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Implementation of a Reliability Shorthand
on the TI-59 Handheld Calculator

by

Hans-Eberhard Peters

October 1982

Thesis Advisor: J.D.Esary

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by

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I. INTRODUCTION

Systems and components can be in either of two states: either they are functioning or they have failed. The ability, that a system stays functioning over a predetermined time interval is called its reliability. It is generally not realistic to assume that a system, say a lightbulb, will fail at a specified time, but rather that T, the time to failure, is a random variable which has a probability distribution that can be specified. The probability distribution for a time to failure is called its life distribution. In this paper we will solely be concerned with one specific type of life distribution which is especially important in reliability theory and practice, the exponential distribution. It has the property that the remaining life of a used component is independent of its age (the "memoryless" property), i.e. a functioning component is always as good as new, the failure rate is constant. The memoryless property is the basis for a reliability shorthand, one that can be implemented on a handheld calculator.

Depending on the size, structure and life distribution of a system, probability statements about its time to failure are in general not easily achieved. Forming the sum of independent life lengths (i.e. convolving the corresponding life distributions) requires knowledge of integral calculus and computations can become rather tedious.

In the case of the exponential distribution, though, computations can be simplified by translating the problem into a simple shorthand notation and using this shorthand as input for some computing device.

In this paper we will show how a reliability shorthand can be implemented on a handheld calculator. Basic structures are used to show how the shorthand can be applied. Two TI-59 programs are provided as a computational aid. Formulas for the convolution of up to four exponential random variables can be found in Appendix A. Appendix B contains a user guide to the TI-59 programs.

II. THE CONCEPT OF A RELIABILITY SHORTHAND

A. BASIC NOTATION

The survival function of a life length can be derived from the distribution function.

Let

T : life length

$F(t) = P(T \leq t)$ be the distribution function of

Then

$\bar{F}(t) = P(T > t)$

$= 1 - F(t)$

is the survival function of T.

In the case of the exponential distribution, $\bar{F}(t) = e^{-\lambda t}$, where λ is the failure rate. Translated into shorthand, the life distribution is denoted

EXP(λ).

B. CONVOLUTION OF DISTRIBUTIONS

When independent random lives are summed up, the corresponding life distributions have to be convolved to determine the probability that the sum of the lives will exceed a specified time t. Let

T_1, T_2 : independent life lengths

$\bar{F}_1(t), \bar{F}_2(t)$: the corresponding survival functions

$f_1(t), f_2(t)$: the corresponding density functions

$T = T_1 + T_2$: the total life length

Then

$$\bar{F}(t) = P(T>t)$$
$$= P(T_1 + T_2 > t)$$
$$= \bar{F}_1(t) + \int_0^t \bar{F}_2(t-s) f_1(s) ds.$$

This means that T will exceed a specified time t when

-either T_1 exceeds t

-or T_1 is smaller than t, say equal to s, and T_2 exceeds t-s.

Integration with respect to s (i.e. summing over all possible values of s) is called the convolution of T_1 and T_2. When T_1 and T_2 are both exponentially distributed with failure rates λ_1 and λ_2 , i.e.

$$\bar{F}_1(t) = e^{-\lambda_1 t}$$
$$\bar{F}_2(t) = e^{-\lambda_2 t},$$

then the survival function of T is

$$\bar{F}(t) = e^{-\lambda_1 t} + \int_0^t e^{-\lambda_2 (t-s)} \lambda_1 e^{-\lambda_1 s} ds.$$

Translated into shorthand, the survival function is denoted

$$EXP(\lambda_1) + EXP(\lambda_2).$$

This shorthand notation is heuristically apparent. We can visualize a 1 component / 1 spare system with $EXP(\lambda_1)$ and $Exp(\lambda_2)$ lives respectively. From component 1 the system has an $EXP(\lambda_1)$ life to begin with. When component 1 fails, the system has an extra $EXP(\lambda_2)$ life.

C. MIXTURE OF DISTRIBUTIONS

1. MIX-Notation

In the previous chapter, we formed the sum of independent random lives, which each had weight one, i.e.

$$T = T_1 + T_2 .$$

Now consider

$$T = \begin{cases} T_1 & \text{with probability } p_1 \\ T_2 & \text{with probability } p_2 \end{cases}$$

where $p_1 + p_2 = 1$.

Let D_1 and D_2 be the probability distributions of the random variables T_1 and T_2 respectively. The corresponding survival functions are $\overline{F}_1(t)$ and $\overline{F}_2(t)$.

Then

$$\overline{F}(t) = p_1 \overline{F}_1(t) + p_2 \overline{F}_2(t) .$$

In shorthand, the mixture of distributions D_1 and D_2 with respect to the mixing probabilities p_1 and p_2 is denoted

$$MIX[\, p_1 D_1 \,,\, p_2 D_2 \,].$$

2. <u>Distributive Law</u>

Now let

$$T = T_3 + T'$$

where

$$T' = \begin{cases} T_1 \text{ with probability } p \\ T_2 \text{ with probability } 1-p. \end{cases}$$

Then

$$T = T_3 + \begin{cases} T_1 \text{ with probability } p \\ T_2 \text{ with probability } 1-p. \end{cases}$$

$$T = \begin{cases} T_3 + T_1 \quad \text{with probability } p \\ T_3 + T_2 \quad \text{with probability } 1-p. \end{cases}$$

The distributive law holds due to the fact that the sum of the mixing probabilities for T_1 and T_2 is equal to one.

The survival function of T can be found by convolution:

$$\bar{F}(t) = \bar{F}_3(t) + \int_0^t (p\bar{F}_1(t-s) + (1-p)\bar{F}_2(t-s))f_3(s)ds.$$

With D_1, D_2, D_3 being the probability distributions for T_1, T_2, T_3, the distributive law can be applied to the shorthand notation:

$$D_3 + \text{MIX}\,[pD_1, (1-p)D_2] = \text{MIX}\,[\ p((D_1 + D_3), (1-p)(D_2 + D_3)\,].$$

Graphically this can be represented as follows:

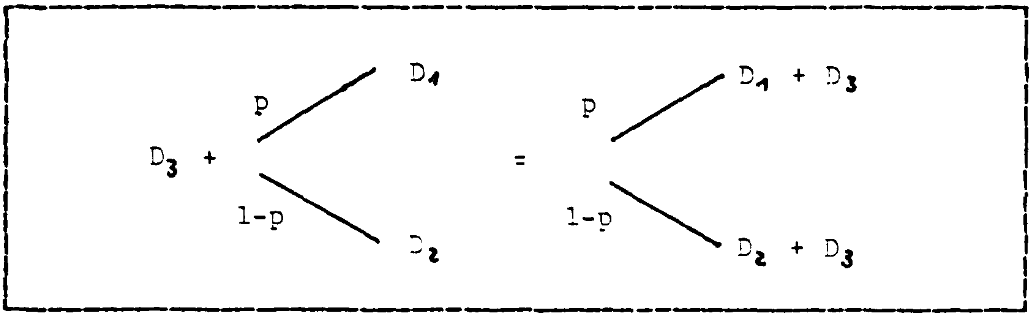


Figure 1: Distributive Property of the MIX-Notation

3. <u>Degeneracy at the Origin</u>

Let

$$P(T=0) = 1.$$

Then the distribution of T is degenerate at zero.

In shorthand notation, such a distribution is called the ZERO-distribution.

Now let $T = T_1 + T_0$

where T_1 and T_0 have probability distributions D_1 and ZERO and survival functions $\bar{F}_1(t)$ and $\bar{F}_0(t)$ respectively.

Then

$$\begin{aligned}\bar{F}(t) &= \bar{F}_1(t) + \int_0^t \bar{F}_0(t-s) f_1(s)\, ds \\ &= \bar{F}_1(t).\end{aligned}$$

The ZERO-distribution doesn't add anything to another distribution, so for instance

$$D_1 + ZERO = D_1$$

$$D_2 + MIX[pD_1, (1-p)ZERO] = MIX[p(D_1 + D_2), (1-p)D_2].$$

III. APPLYING A RELIABILITY SHORTHAND

After this brief survey over the concept of a reliability shorthand we will now show how the shorthand can be applied. To do so we will use basic structures. Part A of this chapter will give examples whose representation in shorthand requires only basic notation described in Chapter II, Parts A and B, whereas Part B of this chapter will give examples whose representation in shorthand makes use of the MIX-notation and the ZERO-distribution.

A. SUMS OF EXPONENTIALS WITH WEIGHT ONE

1. Simple Series System

A series system is a system which is functioning, when all its components are functioning. A two-component series system can be graphically represented as shown in Fig.2 .

Let

T : life of the system

T_1: life of component 1

T_2: life of component 2

$\bar{F}_1(t)$ = survival function of component 1

$= e^{-\lambda_1 t}$

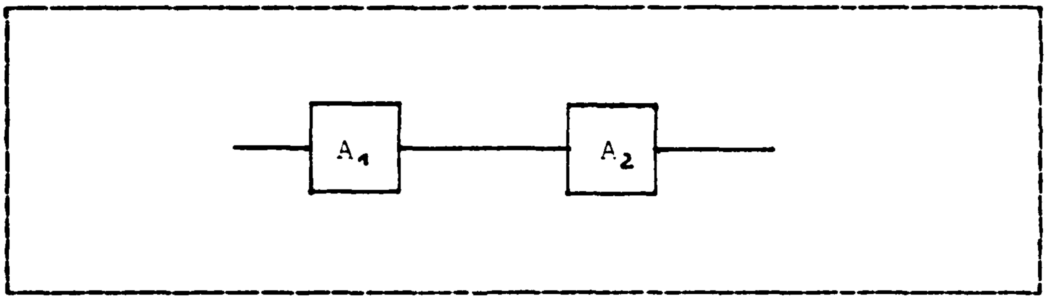


Figure 2: Two-Component Series System

$$\bar{F}_2(t) = \text{survival function of component 2}$$
$$= e^{-\lambda_2 t} \quad .$$

Then

$$T = \min(T_1, T_2)$$
$$\bar{F}(t) = \text{survival function of the system}$$
$$= P(\min(T_1, T_2) > t)$$
$$= P(T_1 > t,\ T_2 > t)$$

Assuming independence of the two components

$$\bar{F}(t) = P(T_1 > t)\ P(T_2 > t)$$
$$= \bar{F}_1(t)\ \bar{F}_2(t)$$
$$= e^{-\lambda_1 t}\ e^{-\lambda_2 t}$$
$$= e^{-(\lambda_1 + \lambda_2) t} \quad .$$

The shorthand notation for this system is

$$\text{EXP}(\lambda_1 + \lambda_2).$$

This is intuitively apparent, as the system has an exponential survival function with failure rate $\lambda_1 + \lambda_2$.

2. Simple Parallel System

A parallel system is a system which is functioning, when at least one of its components is functioning. A two-component parallel system can be graphically represented as follows:

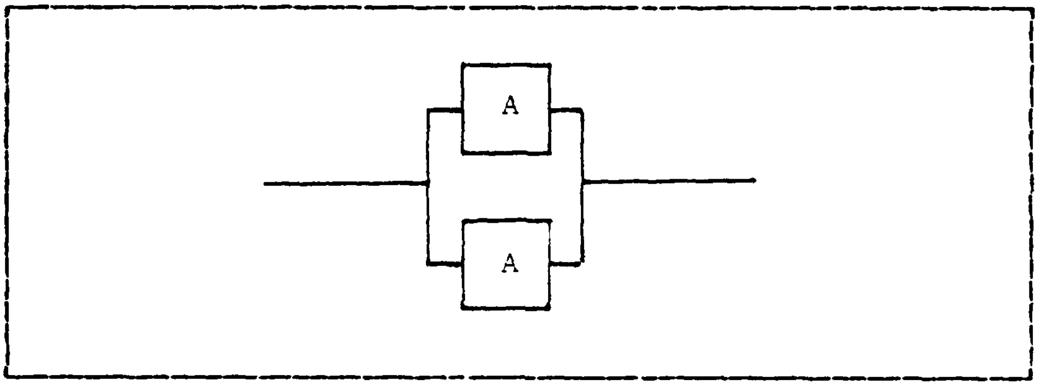


Figure 3: Two-Component Parallel System

Let

$T_1 \sim EXP(\lambda)$, $T_2 \sim EXP(\lambda)$.

Then

$$T = \max(T_1, T_2)$$

$$\bar{F}(t) = P(\max(T_1, T_2) > t)$$

$$= 1 - P(\max(T_1, T_2) \leqslant t)$$

$$= 1 - P(T_1 \leqslant t, T_2 \leqslant t)$$

Assuming independence of the two components,

$$\bar{F}(t) = 1 - P(T_1 \leq t)\, P(T_2 \leq t)$$
$$= 1 - F_1(t)\, F_2(t)$$
$$= 1 - (1-e^{-\lambda t})(1-e^{-\lambda t})$$
$$= 1 - (1-2e^{-\lambda t} + e^{-2\lambda t})$$
$$= 2e^{-\lambda t} - e^{-2\lambda t}.$$

The shorthand notation for the system is

$$EXP(2\lambda) + EXP(\lambda).$$

This follows intuition as the system has an $EXP(2\lambda)$ life to begin with and when one component fails it has an extra $EXP(\lambda)$ life due to the memoryless property of the exponential distribution.

3. Standby-System with Dissimilar Components

Suppose a system consists of two components, one active and one spare. The active component stays in service until it fails and then immediately is replaced by the spare.

Let the time to failure of the two components be $T_1 \sim EXP(\lambda_1)$ and $T_2 \sim EXP(\lambda_2)$ respectively. Then the system time to failure is

$$T = T_1 + T_2$$

and the survival function of the system is

$$\bar{F}(t) = P(T > t)$$

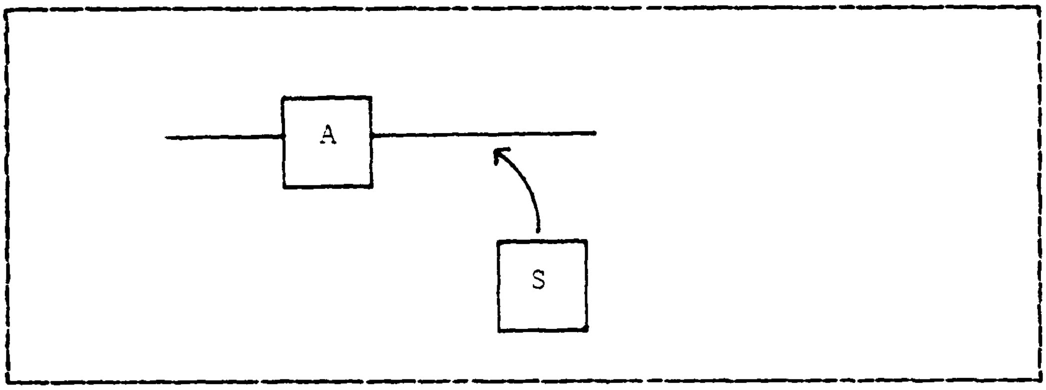


Figure 4: Standby System

$$= \bar{F}_1(t) + \int_0^t \bar{F}_2(t-s) f_1(s)\,ds$$

$$= e^{-\lambda_1 t} + \int_0^t e^{-\lambda_2 (t-s)} \lambda_1 e^{-\lambda_1 s}\,ds$$

$$= \frac{\lambda_1}{\lambda_1 - \lambda_2} e^{-\lambda_2 t} - \frac{\lambda_2}{\lambda_1 - \lambda_2} e^{-\lambda_1 t}$$

The shorthand notation for the system's survival function should be obvious. The system has an EXP(λ_1) life from the active component and an additional EXP(λ_2) life from the spare. So the shorthand notation is

$$\text{EXP}(\lambda_1) + \text{EXP}(\lambda_2).$$

B. SUMS OF EXPONENTIALS WITH WEIGHT BETWEEN ZERO AND ONE

The examples given in the previous chapter only involved exponential lives with weight one. Now we will look at some structures, whose survival function has a shorthand notation which includes the MIX-notation and/or the ZERO-distribution.

1. Parallel System with Dissimilar Failure Rates

The notion of a parallel system has been introduced in Chapter III.A.2 . We now look at the case where

$$T_1 \sim EXP(\lambda_1) \text{ and } T_2 \sim EXP(\lambda_2).$$

Then

$$T = \max(T_1, T_2)$$

$$\begin{aligned}\bar{F}(t) &= P(\max(T_1, T_2) > t) \\ &= 1-P(\max(T_1, T_2) \leqslant t) \\ &= 1-P(T_1 \leqslant t, T_2 \leqslant t)\end{aligned}$$

Assuming independence of the two components

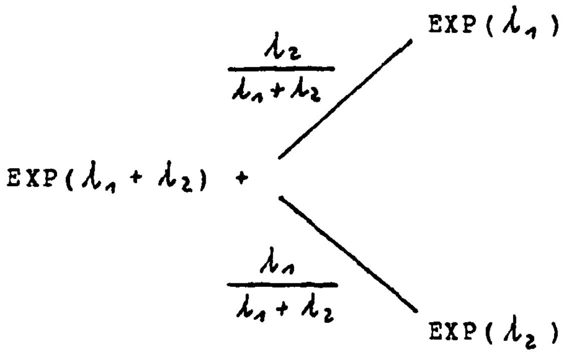
$$\begin{aligned}\bar{F}(t) &= 1-P(T_1 \leqslant t)\ P(T_2 \leqslant t) \\ &= 1-F_1(t)\ F_2(t) \\ &= 1-(1-e^{-\lambda_1 t})(1-e^{-\lambda_2 t}) \\ &= 1-(1-e^{-\lambda_1 t} - e^{-\lambda_2 t} + e^{-(\lambda_1+\lambda_2)t}) \\ &= e^{-\lambda_1 t} + e^{-\lambda_2 t} - e^{-(\lambda_1+\lambda_2)t} .\end{aligned}$$

To find the shorthand notation of the system consider all the ways which lead to the survival of the system:

- either both components survive
- or component 1 fails and component 2 survives
- or component 2 fails and component 1 survives.

If one component fails and one survives, in $\frac{\lambda_2}{\lambda_1+\lambda_2}$ fraction of the cases the survivor will be component 1 and in $\frac{\lambda_1}{\lambda_1+\lambda_2}$ fraction of the cases it will be component 2.

This can graphically be represented as



Making use of the MIX-notation the shorthand notation then is

$$\text{EXP}(\lambda_1 + \lambda_2) + \text{MIX}[\frac{\lambda_2}{\lambda_1+\lambda_2}\text{EXP}(\lambda_1), \frac{\lambda_1}{\lambda_1+\lambda_2}\text{EXP}(\lambda_2)]$$

and using the distributive property it becomes

$$\text{MIX}[\frac{\lambda_2}{\lambda_1+\lambda_2}(\text{EXP}(\lambda_1) + \text{EXP}(\lambda_1+\lambda_2)),$$
$$\frac{\lambda_1}{\lambda_1+\lambda_2}(\text{EXP}(\lambda_2) + \text{EXP}(\lambda_1+\lambda_2))].$$

As a check to see that this shorthand notation represents the survival function of the system, we derive the survival function from the shorthand notation:

$$\bar{F}(t) = \frac{\lambda_2}{\lambda_1+\lambda_2}\left(e^{-\lambda_1 t} + \int_0^t e^{-(\lambda_1+\lambda_2)(t-s)}\lambda_1 e^{-\lambda_1 s}\,ds\right)$$

$$+ \frac{\lambda_1}{\lambda_1+\lambda_2}\left(e^{-\lambda_2 t} + \int_0^t e^{-(\lambda_1+\lambda_2)(t-s)}\lambda_2 e^{-\lambda_2 s}\,ds\right)$$

$$= e^{-\lambda_1 t} + e^{-\lambda_2 t} - e^{-(\lambda_1+\lambda_2)t} .$$

This verifies that the shorthand notation indeed represents the system's survival function.

2. Series System with One Spare

Let us now look at a two-component series system, whose components have dissimilar failure rates with one component having a spare:

Component 1 has the constant failure rate λ_1 and component 2 and the spare have the constant failure rate λ_2. The spare can only replace component 2.

Let

$\bar{F}_1(t)$: the survival function of component 1

$\bar{F}_2(t)$: the survival function of the standby system component 2 with its spare.

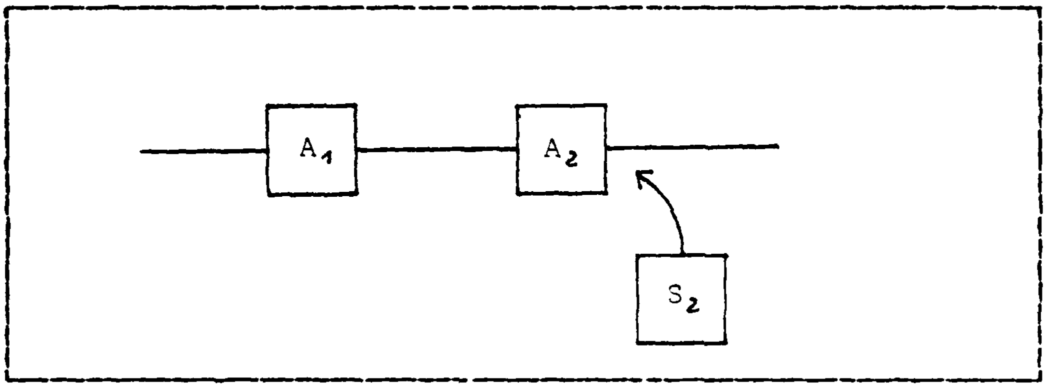


Figure 5: Series System with one Spare

The survival function for a standby system was derived in Chapter II.B. Therefore

$$\begin{aligned}\bar{F}_2(t) &= e^{-\lambda_2 t} + \int_0^t e^{-\lambda_2 (t-s)} \lambda_2 e^{-\lambda_2 s} ds \\ &= e^{-\lambda_2 t} + \lambda_2 e^{-\lambda_2 t} \int_0^t ds \\ &= (1+\lambda_2 t) e^{-\lambda_2 t} .\end{aligned}$$

Now $\bar{F}_1(t) = e^{-\lambda_1 t}$

Then

$$\begin{aligned}\bar{F}(t) &= \bar{F}_1(t)\, \bar{F}_2(t) \\ &= (1+\lambda_2 t)\, e^{-(\lambda_1+\lambda_2)t} .\end{aligned}$$

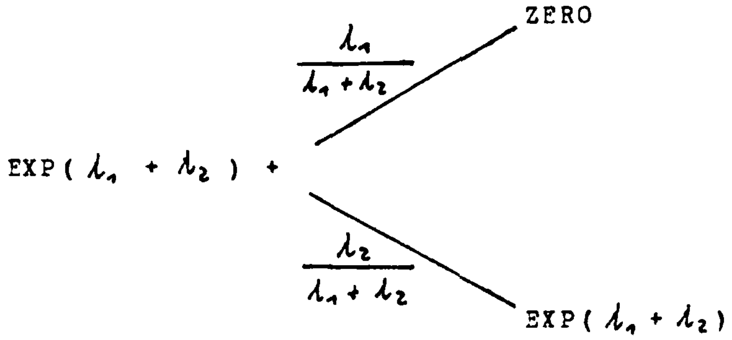
To translate the survival function into shorthand notation, let us consider the ways in which the system can survive:

-either both components survive

-or component 2 fails and its spare survives.

If one component fails, in $\frac{\lambda_1}{\lambda_1+\lambda_2}$ fraction of the time it will be component 1, which means that the system will not survive; in $\frac{\lambda_2}{\lambda_1+\lambda_2}$ fraction of the time the failing component will be component 2.

This can graphically be represented as



Using the MIX-notation the survival function then is

$$\mathrm{EXP}(\lambda_1+\lambda_2) + \mathrm{MIX}\left[\frac{\lambda_1}{\lambda_1+\lambda_2}\mathrm{ZERO}, \frac{\lambda_2}{\lambda_1+\lambda_2}\mathrm{EXP}(\lambda_1+\lambda_2)\right]$$

$$= \mathrm{MIX}\left[\frac{\lambda_1}{\lambda_1+\lambda_2}(\mathrm{ZERO} + \mathrm{EXP}(\lambda_1+\lambda_2)),\right.$$

$$\left.\frac{\lambda_2}{\lambda_1+\lambda_2}(\mathrm{EXP}(\lambda_1+\lambda_2) + \mathrm{EXP}(\lambda_1+\lambda_2))\right]$$

$$= \mathrm{MIX}\left[\frac{\lambda_1}{\lambda_1+\lambda_2}(\mathrm{EXP}(\lambda_1+\lambda_2)),\right.$$

$$\left.\frac{\lambda_2}{\lambda_1+\lambda_2}(\mathrm{EXP}(\lambda_1+\lambda_2) + \mathrm{EXP}(\lambda_1+\lambda_2))\right].$$

To prove, that the shorthand notation does represent the survival function, we derive the latter from the shorthand:

$$\bar{F}(t) = \frac{\lambda_1}{\lambda_1+\lambda_2} e^{-(\lambda_1+\lambda_2)t} + \frac{\lambda_2}{\lambda_1+\lambda_2}\left(e^{-(\lambda_1+\lambda_2)t} +\right.$$

$$\int_0^t e^{-(\lambda_1+\lambda_2)(t-s)} (\lambda_1+\lambda_2)\, e^{-(\lambda_1+\lambda_2)s} ds)$$

$$= (1 + \lambda_2 t)\, e^{-(\lambda_1+\lambda_2)t} .$$

This is the previously found result and this verifies, that the shorthand notation does represent the system's survival function.

3. Two-out-of-Three System

As a last example in this chapter, we will look at a Two-out-of- Three system.

Consider a three component system, whose components have constant failure rates λ_1, λ_2 and λ_3 respectively. The system is functioning, as long as two out of three components are functioning (see Fig. 6).

In other words, the system is functioning as long as there is a path through the system .

Alternatively, the system can be visualized as a parallel-series system (compare Fig. 7).

The survival function of the system is

$$\begin{aligned} \bar{F}(t) = & P(T_1 > t \wedge T_2 > t) + P(T_1 > t \wedge T_3 > t) \\ & + P(T_2 > t \wedge T_3 > t) \\ & - P((T_1 > t \wedge T_2 > t) \wedge (T_1 > t \wedge T_3 > t)) \\ & - P((T_1 > t \wedge T_2 > t) \wedge (T_2 > t \wedge T_3 > t)) \\ & - P((T_1 > t \wedge T_3 > t) \wedge (T_2 > t \wedge T_3 > t)) \end{aligned}$$

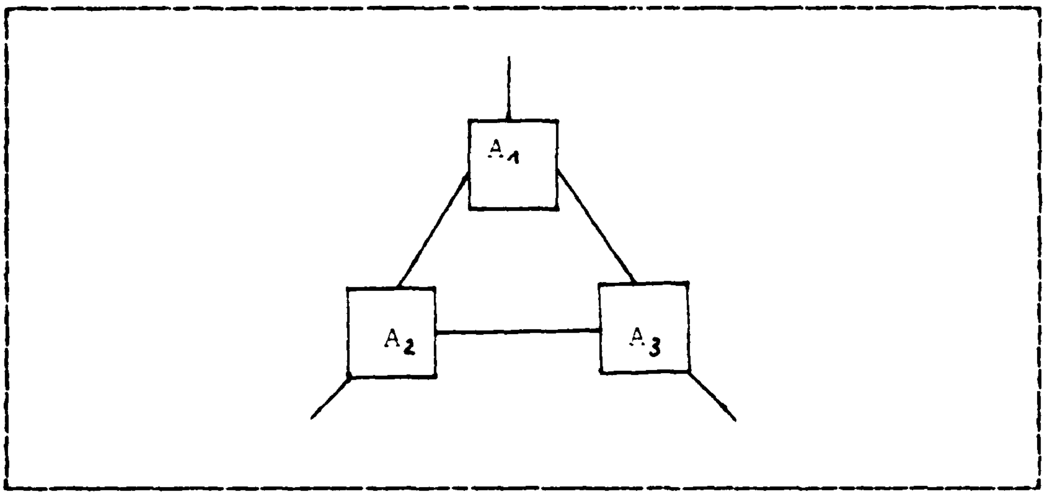


Figure 6: Two-out-of-Three System

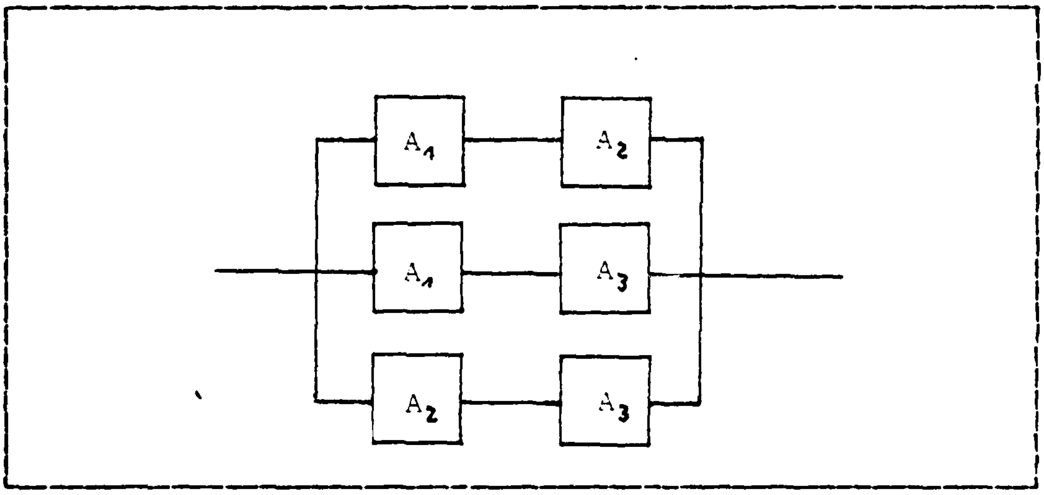


Figure 7: Two-out-of-Three System

$$+ P((T_1 > t \wedge T_2 > t) \wedge (T_1 > t \wedge T_3 > t) \wedge P(T_2 > t \wedge T_3 > t)).$$

Thus

$$\bar{F}(t) = P(T_1 > t \wedge T_2 > t) + P(T_1 > t \wedge T_3 > t)$$
$$+ P(T_2 > t \wedge T_3 > t)$$
$$- 3P(T_1 > t \wedge T_2 > t \wedge T_3 > t)$$
$$+ P(T_1 > t \wedge T_2 > t \wedge T_3 > t)$$

Therefore, and assuming independence of the components,

$$\bar{F}(t) = P(T_1 > t)\, P(T_2 > t) + P(T_1 > T)\, P(T_3 > t)$$
$$+ P(T_2 > t)\, P(T_3 > t)$$
$$- 3P(T_1 > t)\, P(T_2 > t)\, P(T_3 > t)$$
$$+ P(T_1 > t)\, P(T_2 > t)\, P(T_3 > t)$$

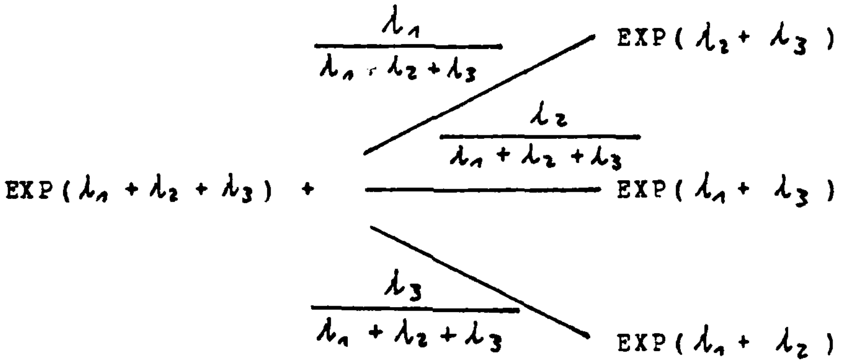
$$= P(T_1 > t)\, P(T_2 > t) + P(T_1 > t)\, P(T_3 > t)$$
$$+ P(T_2 > t)\, P(T_3 > t)$$
$$- 2P(T_1 > t)\, P(T_2 > t)\, P(T_3 > t)$$
$$= e^{-(\lambda_1+\lambda_2)t} + e^{-(\lambda_1+\lambda_3)t} + e^{-(\lambda_2+\lambda_3)t}$$
$$- 2e^{-(\lambda_1+\lambda_2+\lambda_3)t} .$$

Now let us consider all the possible ways, in which the system can survive:

- either all components survive
- or component 1 fails and component 2 and 3 survive
- or component 2 fails and component 1 and 3 survive

- or component 3 fails and component 1 and 2 survive.

If a component fails and the other two survive, in $\frac{\lambda_i}{\lambda_1 + \lambda_2 + \lambda_3}$ fraction of the time it will be component i , i = 1,2,3 . This can graphically be represented as



The shorthand notation then is

$$
\begin{aligned}
&\text{EXP}(\lambda_1 + \lambda_2 + \lambda_3) + \text{MIX}[\frac{\lambda_1}{\lambda_1 + \lambda_2 + \lambda_3}\text{EXP}(\lambda_2 + \lambda_3), \\
&\qquad \frac{\lambda_2}{\lambda_1 + \lambda_2 + \lambda_3}\text{EXP}(\lambda_1 + \lambda_3), \\
&\qquad \frac{\lambda_3}{\lambda_1 + \lambda_2 + \lambda_3}\text{EXP}(\lambda_1 + \lambda_2)],
\end{aligned}
$$

$$
\begin{aligned}
= \text{MIX}[&\frac{\lambda_1}{\lambda_1 + \lambda_2 + \lambda_3}(\text{EXP}(\lambda_2 + \lambda_3) + \text{EXP}(\lambda_1 + \lambda_2 + \lambda_3)), \\
&\frac{\lambda_2}{\lambda_1 + \lambda_2 + \lambda_3}(\text{EXP}(\lambda_1 + \lambda_3) + \text{EXP}(\lambda_1 + \lambda_2 + \lambda_3)), \\
&\frac{\lambda_3}{\lambda_1 + \lambda_2 + \lambda_3}(\text{EXP}(\lambda_1 + \lambda_2) + \text{EXP}(\lambda_1 + \lambda_2 + \lambda_3))\ ].
\end{aligned}
$$

Again, as a check that the shorthand notation represents the survival function, let us derive the survival function from the shorthand notation:

$$F(t) = \frac{\lambda_1}{\lambda_1+\lambda_2+\lambda_3}\Big[e^{-(\lambda_2+\lambda_3)t} + \int_0^t e^{-(\lambda_1+\lambda_2+\lambda_3)(t-s)}(\lambda_2+\lambda_3)e^{-(\lambda_2+\lambda_3)s}\,ds\Big]$$

$$+ \frac{\lambda_2}{\lambda_1+\lambda_2+\lambda_3}\Big[e^{-(\lambda_1+\lambda_3)t} + \int_0^t e^{-(\lambda_1+\lambda_2+\lambda_3)(t-s)}(\lambda_1+\lambda_3)e^{-(\lambda_1+\lambda_3)s}\,ds\Big]$$

$$+ \frac{\lambda_3}{\lambda_1+\lambda_2+\lambda_3}\Big[e^{-(\lambda_1+\lambda_2)t} + \int_0^t e^{-(\lambda_1+\lambda_2+\lambda_3)(t-s)}(\lambda_1+\lambda_2)e^{-(\lambda_1+\lambda_2)s}\,ds\Big]$$

$$= e^{-(\lambda_2+\lambda_3)t} + e^{-(\lambda_1+\lambda_3)t} + e^{-(\lambda_1+\lambda_2)t} - 2e^{-(\lambda_1+\lambda_2+\lambda_3)t}$$

The result again proves that the shorthand notation indeed represents the survival function of the system.

IV. IMPLEMENTING THE SHORTHAND ON THE TI-59

The concept of a reliability shorthand is introduced in the course "Reliability and Weapons System Effectiveness Measurements", OA 4302, at the Naval Postgraduate School, Monterey. Most students taking the course are in the Operations Research (OR) - Curriculum.

The choice of the TI-59 as the computing device, on which the shorthand was to be implemented, was based on the fact, that each student in the OR-Curriculum is issued a TI-59 for use in basic probability and statistics courses. Thus almost every student at the Naval Postgraduate School, who is introduced to the shorthand, is familiar with the TI-59 and has access to such a calculator.

A program, that uses the shorthand notation, times to failure and failure rates as input, should

- calculate the survival probability of basic structures / small systems and
- require moderate computation time.

To achieve these requirements it was decided to incorporate all solutions for the convolution of up to four exponential random variables in the program. The formulas that were used are given in Appendix A.

Two programs are provided in this paper.

Program 1 can be used when all rates are dissimilar or all are the same. It uses the formulas on pages 37 and 38 only.

Program 2 can be used for the general case. It makes use of all the formulas given in Appendix A. The program includes a sorting routine that determines the applicable formula from the entered failure rates.

A user guide to the two programs is provided in Appendix B.

V. SUMMARY

There is a reliability shorthand that denotes the survival function of a system, assuming that the failure rates of all components are constant.

This shorthand can be implemented on the TI-59 handheld calculator. With failure rates, time to failure and shorthand as input the TI-59 calculates the survival probability of the system.

Knowledge of calculus is not necessary to use this method, whereas the standard procedure, finding the survival probability by convolution, requires knowledge of integral calculus.

The choice of the TI-59 as the computing device for the implementation of the shorthand, though, implied limitations; the number of failure rates is limited due to the limited storage capacity of the TI-59, and computing times are comparatively long. The TI-59 can therefore only be used for smaller systems, preferably for the solution of classroom problems.

For the solution of larger problems, the shorthand should be implemented on a state-of-the-art personal computer using a general algorithm for the convolution of any number of exponential random variables.

APPENDIX A

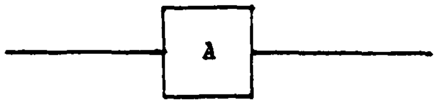
CONVOLUTION FORMULAS

Appendix A contains formulas for the convolution of up to four exponential random variables.

For the two special cases, when all random variables have the same failure rate and all have different failure rates, general formulas for the convolution of any number of exponential random variables are given.

These formulas are used in the two TI-59 programs provided in Appendix B.

System:



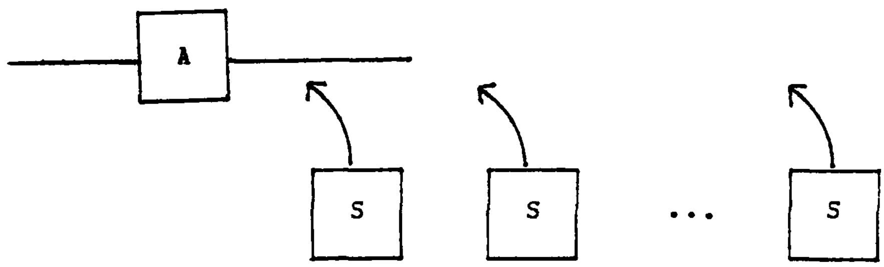
Shorthand: EXP(λ)

Survival Function: $\bar{F}(t) = e^{-\lambda t}$

Description:

A single active component with constant failure rate λ.

System:



Shorthand: $EXP(\lambda) + EXP(\lambda) + \ldots + EXP(\lambda)$

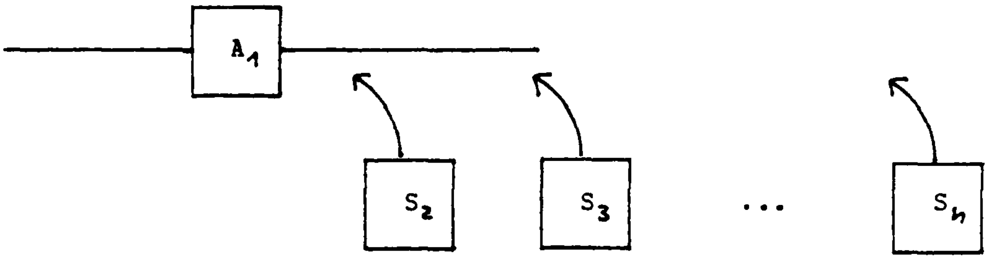
Survival Function: $$\bar{F}(t) = \left(\frac{(\lambda t)^0}{0!} + \frac{(\lambda t)^1}{1!} + \cdots + \frac{(\lambda t)^{n-1}}{(n-1)!} \right) e^{-\lambda t}$$

$$= \sum_{i=1}^{n} \frac{(\lambda t)^{i-1}}{(i-1)!} e^{-\lambda t}$$

Description:

A single active component with constant failure rate is supported by n-1 identical spares.

System:



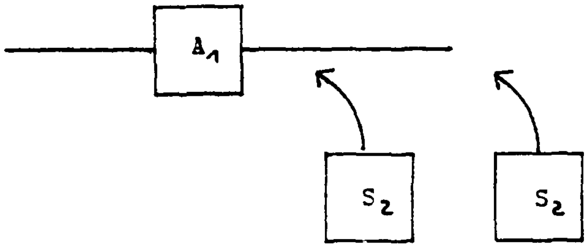
Shorthand: $EXP(\lambda_1) + EXP(\lambda_2) + \ldots + EXP(\lambda_n)$

Survival Function: $$\bar{F}(t) = \sum_{i=1}^{n} \left(\prod_{j \neq i} \frac{\lambda_j}{\lambda_j - \lambda_i} e^{-\lambda_i t} \right)$$

Description:

A single active component with constant failure rate is supported by n-1 spares. The active component and the spares have all constant, but dissimilar failure rates.

System:



Shorthand: $EXP(\lambda_1) + EXP(\lambda_2) + EXP(\lambda_2)$

Survival Function: $\bar{F}(t) = Ae^{-\lambda_1 t} + (B + Ct)\, e^{-\lambda_2 t}$

where $A = \dfrac{\lambda_2^2}{(\lambda_2 - \lambda_1)^2}$

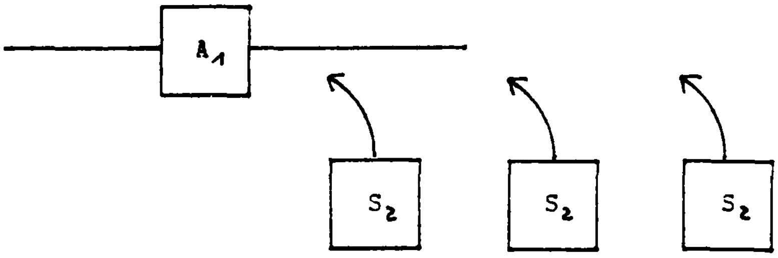
$B = 1 - A$

$C = \dfrac{\lambda_1 \lambda_2}{\lambda_1 - \lambda_2}$

Description:

A single active component with constant failure rate λ_1 is supported by two spares with identical constant failure rate λ_2.

System:



Shorthand: $EXP(\lambda_1) + EXP(\lambda_2) + EXP(\lambda_2) + EXP(\lambda_2)$

Survival Function: $\bar{F}(t) = Ae^{-\lambda_1 t} + (B + Ct + Dt^2)e^{-\lambda_2 t}$

where $A = \dfrac{\lambda_2^{\,3}}{(\lambda_2 - \lambda_1)^3}$

$B = 1 - A$

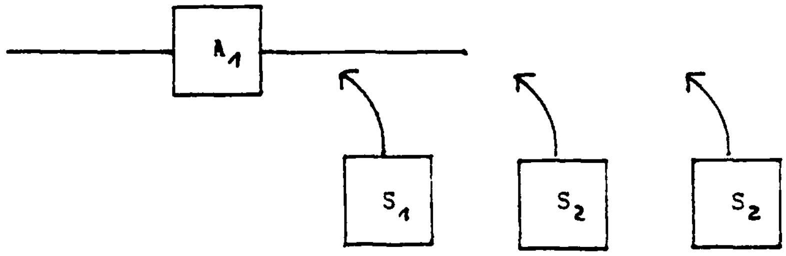
$C = \lambda_2 - \dfrac{\lambda_2^{\,3}}{(\lambda_1 - \lambda_2)^2}$

$D = \dfrac{\lambda_1 \lambda_2^{\,2}}{2(\lambda_1 - \lambda_2)}$

Description:

A single active component with constant failure rate λ_1 is supported by three spares with identical constant failure rate λ_2.

System:



Shorthand: $EXP(\lambda_1) + EXP(\lambda_1) + EXP(\lambda_2) + EXP(\lambda_2)$

Survival Function: $F(t) = (A + Bt)e^{-\lambda_1 t} + (C + Dt)e^{-\lambda_2 t}$

where $A = \dfrac{\lambda_2^3 - 3\lambda_2^2 \lambda_1}{(\lambda_2 - \lambda_1)^3}$

$B = \dfrac{\lambda_1 \lambda_2^2}{(\lambda_2 - \lambda_1)^2}$

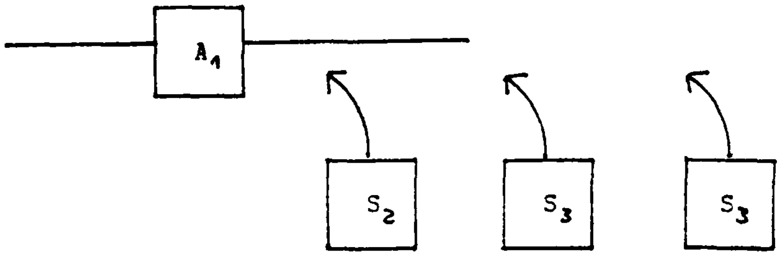
$C = 1 - A$

$D = \dfrac{\lambda_1^2 \lambda_2}{(\lambda_1 - \lambda_2)^2}$

Description:

A single active component with constant failure rate λ_1 is supported by one identical spare and two spares with dissimilar, constant failure rate λ_2.

System:



Shorthand: $EXP(\lambda_1) + EXP(\lambda_2) + EXP(\lambda_3) + EXP(\lambda_3)$

Survival Function: $\bar{F}(t) = Ae^{-\lambda_1 t} + Be^{-\lambda_2 t} + (C + Dt)e^{-\lambda_3 t}$

where

$$A = \frac{\lambda_2 \lambda_3^2}{(\lambda_2-\lambda_1)(\lambda_3-\lambda_1)^2}$$

$$B = \frac{\lambda_1 \lambda_3^2}{(\lambda_1-\lambda_2)(\lambda_3-\lambda_2)^2}$$

$$C = \frac{\lambda_1 \lambda_2}{(\lambda_1-\lambda_3)(\lambda_2-\lambda_3)} + \frac{\lambda_1\lambda_2\lambda_3}{(\lambda_1-\lambda_2)}\left(\frac{1}{(\lambda_1-\lambda_3)^2} - \frac{1}{(\lambda_2-\lambda_3)^2}\right)$$

$$D = \frac{\lambda_1 \lambda_2 \lambda_3}{(\lambda_1-\lambda_3)(\lambda_2-\lambda_3)}$$

Description:

A single active component with constant failure rate λ_1 has three spares. One spare has constant failure rate λ_2, two spares are identical with constant failure rate λ_3.

APPENDIX B

USER GUIDE TO TI-59 PROGRAMS

Appendix B contains a user guide to two TI-59 programs, which use reliability shorthand and failure rates as input to compute the survival probability of a system.

PROGRAM 1 is designed for the two special cases where the reliability shorthand is of the form

$$EXP(\lambda) + EXP(\lambda) + \ldots + EXP(\lambda)$$

or

$$EXP(\lambda_1) + EXP(\lambda_2) + \ldots + EXP(\lambda_n).$$

In the first case the number of terms is not limited, whereas in the second case the number of terms is limited to 40 due to limited storage capacity of the TI-59. In this case the number of terms can be increased to 70 by entering 9 in the display and pressing 2nd Op 1 7 .

PROGRAM 2 is designed to solve problems of the kind, that were introduced in Chapter III.B. . Due to limited memory of the TI-59 the number of exponential terms under one weight in shorthand notation is limited to four.

All results will be printed, if the TI-59 is connected to a TI PC-100A or TI PC-100C printer.

PROGRAM 1 : Procedure

1. Use any library module. Read in program 1 (side 1 of the magnetic card)
2. Press 2nd C' to initialize.
3. Enter n, the number of exponential terms to be convolved, in the display and press A.
4. Enter time t and press B.
5. Enter λ_i and press C . When all failure rates are the same, enter λ only once.
6. a) To find the survival probability of the system, when all failure rates are the same, press 2nd A'.
   b) To find the survival probability of the system, when all failure rates are dissimilar, press 2nd B'.

PROGRAM 1 : Sample Problems

1. Find the survival probability of a parallel system

   (compare Chapter III.A.2)

   a) $\lambda = .3$, $t = 7$, $n = 2$

   b) Shorthand notation:

      EXP(.6) + EXP(.3)

   c)

Enter	Comment	Press	Display
	Initialize	C'	0
2	n	A	0
7	t	B	7
.6	2λ	C	.3
.3	λ	C	.3
	$\bar{F}(t)$	B'	.2299172797

calculation takes 13 seconds

2. Find the survival probability of a standby-system with dissimilar components (compare Chapter III.A.3) .

a) $\lambda_1 = .4$, $\lambda_2 = .5$, $t = 6$, $n = 2$

b) Shorthand notation:

EXP(.4) + EXP(.5)

c)

Enter	Comment	Press	Display
	Initialize	C'	0
2	n	A	0
6	t	B	6
.4	λ_1	C	.4
.5	λ_2	C	.5
	$\overline{F}(t)$	B'	.254441493

calculation takes 13 seconds

3. Find the survival probability of a standby-system with one active component and four similar spares.

a) $\lambda = .3$, $t = 7$, $n = 5$

b) Shorthand notation:

EXP(.3) + EXP(.3) + EXP(.3) + EXP(.3) + EXP(.3)

c)

Enter	Comment	Press	Display
	Initialize	C'	0
5	n	A	0
7	t	B	7
.3	λ	C	.3
	$\overline{F}(t)$	A'	.9378738848

calculation takes 9 seconds

PROGRAM 2 : Procedure

CASE I : To find the convolution of up to four exponential random variables.

1. Use any library module.

   Re-Partition (enter 2 in the display, press 2nd Op 17).

   Read in all four sides of the magnetic card.

2. Press 2nd C' to initialize.

3. Enter n, the number of exponential terms to be convolved, in the display and press A.

4. Enter time t and press B.

5. Enter λ_i and press C (n entries) .

   REMARK: Failure rates, which appear only once in the expression, have to be entered before failure rates, that appear several times.

6. To find the survival probability of the system press E.

PROGRAM 2, CASE I : Sample Problems

(1) Shorthand notation

$EXP(\lambda_1) + EXP(\lambda_2) + EXP(\lambda_2)$

Sample values : $\lambda_1 = .3$, $\lambda_2 = .4$, $t = 7$

Procedure :

Enter	Comment	Press	Display
	Initialize	C'	0
3	n	A	0
7	t	B	7
.3	λ_1	C	.3
.4	λ_2	C	.4
.4	λ_2	C	.4
	$\overline{F}(t)$	E	.5363473866

calculation takes 14 seconds

(2) Shorthand notation

EXP(λ_1) + EXP(λ_2) + EXP(λ_2) + EXP(λ_2)

Sample values : λ_1 = .2 , λ_2 = .4 , t = 3

Procedure :

Enter	Comment	Press	Display
	Initialize	C'	0
4	n	A	0
3	t	B	3
.2	λ_1	C	.2
.4	λ_2	C	.4
.4	λ_2	C	.4
.4	λ_2	C	.4
	$\bar{F}(t)$	E	.9809746099

calculation takes 20 seconds

(3) Shorthand notation

EXP(λ_1) + EXP(λ_1) + EXP(λ_2) + EXP(λ_2)

Sample values : λ_1 = .4 , λ_2 = .3 , t = 5

Procedure :

Enter	Comment	Press	Display
	Initialize	C'	0
4	n	A	0
5	t	B	5
.4	λ_1	C	.4
.4	λ_1	C	.4
.3	λ_2	C	.3
.3	λ_2	C	.3
	$\bar{F}(t)$	E	.9029040721

calculation takes 20 seconds

(4) Shorthand notation

EXP(λ_1) + EXP(λ_2) + EXP(λ_3) + EXP(λ_3)

Sample values : λ_1 = .1 , λ_2 = .3 , λ_3 = .5 ,

t = 10

Procedure :

Enter	Comment	Press	Display
	Initialize	C'	0
4	n	A	0
10	t	B	10
.1	λ_1	C	.1
.3	λ_2	C	.3
.5	λ_3	C	.5
.5	λ_3	C	.5
	$\bar{F}(t)$	E	.7312684703

calculation takes 25 seconds

PROGRAM 2 : Procedure

CASE II : to solve problems of the kind, that were introduced in Chapter III.B. .

1. Derive the system's shorthand notation. Find either the
   - graphical representation or
   - the MIX-notation .
2. Use any library module.

   Re-Partition (enter 2 in the display, press 2nd Op 17).

   Read in all four sides of the magnetic card.
3. Press 2nd C' to initialize.
4. Enter time t and press B.
5. Repeat the following steps for each path of the graphical representation, i.e. for each convolution in the MIX-notation.

   a) Enter n, the number of exponential terms to be convolved, in the display and press A.

   b) Enter λ_i and press C.

      REMARK: Failure rates, which appear only once in the expression, have to be entered before failure rates, that appear several times.

   c) Enter p_i, the weight in the ith path, and press D.

   d) To find the part of the system's survival probability, that is contributed by the ith path, press E.

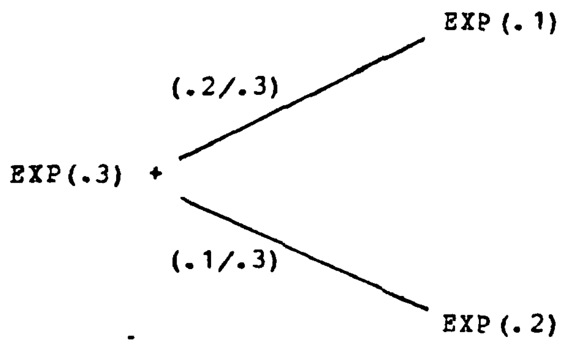
6. To find the survival probability of the system press 2nd E'.

PROGRAM 2, CASE II : Sample Problems

1. Find the survival probability of a parallel system with dissimilar failure rates (compare Chapter III.B.1).

   a) $\lambda_1 = .1$, $\lambda_2 = .2$, $t = 2$

   b) Shorthand notation



$\bar{F}(t)$ = MIX[(.2/.3)(EXP(.1) + EXP(.3),

(.1/.3)(EXP(.2) + EXP(.3)].

c)

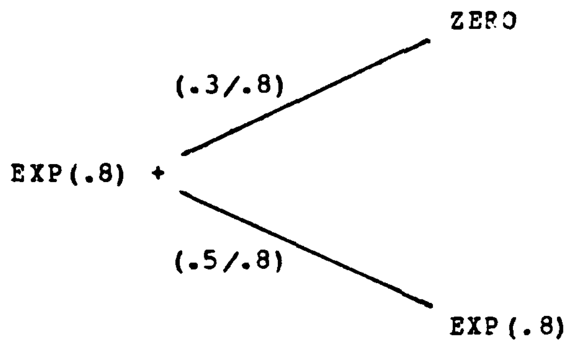
Procedure :

Enter	Comment	Press	Display
	Initialize	C'	0
2	t	B	2
2	n_1	A	0
.1	λ_1	C	.1
.3	$\lambda_1 + \lambda_2$	C	.3
(.2/.3)	P_1	D	.6666666667
		E	.635793541
2	n_2	A	0
.2	λ_2	C	.2
.3	$\lambda_1 + \lambda_2$	C	.3
(.1/.3)	P_2	D	.3333333333
		E	.304445622
	$\bar{F}(t)$	E'	.940239163

2. Find the survival probability of a series system with one spare as introduced in Chapter III.B.2 .

a) $\lambda_1 = .3$, $\lambda_2 = .5$, $t = 7$

b) Shorthand notation



$\bar{F}(t)$ = MIX[(.3/.8) (EXP(.8),

(.5/.8) (EXP(.8) + EXP(.8)].

c)

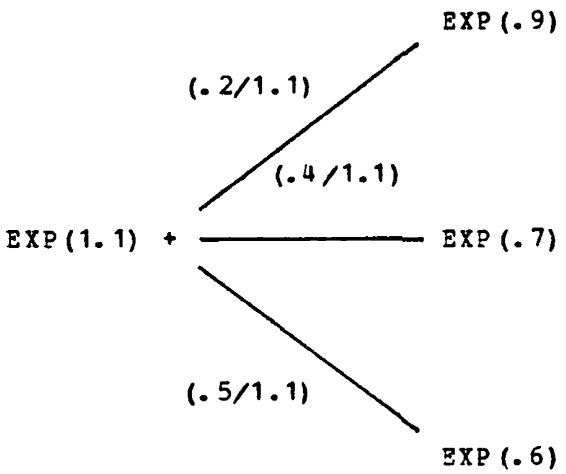
Procedure :

Enter	Comment	Press	Display
	Initialize	C'	0
7	t	B	7
1	n_1	A	0
.8	$\lambda_1 + \lambda_2$	C	.8
(.3/.8)	P_1	D	.375
		E	.0013866989
2	n_2	A	0
.8	$\lambda_1 + \lambda_2$	C	.8
.8	$\lambda_1 + \lambda_2$	C	.8
(.5/.8)	P_2	D	.625
		E	.0152536878
	$\overline{F}(t)$	E'	.0166403867

3. Find the survival probability of a Two-out-of-Three System as introduced in Chapter III.B.3 .

a) $\lambda_1 = .2$, $\lambda_2 = .4$, $\lambda_3 = .5$, $t = 9$

b) Shorthand notation



$\bar{F}(t)$ = MIX[(.2/1.1) (EXP(.9) + EXP(1.1)),

(.4/1.1) (EXP(.7) + EXP(1.1)),

(.5/1.1) (EXP(.6) + EXP(1.1))].

c)

Procedure :

Enter	Comment	Press	Display
	Initialize	C'	0
9	t	B	9
2	n_1	A	0
1.1	$\lambda_1 + \lambda_2 + \lambda_3$	C	1.1
.9	$\lambda_2 + \lambda_3$	C	.9
(.2/1.1)	p_1	D	.1818181818
		E	.0002624871
2	n_2	A	0
1.1	$\lambda_1 + \lambda_2 + \lambda_3$	C	1.1
.7	$\lambda_1 + \lambda_3$	C	.7
(.4/1.1)	p_2	D	.3636363636
		E	.0018043754
2	n_3	A	0
1.1	$\lambda_1 + \lambda_2 + \lambda_3$	C	1.1
.6	$\lambda_1 + \lambda_2$	C	.6
(.5/1.1)	p_3	D	.4545454545
		E	.0044892129
	$\bar{F}(t)$	E'	.0065560755

COMPUTER LISTINGS

PROGRAM 1

```
000  91  R/S
001  76  LBL
002  18  C'
003  29  CP
004  25  CLR
005  47  CMS
006  81  RST
007  76  LBL
008  11   A
009  42  STO
010  00   00
011  75   -
012  01   1
013  95   =
014  42  STO
015  09   09
016  29  CP
017  02   2
018  00   0
019  42  STO
020  08   08
021  01   1
022  42  STO
023  17   17
024  00   0
025  42  STO
026  18   18
027  91  R/S
028  76  LBL
029  12   B
030  42  STO
031  01   01
032  91  R/S
033  76  LBL
034  13   C
035  72  ST*
036  08   08
037  69  OP
038  28   28
039  91  R/S
040  76  LBL
041  16  A'
042  43  RCL
043  20   20
044  65   ×
045  43  RCL
046  01   01
047  95   =
048  42  STO
049  02   02
050  00   0
051  01   1
052  42  STO
053  04   04
054  42  STO
055  05   05
056  42  STO
057  18   18
058  76  LBL
059  28  LOG
060  29  CP
061  43  RCL
062  09   09
063  67   EQ
064  29  CP
065  43  RCL
066  02   02
067  45  YX
068  43  RCL
069  05   05
070  55   ÷
071  43  RCL
072  04   04
073  95   =
074  44  SUM
075  18   18
076  69  OP
077  39   39
078  69  OP
079  25   25
080  43  RCL
081  05   05
082  49  PRD
083  04   04
084  61  GTO
085  28  LOG
086  76  LBL
087  29  CP
088  43  RCL
089  02   02
090  94  +/-
091  22  INV
092  23  LNX
093  65   ×
094  71  SBR
095  47  CMS
096  95   =
097  99  PRT
098  91  R/S
099  76  LBL
100  17  B'
101  02   2
102  00   0
103  42  STO
104  08   08
105  76  LBL
106  39  COS
107  02   2
108  00   0
109  42  STO
110  06   06
111  01   1
112  42  STO
113  16   16
114  76  LBL
115  38  SIN
116  29  CP
117  73  RC*
118  06   06
119  67   EQ
```

PROGRAM 1 continued

120	36	PGM
121	73	RC*
122	08	08
123	32	X:T
124	73	RC*
125	06	06
126	22	INV
127	67	EQ
128	30	TAN
129	69	OP
130	26	26
131	61	GTO
132	38	SIN
133	76	LBL
134	30	TAN
135	49	PRD
136	16	16
137	75	-
138	73	RC*
139	08	08
140	95	=
141	35	1/X
142	49	PRD
143	16	16
144	43	RCL
145	00	00
146	32	X:T
147	43	RCL
148	06	06
149	75	-
150	01	1
151	09	9
152	95	=
153	77	GE
154	36	PGM
155	69	OP
156	26	26
157	61	GTO
158	38	SIN
159	76	LBL
160	36	PGM
161	73	RC*
162	08	08
163	94	+/-
164	65	×
165	43	RCL
166	01	01
167	95	=
168	22	INV
169	23	LNX
170	65	×
171	43	RCL
172	16	16
173	95	=
174	44	SUM
175	18	18
176	43	RCL
177	00	00
178	32	X:T
179	43	RCL
180	08	08
181	75	-
182	01	1
183	09	9
184	95	=
185	67	EQ
186	37	P/R
187	69	OP
188	28	28
189	61	GTO
190	39	COS
191	76	LBL
192	37	P/R
193	71	SBR
194	47	CMS
195	99	PRT
196	91	R/S
197	76	LBL
198	47	CMS
199	43	RCL
200	18	18
201	92	RTN

LABEL ADRESSES

002	18	C'
008	11	A
029	12	B
034	13	C
041	16	A'
059	28	LOG
087	29	CP
100	17	B'
106	39	COS
115	38	SIN
134	30	TAN
160	36	PGM
192	37	P/R
198	47	CMS

PROGRAM 2

000	86	STF
001	00	00
002	61	GTO
003	15	E
004	76	LBL
005	18	C'
006	29	CP
007	25	CLR
008	47	CMS
009	91	R/S
010	76	LBL
011	11	A
012	42	STO
013	00	00
014	75	-
015	01	1
016	95	=
017	42	STO
018	09	09
019	29	CP
020	01	1
021	00	0
022	42	STO
023	08	08
024	01	1
025	42	STO
026	17	17
027	00	0
028	42	STO
029	18	18
030	91	R/S
031	76	LBL
032	12	B
033	42	STO
034	01	01
035	91	R/S
036	76	LBL
037	13	C
038	72	ST*
039	08	08
040	69	OP
041	28	28
042	91	R/S
043	76	LBL
044	14	D
045	49	PRD
046	17	17
047	91	R/S
048	76	LBL
049	16	A'
050	43	RCL
051	10	10
052	65	×
053	43	RCL
054	01	01
055	95	=
056	42	STO
057	02	02
058	00	0
059	01	1
060	42	STO
061	04	04
062	42	STO
063	05	05
064	42	STO
065	18	18
066	76	LBL
067	28	LOG
068	29	CP
069	43	RCL
070	09	09
071	67	EQ
072	29	CP
073	43	RCL
074	02	02
075	45	YX
076	43	RCL
077	05	05
078	55	÷
079	43	RCL
080	04	04
081	95	=
082	44	SUM
083	18	18
084	69	OP
085	39	39
086	69	OP
087	25	25
088	43	RCL
089	05	05
090	49	PRD
091	04	04
092	61	GTO
093	28	LOG
094	76	LBL
095	29	CP
096	43	RCL
097	02	02
098	94	+/-
099	22	INV
100	23	LNX
101	65	×
102	71	SBR
103	47	CMS
104	95	=
105	91	R/S
106	76	LBL
107	17	B'
108	01	1
109	00	0
110	42	STO
111	08	08
112	76	LBL
113	39	COS
114	01	1
115	00	0
116	42	STO
117	06	06
118	01	1
119	42	STO

PROGRAM 2 continued

120	16	16
121	76	LBL
122	38	SIN
123	29	CP
124	73	RC*
125	06	06
126	67	EQ
127	36	PGM
128	73	RC*
129	08	08
130	32	X:T
131	73	RC*
132	06	06
133	22	INV
134	67	EQ
135	30	TAN
136	69	OP
137	26	26
138	61	GTO
139	38	SIN
140	76	LBL
141	30	TAN
142	49	PRD
143	16	16
144	75	-
145	73	RC*
146	08	08
147	95	=
148	35	1/X
149	49	PRD
150	16	16
151	43	RCL
152	00	00
153	32	X:T
154	43	RCL
155	06	06
156	75	-
157	09	9
158	95	=
159	77	GE
160	36	PGM
161	69	OP
162	26	26
163	61	GTO
164	38	SIN
165	76	LBL
166	36	PGM
167	73	RC*
168	08	08
169	94	+/-
170	65	×
171	43	RCL
172	01	01
173	95	=
174	22	INV
175	23	LNX
176	65	×
177	43	RCL
178	16	16
179	95	=
180	44	SUM
181	18	18
182	43	RCL
183	00	00
184	32	X:T
185	43	RCL
186	08	08
187	75	-
188	09	9
189	95	=
190	67	EQ
191	37	P/R
192	69	OP
193	28	28
194	61	GTO
195	39	COS
196	76	LBL
197	37	P/R
198	71	SBR
199	47	CMS
200	91	R/S
201	76	LBL
202	47	CMS
203	43	RCL
204	18	18
205	65	×
206	43	RCL
207	17	17
208	95	=
209	99	PRT
210	44	SUM
211	19	19
212	92	RTN
213	76	LBL
214	22	INV
215	43	RCL
216	11	11
217	33	X²
218	55	÷
219	53	(
220	43	RCL
221	11	11
222	75	-
223	43	RCL
224	10	10
225	54	)
226	33	X²
227	95	=
228	42	STO
229	07	07
230	65	×
231	53	(
232	43	RCL
233	10	10
234	94	+/-
235	65	×
236	43	RCL
237	01	01
238	54	)
239	22	INV

PROGRAM 2 continued

Step	Code	Key
240	23	LNX
241	95	=
242	42	STD
243	18	18
244	43	RCL
245	07	07
246	94	+/-
247	85	+
248	01	1
249	85	+
250	43	RCL
251	10	10
252	65	×
253	43	RCL
254	11	11
255	55	÷
256	53	(
257	43	RCL
258	10	10
259	75	-
260	43	RCL
261	11	11
262	54	)
263	65	×
264	43	RCL
265	01	01
266	95	=
267	65	×
268	53	(
269	43	RCL
270	11	11
271	94	+/-
272	65	×
273	43	RCL
274	01	01
275	54	)
276	22	INV
277	23	LNX
278	95	=
279	44	SUM

Step	Code	Key
280	18	18
281	71	SBR
282	47	CMS
283	91	R/S
284	76	LBL
285	23	LNX
286	43	RCL
287	11	11
288	45	Y^X
289	03	3
290	55	÷
291	53	(
292	43	RCL
293	11	11
294	75	-
295	43	RCL
296	10	10
297	54	)
298	45	Y^X
299	03	3
300	95	=
301	42	STD
302	07	07
303	65	×
304	53	(
305	43	RCL
306	10	10
307	94	+/-
308	65	×
309	43	RCL
310	01	01
311	54	)
312	22	INV
313	23	LNX
314	85	+
315	53	(
316	43	RCL
317	07	07
318	94	+/-
319	85	+

Step	Code	Key
320	01	1
321	85	+
322	53	(
323	43	RCL
324	11	11
325	75	-
326	43	RCL
327	11	11
328	45	Y^X
329	03	3
330	55	÷
331	53	(
332	43	RCL
333	10	10
334	75	-
335	43	RCL
336	11	11
337	54	)
338	33	X^2
339	54	)
340	65	×
341	43	RCL
342	01	01
343	85	+
344	43	RCL
345	10	10
346	65	×
347	43	RCL
348	11	11
349	33	X^2
350	55	÷
351	53	(
352	02	2
353	65	×
354	53	(
355	43	RCL
356	10	10
357	75	-
358	43	RCL
359	11	11

PROGRAM 2 continued

360	54	)	400	55	÷	440	65	×
361	54	)	401	53	(	441	43	RCL
362	65	×	402	53	(	442	01	01
363	43	RCL	403	43	RCL	443	54	)
364	01	01	404	12	12	444	22	INV
365	33	X²	405	75	-	445	23	LNX
366	54	)	406	43	RCL	446	95	=
367	65	×	407	10	10	447	42	STO
368	53	(	408	54	)	448	18	18
369	43	RCL	409	65	×	449	53	(
370	11	11	410	33	X²	450	43	RCL
371	94	+/-	411	54	)	451	07	07
372	65	×	412	95	=	452	94	+/-
373	43	RCL	413	42	STO	453	85	+
374	01	01	414	07	07	454	01	1
375	54	)	415	85	+	455	85	+
376	22	INV	416	43	RCL	456	43	RCL
377	23	LNX	417	10	10	457	10	10
378	95	=	418	65	×	458	33	X²
379	44	SUM	419	43	RCL	459	65	×
380	18	18	420	12	12	460	43	RCL
381	71	SBR	421	33	X²	461	12	12
382	47	CMS	422	55	÷	462	55	÷
383	91	R/S	423	53	(	463	53	(
384	76	LBL	424	43	RCL	464	43	RCL
385	24	CE	425	12	12	465	10	10
386	43	RCL	426	75	-	466	75	-
387	12	12	427	43	RCL	467	43	RCL
388	65	×	428	10	10	468	12	12
389	33	X²	429	54	)	469	54	)
390	75	-	430	33	X²	470	33	X²
391	03	3	431	65	×	471	65	×
392	65	×	432	43	RCL	472	43	RCL
393	43	RCL	433	01	01	473	01	01
394	12	12	434	95	=	474	54	)
395	33	X²	435	65	×	475	65	×
396	65	×	436	53	(	476	53	(
397	43	RCL	437	43	RCL	477	43	RCL
398	10	10	438	10	10	478	12	12
399	95	=	439	94	+/-	479	94	+/-

PROGRAM 2 continued

480	65	×
481	43	RCL
482	01	01
483	54	)
484	22	INV
485	23	LNX
486	95	=
487	44	SUM
488	18	18
489	71	SBR
490	47	CMS
491	91	R/S
492	76	LBL
493	25	CLR
494	43	RCL
495	11	11
496	65	×
497	43	RCL
498	12	12
499	33	X²
500	95	=
501	55	÷
502	53	(
503	43	RCL
504	11	11
505	75	-
506	43	RCL
507	10	10
508	54	)
509	55	÷
510	53	(
511	43	RCL
512	12	12
513	75	-
514	43	RCL
515	10	10
516	54	)
517	33	X²
518	95	=
519	65	×
520	53	(
521	43	RCL
522	10	10
523	94	+/-
524	65	×
525	43	RCL
526	01	01
527	54	)
528	22	INV
529	23	LNX
530	95	=
531	42	STO
532	18	18
533	43	RCL
534	10	10
535	65	×
536	43	RCL
537	12	12
538	33	X²
539	55	÷
540	53	(
541	43	RCL
542	10	10
543	75	-
544	43	RCL
545	11	11
546	54	)
547	55	÷
548	53	(
549	43	RCL
550	12	12
551	75	-
552	43	RCL
553	11	11
554	54	)
555	33	X²
556	95	=
557	65	×
558	53	(
559	43	RCL
560	11	11
561	94	+/-
562	65	×
563	43	RCL
564	01	01
565	54	)
566	22	INV
567	23	LNX
568	95	=
569	44	SUM
570	18	18
571	43	RCL
572	10	10
573	65	×
574	43	RCL
575	11	11
576	55	÷
577	53	(
578	43	RCL
579	10	10
580	75	-
581	43	RCL
582	12	12
583	54	)
584	55	÷
585	53	(
586	43	RCL
587	11	11
588	75	-
589	43	RCL
590	12	12
591	54	)
592	95	=
593	42	STO
594	07	07
595	65	×
596	43	RCL
597	10	10
598	65	×
599	43	RCL

PROGRAM 2 continued

```
600  11   11        640  12   12        680  86  STF
601  65   ×         641  65   ×         681  40  IND
602  43  RCL        642  43  RCL        682  00   00
603  12   12        643  01   01        683  00   0
604  55   ÷         644  95   =         684  42  STD
605  53   (         645  65   ×         685  07   07
606  43  RCL        646  53   (         686  43  RCL
607  10   10        647  43  RCL        687  11   11
608  75   -         648  12   12        688  32  X:T
609  43  RCL        649  94  +/-        689  87  IFF
610  11   11        650  65   ×         690  01   01
611  54   )         651  43  RCL        691  16  A'
612  65   ×         652  01   01        692  87  IFF
613  53   (         653  54   )         693  02   02
614  53   (         654  22  INV        694  48  EXC
615  43  RCL        655  23  LNX        695  87  IFF
616  10   10        656  95   =         696  03   03
617  75   -         657  44  SUM        697  49  PRD
618  43  RCL        658  18   18        698  87  IFF
619  12   12        659  71  SBR        699  04   04
620  54   )         660  47  CMS        700  50  IXI
621  33  X²         661  91  R/S        701  76  LBL
622  35  1/X        662  76  LBL        702  48  EXC
623  75   -         663  10  E'         703  43  RCL
624  53   (         664  98  ADV        704  10   10
625  43  RCL        665  43  RCL        705  67   EQ
626  11   11        666  19   19        706  16  A'
627  75   -         667  99  PRT        707  61  GTO
628  43  RCL        668  91  R/S        708  17  B'
629  12   12        669  76  LBL        709  76  LBL
630  54   )         670  15   E         710  49  PRD
631  33  X²         671  87  IFF        711  43  RCL
632  35  1/X        672  00   00        712  10   10
633  54   )         673  60  DEG        713  67   EQ
634  95   =         674  81  RST        714  16  A'
635  85   +         675  76  LBL        715  43  RCL
636  43  RCL        676  60  DEG        716  12   12
637  07   07        677  22  INV        717  67   EQ
638  65   ×         678  86  STF        718  22  INV
639  43  RCL        679  00   00        719  61  GTO
```

PROGRAM 2 continued **LABEL ADRESSES**

```
720   17  B'
721   76  LBL
722   50  I×I
723   43  RCL
724   10   10
725   22  INV
726   86  STF
727   40  IND
728   00   00
729   22  INV
730   67   EQ
731   57  ENG
732   01   1
733   42  STO
734   07   07
735   76  LBL
736   57  ENG
737   43  RCL
738   12   12
739   22  INV
740   67   EQ
741   58  FIX
742   02   2
743   44  SUM
744   07   07
745   86  STF
746   40  IND
747   07   07
748   87  IFF
749   03   03
750   59  INT
751   22  INV
752   86  STF
753   40  IND
754   07   07
755   76  LBL
756   58  FIX
757   43  RCL
758   12   12
759   32  X:T
```

```
760   43  RCL
761   13   13
762   22  INV
763   67   EQ
764   59  INT
765   04   4
766   44  SUM
767   07   07
768   76  LBL
769   59  INT
770   86  STF
771   40  IND
772   07   07
773   87  IFF
774   00   00
775   17  B'
776   87  IFF
777   03   03
778   16  A'
779   87  IFF
780   04   04
781   25  CLR
782   87  IFF
783   05   05
784   24  CE
785   87  IFF
786   06   06
787   23  LNX
788   91  R/S
```

```
005   18  C'
011   11   A
032   12   B
037   13   C
044   14   D
049   16  A'
067   28  LOG
095   29  CP
107   17  B'
113   39  COS
122   38  SIN
141   30  TAN
166   36  PGM
197   37  P/R
202   47  CMS
214   22  INV
285   23  LNX
385   24  CE
494   25  CLR
664   10  E'
671   15   E
677   60  DEG
700   48  EXC
708   49  PRD
720   50  I×I
734   57  ENG
754   58  FIX
767   59  INT
```

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