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THESIS

AN INTRODUCTION TO A RELIABILITY SHORTHAND

by

John J. Repicky, Jr.

March 1981

Thesis Advisor: James D. Esary

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After describing the possible ways a system can survive a mission, the practitioner of this shorthand can use stock formulas to obtain a system's survival function. Then simple insertion of the failure rates of the system's components into the known equations results in the system's reliability.

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An Introduction to a Reliability Shorthand

by

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Submitted in partial fulfillment of the
requirement for the degree of

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ABSTRACT

The determination of a system's life distribution usually requires the synthesis of a mixture of system survival modes. In order to alleviate the normal non-trivial calculations, this paper presents the concept of a reliability shorthand.

After describing the possible ways a system can survive a mission, the practitioner of this shorthand can use stock formulas to obtain a system's survival function. Then simple insertion of the failure rates of the system's components into the known equations results in the system's reliability.

Simple examples show the convenience of this shorthand. The Ti-59 is demonstrated to be a useful tool, adequate to implement the methodology.

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I. INTRODUCTION

It is generally accepted that the reliability of a system is the probability that the system will operate adequately for a given period of time in its intended application. The determination of a system's life distribution usually requires the synthesis of a mixture of modes in which the system can survive. One can assuredly state that the calculations can be non-trivial.

This paper will present the concept of a reliability shorthand which can greatly simplify the degree of mathematical difficulty usually encountered in determining the reliability of a system. After describing the possible ways a system can survive a mission, the practitioner of this reliability shorthand methodology can specialize a standard formula to obtain a system's survival function. Then simple insertion of the failure rates of the system's components into known preformulated equations results in the system's reliability.

The convenience of this methodology is demonstrated through several simple examples. The reliability shorthand for many systems is catalogued in Appendix A as a ready reference. In Appendix B is a Ti-59 program which allows for the easy calculation of a system's reliability for two general cases of the shorthand methodology.

The concept of a reliability shorthand was first introduced in the Operations Research course 0A4662, 'Reliability and Weapons System Effectiveness Measurement'. The concept has evolved with each presentation of the course. It is hoped this paper will be a beneficial tutorial aid for the students of that course, and act as an introduction to the topic for the interested reader.

II. RELIABILITY SHORTHAND

As a convenient shorthand we will use the convention that the expression $EXP(\lambda_1) + EXP(\lambda_2)$ denotes the distribution for a random variable $T_1 + T_2$, where T_1, T_2 are independent, T_1 has an $EXP(\lambda_1)$ distribution, and T_2 has an $EXP(\lambda_2)$ distribution. The life distribution of many systems can be usefully described using this shorthand.

In the following examples we typically suppose that the components of the systems fail independently and have exponential life distributions.

A. A SYSTEM HAVING TWO ACTIVE COMPONENTS IN SERIES

A two component series system functions if, and only if, both active components, A_1 and A_2, function. The life of the system, T, would be the minimum of the two component lives, $T = \min(T_1, T_2)$.

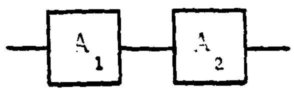


FIGURE 1: TWO ACTIVE COMPONENTS IN SERIES

We will assume $T_1 \sim EXP(\lambda_1)$, $T_2 \sim EXP(\lambda_2)$, and T_1, T_2 are independent. The system's survival function is

$$\begin{aligned}\bar{F}(t) &= P(T>t) \\ &= P(\min(T_1, T_2)>t) \\ &= P(T_1>t, T_2>t).\end{aligned}$$

Using the assumptions of independence and components having exponential life distributions we obtain

$$\begin{aligned}\bar{F}(t) &= P(T_1 > t)\; P(T_2 > t) \\ &= \bar{F}_1(t)\; \bar{F}_2(t) \\ &= e^{-\lambda_1 t}\, e^{-\lambda_2 t} \\ &= e^{-(\lambda_1 + \lambda_2)t}.\end{aligned}$$

The life distribution of the system is $T \sim EXP(\lambda_1 + \lambda_2)$.

When $\lambda_1 = \lambda_2 = \lambda$, then $\bar{F}(t) = e^{-2\lambda t}$ and $T \sim EXP(2\lambda)$. Our shorthand notation $EXP(2\lambda)$ represents the life distribution of a system where two identical components must both function for the system to survive.

B. A STANDBY SYSTEM HAVING ONE ACTIVE AND ONE SPARE COMPONENT

An active component, A, is to complete a mission of duration t. A spare component, S, is available to automatically replace the active component should it fail. The life of the active component is T_1. The life of the spare component is T_2. The life of the system is $T = T_1 + T_2$.

In determining the survival function of this system, we first describe how the system can survive to successfully complete a mission of duration t. Component A can live to time t with the spare never being utilized, or component A can fail at some intermediate time s. Then the spare component automatically replaces the failed component, and component S must live from time s to time t to successfully complete the mission.

With T_1, T_2 independent, the survival function of the system can be represented as:

$$\bar{F}(t) = \bar{F}_1(t) + \int_0^t \bar{F}_2(t-s)\, f_1(s)ds,$$

where $\bar{F}_1(t)$ is the probability of component A living to time t, $f_1(s)$ is the likelihood that component A fails at some time s, and $\bar{F}_2(t-s)$ is the probability that component S lives from time s to time t.

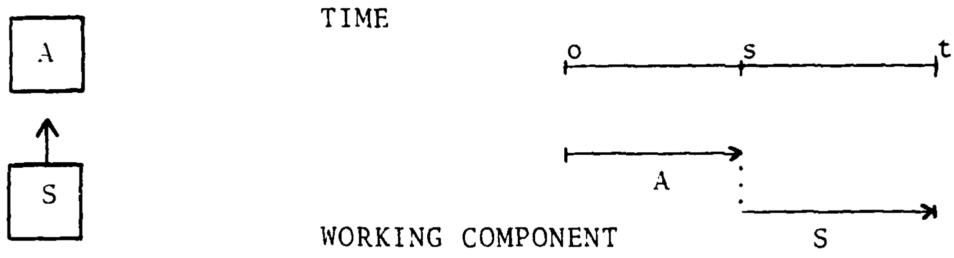


FIGURE 2: A SINGLE ACTIVE COMPONENT WITH ONE SPARE COMPONENT

1. Identical Components

If the active and spare components are identical, then $T_1 \sim EXP(\lambda)$, $T_2 \sim EXP(\lambda)$, T_1, T_2 are independent, and $T = T_1 + T_2$. The survival function is now expressed as

$$\begin{aligned}\bar{F}(t) &= e^{-\lambda t} + \int_0^t e^{-\lambda(t-s)}(\lambda e^{-\lambda s})ds \\ &= e^{-\lambda t} + \int_0^t e^{-\lambda t} e^{\lambda s} \lambda e^{-\lambda s} ds \\ &= e^{-\lambda t} + e^{-\lambda t}\int_0^t \lambda\, ds \\ &= e^{-\lambda t} + e^{-\lambda t}(\lambda t) \\ &= (1 + \lambda t)e^{-\lambda t}.\end{aligned}$$

The shorthand notation for this survival function is EXP(λ) + EXP(λ). Visualize this as a system having one EXP (λ) component, and upon that component's failure the system has a completely new and identical EXP(λ) component because of the spare.

2. Dissimilar Components

If the active and spare components are dissimilar, then $T_1 \sim EXP(\lambda_1)$, $T_2 \sim EXP(\lambda_2)$, T_1, T_2 are independent, and $T = T_1 + T_2$. The formulation of the survival function for this system is identical to the case of similar components, except for the change in failure rates. The survival function is

$$\begin{aligned}
\bar{F}(t) &= e^{-\lambda_1 t} + \int_0^t e^{-\lambda_2(t-s)} \lambda_1 e^{-\lambda_1 s} ds \\
&= e^{-\lambda_1 t} + \int_0^t e^{-\lambda_2 t} e^{\lambda_2 s} \lambda_1 e^{-\lambda_1 s} ds \\
&= e^{-\lambda_1 t} + \lambda_1 e^{-\lambda_2 t} \int_0^t e^{-(\lambda_1-\lambda_2)s} ds \\
&= e^{-\lambda_1 t} + \lambda_1 e^{-\lambda_2 t} \left(\frac{1}{\lambda_1-\lambda_2}\right) \int_0^t (\lambda_1-\lambda_2) e^{-(\lambda_1-\lambda_2)s} ds \\
&= \frac{\lambda_1-\lambda_2}{\lambda_1-\lambda_2} e^{-\lambda_1 t} + \frac{\lambda_1}{\lambda_1-\lambda_2} e^{-\lambda_2 t}(1-e^{-(\lambda_1-\lambda_2)t}) \\
&= \frac{(\lambda_1-\lambda_2)e^{-\lambda_1 t} + \lambda_1 e^{-\lambda_2 t}(1-e^{-(\lambda_1-\lambda_2)t})}{\lambda_1-\lambda_2} \\
&= \frac{\lambda_1 e^{-\lambda_1 t} - \lambda_2 e^{-\lambda_1 t} + \lambda_1 e^{-\lambda_2 t} - \lambda_1 e^{-\lambda_2 t} e^{-(\lambda_1-\lambda_2)t}}{\lambda_1-\lambda_2} \\
&= \frac{\lambda_1 e^{-\lambda_2 t} - \lambda_2 e^{-\lambda_1 t}}{\lambda_1-\lambda_2} .
\end{aligned}$$

The shorthand notation for this survival function is $EXP(\lambda_1) + EXP(\lambda_2)$. As the active component fails a new component takes its place to complete the same task, however, the new component has a different failure rate than that of the initial component.

C. A SYSTEM HAVING TWO ACTIVE COMPONENTS IN PARALLEL

A two component parallel system functions if, and only if, at least one component functions. The life of the system, T, would be the maximum of the two component lives, $T = \max(T_1, T_2)$.

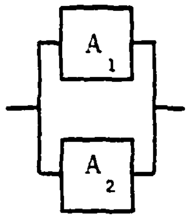


FIGURE 3: TWO ACTIVE COMPONENTS IN PARALLEL

Now assume $T_1 \sim EXP(\lambda_1)$, $T_2 \sim EXP(\lambda_2)$, and T_1, T_2 are independent. The survival function of the parallel system is

$$\bar{F}(t) = P(\max(T_1, T_2) > t)$$
$$= 1 - P(\max(T_1, T_2) \leq t)$$
$$= 1 - P(T_1 \leq t, T_2 \leq t).$$

Using the assumption of independence

$$\bar{F}(t) = 1 - [P(T_1 \leq t)\ P(T_2 \leq t)]$$
$$= 1 - [(1 - \bar{F}_1(t)\ (1 - \bar{F}_2(t)\ )]$$
$$= 1 - [1 - \bar{F}_1(t) - \bar{F}_2(t) + \bar{F}_1(t)\bar{F}_2(t)]$$
$$= \bar{F}_1(t) + \bar{F}_2(t) - \bar{F}_1(t)\ \bar{F}_2(t).$$

Using the assumption that the components have exponential life distributions, the resulting life distribution is

$$\bar{F}(t) = e^{-\lambda_1 t} + e^{-\lambda_2 t} - e^{-(\lambda_1 + \lambda_2)t}.$$

When $\lambda_1 = \lambda_2 = \lambda$, the survival function is $\bar{F}(t) = e^{-\lambda t} + e^{-\lambda t} - e^{-(\lambda + \lambda)t} = 2e^{-\lambda t} - e^{-(\lambda_1 + \lambda_2)t}$. The life of the parallel system begins with both active components functioning together for system survival. The time until one of the components fails has the distribution EXP(λ). When one of the components fails, the memoryless property of the exponential distribution provides that the surviving component has an additional EXP(λ) life with which to complete the mission. The shorthand notation for the survival function of the simple parallel system of identical components is EXP(2λ) + EXP(λ).

Now we will demonstrate the ease of using the reliability shorthand, compared to alternative calculations for determining a system's reliability. Recall that EXP(λ_1) + EXP(λ_2) is the shorthand notation for the survival function

$$\bar{F}(t) = \frac{\lambda_1 e^{-\lambda_2 t} - \lambda_2 e^{-\lambda_1 t}}{\lambda_1 - \lambda_2} .$$

Noting that the parallel system is described by EXP(2λ) + EXP(λ), we can see the simplicity of substituting 2λ for λ_1 and λ for λ_2 into the known survival function equation. The resulting survival function is

$$\bar{F}(t) = \frac{(2\lambda)e^{-(\lambda)t} - (\lambda)e^{-(2\lambda)t}}{(2\lambda) - (\lambda)}$$

$$= \frac{\lambda(2e^{-\lambda t} - e^{-2\lambda t})}{\lambda}$$

$$= 2\,e^{-\lambda t} - e^{-2\lambda t} .$$

The survival functions are equivalent using either method, however, the shorthand methodology uses merely substitution and simple mathematics.

D. A STANDBY SYSTEM HAVING TWO ACTIVE COMPONENTS IN SERIES WITH ONE SPARE COMPONENT

Consider a system which has two identical components in series with a similar component as a standby spare which automatically replaces the first component that fails.

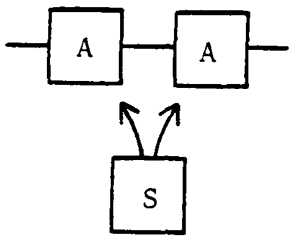


FIGURE 4: TWO ACTIVE COMPONENTS IN SERIES WITH ONE SPARE COMPONENT

The system has component times to failure $T_1 \sim EXP(\lambda)$, $T_2 \sim EXP(\lambda)$ and spare component time to failure $T_3 \sim EXP(\lambda)$, with T_1, T_2, T_3 independent. This system can complete its mission of duration t in two possible ways. It can survive if the original components live to time t and the spare component is never needed. Alternatively, one of the active components could fail at some intermediate time s, causing the system to fail. At that time the surviving component is like new and the spare component replaces the failed component creating a brand new series system to complete the mission from time s to time t.

In determining the system's survival function using reliability shorthand, we recall that a two component series system has an EXP(2λ) life distribution. With the spare component replacement the system accomplishes the task as if it had two independent series systems to function consecutively. The shorthand notation is simply EXP(2λ) + EXP(2λ).

Recall that the shorthand notation EXP(λ) + EXP(λ) represents the life distribution where the survival function is $\bar{F}(t) = (1 + \lambda t)e^{-\lambda t}$. To determine the survival function of EXP(2λ) + EXP(2λ) we substitute 2λ for λ into the known formula and obtain

$$\bar{F}(t) = (1 + 2\lambda t)e^{-2\lambda t}.$$

The usual method of determining the survival function is slightly more involved. The system can survive if the original series system lives to time t with no spare required. If one of the original components fails at some intermediate time s, then the spare component and the surviving component combine as a new series system. Both of the components of the new series system must live from time s to time t for the system to complete the mission. We formulate the survival function as

$$\begin{aligned}\bar{F}(t) &= e^{-2\lambda t} + \int_0^t e^{-\lambda(t-s)}e^{-\lambda(t-s)}2\lambda e^{-2\lambda s}ds \\ &= e^{-2\lambda t} + \int_0^t e^{-\lambda t}e^{\lambda s}e^{-\lambda t}e^{\lambda s}2\lambda e^{-2\lambda s}ds \\ &= e^{-2\lambda t} + e^{-2\lambda t}\int_0^t 2\lambda\, ds \\ &= e^{-2\lambda t} + e^{-2\lambda t}(2\lambda t) \\ &= (1 + 2\lambda t)e^{-2\lambda t}\ .\end{aligned}$$

The results are identical but the difference in mathematical difficulty is obvious. To easily determine a system's life distribution one need only be able to describe how the system successfully survives a mission, and then take advantage of the simple reliability shorthand methodology. In the next chapter we will expand this notation to include mixing of distributions.

III. MIXING DISTRIBUTIONS

In previous cases of systems utilizing spare components we assumed that those spare components would automatically and successfully replace failed components. Successful replacement occurred with probability equal to one. Perfect equipment in real life does not exist. We will assume that switchover and replacement by a spare component occurs with probability p, where $0<p<1$. No transfer occurs with probability $1-p$.

A. MIX NOTATION

For general application let D_1 and D_2 represent the probability distributions of the independent random times to failure T_1 and T_2. Let $D_1 + D_2$ stand for the distribution of the sum $T_1 + T_2$. Now let the notation

$$MIX(p_1 D_1, p_2 D_2)$$

denote the mixture of the distributions D_1 and D_2 with respect to the mixing probabilities p_1 and p_2, where $p_1 + p_2 = 1$. This mixture of distributions has the survival function

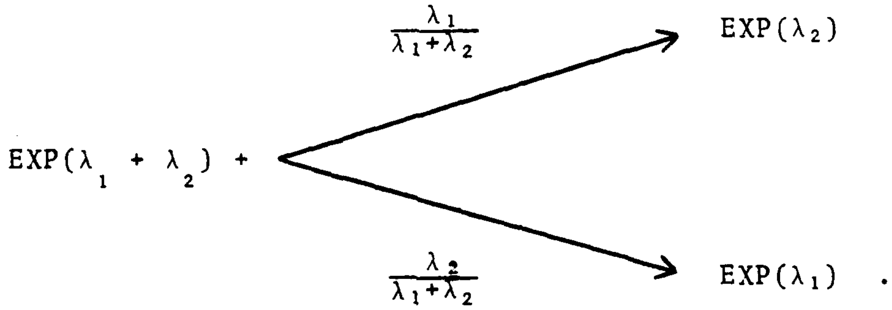
$$\bar{F}(t) = p_1 \bar{F}_1(t) + p_2 \bar{F}_2(t),$$

where $\bar{F}_1(t)$ and $\bar{F}_2(t)$ are the survival functions for D_1 and D_2.

1. A System having two Active Components in Parallel

A simple parallel system continues to survive as long as either active component still functions, regardless of the order in which they fail. Assume component A_1 has life $T_1 \sim EXP(\lambda_1)$, component A_2 has life $T_2 \sim EXP(\lambda_2)$, T_1, T_2 are independent, and T = maximum (T_1, T_2).

From what we know of parallel systems, the life distribution is $EXP(\lambda_1 + \lambda_2) + EXP(\lambda_1)$ if component A_2 is the first to fail, or it is $EXP(\lambda_1 + \lambda_2) + EXP(\lambda_2)$ if component A_1 fails first. The probability that A_1 fails before A_2 is $\frac{\lambda_1}{\lambda_1+\lambda_2}$, and that A_2 fails before A_1 is $\frac{\lambda_2}{\lambda_1+\lambda_2}$. The system life distribution is described by the branching representation



Using the MIX notation this life distribution is

$$EXP(\lambda_1 + \lambda_2) + MIX[(\tfrac{\lambda_1}{\lambda_1+\lambda_2})EXP(\lambda_2),\ (\tfrac{\lambda_2}{\lambda_1+\lambda_2})EXP(\lambda_1)].$$

The survival function for this distribution is

$$\bar{F}(t) = e^{-(\lambda_1+\lambda_2)t} + \int_0^t \left[\frac{\lambda_1}{\lambda_1+\lambda_2} e^{-\lambda_2(t-s)} + \frac{\lambda_2}{\lambda_1+\lambda_2} e^{-\lambda_1(t-s)}\right] \left[(\lambda_1+\lambda_2)\, e^{-(\lambda_1+\lambda_2)s}\right]ds.$$

Applying techniques used previously the survival function becomes

$$\begin{aligned} \bar{F}(t) &= e^{-(\lambda_1+\lambda_2)t} + e^{-\lambda_2 t}\int_0^t \lambda_1 e^{-\lambda_1 s}ds + e^{-\lambda_1 t}\int_0^t \lambda_2 e^{-\lambda_2 s}ds \\ &= e^{-(\lambda_1+\lambda_2)t} + e^{-\lambda_2 t}(1-e^{-\lambda_1 t}) + e^{-\lambda_1 t}(1-e^{-\lambda_2 t}) \\ &= e^{-\lambda_1 t} + e^{-\lambda_2 t} - e^{-(\lambda_1+\lambda_2)t}. \end{aligned}$$

This demonstrates that this MIX notation does represent the parallel system's survival function

$$\bar{F}(t) = \bar{F}_1(t) + \bar{F}_2(t) - \bar{F}_1(t)\bar{F}_2(t).$$

2. <u>Distributive Property</u>

The MIX notation has an algebraic distributive property. Notationally we have

$$D_3 + MIX(p_1D_1, p_2D_2) = MIX[p_1(D_1 + D_3), p_2(D_2 + D_3)].$$

A graphic representation of the distributive property is shown in figure 5.

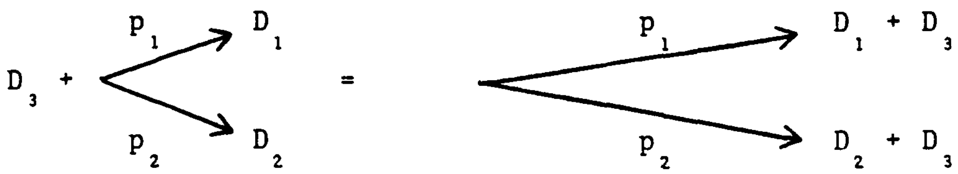


FIGURE 5: DISTRIBUTIVE PROPERTY OF THE MIX NOTATION

For our parallel system example note that

$D_1 = EXP(\lambda_2)$, $D_2 = EXP(\lambda_1)$, $p_1 = \frac{\lambda_1}{\lambda_1+\lambda_2}$, $p_2 = \frac{\lambda_2}{\lambda_1+\lambda_2}$,

and $D_3 = EXP(\lambda_1+\lambda_2)$. Using these values we see that

$EXP(\lambda_1+\lambda_2) + MIX[(\frac{\lambda_1}{\lambda_1+\lambda_2})EXP(\lambda_2), (\frac{\lambda_2}{\lambda_1+\lambda_2})EXP(\lambda_1)]$ is equivalent to

$MIX[(\frac{\lambda_1}{\lambda_1+\lambda_2})[EXP(\lambda_1+\lambda_2) + EXP(\lambda_2)], (\frac{\lambda_2}{\lambda_1+\lambda_2})[EXP(\lambda_1+\lambda_2) + EXP(\lambda_1)]]$. Note that this is of the form $MIX(p_1D_1, p_2D_2)$.

The latter MIX notation, which combines known distributions, is easier to use computationally than the MIX notation previously given. Utilizing the known distribution of $EXP(\lambda_1) + EXP(\lambda_2)$, which has the survival function

$$\bar{F}(t) = \frac{\lambda_1 e^{-\lambda_2 t} - \lambda_2 e^{-\lambda_1 t}}{\lambda_1 - \lambda_2},$$

we can easily convert our MIX notation to determine the parallel system's life distribution.

Substituting this survival function into our MIX notation for the parallel system we obtain

$$\bar{F}(t) = \left(\frac{\lambda_1}{\lambda_1+\lambda_2}\right)\left[\frac{(\lambda_1+\lambda_2)e^{-(\lambda_2)t} - (\lambda_2)e^{-(\lambda_1+\lambda_2)t}}{(\lambda_1+\lambda_2) - (\lambda_2)}\right]$$

$$+ \left(\frac{\lambda_2}{\lambda_1+\lambda_2}\right)\left[\frac{(\lambda_1+\lambda_2)e^{-(\lambda_1)t} - (\lambda_1)e^{-(\lambda_1+\lambda_2)t}}{(\lambda_1+\lambda_2) - (\lambda_1)}\right].$$

By cancelling the λ_1's in the first term and the λ_2's in the second term, then dividing both terms by the denominator, $(\lambda_1+\lambda_2)$, the survival function reduces to that of the simple parallel system

$$\bar{F}(t) = e^{-\lambda_1 t} + e^{-\lambda_2 t} - e^{-(\lambda_1+\lambda_2)t}.$$

As seen in the previous section the alternative method of determining the system's survival function takes the general form

$$\bar{F}(t) = \bar{F}_3(t) + \int_0^t p_1 \bar{F}_1(t-s) f_3(s)ds + \int_0^t p_2 \bar{F}_2(t-s) f_3(s)ds.$$

The reliability shorthand methodology would appear to be preferable.

3. A Standby System Having one Active and a Possible Spare Component

An active component, A, is replaced when it fails by a spare component, S, with probability p. The system has an active component time to failure $T_1 \sim EXP(\lambda)$, a spare component time to failure $T_2 \sim EXP(\lambda)$, and T_1, T_2 are independent.

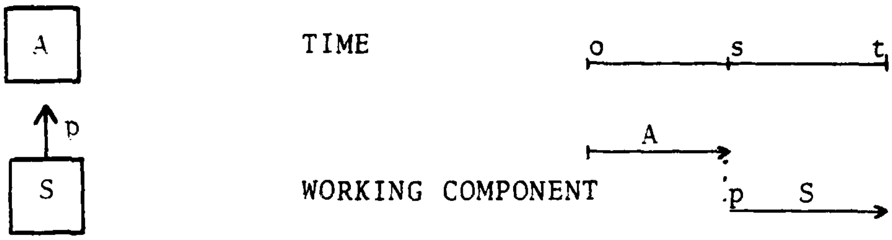


FIGURE 6: A SINGLE ACTIVE COMPONENT POSSIBLY HAVING ONE SPARE COMPONENT

The life of the system is $T = T_1$ with probability $1-p$, or it is $T = T_1 + T_2$ with probability p. The shorthand method of determining the system's life distribution is to view the survival function as a combination of two possible distributions. If no switchover occurs the life T could be T_1 having $\bar{F}_1(t) = e^{-\lambda t}$, or if switchover occurs T could be $T_1 + T_2$ having $\bar{F}_2(t) = (1 + \lambda t)e^{-\lambda t}$. The survival functions $\bar{F}_1(t)$ and $\bar{F}_2(t)$ occur with probabilities 1-p and p, respectively. The life distribution is a mixture of the possible distributions where

$$\bar{F}(t) = (1 - p)\ \bar{F}_1(t) + p\bar{F}_2(t).$$

Thus the system's survival function is

$$\begin{aligned}\bar{F}(t) &= (1-p)e^{-\lambda t} + p(1+\lambda t)e^{-\lambda t} \\ &= e^{-\lambda t} - pe^{-\lambda t} + pe^{-\lambda t} + p\lambda te^{-\lambda t} \\ &= e^{-\lambda t} + p\lambda te^{-\lambda t} \\ &= (1+p\lambda t)e^{-\lambda t}.\end{aligned}$$

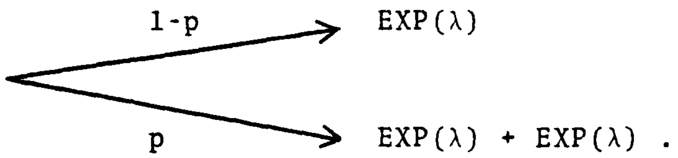
The alternate method of determining the life distribution of the system is to derive its survival function in terms of its possible ways of mission success. The original component could survive to time t with no spare component required, or the original component could fail at some intermediate time s. The spare component then replaces the original component with probability p, and it must live from time s to time t to successfully complete the mission. The system's survival function is then

$$\begin{aligned}\bar{F}(t) &= e^{-\lambda t} + \int_0^t p e^{-\lambda(t-s)} \lambda e^{-\lambda s} ds \\ &= e^{-\lambda t} + \int_0^t p e^{-\lambda t} e^{\lambda s} \lambda e^{-\lambda s} ds \\ &= e^{-\lambda t} + p e^{-\lambda t} \int_0^t \lambda \, ds \\ &= e^{-\lambda t} + p e^{-\lambda t} (\lambda t) \\ &= (1 + p\lambda t) e^{-\lambda t}.\end{aligned}$$

Using the MIX notation we need only write

$MIX[(1-p)EXP(\lambda), p(EXP(\lambda) + EXP(\lambda))]$.

the graphic representation is



The convenience of the shorthand methodology is again demonstrated.

B. DEGENERACY AT ZERO

Let ZERO be the name for the distribution of a random variable that is degenerate at zero. If $p[T_0=0] = 1$, then we say T_0 has the distribution ZERO, or $T_0 \sim$ ZERO. The survival function for T_0 is as shown in Figure 7.

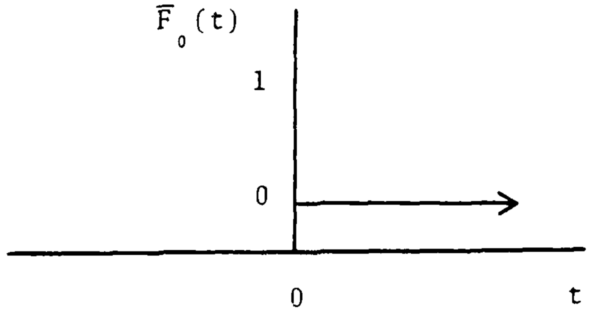


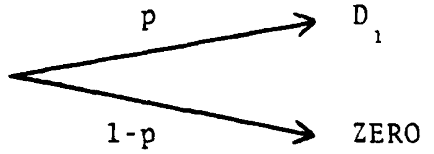
FIGURE 7: SURVIVAL FUNCTION OF THE ZERO DISTRIBUTION

The notion of a ZERO distribution compliments the MIX notation. Assume D_1 is the distribution of a nonnegative random variable T_1 which has the survival function $\bar{F}_1(t)$ and the density $f_1(t)$, where $t \geq 0$. We can then visualize the survival of a component as a combination of $\bar{F}_1(t)$ and $\bar{F}_0(t)$. The survival function is

$$\bar{F}(t) = \bar{F}_1(t) + \int_0^t \bar{F}_0(t-s) f_1(s)\, ds.$$

Since $\bar{F}_0(t-s) = 0$ for the ZERO distribution, the survival function, $\bar{F}(t)$, is simply $\bar{F}_1(t)$. The ZERO distribution adds nothing to another distribution's density, $D_1 + \text{ZERO} = D_1$.

In the MIX notation we could have $\text{MIX}(pD_1, (1-p)\text{ZERO})$ represent the survival function of a distribution. This would be graphically represented as



The survival function for this notation is

$$\bar{F}(t) = p\,\bar{F}_1(t) + (1-p)\bar{F}_0(t)$$
$$= p\,\bar{F}_1(t) + (1-p)\,(0)$$
$$= p\,\bar{F}_1(t)$$

The probability p need not be 1 since a system may not work when it is turned on.

For an example of the ZERO distribution's utilization, let us take the standby system composed of a single active component having a spare component for replacement. In section II-A we saw that T was T_1 with probability 1-p, or T was $T_1 + T_2$ with probability p. In our MIX notation this would be

MIX(p[EXP(λ) + EXP(λ)], (1-p)[EXP(λ)]).

If it were not for the ZERO distribution our distributive property would not hold. With the ZERO distribution this MIX notation can be reexpressed as

EXP(λ) + MIX[pEXP(λ), (1-p)ZERO].

Figure 8 graphically represents this equivalence, keeping in mind that EXP(λ) + ZERO = EXP(λ).

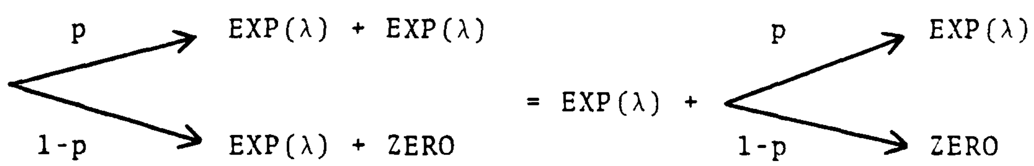


FIGURE 8: DISTRIBUTIVE PROPERTY INCORPORATING THE ZERO DISTRIBUTION

IV. SUMMARY

By learning a simple style of notation and applying it to the survivability of a system, the reliability practitioner can determine the life distribution of the system with non-calculus mathematics.

Appendix A is provided as a start for a ready reference catalogue of systems and their reliability shorthand.

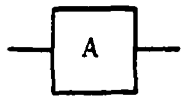
Through the use of computers we can reduce the burden of calculating the survival functions for many systems. Two of the general reliability shorthand cases have been programmed for the Ti-59 and they are presented in Appendix B. The examples provided in that section will demonstrate the computational convenience of the shorthand methodology.

The total scope and depth of the reliability shorthand methodology is yet to be investigated. Computationally, cases requiring the convolution of identical failure rates and distinct failure rates both have known survival function algorithms. Further study is required to determine if there is a useable algorithm which will permit the combination of both cases. This paper was designed to introduce this concept and its known properties to those already familiar with reliability. After seeing the convenience and benefit of the reliability shorthand methodology it is hoped the reader's interest will be further stimulated.

APPENDIX: A

This section contains several examples of the more common systems and their associated reliability shorthand. The format facilitates the addition of other systems in order to build a more thorough ready reference catalogue.

SYSTEM:



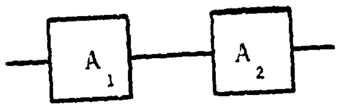
SHORTHAND: EXP(λ)

SURVIVAL FUCTION: $\bar{F}(t) = e^{-\lambda t}$

DESCRIPTION:

A single active component having an exponential life distribution.

SYSTEM:



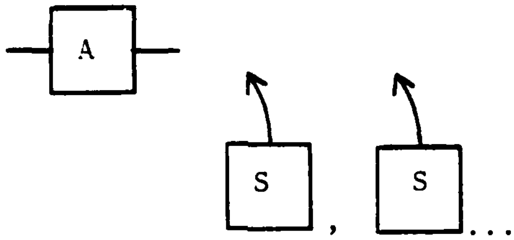
SHORTHAND: $\mathrm{EXP}(\lambda_1 + \lambda_2)$

SURVIVAL FUNCTION: $\bar{F}(t) = e^{-(\lambda_1 + \lambda_2)t}$

DESCRIPTION:

A two component series system which requires both components to function for the system to survive.

SYSTEM:



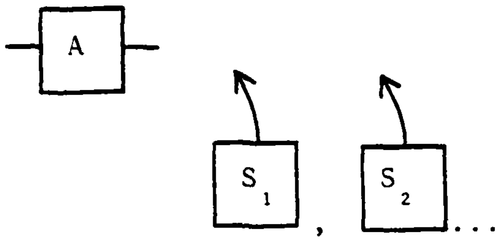
SHORTHAND: $EXP_1(\lambda) + EXP_2(\lambda) + \ldots + EXP_n(\lambda)$

SURVIVAL FUNCTION: $$\bar{F}(t) = \sum_{i=1}^{n} \frac{(\lambda t)^{i-1}}{(i-1)!} e^{-\lambda t}$$

DESCRIPTION:

A single active component has n-1 identical spare components. As each component fails it is replaced by a new identical component which allows the system to survive. The system has n consecutive EXP(λ) lives.

SYSTEM:



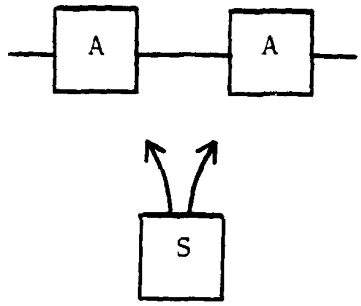
SHORTHAND: $EXP(\lambda_1) + EXP(\lambda_2) + \ldots + EXP(\lambda_n)$

SURVIVAL FUNCTION: $\bar{F}(t) = \sum_{i=1}^{n} \frac{\Pi_{j \neq i} \lambda_j}{\Pi_{j \neq i}(\lambda_j - \lambda_i)} e^{-\lambda_i t}$

DESCRIPTION:

A single active component has n-1 dissimilar spare components. As each component fails it is replaced by a new component which allows the system to survive. Each of the n components has a different failure rate, and the system has n consecutive $EXP(\lambda_i)$ lives.

SYSTEM:



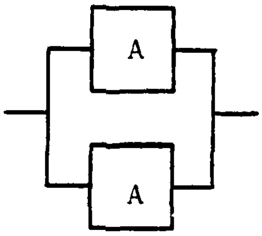
SHORTHAND: $EXP(2\lambda) + EXP(2\lambda)$

SURVIVAL FUNCTION: $\bar{F}(t) = (1 + 2\lambda t)e^{-2\lambda t}$

DESCRIPTION:

A series system composed of two identical active components has another identical component available as a spare. The original series system has a $EXP(2\lambda)$ life. When either component fails and the spare takes its place, the system has a new $EXP(2\lambda)$ life.

SYSTEM:



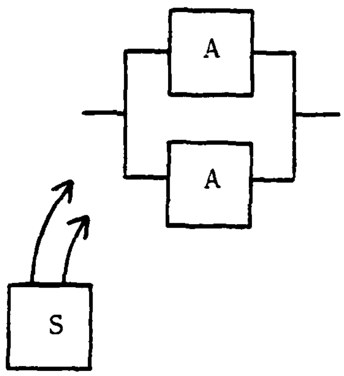
SHORTHAND: EXP(2λ) + EXP(λ)

SURVIVAL FUNCTION: $\bar{F}(t) = 2e^{-\lambda t} - e^{-2\lambda t}$

DESCRIPTION:

The parallel system has two identical active components functioning together with an EXP(2λ) life for system survival. When either component fails the surviving component is as if new with an EXP(λ) life. This new component alone functions for system survival.

SYSTEM:



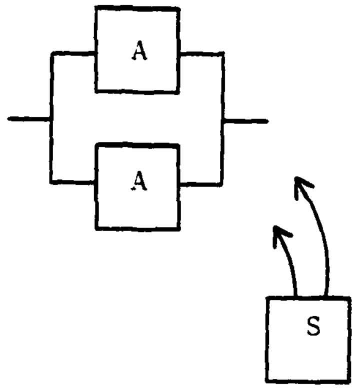
SHORTHAND: EXP(2λ) + EXP(2λ) + EXP(λ)

SURVIVAL FUNCTION: $\bar{F}(t) = 4e^{-\lambda t} - 3e^{-2\lambda t} - 2\lambda t e^{-2\lambda t}$

DESCRIPTION:

A parallel system composed of two identical components has a similar component as a spare which will replace the first component that fails. The original system functions with a EXP(2λ) life until a component fails. When it is replaced a new parallel system exists which has a life of EXP(2λ) + EXP(λ).

SYSTEM:



SHORTHAND: EXP(2λ) + EXP(λ) + EXP(λ)

SURVIVAL FUNCTION: $\bar{F}(t) = e^{-2\lambda t} + 2\lambda t e^{-\lambda t}$

DESCRIPTION:

A parallel system composed of two identical components has a similar component as a spare which will replace the last component that fails. The original system functions with an EXP(2λ) + EXP(λ) life until both components have failed. When the last component is replaced the system survives by the new component which has an EXP(λ) life.

APPENDIX: B

INTRODUCTION

There are two general cases in reliability shorthand where the aid of a programmable calculator greatly simplifies the tedious calculation of a system's survival function.

Case one in reliability shorthand is of the form $EXP(\lambda_1) + EXP(\lambda_2) + \ldots + EXP(\lambda_n)$, and each of the n failure rates are different. When the system description is of this form, $\sum_{i=1}^{n} EXP(\lambda_i)$, the survival function is $\overline{F}(t) = \sum_{i=1}^{n} \frac{\Pi_{j \neq i} \lambda_j}{\Pi_{j \neq i}(\lambda_j - \lambda_i)} e^{-\lambda_i t}$.

Case two in reliability shorthand is of the form $EXP(\lambda) + EXP_2(\lambda) + \ldots + EXP_n(\lambda)$, and each of the n failure rates are identical. When the system description is of this form, $\sum_{i=j}^{n} EXP_i(\lambda)$, the survival function is $\overline{F}(t) = \sum_{i=j}^{n} \frac{(\lambda t)^{i-1}}{(i-1)!} e^{-\lambda t}$

USER PROCEDURES

1. Use any library module and read in side one of the magnetic card.
2. For case one the survival function is found using Label A. Enter t in R_{00}, n in R_{01}, and the n different failure rates in R_{13} through $R_{13+(n-1)}$. The order of the λ_i's does not matter. Press [A] for the system reliability.
3. For case two, the survival function is found using Label B. Enter t in R_{00}, n in R_{01}, and λ in R_{13}. Press [B] for the system reliability.

4. The maximum n for case two is not limited. The maximum for case one is limited to 47 due to the partioning 479.59. Using 9 2nd OP 17 the maximum n can be increased to 77.

LABELS USED

A	A'	sin
B	B'	cos
C	C'	tan
D	D'	
	E'	

STORAGE REGISTER CONTENTS

00	t	08	used
01	n	09	used
02	used	10	used
03	used	11	used
04	used	12	used
06	used	13	λ
07	used	13-59	λ_i

EXAMPLE RUN TIMES

n	Case one - LBL A	Case two - LBL B
1	8 seconds	3 seconds
2	18 seconds	5 seconds
3	34 seconds	7 seconds
4	55 seconds	10 seconds
5	80 seconds	12 seconds

SAMPLE PROBLEMS

CASE ONE:

Reliability shorthand: $EXP(\lambda_1) + EXP(\lambda_2) + EXP(\lambda_3)$

Longhand form:

$$\overline{F}(t) = \frac{\lambda_2\lambda_3}{(\lambda_2-\lambda_1)(\lambda_3-\lambda_1)} e^{-\lambda_1 t} + \frac{\lambda_1\lambda_3}{(\lambda_1-\lambda_2)(\lambda_3-\lambda_2)} e^{-\lambda_2 t} + \frac{\lambda_1\lambda_2}{(\lambda_1-\lambda_3)(\lambda_2-\lambda_3)} e^{-\lambda_3 t}$$

Sample values: $t = 2$, $n = 3$, $\lambda_1 = .5$, $\lambda_2 = .6$, $\lambda_3 = .7$

Procedure:

1) Enter sample values, t=2 STO 00, n=3 STO 01, λ_1=.5 STO 13, λ_2=.6 STO 14, and λ_3=.7 STO 15.

2) Press [A] and $\overline{F}(t)$ is displayed. $\overline{F}(t)$=.88262530

CASE TWO:

Reliability shorthand: $EXP_1(\lambda)+EXP_2(\lambda)+EXP_3(\lambda)+EXP_4(\lambda)+EXP_5(\lambda)$

Longhand form:

$$\overline{F}(t) = \left(\frac{(\lambda t)^0}{0!} + \frac{(\lambda t)^1}{1!} + \frac{(\lambda t)^2}{2!} + \frac{(\lambda t)^3}{3!} + \frac{(\lambda t)^4}{4!}\right)e^{-\lambda t}$$

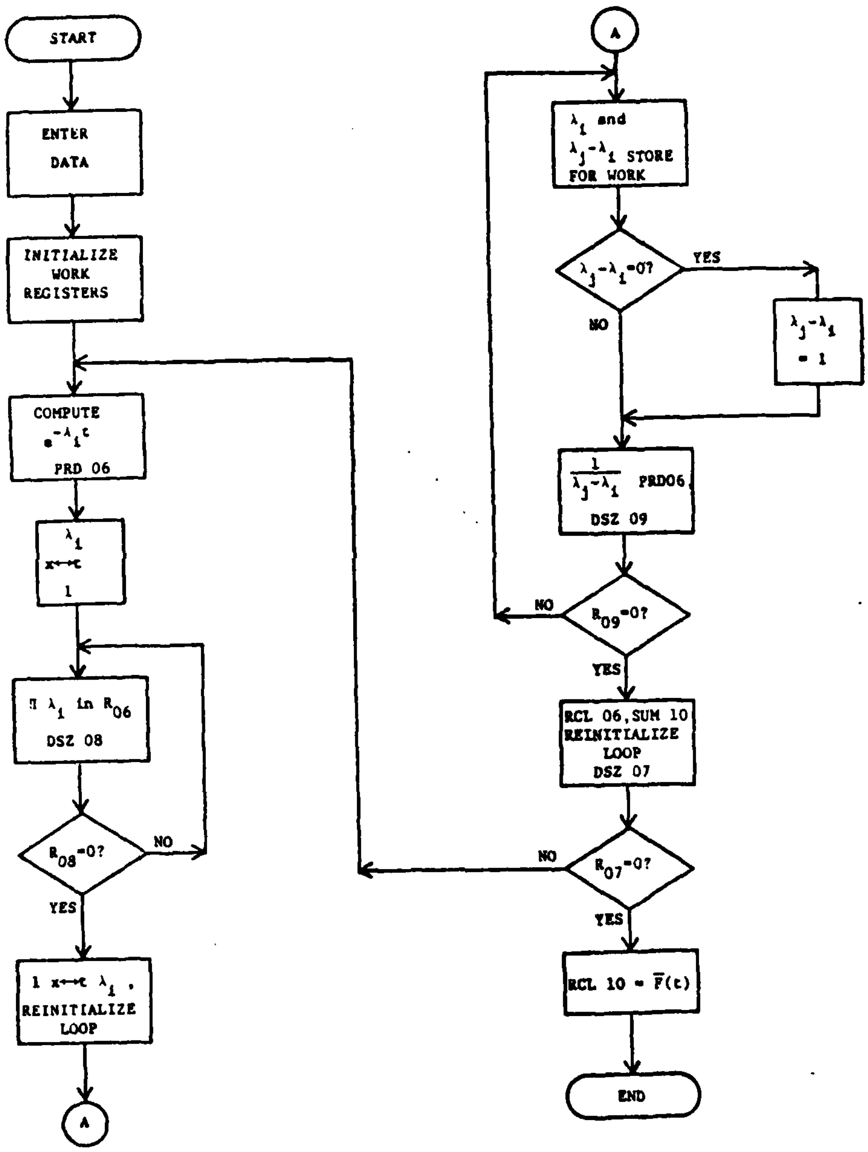
Sample values: $t=2$, $n=5$, and $\lambda=.5$

Procedure:

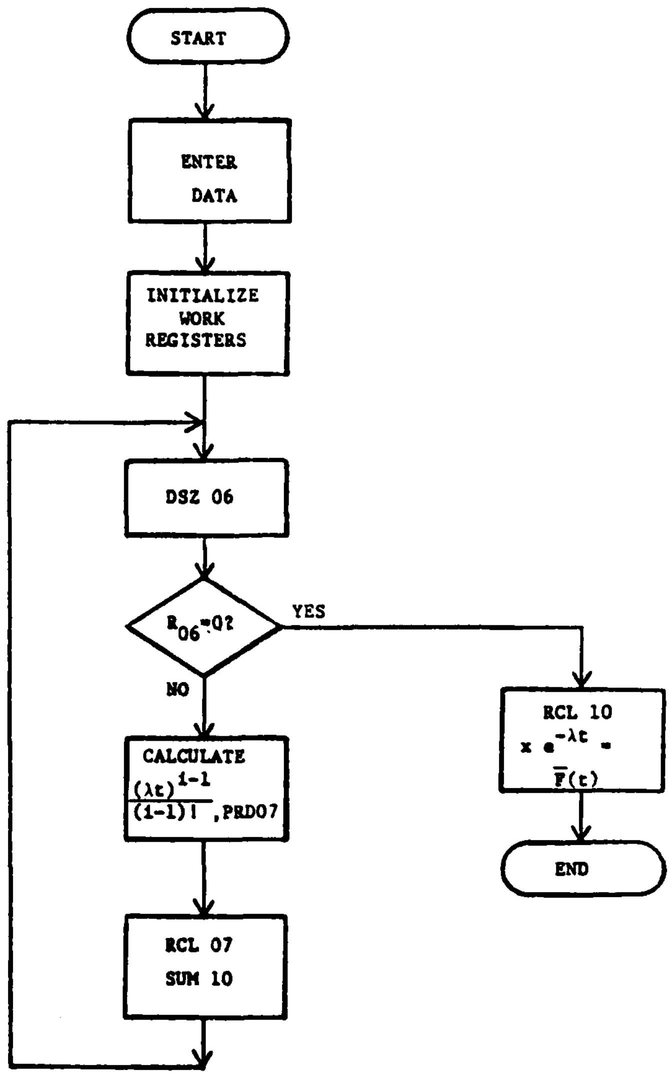
1) Enter sample value, t=2 STO 00, n=5 STO 01, and λ=.5 STO 13

2) Press [B] and $\overline{F}(t)$ is displayed. $\overline{F}(t)$=.9963401532

FLOWCHART: LABEL A, CASE 1



FLOWCHART: LABEL B, CASE 2



COMPUTER LISTING: LABEL A

```
000  76  LBL
001  11  A
002  01  1
003  22  INV
004  23  LNX
005  42  STO
006  02  02
007  01  1
008  42  STO
009  06  06
010  32  X:T
011  00  0
012  42  STO
013  10  10
014  42  STO
015  11  11
016  42  STO
017  12  12
018  43  RCL
019  01  01
020  42  STO
021  07  07
022  42  STO
023  08  08
024  42  STO
025  09  09
026  85  +
027  01  1
028  02  2
029  95  =
030  42  STO
031  03  03
032  42  STO
033  04  04
034  42  STO
035  05  05
036  76  LBL
037  16  A'
038  43  RCL
039  02  02
040  45  YX
041  53  (
042  73  RC*
043  03  03
044  94  +/-
045  65  X
046  43  RCL
047  00  00
048  54  )
049  95  =
050  49  PRD
051  06  06
052  73  RC*
053  03  03
054  32  X:T
055  01  1
056  72  ST*
057  03  03
058  76  LBL
059  17  B'
060  73  RC*
061  04  04
062  49  PRD
063  06  06
064  97  DSZ
065  04  04
066  38  SIN
067  76  LBL
068  38  SIN
069  97  DSZ
070  08  08
071  17  B'
072  43  RCL
073  01  01
074  42  STO
075  08  08
076  85  +
077  01  1
078  02  2
079  95  =
080  42  STO
081  04  04
082  00  0
083  32  X:T
084  72  ST*
085  03  03
086  42  STO
087  11  11
088  76  LBL
089  18  C'
090  73  RC*
091  05  05
092  75  -
093  43  RCL
094  11  11
095  95  =
096  42  STO
097  12  12
098  67  EQ
099  19  D'
100  76  LBL
101  10  E'
102  43  RCL
103  12  12
104  35  1/X
105  49  PRD
106  06  06
107  97  DSZ
108  05  05
109  39  COS
110  76  LBL
111  39  COS
112  97  DSZ
113  09  09
114  18  C'
115  43  RCL
116  01  01
117  42  STO
118  09  09
119  85  +
120  01  1
121  02  2
122  95  =
123  42  STO
124  05  05
125  43  RCL
126  06  06
127  44  SUM
128  10  10
129  01  1
130  42  STO
131  06  06
132  97  DSZ
133  03  03
134  30  TAN
135  76  LBL
136  30  TAN
137  97  DSZ
138  07  07
139  16  A'
140  43  RCL
141  10  10
142  92  RTN
143  76  LBL
144  19  D'
145  01  1
146  42  STO
147  12  12
148  61  GTO
149  10  E'
150  92  RTN
```

COMPUTER LISTING: LABEL B

151	76	LBL
152	12	B
153	01	1
154	22	INV
155	23	LNX
156	42	STO
157	02	02
158	01	1
159	42	STO
160	07	07
161	42	STO
162	10	10
163	43	RCL
164	01	01
165	42	STO
166	06	06
167	76	LBL
168	14	D
169	97	DSZ
170	06	06
171	13	C
172	43	RCL
173	10	10
174	95	=
175	65	×
176	53	(
177	43	RCL
178	02	02
179	45	Y^X
180	53	(
181	43	RCL
182	13	13
183	94	+/-
184	65	×
185	43	RCL
186	00	00
187	54	)
188	54	)
189	95	=
190	92	RTN
191	76	LBL
192	13	C
193	53	(
194	43	RCL
195	00	00
196	65	×
197	43	RCL
198	13	13
199	54	)
200	55	÷
201	53	(
202	43	RCL
203	01	01
204	75	-
205	43	RCL
206	06	06
207	54	)
208	95	=
209	49	PRD
210	07	07
211	43	RCL
212	07	07
213	44	SUM
214	10	10
215	61	GTO
216	14	D
217	92	RTN

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