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ESTIMATING THE DEMAND UNCERTAINTY IN SINGLE-PERIOD INVENTORY PROBLEMS USING THE GOMPERTZ CURVE AND THE SCHMEISER-DEUTSCH DISTRIBUTION

by

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September 1981

Thesis Advisor: G. F. Lindsay

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Estimating the Demand Uncertainty in Single-Period Inventory Problems Using the Gompertz Curve and the Schmeiser-Deutsch Distribution

by

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I. INTRODUCTION

The perishable product is beginning to receive attention in the inventory problem. Often, it has a fixed lifetime after which it becomes useless to satisfy demands. Newspapers, medical drugs, human blood, photographic films and some foodstuffs are common examples.

An inventory problem for perishable products is often treated as a single-period inventory problem, sometimes called a newsboy problem. The newsboy problem was introduced by Morse and Kimball in 1950 and the development of it has been presented in several journals and publications. A brief history of it was given by Masuda [Ref. 1].

A well-known characterization of the newsboy problem is:

> The newsboy must decide how many papers to purchase for resale during the day. If he buys too many papers--that is, more papers than people will purchase from him--he will have papers left at the end of the day and will incur a loss on each, since old newspapers have little salvage value. If, on the other hand, he purchases too few papers for resale, he will sell out early and his later customers will not be able to buy from him; he thus incurs a cost (lost profit) for each paper he could have sold, but didn't. The newsboy's problem is to decide how many papers to have on hand at the beginning of the day so as to minimize the losses (due to surplus or shortage) at the end of the day. [Ref. 2]

Finding good solutions to the newsboy problem depends upon the amount of information about the magnitude of demand X.

In recent research dealing with newsboy ordering decisio where little information existed about demand, it was assumed that at the end of each period the demand for that period would be known [Refs. 1,3,4]. In order for this to happen, one would need unlimited supply or some way to measure the demand from unsatisfied customers. In practice, this is often not the case. For example, when the newsstand is empty, the later customers will not stop at that newsstand. Accordingly, we do not know how many more customers would like to buy that newspaper. Thus, we may have frequency data on demand only up to the value of supply.

This thesis is intended to provide methods permitting solutions to the newsboy problem when we only have demand frequencies up to the supply value.

In Chapter II, two procedures will be proposed to estimate the maximum demand. The first procedure uses the Gompertz Curve since the nature of the demand cumulative distribution function (c.d.f.) can be viewed as a growth curve with the market saturate as an asymptote. The second procedure suggested is the four-parameter Schmeiser-Deutsch Distribution since it can take on a wide variety of shapes so as to match many distributions. Also, the Schmeiser-Deutsch Distribution was found to be a suitable procedure for the case of limited supply.

Using the estimation of maximum demand together with the unit cost of surplus and the unit cost of outage, the estimation of optimal order quantity is discussed in Chapter III for the "under uncertainty" and "under risk" cases.

Conclusions and applications are given in Chapter IV. A review of the Schmeiser-Deutsch Distribution is presented in Appendix A.

II. ESTIMATION OF MAXIMUM DEMAND

Care should be taken to estimate the maximum demand since the optimal ordering quantity for uncertainty depends on it. In this chapter, two alternatives are suggested to estimate the maximum demand when we have unlimited supply. Then, we will show how one of these methods may be used to estimate maximum demand when there is no data on unsatisfied demand (the case of limited supply).

The procedures that appear suitable and realistic are (1) using the Gompertz Curve and (2) using the Schmeiser-Deutsch Distribution. We will discuss the Gompertz Curve first.

A. USING THE GOMPERTZ CURVE TO ESTIMATE THE MAXIMUM DEMAND

In the terminology of economics, the demand c.d.f. can be considered as a "growth curve" since it has a "market saturate" as an upper asymptote. The Gompertz Curve is the most generally used growth curve. Its formula

$$Y = ka^{b^X} , \qquad (1)$$

where X is the time series and Y is the sales amount, depends on the values of the parameters k, a and b (or their logarithms), it may take on any one of a variety of shapes [Ref. 5].

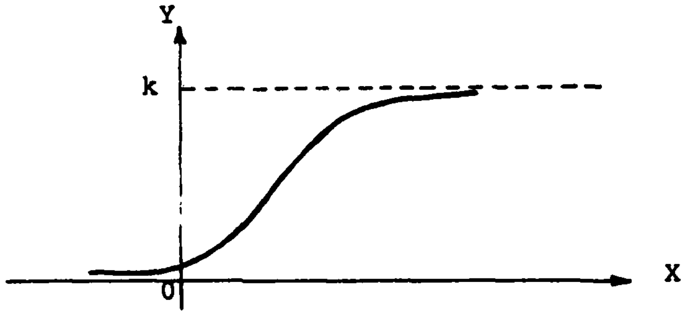

Figure 1. The Gompertz Curve with $0 < a < 1$, $0 < b < 1$

We are interested in using the Gompertz Curve with $0 < a < 1$ and $0 < b < 1$ to approximate the c.d.f. of demand. This form of the Gompertz Curve is known to be asymptotic to k. When we fit the Gompertz Curve to the observed cumulative frequency of the data, k should theoretically be 1.0. In practice, however, it may depart slightly from 1.0. To compensate for this, we use the following form to approximate the c.d.f.:

$$\frac{Y}{k} = a^{b^X}$$

The following description is taken from [Ref. 5]:

The parameters k, a and b are obtained as follows:

1) Be certain that the number of observations is divisible by 3, i.e., there are 3N periods of base data.

2) Take the common logarithms of each Y.

3) Add the first N log Y's to obtain $\Sigma_1 \log Y$; then the second N data points to obtain $\Sigma_2 \log Y$; and the last N points to obtain $\Sigma_3 \log Y$.

4) Set the smallest observation to be unity.

5) Substitute in the following formulas:

$$b^N = \frac{\Sigma_3 \log Y - \Sigma_2 \log Y}{\Sigma_2 \log Y - \Sigma_1 \log Y} , \qquad (2)$$

$$\log a = (\Sigma_2 \log Y - \Sigma_1 \log Y) \cdot \frac{(b-1)}{(b^N-1)^2} , \qquad (3)$$

and

$$\log k = \frac{1}{N} \cdot (\Sigma_1 \log Y - (\frac{b^N-1}{b-1}) \cdot \log a) . \qquad (4)$$

With values of k, a and b, one can compute Y for any value of X using the formula of the Gompertz Curve. Also, the formula can be derived from the Gompertz Curve to generate X from the Gompertz Curve as:

$$X = \frac{\log(\frac{\log Y - \log k}{\log a})}{\log b} \qquad (5)$$

In this chapter, we wish to estimate the maximum demand. If the demand distribution has an infinite upper tail, we can not hope to estimate the 100^{th} percentile of demand. We will be content, therefore, with the 99^{th} percentile of demand. This is found by setting $(Y/k) = .99$ or $Y = .99k$ in equation (5), i.e.,

$$X_{max} \text{ will be } = \frac{\log(\frac{-.0044}{\log a})}{\log b} , \qquad (6)$$

which may be considered to be the estimator of the maximum demand if the smallest observed X is unity. (If not, the maximum demand should be adjusted to be X plus the smallest X minus one.)

As an example assume that the demands for newspapers have been 10, 6, 9, 7, 5, 13, 11, 7, 8, 8. Using the Gompertz Curve to estimate the maximum demand, we can preceded as follows:

Let X be the demand for newspapers, and Y be the cumulative relative frequency of X.

TABLE I

DEMAND AND ITS CUMULATIVE RELATIVE FREQUENCY

X	Y
5	0.1
6	0.2
7	0.4
8	0.6
9	0.7
10	0.8
11	0.9
12	0.9
13	1.0

Since Y is a growth curve, it can be approximated by the Gompertz Curve, $Y = ka^{b^X}$. To determine the appropriate values of k, a and b using the method described above:

1) From these data, there are 9 distinct values between the smallest observation (5) and the largest observation (13), and 9 is divisible by 3. (Here, N = 9/3 = 3.)

2) In the third column of Table II, Y's are converted to log Y's.

3) Values of $\Sigma_1 \log Y$, $\Sigma_2 \log Y$ and $\Sigma_3 \log Y$ are shown in the fourth column of Table II.

4) Since the smallest observed X is not 1.0, the maximum demand should be adjusted by adding the difference between the smallest observed X and 1.0. This is because the Gompertz Curve assumes the smallest observation is unity.

5) From equation (2);

$$b^N = ((-.0915)-(-.4737))/((-.4737)-(-2.0969))$$

$$= 0.2355 .$$

Therefore,

$$b = 0.6175 .$$

From equation (3);

$$\log a = (-.4737-(-2.0969))(.6175-1)/(.2355-1)$$

$$= -1.0622 .$$

Therefore,

$$a = 0.0867 .$$

From equation (4);

$$\log k = \frac{1}{3}(-2.0969-(.2355-1)(-1.0622)/(.675-1))$$

$$= 0.0087.$$

Therefore,

$$k = 1.0203 .$$

I.e., the Gompertz Curve is $Y = (1.0203)(0.0867)^{(0.6175)^X}$.

Using the 99th percentile, the maximum demand is

$$X_{max} = 11.3964 + (5-1) = 15.3964$$

$$\doteq 15.$$

TABLE II

COMPUTATIONS IN FINDING A GOMPERTZ CURVE

X	Y	Log Y	ΣLog Y
5	0.1	-1.0000	
6	0.2	-0.6990	$\Sigma_1 \log Y = -2.0969$
7	0.4	-0.3979	
8	0.6	-0.2218	
9	0.7	-0.1549	$\Sigma_2 \log Y = -0.4737$
10	0.8	-0.0969	
11	0.9	-0.0458	
12	0.9	-0.0458	$\Sigma_3 \log Y = -0.0915$
13	1.0	0	

A program for the TI-59 to determine the maximum demand when the smallest observed X is unity, and to determine the parameters (k, a and b), is given in Appendix B (Program B1).

Experimentation with simulated data seems to indicate that the Gompertz Curve procedure works well when supply is unlimited, but not nearly so well when supply is limited.

B. USING THE SCHMEISER-DEUTSCH DISTRIBUTION TO ESTIMATE THE MAXIMUM DEMAND

K. Pearson pointed out that a theoretical distribution must have at least four free parameters, in order to fit the four characteristics (location, dispersion, skewness, and kurtosis) adequately [Ref. 6]. Kottas and Lau recommends the four-parameter Schmeiser-Deutsch Distribution as most suitable [Ref. 6]. The details of the Schmeiser-Deutsch Distribution are given in Appendix A.

The Schmeiser-Deutsch Distribution has a finite upper tail. Furthermore, if X is a random variable having the Schmeiser-Deutsch Distribution, there exists a closed-form expression for the maximum value of X. This expression is $a+b(1-d)^c$, where a, b, c and d are the four free parameters.

Using the data in the previous example, we will employ the Schmeiser-Deutsch Distribution to estimate the maximum demand. We first estimate the parameters using the algorithm in Appendix A:

1) The mode $m = (7+8)/2 = 7.5$.

   Set $a = 7.5$.

2) Set $d = (Y(7)+Y(8))/2 = 0.5$.

3) Select the two desired quantiles which satisfy equation (9), namely,

$$p_1 = 0.2 , \quad x_1 = 6$$

and

$$p_2 = 0.9 , \quad x_2 = 11$$

Substitute in equation (10), yielding c = 2.9453.

4) Substitute in equation (11), yielding b = 52.0120.

Then, the maximum demand is

$$X_{max} = a + b(1-d)^c$$

$$= 14.2529 \doteq 14.$$

This compares very well with the result obtained by using the Gompertz Curve which was X_{max} = 15.3964. For these data, there is no substantial difference between these two alternative methods of estimating X_{max}.

A program for the TI-59 to determine the maximum demand and the four parameters of the Schmeiser-Deutsch Distribution is given in Appendix B (Program B2).

C. ESTIMATING MAXIMUM DEMAND WHEN THERE IS NO DATA ON UNSATISFIED DEMAND

In the previous work it was implicitly assumed that the value of demand for each period was obtainable even if demand is greater than supply. This rarely happens in the real world unless backorders are taken. Speaking of the "newsboy problem", the newsboy can collect the data in the range of demand from zero to ordering quantity (Q) only. When the newspapers are sold out, he will not know how many more were in demand for that day. If he is recording the sales, his observed data on that day will be X = Q even if the demand is actually $X \geq Q$.

The Schmeiser-Deutsch Distribution can be easily used to investigate the case when data for demands greater than the quantity Q are not available. This is because its parameters can be estimated using only the mode and two selected distinct percentiles.

As an example, suppose that the newsboy orders only 10 newspapers for each day. His sales data for a 10 day period are 10, 6, 9, 7, 5, 10, 10, 7, 8, 8. (Instead of demand, which was 10, 6, 9, 7, 5, 13, 11, 7, 8, 8.) The sales data and their cumulative relative frequency will be:

TABLE III

SALES DATA AND THEIR CUMULATIVE RELATIVE FREQUENCY

X	Y(X)
5	0.1
6	0.2
7	0.4
8	0.6
9	0.7
10	1.0

Since Pr(X = 10) is actually Pr $(X \geq 10)$, we will ignore this data point.

Following the algorithm in Appendix A for the rest of the data points:

1) Set a = 7.5.

2) Set d = 0.5.

3) Select $p_1 = 0.6$, $x_1 = 8$, $p_2 = 0.7$, $x_2 = 9$ which satisfy equation (9).

Substitute in equation (10), yielding c = 1.5850.

4) Substitute in equation (11), yielding b = 19.2279.

From this, the maximum demand is $X_{max} = 13.9093$ which is close to X_{max} when the newsboy has unlimited supply.

With this X_{max}, one can determine Q*. Procedures to do this are discussed in the next chapter.

III. ESTIMATION OF OPTIMAL ORDER QUANTITY

In the previous chapter it was shown how to estimate the maximum demand (X_{max}). In this chapter, the estimate of the optimal order quantity (Q*) for the case of uncertainty and also for the case of risk will be discussed.

A. OPTIMAL ORDER QUANTITY UNDER UNCERTAINTY

When data in hand is not sufficient to estimate the possible future values of demand or when the distribution of demand is not known, the problem of deciding on Q falls into the class called decisions under uncertainty.

It has been shown that the optimal decisions under uncertainty (Q*) are [Ref. 1]:

$$Q^* = \left(\frac{c_o}{c_s + c_o}\right) \cdot X_{max} , \quad \text{for a continuous demand ,}$$

and

$$Q^* < \left(\frac{c_o}{c_s + c_o}\right)(X_{max} + 1) < Q^* + 1 ,$$

for a discrete demand, where c_s is the unit cost of surplus and c_o is the unit cost of outage.

Since one consequence of each decision is another observation of demand, one begins under uncertainty and uses the uncertainty optimal order quantity until demand data is adequate to estimate the distribution of demand and change to the optimum order quantity under risk.

B. OPTIMAL ORDER QUANTITY UNDER RISK

When one says "under risk", one means that the probability distribution of demand is known or, at least, some estimate of the probability distribution of demand is available. The latter case is possible using the Schmeiser-Deutsch Distribution suggested in Appendix A.

The optimal order quantity under risk is the value of Q* such that

$$F(Q^*) = \frac{c_o}{c_s + c_o}, \quad \text{for a continuous demand distribution,}$$

and

$$F(Q^*-1) < \frac{c_o}{c_s + c_o} < F(Q^*),$$

for a discrete demand distribution.

It should be noted here that all one really needs to estimate the probability distribution of demand is a value for the $[c_o/(c_o+c_s)]^{th}$ quantile. The inverse c.d.f. of the Schmeiser-Deutsch Distribution can easily take care of this problem.

As an example, suppose that the newsboy purchased the newspapers at the cost of 10¢ each and he can sell them for 20¢ each, with leftovers having a salvage value of 5¢ each.

Let p be the ratio $[c_o/(c_o+c_s)]$. Therefore, $p = 2/3$.

Consider the data from the previous example at the end of Chapter II. These data are adequate for a decision under risk. The p^{th} quantile can be estimated by the inverse c.d.f. of the Schmeiser-Deutsch distribution,

$$F^{-1}(p) = \begin{cases} a-b(d-p)^c & \text{if} \quad p \le d \\ a+b(p-d)^c & \text{if} \quad p > d , \end{cases}$$

where the four parameters were found to be a = 7.5, b = 19.2279, c = 1.5850 and d = 0.5.

Therefore, $F^{-1}(p) = 8.623$, which is the value of Q* for the case of continuous demand.

For the case of discrete demand, the optimal order quantity under risk is the value of Q* such that $F(Q^*-1) < p < F(Q^*)$, and then Q* = 9 in this example.

In the next chapter we will give conclusions and applications from this work.

IV. CONCLUSIONS AND APPLICATIONS

One of the important requirements needed to obtain the optimal order quantity in newsboy problems is the nature of the demand distribution. This thesis suggested some procedures which can be used to estimate the demand distribution even if data on unsatisfied demand are not available. In the case where it is hard to tell what common distribution the demand should belong to, the Schmeiser-Deutsch Distribution was felt to be useful.

It is fitting at this point to explain how the procedures from this and other research on the newsboy problems would be used.

Suppose we are starting a sequence of single-period inventory ordering decisions but have no information on the distribution of demand.

One way of employing these results would involve three phases:

1) For the decisions at time period 1 through some period m, the newsboy would probably rely upon a guess at the value of the maximum demand (X_{max}). Considering the unit cost of surplus (c_s) and the unit cost of outage (c_o), he has two alternatives:

(a) In case of $c_s << c_o$, use $Q^* = X_{max}$.

(b) Otherwise, use the uncertainty decision rule.

$$Q^* = [c_o/(c_s + c_o)] \cdot X_{max} .$$

2) For time periods (m+1) through some later period n, he would use $Q^* = [c_o/(c_o + c_s)] \cdot X_{max}$ with the value of X_{max} obtained from the Gompertz Curve or the Schmeiser-Deutsch Distribution, using data from the first m periods. Clearly, he may wish to revise this estimate as more data become available.

3) After time period n, he can switch from treating the decision as a decision under uncertainty to a decision under risk, using the inverse c.d.f. of the Schmeiser-Deutsch Distribution to estimate the value of $[c_o/(c_o + c_s)]^{th}$ quantile, which is known to be Q* for continuous demand.

It should be noted here that because of the data needed, m must provide at least three periods where he did not sell out, and from the work of Yong-u Sok, n should provide at least five periods where he did not sell out [Ref. 4].

From the work outlined above we have a reasonable means of starting a sequence of newsboy decisions when there is no information about the demand distribution and lost sales are unknown. It is hoped that the procedures suggested in this paper will be useful to those interested in single-period inventory problems and their solutions.

APPENDIX A

THE SCHMEISER-DEUTSCH DISTRIBUTION

In this appendix, the Schmeiser-Deutsch Distribution will be briefly reviewed. For a further study [Ref. 6] and [Ref. 7] are suggested.

Due to the lack of convenient methods of generating random values from distributions having more arbitrary shapes, the Schmeiser-Deutsch Distribution was developed and found suitable for generating random values from many common statistical distributions.

The Schmeiser-Deutsch Distribution can take on a wide variety of shapes, which makes it useful for many applications. Distributions ranging from Bernoulli trials through U-shaped distributions to the uniform distribution, as well as heavier tailed distributions, are obtainable. Also, shapes ranging from symmetry to maximum skewness, as measured by the third moment, are possible. The Schmeiser-Deutsch Distribution can have values for the first four moments so as to match any of the well-known distributions, and straightforward parameter determination techniques exist.

A most favorable characteristic of the Schmeiser-Deutsch Distributions is the existence of simple closed-form expressions for both the c.d.f. and the inverse c.d.f., being respectively,

$$F(x) = p = \begin{cases} d-[(a-x)/b]^{1/c} & \text{if } (a-bd^c) \le x \le a \\ d+[(x-a)/b]^{1/c} & \text{if } a \le x \le a+b(1-d)^c \end{cases} \quad (7)$$

and

$$x = F^{-1}(p) = \begin{cases} a-b(d-p)^c & \text{if } p \le d \\ a+b(p-d)^c & \text{if } p > d \end{cases} \quad (8)$$

where a, b, c and d are the four parameters of the distribution, and p is a percentile [Ref. 7].

"The location and the spread of the distribution are determined by a and b, respectively. The shape of the distribution, often measured in terms of skewness and tail-weight (or peakness), is determined by $-(d-p)^c$ in case of $p \le d$, or $(p-d)^c$ in case of $p > d$. This factor is a transformation from the rectangular shape of the uniform distribution to the shape of interest. The $d = 0.5$ yields a symmetric distribution. If $c > 1$, then skew is to the right for $d < 0.5$ and to the left for $d > 0.5$. (If $c < 1$, the direction of skew is reversed.)" [Ref. 7]

"The distribution possesses two characteristics which may adversely affect its usefulness for specific modeling purposes. The first characteristic is that the value of the density function evaluated at the mode, $f(x = a)$, assumes only three values: zero for $0 \le c < 1$, one for $c = 1$, and infinity for $c > 1$. The second characteristic is the truncated tails of the distribution." [Ref. 7]

"Despite the truncated tails, the distribution's versatility in assuming a wide variety of shapes makes it a reasonable model for a wide range of processes. This is due to the ability to assume the essential features of a process, which often makes the truncation insignificant." [Ref. 7]

<u>Parameter Determination</u> [Ref. 7]

The mode and quantiles are used in estimating parameters for the Schmeiser-Deutsch Distribution. The Schmeiser-Deutsch Distribution is useful in two common situations. The first is when full information is not available, but such quantities as the "most likely value" and the "minimum" and "maximum" values can be estimated. The second situation is when data is in histogram form where F(m) is the percentile of the mode m. The two desired quantiles, $p_1 = F(x_1)$ and $p_2 = F(x_2)$, must satisfy either of the following two expressions:

$$\begin{aligned} &(|a-x_1| < |x_2-a| \quad \text{and} \quad |d-p_1| < |p_2-d|) \\ &(|a-x_1| < |x_2-a| \quad \text{and} \quad |d-p_1| < |p_2-d|) \end{aligned} \qquad (9)$$

Although computations to determine the four parameters can be easily handled by computer, much attention has been given in this thesis to develop "manual procedures", meaning manual computational capability. The TI-59 handheld calculator (characterized by low price and surprising capability and compactness) can be considered as a "manual" tool, and a program for the TI-59 to determine the four parameters is given in Appendix B.

An algorithm for estimating the parameters is as follows: [Ref. 7]

1) Set $a = m$

2) Set $d = F(m)$

3) Set

$$c = \ln\left|\frac{a-x_1}{a-x_2}\right| / \ln\left|\frac{d-p_1}{d-p_2}\right| \qquad (10)$$

4) Set $b = |a-x_1|/|d-p_1|^c$ (11)

5) Due to the existence of the inverse c.d.f. in closed-form, the value of X corresponding to a percentile p of the distribution may be found by inserting p into equation (8).

Furthermore, the range of X can be calculated easily by substituting $p = 0$ and $p = 1$ at Step 5), or obtained directly from equation (7) since $X_{min} = a-bd^c$ and $X_{max} = a+b(1-d)^c$. At this point, one should determine whether or not the obtained distribution is an adequate model by plotting the p.d.f. or c.d.f. [Ref. 7].

APPENDIX B

TI-59 Programs

This appendix presents two TI-59 programs to determine the maximum demand and the parameters. The first program is using the Gompertz Curve with the restriction that the smallest X is unity. The second program is using the Schmeiser-Deutsch Distribution with the two desired quantiles satisfying equation (9).

Program B_1

1. Program Description

The program will estimate the maximum demand using the Gompertz Curve, and will also determine the parameters (k, a and b) in $Y = ka^{b^X}$.

2. User Instruction

Step	Procedure	Enter	Press	Display
1	Enter n	n	[A]	n
2	Enter $Y_1,\ldots,Y_{3n}$	Y_1	[R/S]	Y_1
		.	.	.
		.	.	.
		.	.	.
		Y_n	[R/S]	$\Sigma_1 \log Y$
		Y_{n+1}	[R/S]	Y_{n+1}
		.	.	.
		.	.	.
		.	.	.
		Y_{2n}	[R/S]	$\Sigma_2 \log Y$

Step	Procedure	Enter	Press	Display
		Y_{2n+1}	[R/S]	Y_{2n+1}
		.	.	.
		.	.	.
		.	.	.
		Y_{3n}	[R/S]	$\Sigma_3 \log Y$
3	Compute X_{max}		[B]	X_{max}
4	Review parameters (optional)			
	k		[C]	k
	a		[D]	a
	b		[E]	b

The maximum demand shown on the display is for the case when the smallest observed X is unity. Otherwise, the maximum demand should be adjusted by adding $X_{min}-1$, where X_{min} is the smallest observed X.

```
000  76  LBL
001  11   A
002  47  CMS
003  42  STO
004  57   57
005  42  STO
006  58   58
007  42  STO
008  59   59
009  35  1/X
010  42  STO
011  04   04
012  01   1
013  42  STO
014  56   56
015  43  RCL
016  57   57
017  91  R/S
018  42  STO
019  55   55
020  28  LOG
021  44  SUM
022  01   01
023  43  RCL
024  57   57
025  32  X:T
026  43  RCL
027  56   56
028  67   EQ
029  16  A'
030  01   1
031  94  +/-
032  44  SUM
033  57   57
034  43  RCL
035  55   55
036  61  GTO
037  00   00
038  17   17
039  76  LBL
040  16  A'
041  43  RCL
042  01   01
043  76  LBL
044  18  C'
045  91  R/S
046  42  STO
047  55   55
048  28  LOG
049  44  SUM
050  02   2
051  43  RCL
052  58   58
053  32  X:T
054  43  RCL
055  56   56
056  67   EQ
057  17  B'
058  01   1
059  94  +/-
060  44  SUM
061  58   58
062  43  RCL
063  55   55
064  61  GTO
065  18  C'
066  76  LBL
067  17  B'
068  43  RCL
069  02   02
070  76  LBL
071  19  D'
072  91  R/S
073  42  STO
074  55   55
075  28  LOG
076  44  SUM
077  03   03
078  43  RCL
079  59   59
080  32  X:T
081  43  RCL
082  56   56
083  67   EQ
084  10  E'
085  01   1
086  94  +/-
087  44  SUM
088  59   59
089  43  RCL
090  55   55
091  61  GTO
092  19  D'
093  76  LBL
094  10  E'
095  43  RCL
096  03   03
097  91  R/S
098  76  LBL
099  12   B
100  53   (
101  53   (
102  43  RCL
103  03   03
104  75   -
105  43  RCL
106  02   02
107  54   )
108  55   ÷
109  53   (
110  43  RCL
111  02   02
112  75   -
113  43  RCL
114  01   01
115  54   )
116  54   )
117  42  STO
118  05   05
119  45  YX
120  43  RCL
121  04   04
122  95   =
123  42  STO
124  06   06
125  28  LOG
126  95   =
127  42  STO
128  07   07
129  43  RCL
130  06   06
131  75   -
132  01   1
133  95   =
134  42  STO
135  06   06
136  43  RCL
137  05   05
138  75   -
139  01   1
140  95   =
141  42  STO
142  05   05
143  53   (
144  43  RCL
145  02   02
146  75   -
147  43  RCL
148  01   01
149  54   )
```

```
150  65   ×
151  43  RCL
152  06   06
153  55   ÷
154  43  RCL
155  05   05
156  33  X²
157  95   =
158  42  STO
159  08   08
160  65   ×
161  43  RCL
162  05   05
163  55   ÷
164  43  RCL
165  06   06
166  95   =
167  94  +/-
168  85   +
169  43  RCL
170  01   01
171  95   =
172  65   ×
173  43  RCL
174  04   04
175  95   =
176  42  STO
177  09   09
178  00   0
179  93   .
180  09   9
181  09   9
182  28  LOG
183  55   ÷
184  43  RCL
185  08   08
186  95   =
187  28  LOG
188  55   ÷
189  43  RCL
190  07   07
191  95   =
192  91  R/S
193  76  LBL
194  13   C
195  43  RCL
196  09   09
197  27  INV
198  23  LNX
199  91  R/S
200  76  LBL
201  14   D
202  43  RCL
203  08   08
204  27  INV
205  23  LNX
206  91  R/S
207  76  LBL
208  15   E
209  43  RCL
210  07   07
211  27  INV
212  23  LNX
213  91  R/S
214  00   0
```

Program B2

1. Program Description

Estimate the maximum demand using the Schmeiser-Deutsch Distribution. Also determine the parameters (a, b, c and d) in

$$F(x) = \begin{cases} d - \left[\frac{a-x}{b}\right]^{1/c} & \text{if} \quad a-bd^c \le x \le a \\ d + \left[\frac{x-a}{b}\right]^{1/c} & \text{if} \quad a \le x \le a+b(1-d)^c \end{cases}$$

2. User Instruction

Step	Procedure	Enter	Press	Display
1	Enter mode m	m	[R/S]	m
2	Enter F(m)	F(m)	[R/S]	F(m)
3	Enter p_1	p_1	[R/S]	p_1
4	Enter x_1	x_1	[R/S]	x_1
5	Enter p_2	p_2	[R/S]	p_2
6	Enter x_2	x_2	[R/S]	x_2
7	Compute X_{max}		[E]	X_{max}
8	Review parameters (optional)			
	a		[A]	a
	b		[B]	b
	c		[C]	c
	d		[D]	d

000	42	STO	050	53	(	100	02	2
001	01	01	051	43	RCL	101	85	+
002	91	R/S	052	04	04	102	43	RCL
003	42	STO	053	75	-	103	01	01
004	04	04	054	43	RCL	104	95	=
005	91	R/S	055	07	07	105	91	R/S
006	42	STO	056	54	)	106	76	LBL
007	05	05	057	54	)	107	11	A
008	91	R/S	058	50	I×I	108	43	RCL
009	42	STO	059	23	LNX	109	01	01
010	06	06	060	54	)	110	91	R/S
011	91	R/S	061	42	STO	111	76	LBL
012	42	STO	062	03	03	112	12	B
013	07	07	063	53	(	113	43	RCL
014	91	R/S	064	53	(	114	02	02
015	42	STO	065	43	RCL	115	91	R/S
016	08	08	066	01	01	116	76	LBL
017	91	R/S	067	75	-	117	13	C
018	76	LBL	068	43	RCL	118	43	RCL
019	15	E	069	06	06	119	03	03
020	53	(	070	54	)	120	91	R/S
021	53	(	071	50	I×I	121	76	LBL
022	53	(	072	55	÷	122	14	D
023	43	RCL	073	53	(	123	43	RCL
024	01	01	074	53	(	124	04	04
025	75	-	075	43	RCL	125	91	R/S
026	43	RCL	076	04	04	126	00	0
027	06	06	077	75	-	127	00	0
028	54	)	078	43	RCL	128	00	0
029	55	÷	079	05	05	129	00	0
030	53	(	080	54	)	130	00	0
031	43	RCL	081	50	I×I	131	00	0
032	01	01	082	45	Y^x	132	00	0
033	75	-	083	43	RCL	133	00	0
034	43	RCL	084	03	03	134	00	0
035	08	08	085	54	)	135	00	0
036	54	)	086	54	)	136	00	0
037	54	)	087	42	STO	137	00	0
038	50	I×I	088	02	02	138	00	0
039	23	LNX	089	53	(	139	00	0
040	55	÷	090	01	1	140	00	0
041	53	(	091	75	-	141	00	0
042	53	(	092	43	RCL	142	00	0
043	43	RCL	093	04	04	143	00	0
044	04	04	094	54	)	144	00	0
045	75	-	095	45	Y^x	145	00	0
046	43	RCL	096	43	RCL	146	00	0
047	05	05	097	03	03	147	00	0
048	54	)	098	65	×	148	00	0
049	55	÷	099	43	RCL	149	00	0

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