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DIRECT PROBLEM GEODETIC COMPUTATION USING A PROGRAMMABLE POCKET CALCULATOR

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Abstract

By exploiting the programming potential of modern calculators, geodesic accuracy can be readily and quickly achieved by numerical integration. Tests of the Texas T.I. 58 program included, gave results on a 132 kilometre line which agreed with test data to better than 0″·0001. Typical lines of approximately 30 kilometres were calculated in less than ten minutes.

Introduction

For the occasional user, the problems of access to a computer for developing or running a program, can often be overcome by using the later generation of programmable calculators.

A recent requirement for geodetic computation prompted further investigation of my own calculator, a Texas T.I.58, which has programming facilities, until quite recently the preserve of a computer or the larger table-top scientific calculators.

Many of the equations for spheroidal geodetic computation can be readily programmed, but in general all are developed through expansion of series and are therefore long and complex, or short and approximate.

To achieve precise results for the direct case computation, the algorithm devised by Kivioja [1] proved particularly suitable, using straightforward equations to achieve geodesic accuracy by numerical integration. Centimetre accuracy can be readily achieved.

Kivioja's Algorithm

The theory has been extensively documented in ref. [1] but briefly is as follows:

If a geodesic, length S, is divided into n equal increments length dS, then providing dS is small the elemental triangle ABC (Figure 1) may be considered plane. Also the sides AB, BC can be readily determined.

Using plane trigonometry the following straightforward expressions are derived,

$$d\lambda = \frac{dS \sin \alpha}{\nu \cos \phi} \qquad (1)$$

$$d\phi = \frac{\cos \alpha \, dS}{\rho} \qquad (2)$$

These enable $d\phi$ and $d\lambda$ to be computed for each increment.

To keep each element in the correct azimuth, use is made of Clairaut's constant

$$K = \nu \cos \phi \sin \alpha \qquad (3)$$

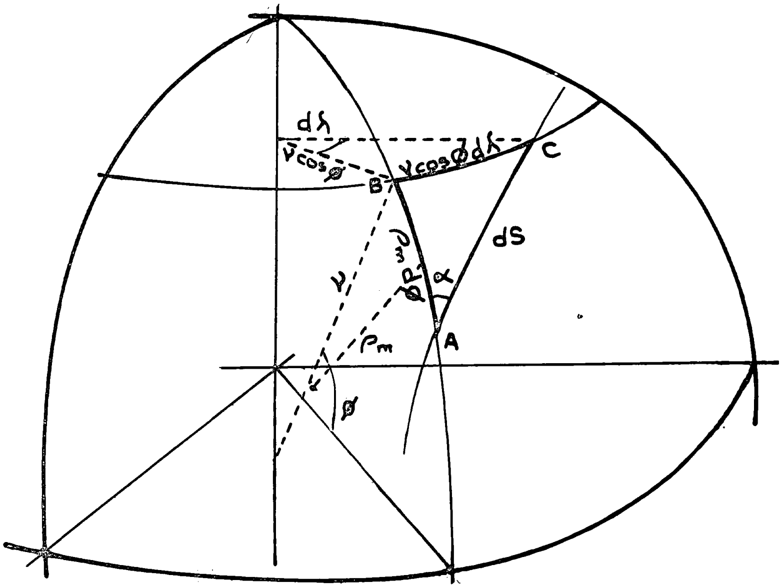

Fig. 1

from which the azimuth of the next increment may be computed using

$$\sin\alpha = \frac{K}{\nu\cos\phi} \tag{4}$$

Kivioja's algorithm uses the mean values of ρ, ν, ϕ and α for each element to give better approximations to a plane triangle.

The algorithm is as follows:

1. input ϕ_1, λ_1, α_{12}, S_{12}, n
2. $ds = S/n$
3. calculate: ρ_i, ν_i,
4. K_r
5. then $\frac{1}{2}d\phi$ (approx) using $\frac{1}{2}ds$
6. $\phi_m = \phi_i + \frac{1}{2}d\phi$
7. using ϕ_m, calculate ρ_m, ν_m, α_m
8. then $d\phi_r$, $d\lambda_r$
9. then $\phi_{i+1} = \phi_i + d\phi_r$, $\lambda_{i+1} = \lambda_i + d\lambda_r$
10. calculate ρ_{i+1}, ν_{i+1}, α_{i+1}
11. repeat from 4. to last increment
12. then $\phi_2 = \phi_{i+1}$, $\lambda_2 = \lambda_{i+1}$, $\alpha_{21} = \alpha_{i+1} + 180$

where suffices: i = beginning of increment

m = mean

$i+1$ = end of increment

r = increment number.

The Program

Subroutines are used for each of the four equations above, also for ρ and ν. The main program calls subroutines as required, summing $d\phi$ and $d\lambda$ at the end of each loop.

After removing the almost mandatory syntax errors one problem remained. After the first increment, the azimuth of the remainder are determined from equation (4) through the arcsine. If the azimuth fell in the second or third quadrant, the remaining computation proceeded in the first or fourth. This problem was overcome by using a test sequence on the input azimuth to set a flag if it fell in the second or third quadrant. If the flag is set, 180° is added to the computed azimuth at the end of the α subroutine. The program is listed in the appendix.

Use of the Program

To accommodate the program the calculator must be repartitioned.

1. Enter 2, press 2nd Op 17 (display—319.20)

2. Spheroid data may be input using a and b, or a and 1/f, (1/f to two decimal places to retain subsequent precision).
The spheroid may be changed as required.

2.1. enter a (metres), press D (display—a)

2.2. enter b (metres), press R/S (display—o., or previous 't' register contents).
Alternatively,

2.1. enter a (metres), press E (display—a)

2.2. enter 1/f (display as 2.2 above)

3. Station data Input.

The latitude, longitude and azimuth of the initial station are input as in the following example:

32° 25′ 54″.243, enter 32.2554243

If the latitude is South, or longtitude West, enter as a negative value.

3.1 enter latitude ϕ_1, press A. (display in decimal degrees)

3.2 enter longitude λ_1, press R/S. (display in decimal degrees)

3.3 enter azimuth α_{12}, press R/S. (display depends on test routine)

3.4 enter spheroid distance, S (metres). (display—S)

3.5 enter number of increments n, such that dS is approximately one to two kilometres, press R/S. (n must be an integer).

The computation now proceeds taking approximately 23 seconds for each increment. After completion the display shows the latitude ϕ_2, as in 3 above, in degrees, minutes and seconds.

3.6 Press R/S, display—longitude λ_2.

3.7 Press R/S, display—reverse azimuth α_{21}. (If the displayed value is greater than 360°, subtract 360).

4. For a new line, repeat steps 3.1 to 3.7.

Alternatively, for a new line attached to the last station:

4.1 enter $\alpha_{r,r+1}$, press B.

4.2 enter S, press R/S.

4.3 enter n, press R/S, display—ϕ_{r+1}.

4.4 press R/S, display—λ_{r+1}

4.5 press R/S, display—$\alpha_{r+1,r}$

The program was tested using a 132 km line from **[1]**, also a six station closed traverse from **[2]**. In all cases, the maximum difference noted did not exceed 0″·0001, equivalent to approximately 3 mm.

The program functions in all quadrants and with any azimuth, except the special case where the azimuth is, or reaches 90° or 270°. Here the latitude increment becomes zero (Equation 2) and the azimuth again 90° (270°). The closed loop then repeats.

Unfortunately the capacity of the T.I.58 is inadequate to compute the inverse case using Kivioja's algorithm. It can however be accommodated using the Gauss Mid-Latitude formulae.

Acknowledgements

My thanks to Carl Calvert, a fellow student who laboriously computed test data for the subroutines as an independent check. Also to L. G. Small, Senior Lecturer, North East London Polytechnic, for his assistance and encouragement.

Footnote

For users of the more powerful Hewlett-Packard 67 or 97 calculators, L. A. Kivioja has advised that programs for the direct and inverse case are to be published in the March issue of *Surveying and Mapping*.

References

1. Kivioja, L. A., "Computation of Geodetic Direct and Indirect problems by Computers Accumulating Increments from Geodetic Line Elements", *Bulletin Geodisique*, No. 99, March 1971, Paris.

2. Hollwey, J. R., Small, L. G. and Cross, P. A. "Geodetic Appreciation", Lecture notes, North East London Polytechnic. 1979.

APPENDIX

LOC	CODE	KEY	COMMENTS	LOC	CODE	KEY	COMMENTS	LOC	CODE	KEY	COMMENTS
0	76	2nd LBL	data	5	00	0		110	43	RCL	
1	14	D	input	6	95	=		1	13	13	
2	42	STO		7	77	2nd $x \geqslant t$		2	42	STO	
3	04	04		8	35	$1/x$		3	02	02	
4	91	R/S		9	61	GTO		4	71	SUBR	
5	33	x^2		60	23	lnx		5	16	2nd A	
6	55			1	76	2nd LBL		6	71	SUBR	
7	43	RCL		2	35	$1/x$		7	19	2nd D	
8	04	4		3	22	INV		8	97	2nd DSZ	
9	33	x^2		4	86	2nd St flg		9	03	3	
10	95	=		5	01	1		120	13	C	
1	94	+/−		6	76	2nd LBL		1	43	RCL	
2	85	+		7	23	lnx		2	13	13	
3	01	1		8	91	R/S		3	22	INV 2nd	
4	95	=		9	42	STO		4	88	DMS	
5	42	STO		70	12	12		5	91	R/S	
6	01	01		1	91	R/S		6	43	RCL	
7	01	1		2	42	STO		7	10	10	
8	08	8		3	03	03		8	22	INV 2nd	
9	00	0		4	35	$1/x$		9	88	DMS	
20	42	STO		5	65	×		130	91	R/S	
1	00	00		6	43	RCL		1	43	RCL	
2	55			7	12	12		2	07	7	
3	89	2nd π		8	95	=		3	85	+	
4	95	=		9	42	STO		4	43	RCL	
5	42	STO		80	12	12		5	00	0	
6	08	08		1	71	SUBR	program	6	95	=	
7	09	9		2	16	2nd A		7	22	INV 2nd	
8	00	0		3	76	2nd LBL		8	88	DMS	
9	32	$x \quad t$		4	13	C		9	22	INV	
30	91	R/S		5	71	SUBR		140	86	2nd St flg	
1	76	2nd LBL		6	17	2nd B		1	01	1	
2	11	A		7	43	RCL		2	91	R/S	
3	88	2nd DMS		8	12	12		3	76	2nd LBL	r, ρ
4	42	STO		9	55			4	16	2nd A	subr.
5	02	02		90	02	2		5	53	(	
6	42	STO		1	95	=		6	53	(	
7	13	13		2	71	SUBR		7	53	(	
8	91	R/S		3	18	2nd C		8	53	(	
9	88	2nd DMS		4	44	SUM		9	01	1	
40	42	STO		5	02	02		150	75	—	
1	10	10		6	71	SUBR		1	53	(	
2	91	R/S		7	16	2nd A		2	43	RCL	
3	76	2nd LBL		8	71	SUBR		3	01	1	
4	12	B		9	19	2nd D		4	65	×	
5	88	2nd DMS		100	43	RCL		5	43	RCL	
6	42	STO		1	12	12		6	02	2	
7	07	07		2	71	SUBR		7	38	2nd SIN	
8	22	INV		3	18	2nd C		8	33	x^2	
9	77	2nd $x \geqslant t$		4	44	SUM		159	54	)	
50	23	lnx		5	13	13					
1	86	2nd St flg		6	71	SUBR					
2	01	1		7	10	2nd E					
3	75	—		8	44	SUM					
4	43	RCL		9	10	10					

APPENDIX-*continued*

LOC	CODE	KEY	COMMENTS	LOC	CODE	KEY	COMMENTS	LOC	CODE	KEY	COMMENTS
160	54	)		5	18	2nd C	$d\phi$ subr	270	43	RCL	
1	34	$\sqrt{x}$		6	53	(		1	12	12	
2	54	)		7	24	CE		2	65	×	
3	42	STO		8	65	×		3	43	RCL	
4	11	11		9	43	RCL		4	08	8	
5	35	1/x		220	07	7		5	55		
6	65	×		1	39	2nd COS		6	43	RCL	
7	43	RCL		2	65	×		7	05	5	
8	04	4		3	43	RCL		8	65	×	
9	54	)		4	08	8		9	43	RCL	
170	42	STO		5	55			280	07	7	
1	05	05		6	43	RCL		1	38	2nd SIN	
2	53	(		7	06	6		2	55		
3	53	(		8	54	)		3	43	RCL	
4	43	RCL		9	92	INV SUBR		4	02	02	
5	11	11		230	76	2nd LBL	x	5	39	2nd COS	
6	45	y^x		1	19	2nd D	subr	6	54	)	
7	03	3		2	53	(		7	92	INV SUBR	
8	54	)		3	53	(		8	76	2nd LBL	data
9	35	1/x		4	53	(		9	15	E	input
180	65	×		5	43	RCL		290	42	STO	
1	53	(		6	09	9		1	04	04	
2	43	RCL		7	55			2	91	R/S	
3	04	4		8	53	(		3	35	1/x	
4	65	×		9	43	RCL		4	75	—	
5	53	(		240	05	5		5	01	1	
6	01	1		1	65	×		6	95	=	
7	75	—		2	43	RCL		7	33	x^2	
8	43	RCL		3	02	2		8	61	GTO	
9	01	1		4	39	2nd COS		9	00	0	
190	54	)		5	54	)		300	11	11	
1	54	)		6	54	)					
2	54	)		7	22	INV 2nd					
3	54	)		8	38	SIN					
4	42	STO		9	54	)					
5	06	06		250	87	2nd if flg					
6	92	INV SUBR		1	01	1					
7	76	2nd LBL	K	2	33	x^2					
8	17	2nd B	subr	3	61	GTO					
9	53	(		4	34	$\sqrt{x}$					
200	43	RCL		5	76	2nd LBL					
1	05	5		6	33	x^2					
2	65	×		7	94	+/−					
3	43	RCL		8	85	+					
4	07	7		9	43	RCL					
5	38	2nd SIN		260	00	0					
6	65	×		1	54	)					
7	43	RCL		2	76	2nd LBL					
8	02	2		3	34	$\sqrt{x}$					
9	39	2nd COS		4	42	STO					
210	54	)		5	07	07					
1	42	STO		6	92	INV SUBR					
2	09	09		7	76	2nd LBL	$d\lambda$				
3	92	INV SUBR		8	10	2nd E	subr				
4	76	2nd LBL		9	53	(					