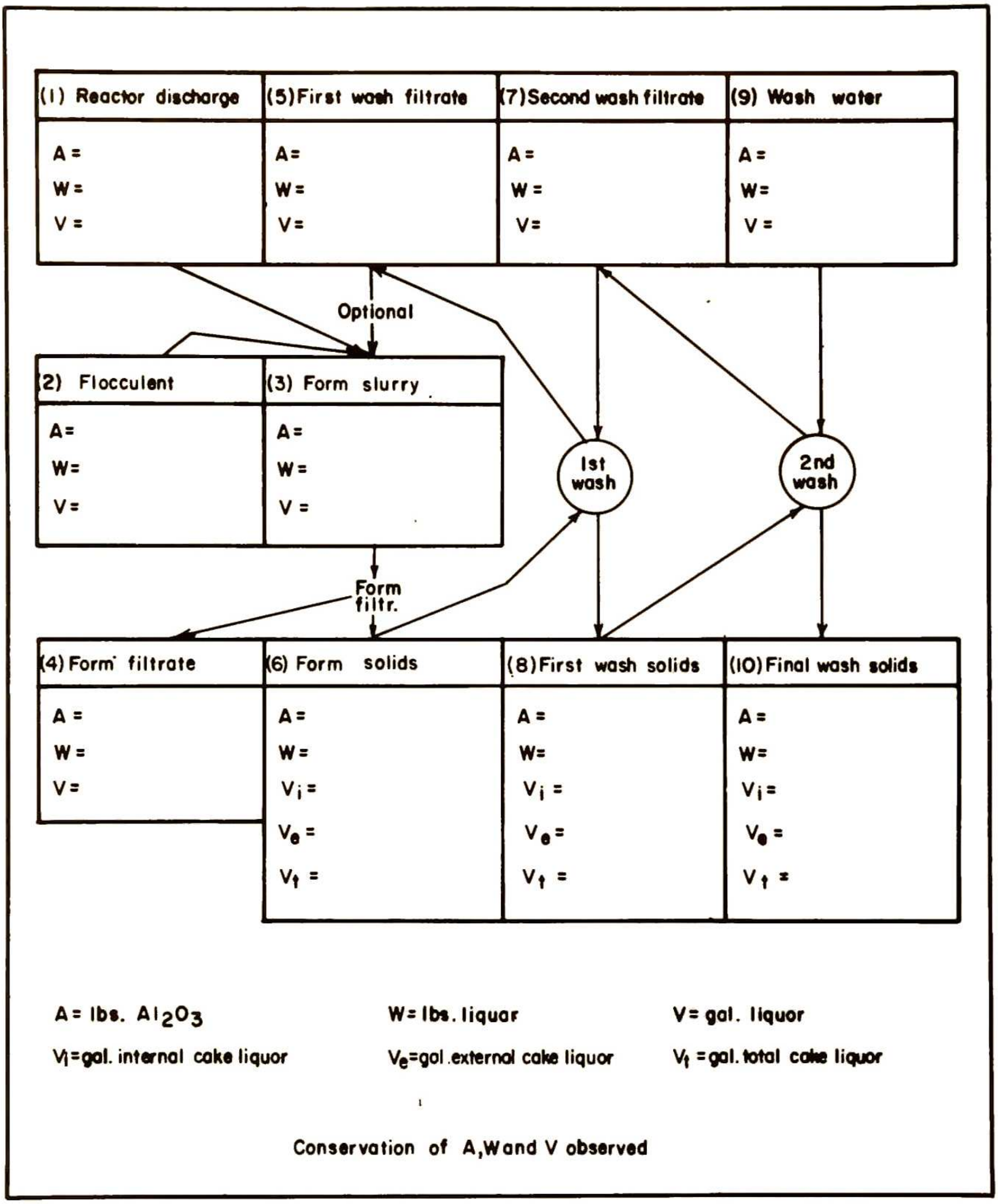

FIGURE 5.—Sample filtration circuit material balance form.

At this point, the form cake goes to the first wash and the figure 3 calculations are performed. First, N and f (for j = 1) must be calculated

$$N = \frac{V_{\text{wash liquor}}}{V_{\text{external cake liquor}}} \text{ and}$$

$$f = 1 - e^{-N}.$$

Then the wash liquor alumina is

$$A_1 = \frac{N}{N\text{-}f}(A_2 - f A_{oe}).$$

Now the first wash solids alumina weight must be (by material balance)

$$A_3 = A_0 + A_1 - A_2.$$

At this point, the cake undergoes voids shrinkage to give a new V_i

$$V_{3i} = V_{oi} - \frac{k}{V_t}(A_0 - A_3).$$

Now if the current wash liquor, A_1, is not the last wash (wash water), the foregoing calculations are repeated beginning with internal-external alumina apportionment for the form cake. If, however, this is the last wash, A_1 is compared to the known alumina content, A_f. If the two are not identical (within 0.0005 lb), a new A_{1st} wash filtrate, A_2, value must be applied for another series of calculations.

Generally, after two sets of calculations, the material balance will not be obtained. Therefore, the following equation is used to determine the next A_2 estimate:

$$A_2 = A_2'' - \frac{(A_f'' - A_f)(A_2'' - A_2')}{(A_f'' - A_f')},$$

where the singly primed variables are values from the first calculation set and doubly primed variables are from the second set.

SAMPLE CALCULATIONS

First Material Balance Trial

Using the data from table 1, along with the assumption that $A_{1st\ wash\ filtrate} = 16.00$ lb, the form slurry alumina must be

$$A_{form\ slurry} = 80.75 + (16.00)(1) = 96.75 \text{ lb.}$$

The two liquor fractions after form filtration are then

$$A_{form\ filtrate} = \frac{90.16}{72.4 + 2.0 + 28.52}(96.75) = 84.75 \text{ lb and}$$

$$A_{form\ cake} = 96.75 - 84.75 = 12.00 \text{ lb.}$$

The form cake liquor fractions are then

$$A_{i\ form\ cake} = \frac{9.00}{12.76} \times 12.00 = 8.46 \text{ lb and}$$

$$A_{e\ form\ cake} = 12.76 - 8.46 = 3.54 \text{ lb.}$$

For the next step, the wash ratio, N, and salt-free wash liquor alumina removal fraction, f, are

$$N = \frac{28.52}{3.76} = 7.585$$

$$f = 1 - e^{-7.585} = 0.99949.$$

The second wash filtrate alumina is then

$$A_1 = A_{2d\ wash\ filtrate} = \frac{7.585\,(16.00 - 0.99949 \times 3.54)}{7.585 - 0.99949}$$

$$= 14.35 \text{ lb.}$$

Then the first wash cake solids alumina is

$$A_{washed\ cake} = 12.00 + 14.35 - 16.00 = 10.35 \text{ lb.}$$

After voids shrinkage, the new V_i is

$$V_{i\ washed\ cake} = \frac{9.00 - 5\,(12.00 - 10.35)}{12.76} = 8.357 \text{ gal.}$$

Now there is one more wash; therefore, the new cake aluminas are

$$A_{i\ 1st\ wash\ cake} = \frac{8.357}{12.76} \times 10.35 = 6.78 \text{ lb and}$$

$$A_{e\ 1st\ wash\ cake} = 10.35 - 6.78 = 3.57 \text{ lb.}$$

With a new external cake liquor volume, $V_e = 12.76 - 8.357 = 4.403$, the new N and f values must be

$$N = 28.52/4.403 = 6.4776 \text{ and}$$

$$f = 1 - e^{-6.4776} = 0.99846.$$

The new wash filtrate (wash water this time) is then

$$A_{wash\ water} = A_f'$$

$$= \frac{6.4776\,(14.35 - 0.99846 \times 3.57)}{6.4776 - 0.99846}$$

$$= 12.76 \text{ lb.}$$

This is, of course, 12.76 lb too much for wash water, and thus the entire calculation set must be repeated for a different value of the first wash filtrate alumina, A_2.

Procedure Following First Balance Trial

The first wash filtrate is now arbitrarily chosen to be 8.00 lb. This time one obtains a wash water alumina value of 2.80 lb, as can be seen from the summaries in table 2.

Now with two iterations completed, equation 14 can be used to obtain a good estimate for the true value of the first wash filtrate alumina

$$A_{1st\ wash\ filtrate} = 8.00 - \frac{(2.80 - 0)(8.00 - 16.00)}{2.80 - 12.76} = 5.75 \text{ lb.}$$

After this series of calculations, the Al_2O_3 weight in the wash water is found to be -0.03 lb. Further use of equation 14 yields $A_{1st\ wash\ filtrate}$ as 5.78, which is the correct value for making the wash water alumina equal to 0.00 lb.

Evaluation by Least Squares Error

Goodness of Fit

Now a least squares error evaluation of this balance can be done. The method used here is the sum of the squares of the fractional errors for the streams

$$\Sigma(\text{error})^2 = \sum_{i-1}^{5}\left(\frac{A_{i\ material\ balance} - A_{i\ analytical}}{A_{i\ analytical}}\right)^2, \quad (17)$$

where the analytical weight can be reliably calculated from

TABLE 2.—Summary of a material balance calculation method, test 1-3 (V_i gal, k = 5 gal²/lb)

Stream	Al_2O_3, Pounds					Square of the error
	1st trial	2d trial	3rd trial	Final trial	Analysis	
1st wash filtrate, A_2	16.00	8.00	5.75	5.78	7.04	0.0320
Feed slurry	96.75	88.75	86.50	86.53	NAP	NAP
Form cake:						
Internal liquor	8.46	7.76	7.57	7.57	10.73	.0000
External liquor	3.54	3.24	3.16	3.16		
2d wash filtrate	14.35	5.48	2.98	3.01	3.15	.0020
1st wash cake:						
Internal liquor	6.78	5.33	4.94	4.94	8.53	.0045
External liquor	3.57	3.16	3.02	3.02		
Wash water, A_f	12.76	2.80	-.03	0	0	NAP
Final cake liquor	ND	ND	ND	4.95	5.55	.0117
Sum of the squares of the error	NAP	NAP	NAP	NAP	NAP	.0502

ND Not determined.
NAP Not applicable.

$$A_{analytical} = V \times \rho \times \frac{\text{pct } Al_2O_3}{100}$$

$$= V \times 8.34\,[1 + 0.02079\,(\text{pct } Al_2O_3)^{1.1}]\left(\frac{\text{pct } Al_2O_3}{100}\right). \quad (18)$$

This gives for the first wash filtrate

$$A_{analytical} = 28.52 \times 8.34\,[1 + 0.02079\,(2.78)^{1.1}]\left(\frac{2.78}{100}\right)$$

$$= 7.04 \text{ lb.}$$

Therefore, the square of the error is

$$(\text{error})^2 = \left(\frac{5.78 - 7.04}{7.04}\right)^2 = 0.0320.$$

The other errors are similarly calculated and presented in table 2. The sum of the squares of the error (SSE) for this material is thus 0.0502. But this SSE is, of course, not a least squares error, but rather just the error introduced on assuming specific values of V_i and k.

Least Squares Fit

To do a least squares fit, the variables V_i and k must be changed until the SSE can no longer be lowered. This is a difficult task to handle; therefore, a TI-59 program was developed to reduce calculation time (see appendix B).

Table 3 summarizes the SSE's obtained for various values of V_i and k. The best least squares fit was for V_i = 9.2 gal and k = 7.5 gal²/lb.

If the shrinkage constant k is set to 0.0, the model is identical to the diffusion model with the number of cells j equal to 1. The least squares SSE is then 0.0902 for V_i = 8.1 gal and the diffusion constant approaches infinity.

When both the shrinkage constant and internal cake volume are 0.0, the model is identical to the PMCS model. The large SSE (2.1011) here compared with that for the shrinking voids model makes it obvious that the PMCS model is unsuitable for test 1-3 (and also for other tests).

MODEL EVALUATION USING MINIPLANT DATA

The shrinking voids model was applied to data from seven different miniplant tests where $AlCl_3$ leach reactor discharges were filtered. The tests utilized three feed sizes—minus 10 mesh, minus 20 mesh, and minus 18 mesh (misted).

Misting is the process by which dust in the feed is suppressed. The as-mined raw kaolin fines are crushed to at least minus 14-mesh size and are then tumbled on an inclined rotating disk, while being moistened by a fine mist of water. The momentary wetting of the outer surfaces of the large particles causes the submicroscopic particles present in the raw crushed kaolin to adhere to larger particles. Repeated impact of the particles against each other

TABLE 3.—Summary of sums of squares of errors (SSE's) for test 1-3 material balance

Vi, gal	k, gal²/lb								
	0	3.0	5.0	6.5	7.0	7.5	8.0	8.5	10.0
11.0	0.9294	0.7832	0.6944	0.6323	0.6126	0.5930	0.5740	0.5555	0.5022
10.5	.6346	.4888	.4048	.3486	.3311	.3143	.2981	.2826	.2397
10.0	.4124	.2798	.2093	.1658	.1531	.1413	.1303	.1202	.0950
9.5	.2544	.1448	.0951	.0697	.0635	.0584	.0544	.0514	.0490
9.3	.2075	.1093	.0694	.0524	.0491	.0471	.0462	.0464	.0542
9.2	.1873	.0953	.0604	.0478	.0461	[1].04558	.0463	.0483	.0612
9.1	.1692	.0836	.0540	.0458	.04563	.0467	.0491	.0528	.0709
9.0	.1531	.0740	[2].0499	.0462	.0471	.0504	.0545	.0598	.0833
8.5	.1010	.0576	.0619	.0818	.0915	.1027	.1153	.1293	.1798
8.0	[3].0913	.0859	.1197	.1634	.1813	.2008	.2219	.2445	.3213
7.0	.1722	.2408	.3296	.4166	.4493	.4835	.5196	.5573	.6796
6.0	.3473	.4749	.6050	.7235	.7666	.8114	.8580	.9061	1.0595
5.0	.5777	.7398	.8922	1.0266	1.0748	1.1248	1.1763	1.2294	1.3970
3.0	1.1122	1.2609	1.3970	1.5159	1.5585	1.6024	1.6477	1.6942	1.8411
0	[4]2.1011	2.0990	2.1128	2.1306	2.1379	2.1459	2.1545	2.1638	2.1952

[1] The least squares SSE, at V_i = 9.2 and k = 7.5.
[2] The SSE for the sample calculation differs from that in table 2 because of roundoff errors.
[3] The least squares SSE (0.0902) for the diffusion model is at V_i = 8.1, k = 0, and j = 1.
[4] The SSE for the PMCS model.

TABLE 4.—Material balances for seven miniplant tests

	Test and feed size						
	1-3, minus 10 mesh	1-4, minus 10 mesh	3-2a, minus 20 mesh	3-3a, minus 18 mesh[1]	3-2b, minus 20 mesh	3-3b, minus 18 mesh[1]	3-4, minus 10 mesh
INPUT DATA							
Number of washes	2	2	2	2	2	2	2
Al_2O_3, weight percent:							
1st wash filtrate	2.78	1.71	2.80	2.42	3.66	3.33	3.55
Form cake	8.31	7.37	7.51	7.75	8.07	8.25	8.38
2d wash filtrate	1.29	.77	1.07	1.06	2.21	2.03	2.81
1st wash cake	6.84	6.41	5.64	5.24	6.05	5.69	8.16
Wash water	0	0	0	0	1.20	1.08	2.30
2d wash cake	4.68	3.78	2.91	3.35	3.32	3.38	5.41
Code	1	1	1	1	1	1	1
Reactor discharge:							
Al_2O_3 pounds	80.75	78.76	76.05	76.65	77.33	76.90	76.90
Liquor gallons	74.40	74.35	73.96	74.67	77.12	74.03	74.40
Wash water do	28.52	42.64	29.21	28.52	29.47	26.49	29.55
Cake liquor, gallons:							
Total	12.76	13.85	12.93	10.86	12.31	10.34	13.20
Internal	9.2	10.0	7.0	6.5	6.8	5.5	10.4
Voids shrinkage constant, k — square gallons per pound	7.5	8.5	-0.5	2.0	5.0	1.5	10.5
SHRINKING VOIDS MODEL MATERIAL BALANCE							
Al_2O_3, pounds:							
1st wash filtrate	5.822	5.408	6.937	5.606	9.341	7.973	9.601
Form cake	10.733	9.964	10.401	8.657	10.010	8.731	10.984
2d wash filtrate	3.231	2.900	2.758	2.520	5.998	4.772	8.032
1st wash cake	8.143	7.457	6.221	5.571	6.666	5.529	9.415
Wash water	0	0	0	0	3.024	2.440	5.963
2d wash cake	4.911	4.556	3.463	3.051	3.693	3.197	7.346
Sum of squares of errors[2]	0.0456	0.4177	0.2100	0.0135	0.0092	0.0009	0.0341

[1] Misted.

[2] Calculated using equation 17.

as they are tumbled on the disk causes the small, dust-like particles to be smeared, or plastically molded onto the larger particles, so that the proportion of fines in the misted raw kaolin is greatly reduced, compared with raw crushed kaolin. Suppression of dust in the feed allows the leached slurry to both settle and filter rapidly. Less flocculant is needed, compared with the same size raw kaolin feed that has not been misted.

The summaries in table 4 show the least squares error sums varying from 0.009 to 0.0456. If these sums are divided by the number of data streams used in calculating them (six streams), and the square root is then taken, the average stream error is seen to range from 1.7 to 8.7 pct. There may be room for improvement, but this error value is much better than that for the PMCS model (64.8 pct) and the diffusion model (13.4 pct) in test 1-3.

Analytical weights and best-fit balance weights are summarized in table 5. Comparison of best-fit (prior material balance) methods and shrinking voids model balances show the average model error (5.7 pct) to be somewhat superior to that for the best-fit average error (7.6 pct). Thus, the shrinking voids model should be extremely useful for predicting material balances and predicting filtration performances in other plants.

Probably the easiest approach to doing the material balance is through the Fortran program in appendix A. The

TABLE 5.—Comparison of best-fit material balance with shrinking voids model material balance

	Test and feed size						
	1-3, minus 10 mesh	1-4, minus 10 mesh	3-2a, minus 20 mesh	3-3a, minus 18 mesh[1]	3-2b, minus 20 mesh	3-3b, minus 18 mesh[1]	3-4, minus 10 mesh
ANALYTICAL WEIGHTS—UNBALANCED							
Al_2O_3, pounds:							
1st wash filtrate	7.04	6.31	7.26	6.07	9.77	7.93	9.48
Form cake	10.73	10.11	9.65	8.41	10.00	8.62	11.21
2d wash filtrate	3.15	2.78	2.67	2.58	5.70	4.69	7.37
1st wash cake	8.53	8.59	6.93	5.36	7.15	5.60	10.86
Wash water	0	0	0	0	3.02	2.44	5.96
2d wash cake	5.55	4.76	3.35	3.27	3.67	3.15	6.75
BEST-FIT MATERIAL BALANCE							
Al_2O_3, pounds:							
1st wash filtrate	5.93	5.81	7.07	6.02	9.41	8.39	9.61
Form cake	10.83	10.13	10.05	8.98	10.26	9.05	11.11
2d wash filtrate	2.94	3.31	3.29	2.39	5.80	5.06	8.47
1st wash cake	7.84	7.63	6.27	5.35	6.65	5.72	9.97
Wash water	0	0	0	0	2.82	2.53	5.95
2nd wash cake	4.90	4.32	2.98	2.96	3.67	3.19	7.45
AVERAGE PERCENT ERROR[2]							
Best-fit	8.5	10.3	11.4	5.6	4.4	4.8	8.2
Diffusion	12.3	12.0	5.9	6.5	6.2	2.2	8.1
Shrinking voids	8.7	8.3	5.9	4.7	3.9	1.2	7.5

[1] Misted.

[2] The overall average percent error is 7.6 for best-fit, 7.6 for diffusion, and 5.7 for the shrinking voids model.

following information must be obtained and organized into a data file, described in appendix A (program lines 1200-1300), before running the program:

1. Number of wash stages.
2. Reactor discharge slurry.
    a. Weight-percent Al_2O_3.
    b. Volume (gallons).
3. Flocculant water volume (gallons).
4. Wash water volume (gallons).
5. Weight-percent Al_2O_3—plant data.
    a. All wash filtrate liquors.
    b. All washed cake liquors.
6. Cake liquor volume V_t (gallons)—plant data.
7. Optional flow code (0 = first wash filtrate not recycled, 1 = recycled).

A sample printout for test 1-3 is given in appendix A ("Run in Material Balance Mode" section).

In order to make the model into a useful predictor, it is now necessary to decide what factors determine the size of internal (V_i and total (V_t)) cake volumes. The most obvious factor is the nature of the solids. The three types of solids fed to the miniplant are characterized by the average cake liquor volumes and shrinkage constants in table 6.

TABLE 6.—Average filtration parameters for miniplant leach residues containing 123 lb of dry solids[1]

Feed size, mesh	V_t, gal	V_i, gal	k, gal²/lb
Minus 10	13.27	9.9	8.8
Minus 20	12.62	6.9	2.3
Minus 18[2]	10.60	6.0	1.8

[1] All constants in the table are directly proportional to the weight of dry solids. For example, 246 lb of dry solids would have the following minus 10-mesh feed parameters: V_t = 26.54, V_i = 19.8, and k = 17.6.

[2] Misted.

It should be noted that porosity measurements (appendix C) show that 123 lb of miniplant cake solids have a physical voids volume of

$$V_i = \frac{1\text{ gal solids}}{2.18 \times 8.34\text{ lb solids}} \times 123\text{ lb solids}\left(\frac{0.546}{1\text{-}0.546}\right)$$

$$= 8.1\text{ gal.}$$

V_i was also shown to be invariant with change in particle size. Therefore, these V_i's should be considered as effective internal volumes rather than physical ones. In any case, these empirical values for each feed may be used to help predict filter performance.

APPLICATION OF THE SHRINKING VOIDS MODEL

Development of Material Balances From Plant Data

Material balance calculation methods are detailed in the previous section. Simple application of the Fortran program used therein provides a balance more reliable than those of existing methods (best-fit, PMCS, and diffusion). The only restriction on the program is that new concentration-density conversions be applied when the solutions are not aqueous $AlCl_3$.

Prediction of Hypothetical Plant Material Balances

With the completion of a material balance derived from miniplant data, the parameters V_t, V_i, and K become known for three different sizes of leached kaolin slurry feeds. If it is now desired to predict belt filtration performance for another configuration using a different amount of wash water, the following information must be specified:

1. Number of wash stages.
2. Reactor discharge slurry.
    a. Weight-percent Al_2O_3.
    b. Volume (gallons).
3. Flocculant water volume (gallons).
4. Wash water volume (gallons).
5. Feed cake parameters for a specified dry solid weight.
    a. V_t gallons—lab or plant data.
    b. V_i gallons—modeling data.
    c. k square gallons per pound—modeling data.
6. Optional flow code (0 = first wash filtrate not recycled, 1 = recycled).

This information is then organized into the data file mentioned in appendix A, prior to running the Fortran program to give a complete material balance.

Tables 7 and 8 summarize shrinking voids model predicted alumina losses and form filtrate (product) alumina concentrations and volumes obtainable using various operating configurations. The model may be also applied to filtration in plants processing other types of slurries. However, the appropriate salt-concentration-to-density conversions in the Fortran program must first be changed (see comments in program, appendix A) because the current program applies only for $AlCl_3$ solutions.

Optimization of Horizontal Belt Filtration Operations

It is a general rule that countercurrent solids-liquid separation systems produce greater solute recoveries when more wash water is used. However, the extra water must often be removed later at additional expense. Recoveries may be also increased by adding extra wash stages—but at increased capital and operating costs. In any case, the final filtration configuration should be

TABLE 7.—Predicted Al_2O_3 filtration losses under various operating configurations, pounds

Number of washes	Feed size, mesh		
	Minus 10	Minus 20	Minus 18[1]
	20 GALLONS OF WASH WATER		
1	8.256	6.035	5.114
2	5.787	3.697	3.084
3	3.834	2.344	1.890
4	2.481	1.523	1.172
5	1.582	1.005	.732
6	.991	.669	.458
	30 GALLONS OF WASH WATER		
1	7.450	5.335	4.574
2	5.080	3.013	2.617
3	3.069	1.685	1.471
4	1.657	.937	.818
5	.802	.518	.451
6	.351	.285	.247
	50 GALLONS OF WASH WATER		
1	6.240	4.343	3.820
2	4.238	2.401	2.129
3	2.506	1.255	1.148
4	1.285	.641	.606
5	.578	.323	.316
6	.235	.162	.163

[1] Misted.

NOTE.—Based on standard slurry feed: 77.62 lb of Al_2O_3, 76.52 gal liquor (which contains 5 gal of flocculant water). See appendix B, step 9, for method used in calculating these numbers on the TI-59.

TABLE 8.—Predicted form filtrate concentrations under various operating configurations, weight-percent Al_2O_3[1]

Number of washes	Feed size, mesh		
	Minus 10	Minus 20	Minus 18[2]
	20 GALLONS OF WASH WATER		
1	8.18	8.36	8.27
2	8.44	8.61	8.48
3	8.65	8.75	8.61
4	8.80	8.84	8.68
5	8.89	8.89	8.73
6	8.95	8.93	8.76
	30 GALLONS OF WASH WATER		
1	7.46	7.61	7.54
2	7.68	7.83	7.72
3	7.88	7.96	7.83
4	8.01	8.03	7.89
5	8.10	8.07	7.92
6	8.14	8.09	7.94
	50 GALLONS OF WASH WATER		
1	6.35	6.46	6.40
2	6.51	6.62	6.54
3	6.65	6.71	6.61
4	6.75	6.76	6.66
5	6.80	6.79	6.68
6	6.83	6.80	6.69

[1] Calculated using weight-percent = 9.655 [(pounds of reactor discharge Al_2O_3 — pounds of Al_2O_3 loss)/(gallons form filtrate)]$^{1/1.1}$.
[2] Misted.

NOTE.—Form filtrate volume was as follows:

Wash water, gallons	Feed size, mesh		
	Minus 10	Minus 20	Minus 18 Misted
20	83.25	83.90	85.92
30	93.25	93.90	95.92
50	113.25	113.90	115.92

chosen only after a series of material balances (like those summarized in tables 7 and 8) have been completed and the overall minimum plant cost determined.

A selection from the three mesh sizes of feeds tested (minus 10, minus 20, and minus 18 misted) will, of course, be decided through the foregoing economic evaluation. However, table 7 shows that minus 18-mesh feed consistently yields the lowest alumina loss and minus 10-mesh feed the greatest. Since feed preparations are probably similar for these feed sizes, this seems to single out minus 18-mesh feed size as optimum.

However, table 8 shows minus 18-mesh feed giving the poorest quality form filtrate product as the product is more dilute than for other feed solids. This poor quality results because the feed's small cake liquor volume, V_t, removes significantly less water through the final wash cake than do other feeds.

Large voids shrinkage constants, k, do not significantly contribute to improved Al_2O_3 recovery even though they represent rapid improvement in washing efficiency. Note, for example, that k is largest for minus 10-mesh feed, which has the poorest recovery.

SUMMARY AND CONCLUSIONS

In order to minimize expensive in-plant testing of a continuous horizontal washing and vacuum countercurrent belt filtration system, a model that can reliably predict system performance must be developed. Application of the PMCS filtration model to the miniplant data gave completely unreliable results. Therefore, a shrinking voids model was proposed in which liquor residing in the cake particle pores is considered not readily washed from the cake. This model predicted increases in washing efficiency as the cake liquor becomes more dilute (and less viscous). The typical error in predicting stream alumina balance sets varied from 1.7 to 8.7 pct, with an average error of 5.7 pct. This method of balance was more effective than that for the diffusion model and for typical metallurgical best-fit methods.

The optimum empirical parameters determined by shrinking voids model balances—V_t, total cake liquor volume, V_i, internal cake liquor volume, and k, voids shrinkage constant—were then used to predict belt filtration recoveries and product liquor concentrations for systems having various numbers of wash stages and various amounts of wash water. It was found that the minus 18-mesh misted solids were the best feed if optimum alumina recoveries are desired. However, it was unclear as to what the optimum wash water volume or number of wash stages should be since this will depend on the economics for other steps in the clay-HCl-miniplant.

APPENDIX A.—MATERIAL BALANCE BY FORTRAN COMPUTER

The Material Balance Program

```
#FILE (BCME)BELFIL ON MRC
1000 C-#####    PROGRAM FOR DOING A BEST-FIT MATERIAL BALANCE ON DATA TAKEN
1010 C-##### FROM A CONTINUOUS COUNTER-CURRENT BELT FILTRATION RUN.
1020 C-#####    THE BALANCE, DONE USING THE SHRINKING VOIDS MODEL, YIELDS 2
1030 C-##### MODEL PARAMETERS, VI AND K, USEFUL IN FUTURE PREDICTIONS OF
1040 C-##### FILTER PERFORMANCE.
1050 C-#####    WITH THESE PARAMETERS KNOWN, THE ONLY KNOWLEDGE REQUIRED
1060 C-##### TO PREDICT FILTRATION PERFORMANCE IN OTHER CONFIGURATIONS IS
1070 C-##### THE TOTAL CAKE LIQUOR VOLUME (VT).
1080 C-##### BELT FILTRATION STREAM NUMBERING SYSTEM IS:
1090 C-##### 1 - REACTOR DISCHARGE SLURRY
1100 C-##### 2 - FLOCCULANT
1110 C-##### 3 - FORM FILTRATION FEED SLURRY LIQUOR
1120 C-##### 4 - FORM FILTRATE
1130 C-##### 5 - 1ST WASH FILTRATE
1140 C-##### 6 - FORM FILTRATION CAKE LIQUOR
1150 C-##### 7 - 1ST WASH LIQUOR = 2ND WASH FILTRATE
1160 C-##### 8 - 1ST WASHED CAKE LIQUOR
1170 C-##### 9 - 2ND WASH LIQUOR = 3RD WASH FILTRATE OR MINIPLANT WASH WATER
1180 C-#####10 - 2ND WASHED CAKE LIQUOR
1190 C-#####11 TO IW   ETC....
1200 C-##### THIS PROGRAM READS DATA FROM FILE 5 = "BOLDAT" WHERE DATA IS
1210 C-##### STORED IN THE FOLLOWING FORMATS:
1220 C-#####    FOR MATERIAL BALANCE MODE
1230 C-##### 200  0, NO. OF WASHES, PCT AL2O3 IN REACTOR DISCHARGE
1240 C-##### 300 PCT AL2O3 IN 1ST WASH FILTRATE LIQUOR
1250 C-##### 305 PCT AL2O3 IN 1ST WASHED CAKE LIQUOR
1260 C-##### 310 PCT AL2O3 IN 2ND WASH FILTRATE LIQUOR
1270 C-##### 315 PCT AL2O3 IN 2ND WASHED CAKE LIQUOR
1280 C-##### 320 ETC. UNTIL ALL PLANT DATA HAS BEEN ENTERED
1290 C-##### 400  GALLONS REACTOR DISCH., GAL. FLOCCULENT ADDED TO DISCH.,
1300 C-#####      GAL. WASH WATER, TOTAL GAL. LIQUOR IN CAKE,
1310 C-#####      A CODE(0 = 1ST WASH FILTRATE NOT RECYCLED, 1=RECYCLED)
1320 C-#####
1330 C-#####    FOR PREDICTOR MODE
1340 C-##### 200  1, NO. OF WASHES, PCT AL2O3 IN REACTOR DISCHARGE
1350 C-##### 300  GAL INTERNAL CAKE LIQUOR, VOIDS SHRINKAGE CONSTANT
1360 C-##### 400  (SAME AS FOR MATERIAL BALANCE ABOVE)
1370 C-#####
1380 C-##### IF SOLUTIONS ARE NOT AQUEOUS ALCL3 THEN DENSITY EQUATIONS
1390 C-##### MUST BE CHANGED AT POINTS MARKED BY "==========".
1400 C-##### LIST OF VARIABLES USED IN PROGRAM:
1410 C-##### ANCON(X)-- MEASURED AL2O3 CONCENTRATION (PCT.) FOR STREAM X
1420 C-##### ANWT(X)--  MEASURED AL2O3 WEIGHT (LBS.) FOR STREAM X
1430 C-##### AWT(X)--   TRIAL VALUE AL2O3 WEIGHT (LBS) FOR STREAM X
1440 C-##### WTLIQ(X)-- WEIGHT (LBS) FOR LIQUOR IN STREAM X
1450 C-##### VOLLIQ(X)--VOLUME (GALLONS) FOR LIQUOR STREAM X
1460 C-##### AIWT(X)--  INTERNAL LIQUOR AL2O3 WEIGHT (LBS), STREAM X
1470 C-##### AEWT(X)--  EXTERNAL LIQUOR AL2O3 WEIGHT (LBS), STREAM X
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1480 C-##### NOWASH-- NUMBER OF WASH STAGES
1490 C-##### VOLFD-- VOLUME OF FEED STREAM, NO. 1
1500 C-##### VOLFL-- VOLUME OF FLOCCULANT ADDITION STREAM
1510 C-##### VOLWW-- VOLUME OF WASH WATER STREAM
1520 C-##### VOLCL-- CAKE LIQUOR VOLUME (INTERNAL + EXTERNAL)
1530 C-##### CODE--  (1 FOR RECYCLE OF 1ST WASH FILTRATE, 0 FOR NO RECYCLE)
1540 C-##### IW-- TOTAL NO. OF STREAMS
1550 C-##### L-- SIZE OF VI - CONST TEST MATRIX
1555 C-##### SMALL L's REDUCE CALCULATION TIME, BUT MAY DECREASE ACCURACY
1560 C-##### VI-- 1ST CAKE INTERNAL VOLUME, REFERENCE VALUE
1570 C-##### VIC--ANY CAKE INTERNAL VOLUME, CALCULATED
1580 C-##### DVI--INCREMENTAL CHANGE IN TRIAL VALUE OF VI
1590 C-##### BVI--BEST VALUE FOR VI
1600 C-##### EK-- VOIDS SHRINKAGE CONSTANT, REFERENCE VALUE
1610 C-##### EKK--VOIDS SHRINKAGE CONSTANT, CURRENT VALUE
1620 C-##### DKK--INCREMENTAL CHANGE IN TRIAL VALUE OF EKK
1630 C-##### BKK--BEST VALUE FOR EKK
1631 C-##### WASHRA-- RATIO OF WASH WATER VOLUME TO CAKE VOLUME
1632 C-##### F-- FRACTION OF AL2O3 REMOVED BY AL2O3-FREE WASH WATER
1640 C-##### AWT5C-- STREAM 5 AL2O3 WT., CURRENT VALUE
1650 C-##### AWT5E-- STREAM 5 AL2O3 WT., EARLIER VALUE
1660 C-##### WWC--    WASH WATER AL2O3 WT., CURRENT VALUE
1670 C-##### WWE--    WASH WATER AL2O3 WT., EARLIER VALUE
1680 C-##### SSE-- SUM OF SQUARES OF ERRORS
1690 C-##### ASE-- AVERAGE SUM OF SQUARE OF ERROR PER STREAM
1700 C-##### BASE--BEST AVG. SUM OF SQUARE OF ERROR
1710 FILE  5(KIND=DISK,TITLE="BOLDAT",FILETYPE=7)
1720        DIMENSION ANCON(50),ANWT(50),AWT(50),WTLIQ(50),
1730       -VOLLIQ(50),AIWT(50),AEWT(50)
1740 C-##### MODE 0 DOES BEST-FIT MATERIAL BALANCE, MODE 1 PREDICTS A BALANCE
1750       READ(5,8) MODE,NOWASH,ANCON(1)
1760     8 FORMAT(2I4,F8.3)
1770       WRITE(6,9) MODE,NOWASH,ANCON(1)
1780     9 FORMAT(5H MODE,I3/I3,35H WASHES, REACTOR DISCH.AL2O3 CONC.=,F8.3)
1790       IW=NOWASH*2+6
1800       IF(MODE.EQ.1) GOTO1200
1810       IF(MODE.NE.0) GOTO 2000
1820       JW=IW
1830       L=6
1840       N=4
1850    10 N=N+1
1860       READ(5,11) ANCON(N)
1870    11 FORMAT(F8.3)
1880       IF(N.LT.IW) GOTO 10
1890       WRITE(6,12)((I,ANCON(I)),I=5,IW)
1900    12 FORMAT(7H STREAM,I3,F8.3,11H PCT. AL2O3)
1910       ANCON(4)=ANCON(6)
1920       ANCON(3)=ANCON(6)
1930       ANCON(2)=0
1940    20 READ(5,21) VOLFD,VOLFL,VOLWW,VOLCL,CODE
1950    21 FORMAT(5F8.3)
1960       WRITE(6,22) VOLFD,VOLFL,VOLWW,VOLCL,CODE
1970    22 FORMAT(/19H VOLUMES IN GALLONS/65H REACTOR DISCHARGE  FLOCCULANT
```

```
1980      -WASH WATER  CAKE LIQUOR      CODE/5F13.2)
1990       IF(CODE.NE.0.0.AND.CODE.NE.1.0) GOTO 2000
2000       IF(IW.LT.8.OR.IW.GT.50) GOTO 2000
2010       IT=0
2020       VOLLIQ(1)=VOLFD
2030       VOLLIQ(2)=VOLFL
2040 C-##### INITIALIZES VOLUMES FOR FILTRATES & CAKE LIQUORS
2050       DO 50 I=6,IW,2
2060          VOLLIQ(I-1)=VOLWW
2070          VOLLIQ(I)=VOLCL
2080   50  CONTINUE
2090       VOLLIQ(3)=VOLFD+VOLFL+VOLLIQ(5)*CODE
2100       VOLLIQ(4)=VOLLIQ(3)-VOLLIQ(6)
2130 C-======== CALCULATES AL203 WTS. USING AL203 CONCENTRATIONS
2140       DO 60 I=1,JW
2150           ANWT(I)=VOLLIQ(I)*.0834*(1+.02079*ANCON(I)**1.1)*ANCON(I)
2160  60   CONTINUE
2170       IF(MODE.EQ.1) GOTO 70
2180       DVI=VOLCL/(L-1)
2190       VI=0
2200       DKK=VOLFD**2/((L-1)*ANWT(1)*NOWASH)
2210       EK=-2*DKK/(L-1)
2220       BASE=99999
2230   70  AWT(1)=ANWT(1)
2240       AWT(2)=0
2250 C-##### BEGINS THE BEST-FIT MATERIAL BALANCE
2260   80  DO 1100 I=1,L
2270          EKK=EK
2280          DO 1000 J=1,L
2290 C-##### VI >= VOLCL IS UNREASONABLE, CAUSING CALCULATION PROBLEMS
2300             IF(VI.GE.VOLCL)VI=.99*VOLCL
2310             AWT(5)=8
2320             AWT5C=9
2330             WWC=ANWT(IW-1)+1
2340             AWT5E=AWT5C+1
2350             WWE=WWC+1
2360             LIM=0
2370 C-##### DOES 1 MATERIAL BALANCE FOR ASSUMED 1ST FILTRATE COMPOSITION
2380  799        LIM=LIM+1
2390             VIC=VI
2400             AWT(3)=AWT(1)+AWT(2)+AWT(5)*CODE
2410             AWT(6)=VOLLIQ(6)*AWT(3)/VOLLIQ(3)
2420             AIWT(6)=VI*AWT(6)/VOLLIQ(6)
2430             AEWT(6)=AWT(6)-AIWT(6)
2440             DO 900 K=7,IW,2
2450                WASHRA=VOLWW/(VOLCL-VIC)
2460                F=1-EXP(-WASHRA)
2470                AWT(K)=WASHRA*(AWT(K-2)-F*AEWT(K-1))/(WASHRA-F)
2480                TEMP=AEWT(K-1)+AWT(K)-AWT(K-2)
2490                VIC=VIC-EKK*(AEWT(K-1)-TEMP)/VOLLIQ(K-1)
2500 C-##### VIC >= VOLCL IS UNREASONABLE, CAUSING CALCULATION PROBLEMS
2510                IF(VIC.GE.VOLCL) GOTO 999
2520               AIWT(K+1)=VIC*(AIWT(K-1)+TEMP)/VOLLIQ(K-1)
```

```
              AEWT(K+1)=AIWT(K-1)+TEMP-AIWT(K+1)
              AWT(K+1)=AIWT(K+1)+AEWT(K+1)
 900       CONTINUE
           AWT5E=AWT5C
           AWT5C=AWT(5)
           WWE=WWC
           WWC=AWT(IW-1)
           ERR=ABS(ANWT(IW-1)-AWT(IW-1))
           IF(ERR.LT.0.0005) GOTO 950
           IF(LIM.GT.20) GOTO 999
C-##### WASH WATER AL203 DIFFERS FROM TRUE AL203 BY MORE THAN 0.0005
C-##### LBS. SO, CHOOSE NEW 1ST FILTRATE ESTIMATE
           AWT(5)=AWT5C-(WWC-ANWT(IW-1))*(AWT5C-AWT5E)/(WWC-WWE)
           GOTO 799
 950       AWT(4)=AWT(3)-AWT(6)
           IT=IT+1
           IF(MODE.EQ.1) GOTO 963
C-##### CALCULATES SUMS OF SQUARES OF ERRORS ON PCT. BASIS
           SSE=0
           NUM=0
           DO 960 II=5,IW
              IF(ANWT(II).EQ.0.0)GOTO 960
              SSE=SSE+((ANWT(II)-AWT(II))/ANWT(II))**2
              NUM=NUM+1
 960       CONTINUE
           ASE=SSE/NUM
           IF(ASE.GT.BASE) GOTO 962
C-##### A LOWER SSE HAS BEEN FOUND
           BASE=ASE
           BVI=VI
           BKK=EKK
 962       IF(MODE.NE.1) GOTO 999
C-##### PRINT THE FINAL RESULTS
 963       WRITE(6,964)
 964       FORMAT(/15X,26HTHE FINAL MATERIAL BALANCE)
           WRITE(6,965) VI,EKK
 965       FORMAT(/27H PARTICLE INTERNAL VOLUME =,F8.3,
     -     30H GALLONS, SHRINKAGE CONSTANT =,F8.3)
           WRITE(6,970)
 970       FORMAT(45H  NO.   LBS. AL203   LBS. LIQUOR  GAL. LIQUOR)
           DO 980 M=1,IW
C-========== CALCULATES WEIGHT OF AQUEOUS ALCL3 SOLUTIONS
              WTLIQ(M)=8.34*VOLLIQ(M)+2.10*AWT(M)
              WRITE(6,985)M,AWT(M),WTLIQ(M),VOLLIQ(M)
 980       CONTINUE
 985       FORMAT(I4,F12.3,2F12.2)
           WRITE(6,990)ASE,SSE
 990       FORMAT(20H AVG. SQ. OF ERROR = ,F9.6,7H  SSE =,F9.5)
           WRITE(6,995) IT
 995       FORMAT(19H NO. OF BALANCES = ,I4)
 999       EKK=EKK+DKK
1000    CONTINUE
        VI=VI+DVI
```

```
3060 1100 CONTINUE
3070      IF(MODE.EQ.1) GOTO 2000
3080 C-##### CHOOSES AREA OF VI - CONST MATRIX FOR NEXT SEARCH
3090      DVI=DVI/2
3100      VI=BVI-(L-1)*DVI/2
3110      DKK=DKK/2
3120      EK=BKK-(L-1)*DKK/2
3130      IF(DVI.GT.0.01.OR.DKK.GT.0.1) GOTO 80
3140 C-##### BEST-FIT HAS BEEN FOUND, PREPARE TO PRINT IT
3150      MODE=1
3160      L=1
3170      VI=BVI
3180      EK=BKK
3190      GOTO 80
3195 C-##### PREDICTOR MODE INITIALIZATION
3200 1200 READ(5,1700) VI,EK
3210 1700 FORMAT(2F8.3)
3220      WRITE(6,1800) VI,EK
3230 1800 FORMAT(23H INTERNAL CAKE VOLUME =,F8.3,17H GALLONS, CONST.=,F8.3)
3240      L=1
3250      JW=1
3260      AWT(IW-1)=0
3270      GOTO 20
3280 2000 STOP
3290      END
```

Run in Material Balance Mode (Test 1-3)

```
LIST BOLDAT
#FILE (BCME)BOLDAT ON MRC
100    0   2  10.483
200    2.78
300    8.31
400    1.29
500    6.84
600    0
700    4.68
800   72.4     2.      28.52   12.76  1.
#
RUN BELFIL
#RUNNING 0588

MODE  0
 2 WASHES, REACTOR DISCH.AL2O3 CONC.=  10.483
STREAM  5    2.780 PCT. AL2O3
STREAM  6    8.310 PCT. AL2O3
STREAM  7    1.290 PCT. AL2O3
STREAM  8    6.840 PCT. AL2O3
STREAM  9    0.000 PCT. AL2O3
STREAM 10    4.680 PCT. AL2O3

VOLUMES IN GALLONS
REACTOR DISCHARGE  FLOCCULANT  WASH WATER  CAKE LIQUOR      CODE
       72.40          2.00        28.52       12.76         1.00
```

The Final Material Balance

```
PARTICLE INTERNAL VOLUME =   9.141 GALLONS, SHRINKAGE CONSTANT =    7.065
 NO.   LBS. AL2O3   LBS. LIQUOR  GAL. LIQUOR
  1       80.747       773.39        72.40
  2        0.000        16.68         2.00
  3       86.595      1040.20       102.92
  4       75.859       911.24        90.16
  5        5.848       250.14        28.52
  6       10.736       128.96        12.76
  7        3.212       244.60        28.52
  8        8.100       123.43        12.76
  9        0.000       237.86        28.52
 10        4.888       116.68        12.76
AVG. SQ. OF ERROR = 0.009120  SSE =  0.04560
NO. OF BALANCES =  289
#ET=43.0 PT=1.5 IO=0.3
```

Run in Predictor Mode (Four Washes)

```
LIST BOLDAT
#FILE (BCME)BOLDAT ON MRC
100    1   4  10.254
200    9.9     8.8
300   71.52    5.        20.     13.27  1.
#
RUN BELFIL
#RUNNING 0679

MODE  1
 4 WASHES, REACTOR DISCH.AL2O3 CONC.=  10.254
INTERNAL CAKE VOLUME =    9.900 GALLONS, CONST.=   8.800

VOLUMES IN GALLONS
REACTOR DISCHARGE  FLOCCULANT  WASH WATER  CAKE LIQUOR      CODE
      71.52           5.00        20.00        13.27         1.00
```

The Final Material Balance

```
PARTICLE INTERNAL VOLUME =   9.900 GALLONS, SHRINKAGE CONSTANT =   8.800
 NO.    LBS. AL2O3    LBS. LIQUOR  GAL. LIQUOR
  1       77.619        759.48        71.52
  2        0.000         41.70         5.00
  3       87.115        987.92        96.52
  4       75.138        852.09        83.25
  5        9.496        186.74        20.00
  6       11.977        135.82        13.27
  7        7.768        183.11        20.00
  8       10.249        132.19        13.27
  9        5.563        178.48        20.00
 10        8.044        127.56        13.27
 11        2.904        172.90        20.00
 12        5.385        121.98        13.27
 13       -0.000        166.80        20.00
 14        2.481        115.88        13.27
AVG. SQ. OF ERROR = 0.000000  SSE =  0.00000
NO. OF BALANCES =    1
#ET=45.9 PT=0.5 IO=0.4
```

APPENDIX B.—MATERIAL BALANCE USING A PROGRAMMABLE CALCULATOR

The balance form in figure 5 is used for this procedure.

1. Input composition is specified (A, W, V) for
   a. Heat exchanger (reactor) discharge
   b. Flocculant
   c. Wash water
2. All wash filtrate volumes are set equal to the wash water volume.
3. An average cake liquor volume is determined from the average value obtained in the miniplant run. See appendix C for methods used to determine this value. Set all cake volumes equal to this value V_t.
4. Make estimates (guesses) for values of
   a. Internal cake liquor volume ($V_i = 0$ to V_t)
   b. Voids shrinkage constant (k = any finite value)
5. Make up the following data table specifying the values and order of data entry into the TI-59 programmable calculator:
   a. Number of washes (1 to 12) .... n
   b. First wash filtrate, analytical wt-pct Al_2O_3 .... a_1
   c. First feed cake (= form cake), analytical wt-pct Al_2O_3 .... a_2
   d. First wash liquor, analytical wt-pct Al_2O_3 .... a_3
   e. Succeeding feed cakes (if applicable)
   f. Succeeding wash liquors (if applicable)
   g. Final feed cake, analytical wt-pct Al_2O_3 .... a_4
   h. Final wash liquor (that is, wash water), analytical wt-pct Al_2O_3 .... a_5
   i. Final product cake, analytical wt-pct Al_2O_3 .... a_6
   j. Circuit code (0 = first wash filtrate not recycled, 1 = recycled) c
   k. Reactor discharge, pounds of Al_2O_3 .... A_2
   l. Reactor discharge, gallons of liquor .... V_2
   m. Wash water, gallons of liquor .... V_f
   n. Total cake liquor, average gallons of liquor .... V_t
   o. Internal cake liquor, gallons (estimated) .... V_i
   p. Voids shrinkage constant, square gallons per pound .... k
6. The TI-59 program summarized at the end of this appendix is run as follows:
   a. Push RST button
   b. Push R/S, enter first data value from step 5 (data table). Repeat until entire table has been entered.
   c. After a few minutes, the analytical weight balances for streams of steps 5b through 5i will be printed out in order. The final SSE value represents the sum of the percent least squares errors for the previous analytical balance values.
7. To find a least squares SSE, different values of V_i and k must be tried. To change these values, enter them in order after the last SSE is read and another material balance will be printed out. Continue to do this until SSE reaches a minimum value.
8. To save printer paper, keep printer off until it has been determined that a minimum SSE has been reached. Then turn the printer back on and enter the appropriate V_i and k values to obtain the optimal Al_2O_3 balance values.
9. At this time, one may want to predict a balance for a filtration system having different values for different reactor discharge alumina (memory 31), reactor liquor volumes (memory 30), wash water volumes (memory 29), cake liquor volumes (memory 28), wash water alumina weights (memory 2), and numbers of wash stages (memory 42). If so, enter these new values into the corresponding memories. If the first wash filtrates in memories 34 and 36 are equal, change the values of one of them. Then enter the number of washes into the register, push A, push A′, and enter values for steps 5o and 5p into calculator and wait for the calculator to stop. Then push RCL 40 to obtain the final cake alumina losses.

Figure B-1 provides a schematic of the program for material balance using the shrinking voids model.

Program Memory Bank

0	
1	$A_{\text{cake analysis}}$ (final)
2	$A_{\text{wash liquor analysis}}$ (wash water)
3	$A_{\text{cake analysis}}$ (next to last)
4	$A_{\text{wash liquor analysis}}$ (next to last)
5-24	Alternate $A_{\text{cake analysis}}$ and $A_{\text{wash liquor analysis}}$ until first stage analysis is reached
25	Not assigned
26	$A_{\text{first wash liquor guess}}$
27	$V_{\text{internal cake liquor guess}}$
28	$V_{\text{total cake liquor}}$
29	$V_{\text{wash water}}$
30	$V_{\text{reactor discharge + flocculant}}$
31	$A_{\text{reactor discharge + flocculant}}$
32	Circuit code
33	A_f', wash water (earlier calculation)
34	A_2', first wash filtrate (earlier calculation)
35	A_f'', wash water (current calculation)
36	A_2'', first wash filtrate (current calculation)
37	$A_{\text{total cake liquor}}$ (previous wash)
38	$A_{\text{external feed cake}}$
39	$A_{\text{internal feed cake}}$
40	$A_{\text{total cake liquor}}$ (present wash)
41	k, voids shrinkage constant
42	Number of washes
43	N, wash ratio
44	f, pure water wash recovery
45	$V_{\text{internal cake liquor}}$
46	$\Sigma(\text{error})^2$
47	Current wash calculation number
48	Memory where current A stored
49	Location of highest numbered A

Belt-Filter Material Balance Modeling

TI-59 Program

```
000  22 INV    050  72 ST*    100  36  36    150  65  x     200  43 RCL
001  58 FIX    051  00  00    101  42 STO    151  43 RCL    201  44  44
002  91 R/S    052  69 OP     102  26  26    152  32  32    202  95  =
003  11  A     053  30  30    103  25 CLR    153  95  =     203  42 STO
004  22 INV    054  71 SBR    104  42 STO    154  42 STO    204  35  35
005  86 STF    055  45 Yx     105  47  47    155  40  40    205  43 RCL
006  01  01    056  43 RCL    106  43 RCL    156  65  x     206  42  42
007  42 STO    057  28  28    107  45  45    157  43 RCL    207  32 X:T
008  00  00    058  95  =     108  42 STO    158  27  27    208  43 RCL
009  42 STO    059  72 ST*    109  27  27    159  55  ÷     209  47  47
010  36  36    060  00  00    110  94 +/-    160  43 RCL    210  67  EQ
011  85  +     061  97 DSZ    111  85  +     161  28  28    211  13  C
012  01  1     062  00  00    112  43 RCL    162  95  =     212  87 IFF
013  95  =     063  61 GTO    113  28  28    163  42 STO    213  00  00
014  42 STO    064  76 LBL    114  95  =     164  39  39    214  38 SIN
015  34  34    065  16  A'    115  35 1/X    165  94 +/-    215  43 RCL
016  76 LBL    066  91 R/S    116  65  x     166  85  +     216  35  35
017  15  E     067  42 STO    117  43 RCL    167  43 RCL    217  19  D'
018  91 R/S    068  45  45    118  29  29    168  40  40    218  76 LBL
019  72 ST*    069  04  4     119  95  =     169  95  =     219  38 SIN
020  00  00    070  02  2     120  42 STO    170  42 STO    220  43 RCL
021  97 DSZ    071  02  2     121  43  43    171  38  38    221  35  35
022  00  00    072  04  4     122  94 +/-    172  43 RCL    222  48 EXC
023  15  E     073  69 OP     123  22 INV    173  40  40    223  26  26
024  91 R/S    074  04  04    124  23 LNX    174  87 IFF    224  94 +/-
025  42 STO    075  43 RCL    125  94 +/-    175  00  00    225  85  +
026  32  32    076  45  45    126  85  +     176  14  D     226  43 RCL
027  91 R/S    077  69 OP     127  01  1     177  19  D'    227  26  26
028  42 STO    078  06  06    128  95  =     178  76 LBL    228  85  +
029  31  31    079  02  2     129  42 STO    179  14  D     229  43 RCL
030  91 R/S    080  06  6     130  44  44    180  01  1     230  38  38
031  42 STO    081  69 OP     131  43 RCL    181  44 SUM    231  85  +
032  30  30    082  04  04    132  31  31    182  47  47    232  43 RCL
033  91 R/S    083  91 R/S    133  85  +     183  43 RCL    233  39  39
034  42 STO    084  42 STO    134  43 RCL    184  26  26    234  95  =
035  29  29    085  41  41    135  26  26    185  75  -     235  48 EXC
036  91 R/S    086  69 OP     136  65  x     186  43 RCL    236  40  40
037  42 STO    087  06  06    137  43 RCL    187  44  44    237  42 STO
038  28  28    088  86 STF    138  32  32    188  65  x     238  37  37
039  43 RCL    089  00  00    139  95  =     189  43 RCL    239  75  -
040  49  49    090  43 RCL    140  65  x     190  38  38    240  43 RCL
041  42 STO    091  49  49    141  43 RCL    191  95  =     241  40  40
042  00  00    092  42 STO    142  28  28    192  65  x     242  95  =
043  76 LBL    093  48  48    143  55  ÷     193  43 RCL    243  65  x
044  61 GTO    094  25 CLR    144  53  (     194  43  43    244  43 RCL
045  71 SBR    095  42 STO    145  43 RCL    195  55  ÷     245  41  41
046  45 Yx     096  46  46    146  30  30    196  53  (     246  55  ÷
047  43 RCL    097  76 LBL    147  85  +     197  43 RCL    247  43 RCL
048  29  29    098  12  B     148  43 RCL    198  43  43    248  28  28
049  95  =     099  43 RCL    149  29  29    199  75  -     249  95  =
```

```
250  22 INV
251  44 SUM
252  27  27
253  43 RCL
254  29  29
255  55  ÷
256  53  (
257  43 RCL
258  28  28
259  75  -
260  43 RCL
261  27  27
262  95  =
263  42 STO
264  43  43
265  94 +/-
266  22 INV
267  23 LNX
268  94 +/-
269  85  +
270  01  1
271  95  =
272  42 STO
273  44  44
274  43 RCL
275  40  40
276  65  x
277  43 RCL
278  27  27
279  55  ÷
280  43 RCL
281  28  28
282  95  =
283  42 STO
284  39  39
285  94 +/-
286  85  +
287  43 RCL
288  40  40
289  95  =
290  42 STO
291  38  38
292  87 IFF
293  00  00
294  14  D
295  43 RCL
296  40  40
297  19 D'
298  43 RCL
299  42  42
300  32 X:T
301  43 RCL
302  47  47
303  22 INV
304  67  EQ
305  14  D
306  03  3
307  06  6
308  03  3
309  06  6
310  01  1
311  07  7
312  69 OP
313  04  04
314  43 RCL
315  46  46
316  69 OP
317  06  06
318  61 GTO
319  16 A'
320  76 LBL
321  13  C
322  22 INV
323  87 IFF
324  00  00
325  17 B'
326  58 FIX
327  03  03
328  43 RCL
329  02  02
330  52 EE
331  22 INV
332  52 EE
333  32 X:T
334  43 RCL
335  35  35
336  52 EE
337  22 INV
338  52 EE
339  22 INV
340  58 FIX
341  22 INV
342  67  EQ
343  47 CMS
344  22 INV
345  86 STF
346  00  00
347  43 RCL
348  36  36
349  19 D'
350  61 GTO
351  12  B
352  76 LBL
353  47 CMS
354  48 EXC
355  36  36
356  48 EXC
357  34  34
358  48 EXC
359  36  36
360  48 EXC
361  35  35
362  48 EXC
363  33  33
364  48 EXC
365  35  35
366  43 RCL
367  36  36
368  75  -
369  53  (
370  43 RCL
371  35  35
372  75  -
373  43 RCL
374  02  02
375  54  )
376  55  ÷
377  53  (
378  43 RCL
379  35  35
380  75  -
381  43 RCL
382  33  33
383  54  )
384  65  x
385  53  (
386  43 RCL
387  36  36
388  75  -
389  43 RCL
390  34  34
391  95  =
392  42 STO
393  36  36
394  61 GTO
395  12  B
396  76 LBL
397  19 D'
398  87 IFF
399  01  01
400  52 EE
401  99 PRT
402  75  -
403  73 RC*
404  48  48
405  95  =
406  55  ÷
407  73 RC*
408  48  48
409  95  =
410  33 X^2
411  44 SUM
412  46  46
413  01  1
414  22 INV
415  44 SUM
416  48  48
417  76 LBL
418  52 EE
419  92 RTN
420  76 LBL
421  17 B'
422  43 RCL
423  35  35
424  99 PRT
425  01  1
426  22 INV
427  44 SUM
428  48  48
429  61 GTO
430  38 SIN
431  76 LBL
432  45 Y^X
433  01  1
434  85  +
435  93  .
436  00  0
437  02  2
438  00  0
439  07  7
440  09  9
441  65  x
442  73 RC*
443  00  00
444  45 Y^X
445  01  1
446  93  .
447  01  1
448  95  =
449  65  x
450  73 RC*
451  00  00
452  65  x
453  93  .
454  00  0
455  08  8
456  03  3
457  04  4
458  65  x
459  92 RTN
460  76 LBL
461  11  A
462  86 STF
463  01  01
464  42 STO
465  42  42
466  65  x
467  02  2
468  85  +
469  02  2
470  95  =
471  42 STO
472  49  49
473  92 RTN
```

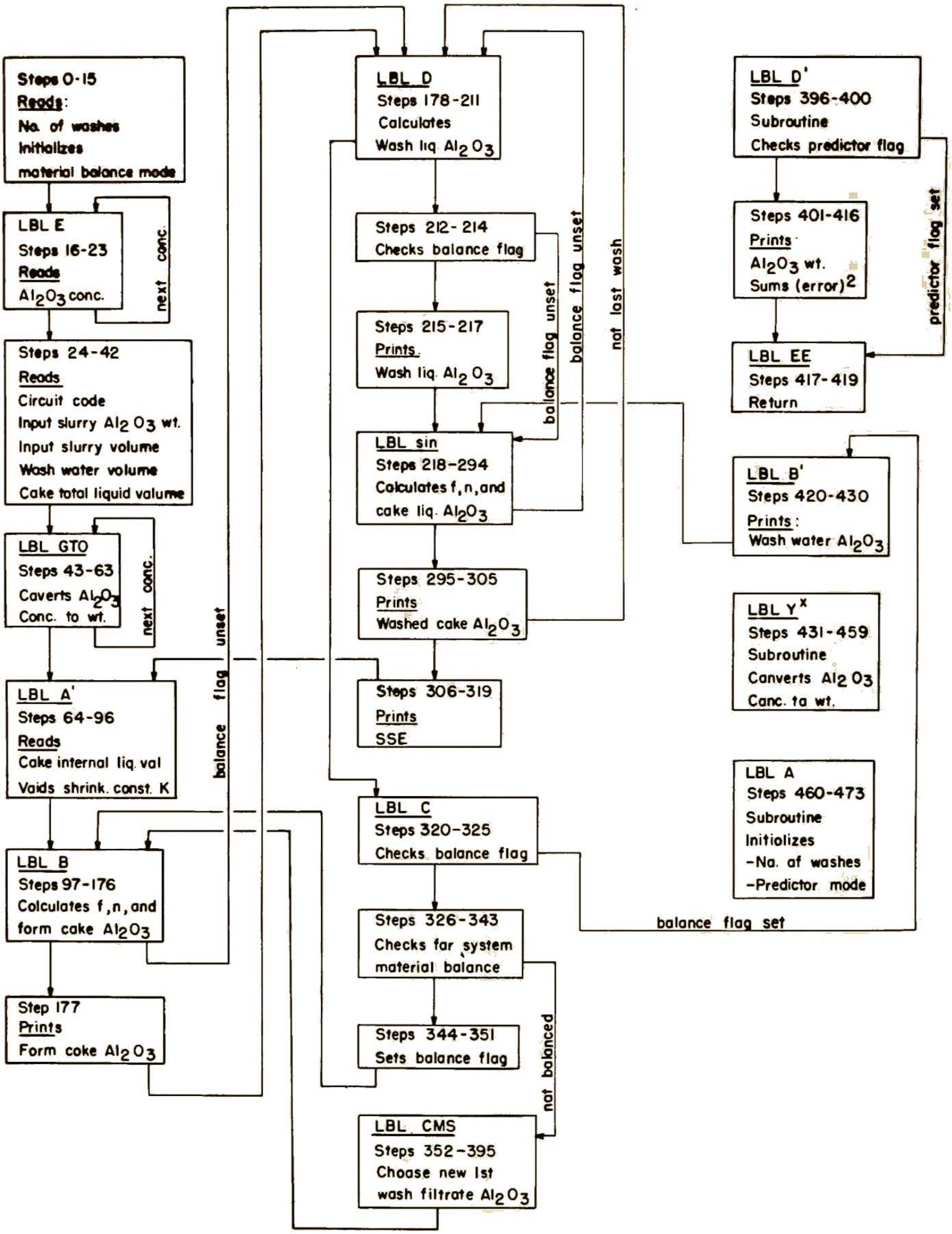

FIGURE B-1.—Schematic of TI-59 program for material balance using the shrinking voids model.

APPENDIX C.—MEASUREMENT OF LEACH RESIDUE POROSITIES AND DENSITIES

The porosity of calcined kaolin clay that has been leached in a stoichiometrically 5-pct-excess aqueous 26 wt-pct HCl was determined as follows:

A sample of the leached solids (still saturated in the leach liquor) was first dried with a cloth dish towel to remove external moisture. Then a weighed amount, W_s, of these solids was dropped into a tared 25-ml pycnometer which was then filled to the mark with leach liquor of known density, ρ_{liquor}. Then using the weight, W_ℓ, of leach liquor added, the volume, V_s, of the original towel-dried solids was found to be

$$V_s = 25 - \frac{W_\ell}{\rho_{liquor}} \text{ milliliters.} \qquad \text{(C-1)}$$

The solids were then thoroughly washed with distilled water and dried in an oven at 125° C to obtain a dry solids weight, W_{ds}. From this weight, the weight, W_i, and volume V_i, of internal liquor (inside the particle pores) were found to be

$$W_i = W_s - W_{ds} \text{ grams and} \qquad \text{(C-2)}$$

$$V_i = \frac{W_i}{\rho_{liquor}} \text{ milliliters.} \qquad \text{(C-3)}$$

This information is then used to find the percent porosity, P, by volume

$$P = 100 \frac{V_i}{V_s}. \qquad \text{(C-4)}$$

Table C-1 summarizes these porosities along with the wet particle densities

$$\rho_s = \frac{W_s}{V_s} \text{ grams per milliliter,} \qquad \text{(C-5)}$$

and an absolute solids density

$$\rho_{ds} = \frac{W_{ds}}{V_{ds}} \text{ grams per milliliter.} \qquad \text{(C-6)}$$

TABLE C-1.—Summary of 1-day-old leach residue densities and porosities (20° C)

	Wet particle density[1] g/cm³	Dry particle absolute density, g/cm³	Dry particle porosity, volume percent
Feed size, mesh:			
Minus 10 plus 14	1.687	2.212	56.24
Minus 14 plus 20	1.619	2.146	53.40
Minus 20 plus 28	1.728	2.181	54.14
Average	1.68	2.18	54.6

[1] $\rho_{liquor} = 1.2776$.

APPENDIX D.—CAKE LIQUOR DENSITY AND VOLUME DETERMINATIONS

The original wet cake is weighed, W_{ws}, and washed with a weight of wash water, W_{ww}. The repulp liquor obtained has a density, ρ_R, and weight-percent Al_2O_3, P_R, as does the original liquor in the cake, density, ρ_o, and weight-percent, P_o. The dry washed cake has a weight, W_{ds}.

The relationship between density and weight-percent Al_2O_3 is

$$\rho_o = 1 + 0.02079\,(P_o)^{1.1}, \qquad \text{(D-1)}$$

$$\rho_R = 1 + 0.02079\,(P_R)^{1.1}. \qquad \text{(D-2)}$$

The weight of the original liquor is

$$W_L = W_{ds} + W_{ws}. \qquad \text{(D-3)}$$

The weight of alumina in the original liquor is

$$W_A = \frac{P_R}{100}\,(W_L + W_{ww}). \qquad \text{(D-4)}$$

The percent alumina in the original liquor is

$$P_o = 100\,\frac{W_A}{W_L}. \qquad \text{(D-5)}$$

To obtain the original liquor density, P_o in equation D-5 is substituted into equation D-1

$$\rho_o = 1 + 0.02079\left(100\,\frac{W_A}{W_L}\right)^{1.1}. \qquad \text{(D-6)}$$

Substitution of the W_L and W_A values of equations D-3 and D-4 gives

$$\rho_o = 1 + 0.02079\left(P_R\left(\frac{W_{ws} - W_{ds} + W_{ww}}{W_{ws} - W_{ds}}\right)\right)^{1.1}. \qquad \text{(D-7)}$$

Solving equation D-2 for P_R gives

$$P_R = \left(\frac{\rho_R - 1}{0.02079}\right)^{1/1.1}. \qquad \text{(D-8)}$$

Finally, substitution of this value into equation D-7 gives

$$\begin{aligned}\rho_o &= 1 + 0.02079\left[\left(\frac{\rho_R - 1}{0.02079}\right)^{1/1.1}\left(\frac{W_{ws} - W_{ds} + W_{ww}}{W_{ws} - W_{ds}}\right)\right]^{1.1}, \qquad \text{(D-9)}\\ &= 1 + (\rho_R - 1)\left(\frac{W_{ws} - W_{ds} + W_{ww}}{W_{ws} - W_{ds}}\right)^{1.1}.\end{aligned}$$

With this density, the liquor volume per weight of cake solids must then be

$$V_t = (W_{ws} - W_{ds})/\rho_o. \qquad \text{(D-10)}$$

APPENDIX E.—NOMENCLATURE

A = Pounds Al_2O_3 in liquor stream (usually subscripted).
A_f = Pounds Al_2O_3 in wash water stream (value usually zero).
C_{eq} = Equilibrium Al_2O_3 concentration in liquor, pounds per gallon.
C_f = Final Al_2O_3 concentration in liquor, pounds per gallon.
C_i = Initial Al_2O_3 concentration in liquor, pounds per gallon.
C_w = Solute concentration in wash liquor, pounds per gallon.
C_1 = Solute concentration in feed cake liquor, pounds per gallon.
C_2 = Solute concentration in washed cake liquor, pounds per gallon.
e = Natural logarithm base, 2.71828. . .
f = Fraction of cake Al_2O_3 removed during wash with salt-free wash liquor.
j = Number of perfect mixing cells.
k = Voids shrinkage constant, square gallons per pound.
N = The wash ratio, V_i/V_o.
R = Residual, a theoretical measure of solute remaining in cake after washing.
tc = Time constant in diffusion model.
V = Gallons of liquor (usually subscripted).
W = Pounds of liquor in stream.

Greek letters

Δ = Change in the quantity that follows.
ρ = Liquor density, pounds per gallon.

Subscripts used with A and V

e = External liquor fraction of cake particle.
i = Internal liquor fraction of cake particle.
t = Total liquor fraction of cake particle.
o = Feed cake.
1 = Wash liquor.
2 = Wash filtrate.
3 = Washed cake.
4 = New feed cake.

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