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Monterey, California

TI-59 PROGRAMS FOR MULTIPLE REGRESSION

by

D. R. Barr

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Programs for conducting multiple regression analyses with up to seven independent variables are given for the TI-59 calculator.

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TI-59 PROGRAMS FOR MULTIPLE REGRESSION

by

D. R. Barr

1. Introduction

The general linear hypothesis model of full rank [Graybill, 1961] can be written as

$$\underset{n\times 1}{Y} = \underset{n\times k}{X}\ \underset{k\times 1}{\beta} + \underset{n\times 1}{\varepsilon}\ , \qquad \varepsilon \sim N(0,\sigma^2 I)\ ,$$

where Y is the vector of n dependent variable values to be observed, X is the "design matrix" of fixed values of the independent variables, β is a vector of unknown fixed coefficients in the model and ε is the unobservable random error vector. The $i^{\underline{th}}$ row in this matrix equation,

$$Y_i = x_{0i}\beta_0 + \beta_1 x_{1i} + \cdots + \beta_k x_{ki} + \varepsilon_i\ ,$$

represents the $i^{\underline{th}}$ dependent value to be observed as a linear function of the $i^{\underline{th}}$ set of independent variable values: $x_{0i}, x_{1i}, \ldots , x_{ki}$.

The solution to the normal equations for estimating β,

$$(x'x)\hat{\beta} = x'y\ ,$$

where y is the $n \times 1$ vector of observed outcomes on Y, can be found using program 02 in the TI-59 master module, provided $x'x$ is of full rank k and $1 < k < 9$. Due to space requirements for computing $x'x$, the program presented here requires $k < 8$. Also, because of optional programs (which run faster) for $k = 2$ (OP12-OP15 and $\Sigma+$) and $k = 3$ (statistics module program 18), the programs presented here are most useful for $3 < k < 8$. There is no limit on the number of observations, n. Tests of hypotheses about components of β can be carried out using a "reduced model" solution, and confidence intervals for linear functions of the coefficients can be obtained using $(x'x)^{-1}$ and $\hat{\sigma}^2$, based on the t-distribution, as outlined in [1]. Tests and confidence intervals for σ^2 can be obtained from $\hat{\sigma}^2$, based on the χ^2 distribution.

2. The Programs

Listings of two programs are given in the appendix. Program #1 serves two purposes:

(a) computation of $x'x$, $x'y$ and $y'y$ as rows of the data matrix $(x \vdots y)$ are entered;

(b) computation of $\hat{\beta}$, $\hat{\beta}'x'y$ and $(x'x)^{-1}$ for the full model.

Program #2 also serves two purposes:

(a) computation of reduced model components $(x'x)_r$ and $(x'y)_r$, and loading them into memory:

(b) computation of $\hat{\beta}_r'(x'y)_r$ for the reduced model.

A multiple regression problem is solved with these programs following the sequence:

(a) load Program #1 (2 banks)

(b) enter rows of the data matrix

(c) store x'x and x'y on cards

(d) compute and record $\hat{\beta}$, $\hat{\beta}'x'y$, $(x'x)^{-1}$, y'y and x'y

(e) load Program #2 (1 bank), along with the stored data

(f) compute $\hat{\beta}_r'(x'y)_r$ for the reduced model.

Steps (c), (e) and (f) are optional, depending on whether it is desired to test hypotheses that various components of β are zero. Components of $\hat{\beta}$ and $(x'x)^{-1}$ can be recalled from memory (or listed on the printer using the "INV LIST" command) as desired.

Entering each row of the data matrix and updating x'x, x'y and y'y requires from 30-60 seconds, depending on k. Computation of $\hat{\beta}$ and $(x'x)^{-1}$ requires from 1 minute to 12 minutes, again depending on the dimension of x'x. In what follows, we give instructions for using the programs in the TI program record format. This is followed by an annotated example printer output (use of the printer is optional). Program listings are given in the Appendix.

3. Instructions

Instructions for running the programs are given in Figures 1 and 2. Using the quantities computed when both programs are used, an "analysis of variance" summary can be constructed. The format of this table, and sources of its entries, are given below, in Figure 3. An hypothesis of the form $H_0:\beta_i = 0$ is tested with the F ratio with the "β_i sum of squares" in the numerator and the "error mean square"

$$\hat{\sigma}^2 = \frac{y'y - \hat{\beta}'x'y}{n - k}$$

in the denominator. Degrees of freedom for this F ratio would be 1 and $n-k$. Note that the sum of squares due to regression in the full model, $\hat{\beta}'x'y$, generally is not partitioned into independent components due to $\beta_0, \beta_1, \ldots, \beta_k$ unless the design is "orthogonal" ($x'x$ is diagonal). Thus, tests of several hypotheses about components of β are generally not statistically independent. See [Searle, 1972] for a discussion of how to proceed if it is desired to test several hypotheses in the non-orthogonal case.

If one wishes to compute the coefficient of determination R^2 (which is not as useful as the related F-ratio), proceed as follows. First, compute the "adjusted total sum of squares," $SST_{(adj)} = y'y - (\Sigma y_i)^2/n$, where $y'y$ is stored in register 78, Σy_i in register 71* and n in register 08 at the first step of Program 1. Then

---

*See Comment in Figure 3.

TITLE MULTIPLE REGRESSION--PGM #1 PAGE 1 OF 1

PROGRAMMER BARR DATE 4/1/80

TI Programmable Program Record

Partitioning (Op 17) S.T.D. Library Module MASTER Printer OPTIONAL Cards 1 (Banks 1,2)

PROGRAM DESCRIPTION

For the general linear hypothesis model $Y = X\beta + \varepsilon$, calculates: x'x, x'y, y'y, |x'x|, $(x'x)^{-1}$, $\hat{\beta}$, $\hat{\beta}$'x'y, Σy_i. Program can handle up to 7 independent variables. Running time increases as the dimension k of x'x increases. Any number n of data points can be accommodated.

USER INSTRUCTIONS

STEP	PROCEDURE	ENTER	PRESS	DISPLAY
1	Initialize	k	E	1
2	Enter rows of (x ⋮ y)	x_{11} → R/S; x_{12} → R/S; . . . ; x_{1n} → R/S; y_1 → R/S		
			B	next row index
	(repeat above sequence until all rows entered).*			
3	Display x'x, x'y, y'y, Σy_i (or use "INV LIST" with printer)			
	#08 - $(07 + k^2)$: columns of x'x (note: #08 is n)			
	#71 - (70 + k): x'y. (NOTE: #71 is Σy_i.)			
	#78: y'y			
	Note: save x'x and x'y on cards if "reduced model" is desired (banks 2,3,4)			
4	Compute $\hat{\beta}$		C	1
	(with printer, |x'x| is printed)			
5	Display $\hat{\beta}$, $\hat{\beta}$'x'y, $(x'x)^{-1}$: Recall (or use "INV LIST" w/printer)			
	#08 - $(07 + k^2)$: Columns of $(x'x)^{-1}$ in permuted order--order of permutation stored in #$(08 + k^2)$ - $(07 + k^2 + k)$			
	(#08 + k^2 + k) - $(07 + k^2 + 2k)$: $\hat{\beta}$			
	#71 - (70 + k): x'y			
	#79: $\hat{\beta}$'x'y			

USER DEFINED KEYS	DATA REGISTERS (INV)		LABELS (Op 08) (see listing)
A	0	0	
B x'x update	1 ALL USED;	1 INTERNAL	
C $\hat{\beta}$ computation	2 PARTITIONING	2 TO 319.79	
D	3	3	
E Initialize	4	4	

*If an error in entry is made, press "SBR 015" and re-enter the data row. This must be done before pressing "B".

FLAGS 0	1	2	5	6	7	8	9

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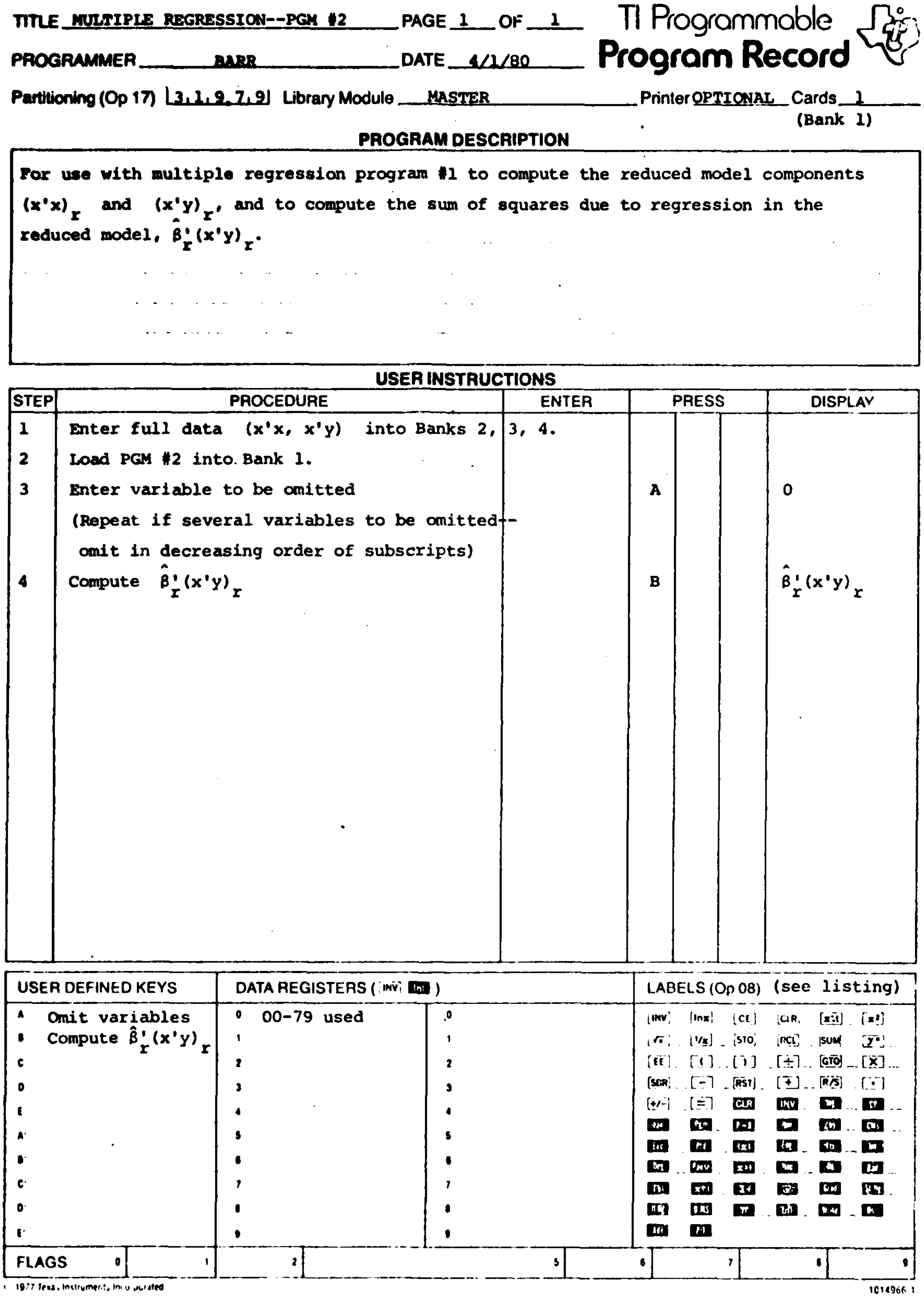
FIGURE 1



TITLE MULTIPLE REGRESSION--PGM #2 PAGE 1 OF 1 **TI Programmable Program Record**

PROGRAMMER BARR DATE 4/1/80

Partitioning (Op 17) 3, 1, 9, 7, 9 Library Module MASTER Printer OPTIONAL Cards 1
(Bank 1)

PROGRAM DESCRIPTION

For use with multiple regression program #1 to compute the reduced model components $(x'x)_r$ and $(x'y)_r$, and to compute the sum of squares due to regression in the reduced model, $\hat{\beta}_r'(x'y)_r$.

USER INSTRUCTIONS

STEP	PROCEDURE	ENTER	PRESS	DISPLAY
1	Enter full data (x'x, x'y) into Banks 2,	3, 4.		
2	Load PGM #2 into Bank 1.			
3	Enter variable to be omitted		A	0
	(Repeat if several variables to be omitted--			
	omit in decreasing order of subscripts)			
4	Compute $\hat{\beta}_r'(x'y)_r$		B	$\hat{\beta}_r'(x'y)_r$

USER DEFINED KEYS		DATA REGISTERS (INV Int)			
A	Omit variables	0	00-79 used	0	
B	Compute $\hat{\beta}_r'(x'y)_r$	1		1	
C		2		2	
D		3		3	
E		4		4	
A'		5		5	
B'		6		6	
C'		7		7	
D'		8		8	
E'		9		9	

LABELS (Op 08) (see listing)

FLAGS 0 1 2 5 6 7 8 9

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FIGURE 2

Index	Source	Degrees of Freedom	Sums of Squares
1.	Regression (full)	k	$\hat{\beta}'x'y$ (Pgm 1)
2_0.	β_0	1	$\hat{\beta}'x'y - \hat{\beta}_r'(x'y)_r$ (Pgms 1 & 2)
2_1.	β_1	1	
$\vdots$	$\vdots$		$\vdots$
3.	Mean	1	$n\bar{y}^2$ (from Σy_i in first component of $x'y$, Pgm 1)*
4.	Error	n-k	$y'y - \beta'x'y$ (Pgm 1)
5.	Total	n	$y'y$

FIGURE 3. AOV Summary

---

*Provided the first column of X is a vector of 1's.

$$R^2 = \frac{SST_{(adj)} - SSE}{SST_{(adj)}} ,$$

where the sum of squares due to error is given by

$$SSE = y'y - \hat{\beta}'x'y = (n-k)\hat{\sigma}^2 .$$

4. Example. (k = 3 example from the TI "Applied Statistics" module booklet; page 5-15)

(1) Read in Program #1, banks 1 and 2,

(2) enter "3', press E,

(3) enter first row of $(x \vdots y)$, as follows (do not forget the "1" corresponding to the constant term):

1/ R/S, 0, R/S, 0, R/S, 17.3, R/S, B (next row index displays),

(repeat this sequence, entering 9 rows of data as in Table 5.2, which we produce in Table 1 for convenience.

row number	a_0	a_1	a_2	dependent variable
1	1	0	0	17.3
2	1	0	1	18.1
3	1	0	2	18.7
4	1	1	0	18.6
5	1	1	1	19.1
6	1	1	2	19.5
7	1	2	0	19.6
8	1	2	1	19.9
9	1	2	2	20.3

Table 1

(4) write banks 2, 3, 4 to save x'x, x'y if tests of hypotheses about coefficients are desired (see listings below). Save y'y (register 78) and Σy_i (register 71),

(5) press C to compute $\hat{\beta}$, $\hat{\beta}'x'y$, $(x'x)^{-1}$,

(6) recall and note desired components, as follows (see listing below)

- a) columns of $(x'x)^{-1}$ in registers 08 to 16, order of columns permuted; permutation stored in 17, 18, 19
- b) $\hat{\beta}$ in registers 20, 21, 22
- c) x'y in registers 71, 72, 73
- d) y'y (total sum of squares) in register 78
- e) $\beta'x'y$ (sum of squares due to regression) in register 79.

(7) To test $H_0:a_0 = 0$ (for example) reduce model to eliminate the first independent variable as follows:

- a) read in data (stored at step 2) in banks 2, 3, 4
- b) read in Program #2, bank 1
- c) enter the variable to be deleted ("1" in this case), press A
- d) when computation is complete, Press B to compute the reduced sum of squares due to regression
- e) when computation is complete, the sum of squares due to regression in the reduced model $\hat{\beta}_r'(x'y)_r$ (where "r" means "reduced"), is displayed and is stored in register 79.

f) The F-ratio for testing $H_0:a_0 = 0$ is formed as follows:

$$\frac{\hat{\beta}'x'y - \hat{\beta}_r'(x'y)_r}{(y'y - \hat{\beta}'x'y)/(n-k)} = 2705.2$$

degrees of freedom are 1 and $(n-k) = 6$.

(7') If it is desired to test other hypotheses of the form $H_0:a_i = 0$, repeat the steps in (7) above, with appropriate entry of "i" at step (7c). (Step (7b) need not be repeated.)

(7") It is desired to test a hypothesis that several coefficients are jointly zero, $H_0:a_i = a_j = \cdots = a_m = 0$, repeat step (7c) for each variable to be deleted, in descending order of magnitude of the subscripts. In this case, the F-ratio shown in (7f) should be divided by the number (say s) of remaining variables, and the F statistic has s and n-k degrees of freedom.

(8) Values obtained for the example and the hypothesis illustrated in step (7) are shown on the attached printouts. A summary AOV table is shown in Table 2.

Source	df	SS	F
Regression: full	3	3259.7161	
reduced	2	2565.8327	$\frac{693.88}{.1539/6} = 27$
due to a_0	1	693.8834	
Total	9	3259.87	
Error	6	(3259.87-3259.7161)=.1539	$\hat{\sigma}^2 = .1539/6$

Table 2. Summary AOV for example.

EXAMPLE RESULTS

At Step 4

		57.	00
		10.	01
		0.	02
		3.	03
		3.	04
		73.	05
		60.	06
	k	3.	07
x'x	n	9.	08
		9.	09
		9.	10
		9.	11
		15.	12
		9.	13
		9.	14
		9.	15
		15.	16
		0.	17
		0.	18
		0.	19
		0.	70
x'y	Σy_i	171.1	71
		176.8	72
		174.1	73
		0.	74
		0.	75
		0.	76
		0.	77
	yy'	3259.87	78
		0.	79

```
Step 5

|x'x|                          324.

                                 9.        00
                                16.        01
                                16.        02
                                19.        03
                                 3.        04
                                 4.        05
                               324.        06
                                 3.        07
                    -.1666666667           08
                            -7.-13         09
                     .1666666667           10
                    -.1666666667           11
x'x^-1               .1666666667           12
                                 0.        13
                     .4444444444           14
                    -.1666666667           15
                    -.1666666667           16
                                 3.        17
Permutation                      2.        18
                                 1.        19
                      17.56111111          20
β̂                            0.95          21
                              0.5          22
                                 0.        23
                                 0.        24
                                 0.        25

                                 0.        70
                             171.1         71
                             176.8         72
                             174.1         73
                                 0.        74
                                 0.        75
                                 0.        76
                                 0.        77
  y'y                      3259.87         78
β̂'x'y                   3259.716111        79
```

Step 7c-7e:

(enter "1",
press "A")

```
                     9.      00
                    16.      01
                     0.      02
                     0.      03
                    73.      04
                    74.      05
                     1.      06
                     2.      07
 reduced            15.      08
   x'x               9.      09
(omit a0)            9.      10
                    15.      11
                     9.      12
                     9.      13
                    15.      14
                     0.      15
                    15.      16
                     0.      17
                     0.      18
                     0.      19
                     0.      20


                     0.      70
 reduced         176.8       71
 x'y             174.1       72
                 174.1       73
                     0.      74
                     0.      75
                     0.      76
                     0.      77
 y'y           3259.87       78
                     0.      79
```

(Press "B")

$|x'x_r|$ 144.

$\hat{\beta}_r'(x'y)_r$ 2565.832708

(displayed and stored in 79)

APPENDIX 1

PROGRAM LISTINGS

Note: Program #1 is recorded with standard partitioning; when run it internally partitions the calculator to 319.79. Program #2 and data cards storing x'x and x'y (if written) are written with 319.79 partitioning.

If it is desired to write Program #1 after it has been run, repartition back to standard format prior to writing (enter 6, press "2nd OP 17").

PROGRAM 1

LABEL ADDRESSES

```
001  15  E
009  18 C'
025  11  A
034  12  B
044  17 B'
104  16 A'
134  13  C
149  33 X²
157  34 ΓX
178  35 1/X
217  23 LNX
226  24 CE
```

MULTIPLE REGRESSION
PROGRAM 1

```
000  76 LBL
001  15  E
002  47 CMS
003  42 STD
004  07  07
005  08  8
006  69 DP
007  17  17
008  76 LBL
009  18 C'
010  73 RC*
011  06  06
012  33 X²
013  44 SUM
014  78  78
015  05  5
016  07  7
017  42 STD
018  00  00
019  69 DP
020  21  21
021  43 RCL
022  01  01
023  91 R/S
024  76 LBL
025  11  A
```

```
026  72 ST*
027  00  00
028  69 DP
029  20  20
030  91 R/S
031  61 GTD
032  11  A
033  76 LBL
034  12  B
035  22 INV
036  86 STF
037  01  01
038  00  0
039  42 STD
040  03  03
041  42 STD
042  04  04
043  76 LBL
044  17 B'
045  05  5
046  07  7
047  85  +
048  43 RCL
049  03  03
050  54  )
051  42 STD
```

```
052  05  05
053  05  5
054  07  7
055  85  +
056  43 RCL
057  04  04
058  54  )
059  42 STD
060  06  06
061  73 RC*
062  05  05
063  65  ×
064  73 RC*
065  06  06
066  54  )
067  32 X:T
068  87 IFF
069  01  01
070  16 A'
071  08  8
072  85  +
073  43 RCL
074  04  04
075  85  +
076  53  (
077  43 RCL
```

```
078  07  07
079  65  X
080  43 RCL
081  03  03
082  54  )
083  54  )
084  42 STO
085  05  05
086  32 X:T
087  74 SM*
088  05  05
089  69 OP
090  24  24
091  43 RCL
092  07  07
093  32 X:T
094  43 RCL
095  04  04
096  22 INV
097  77  GE
098  17 B'
099  86 STF
100  01  01
101  61 GTO
102  17 B'
103  76 LBL
104  16 A'
105  07  7
106  01  1
107  85  +
108  43 RCL
109  03  03
110  54  )
111  42 STO
112  05  05
113  32 X:T
114  74 SM*
115  05  05
116  69 OP
117  23  23
118  43 RCL
119  07  07
120  32 X:T
121  43 RCL
122  03  03
123  67  EQ
124  18 C'
125  00  0
126  42 STO
127  04  04
128  22 INV
129  86 STF
130  01  01
131  61 GTO
132  17 B'
133  76 LBL
134  13  C
135  43 RCL
136  01  01
137  75  -
138  01  1
139  54  )
140  42 STO
141  00  00
142  36 PGM
143  02  02
144  13  C
145  01  1
146  42 STO
147  03  03
148  76 LBL
149  33 X²
150  43 RCL
151  03  03
152  32 X:T
153  00  0
154  42 STO
155  04  04
156  76 LBL
157  34 √X
158  43 RCL
159  07  07
160  33 X²
161  85  +
162  08  8
163  85  +
164  43 RCL
165  04  04
166  54  )
167  42 STO
168  02  02
169  73 RC*
170  02  02
171  67  EQ
172  35 1/X
173  69 OP
174  24  24
175  61 GTO
176  34 √X
177  76 LBL
```

```
178  35 1/X
179  07  7
180  00  0
181  85  +
182  43 RCL
183  03  03
184  54  )
185  42 STO
186  02  02
187  08  8
188  85  +
189  43 RCL
190  07  07
191  85  +
192  43 RCL
193  07  07
194  33 X²
195  85  +
196  43 RCL
197  04  04
198  54  )
199  42 STO
200  05  05
201  73 RC*
202  02  02
203  72 ST*
204  05  05
205  43 RCL
206  07  07
207  32 X:T
208  43 RCL
209  03  03
210  67  EQ
211  23 LNX
212  69 OP
213  23  23
214  61 GTO
215  33 X²
216  76 LBL
217  23 LNX
218  25 CLR
219  36 PGM
220  02  02
221  15  E
222  00  0
223  42 STO
224  01  01
225  76 LBL
226  24 CE
```

```
227  08  8
228  85  +
229  43 RCL
230  07  07
231  33 X²
232  85  +
233  43 RCL
234  07  07
235  85  +
236  43 RCL
237  01  01
238  54  )
239  42 STO
240  02  02
241  07  7
242  01  1
243  85  +
244  43 RCL
245  01  01
246  54  )
247  42 STO
248  05  05
249  73 RC*
250  02  02
251  65  ×
252  73 RC*
253  05  05
254  54  )
255  44 SUM
256  79  79
257  69 OP
258  21  21
259  43 RCL
260  07  07
261  32 X:T
262  43 RCL
263  01  01
264  22 INV
265  77  GE
266  24 CE
267  25 CLR
268  36 PGM
269  02  02
270  17 B'
271  91 R/S
272  00  0
273  00  0
274  00  0
275  00  0
```

PROGRAM 2

LABEL ADDRESSES

```
032  11  A
072  17 B'
114  12  B
147  18 C'
178  19 D'
```

MULTIPLE REGRESSION

PROGRAM 2

```
000  76 LBL        030  92 RTN        060  01  1
001  16 A'         031  76 LBL        061  54  )
002  73 RC*        032  11  A         062  42 STO
003  05  05        033  42 STO        063  04  04
004  72 ST*        034  06  06        064  43 RCL
005  04  04        035  65  ×         065  07  07
006  69 OP         036  43 RCL        066  75  -
007  25  25        037  07  07        067  02  2
008  69 OP         038  54  )         068  54  )
009  24  24        039  85  +         069  42 STO
010  97 DSZ        040  08  8         070  02  02
011  03  03        041  54  )         071  76 LBL
012  16 A'         042  42 STO        072  17 B'
013  92 RTN        043  05  05        073  43 RCL
014  76 LBL        044  75  -         074  07  07
015  10 E'         045  43 RCL        075  75  -
016  43 RCL        046  07  07        076  01  1
017  07  07        047  54  )         077  54  )
018  75  -         048  42 STO        078  42 STO
019  43 RCL        049  04  04        079  03  03
020  06  06        050  71 SBR        080  71 SBR
021  54  )         051  10 E'         081  16 A'
022  65  ×         052  08  8         082  69 OP
023  43 RCL        053  85  +         083  25  25
024  07  07        054  43 RCL        084  69 OP
025  54  )         055  06  06        085  32  32
026  42 STO        056  54  )         086  43 RCL
027  03  03        057  42 STO        087  02  02
028  71 SBR        058  05  05        088  32 X:T
029  16 A'         059  75  -         089  00  0
```

```
090  22 INV
091  77  GE
092  17 B'
093  71 SBR
094  10 E'
095  07  7
096  01  1
097  85  +
098  43 RCL
099  06  06
100  54  )
101  42 STO
102  05  05
103  75  -
104  01  1
105  54  )
106  42 STO
107  04  04
108  43 RCL
109  07  07
110  75  -
111  43 RCL
112  06  06
113  54  )
114  42 STO
115  03  03
116  71 SBR
117  16 A'
118  69 OP
119  37  37
120  91 R/S
121  76 LBL
122  12  B
123  00  0
124  42 STO
125  79  79
126  36 PGM
127  02  02
128  13  C
129  43 RCL
130  07  07
131  42 STO
132  00  00
133  43 RCL
134  03  03
135  85  +
136  43 RCL
137  07  07
138  54  )
139  42 STO
140  01  01
141  76 LBL
142  18 C'
143  73 RC*
144  03  03
145  85  +
146  07  7
147  00  0
148  54  )
149  42 STO
150  04  04
151  73 RC*
152  04  04
153  72 ST*
154  01  01
155  69 OP
156  33  33
157  69 OP
158  31  31
159  97 DSZ
160  00  00
161  18 C'
162  69 OP
163  21  21
164  25 CLR
165  36 PGM
166  02  02
167  15  E
168  07  7
169  01  1
170  42 STO
171  02  02
172  76 LBL
173  19 D'
174  73 RC*
175  01  01
176  65  ×
177  73 RC*
178  02  02
179  54  )
180  44 SUM
181  79  79
182  69 OP
183  21  21
184  69 OP
185  22  22
186  97 DSZ
187  07  07
188  19 D'
189  43 RCL
190  79  79
191  91 R/S
192  00  0
193  00  0
194  00  0
195  00  0
```

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