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6 ON APPROXIMATING THE CIRCULAR COVERAGE FUNCTION.

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ABSTRACT

This paper presents tables and graphs assessing the accuracy of a formula (programmable on a desk calculator) for approximating the circular coverage function of a circular-normal probability distribution. Use of this formula together with the error curves presented here enable one to calculate the circular coverage function with a maximum error on the order of 0.0002 for offsets up to 5σ and covering-circle radii up to 4.5σ .

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ON APPROXIMATING THE CIRCULAR COVERAGE FUNCTION

INTRODUCTION

Consider a circular normal probability distribution with standard deviation σ centered at a point A. The probability of falling within a distance R of a point B r units away from A is known as the circular coverage function $P(R/\sigma, r/\sigma)$. It is given analytically by definite integrals or infinite series that cannot be expressed in closed form:

$$\begin{aligned} P(R', r') &= \int_0^{R'} e^{-(r'^2+t^2)/2} I_0(r't) t dt \\ &= 1 - e^{-(R'^2+r'^2)/2} \sum_{n=0}^{\infty} \left(\frac{r'}{R'}\right)^n I_n(r'R') \\ &= 1 - e^{-(R'^2+r'^2)/2} \sum_{i=0}^{\infty} \frac{1}{i!} \left(\frac{r'^2}{2}\right)^i \sum_{j=1}^i \frac{1}{j!} \left(\frac{R'^2}{2}\right)^j \end{aligned}$$

(reference 1), where $R' = R/\sigma$, $r' = r/\sigma$, and I_n is the modified Bessel function of order n.

EVALUATION OF THE FUNCTION

Extensive tables of this function are available (references 1 through 4 and other references cited in reference 5). However, it is more convenient to use closed-form approximations that can be programmed on a desk calculator if the accuracy desired is not too high. A standard reference handbook (reference 6) gives the following approximation of the circular coverage function:

$$P(R', r') = \begin{cases} f(R'), & \text{if } R' < 1 \\ 1 - Q(x), & \text{if } R' > 1 \end{cases}$$

where

$$f(R') = \frac{2R'^2}{4+R'^2} \exp\left(-\frac{2r'^2}{4+R'^2}\right)$$

$$x = \frac{\left(\frac{R'^2}{2+r'^2}\right)^{1/3} - 1 + \frac{2(2+2r'^2)}{9(2+r'^2)^2}}{\left[\frac{2(2+2r'^2)}{9(2+r'^2)^2}\right]^{1/2}}$$

$$Q(x) = (2\pi)^{-1/2} \int_x^\infty e^{-t^2/2} dt$$

The formula for P can be programmed on a TI-59 calculator using 127 steps (table 1). The approximation used for the normal distribution $Q(x)$ is (reference 6):

$$Q(x) = (2\pi)^{-1/2} e^{-x^2/2} (b_1 t + b_2 t^2 + b_3 t^3 + b_4 t^4 + b_5 t^5) + E(x),$$

where the error $E(x)$ is smaller than 7.5×10^{-8} and

$$t = (1 + px)^{-1}$$

$$p = .2316419$$

$$b_1 = .319381530$$

$$b_2 = -.356563782$$

$$b_3 = 1.781477937$$

$$b_4 = -1.821255978$$

$$b_5 = 1.330274429$$

ACCURACY OF THE APPROXIMATION

Reference 6 gives no source, method of derivation, or statement on accuracy. The purpose of this paper is to evaluate the error made in using the approximation formula.

The approximation for $R' < 1$ is the well-known Carlton formula (reference 1). For $R' < 0.4$, it is accurate to 4 decimal places. For $0.4 < R' < 1$, it is off by no more than .0065 (and usually by no more than .001 - reference 1).

For the cases $R' \geq 1$, direct comparison with the tables (references 2 and 3) reveals that the maximum error is .024 for $1 \leq R' \leq 4.5$ and $0 \leq r' \leq 5$. The error is usually smaller.

Figures 1 and 2 are graphs of the difference between the exact and approximate values of the circular coverage function, with linear interpolation used between the following milestone values:¹

$$R' = .25, .5, .75, .9, 1, 1.1, 1.25, 1.5, 1.75, 2, 2.5, 3, 3.5, 4, 4.5$$
$$r' = 0, .5, 1, 1.5, 2, 2.5, 3, 4, 5.$$

Figure 3 is a graph of the ratio of the approximate value of the circular coverage function to its exact value, with linear interpolation used between the above milestone values. In table 2 we compare exact and

¹Since the formula exhibits a discontinuity at $R' = 1$, the graphs for $.9 < R' < 1$ are not accurate.

approximate values of the circular coverage function for these milestones: values, difference, ratio, and percent increase of exact value over approximate value.

As can be seen from table 2, whenever the exact value is smaller than the approximate value, it is by no more than 2.4%. Further, whenever the exact value is over 11% larger than the approximate value, the exact value is smaller than .16. If over 25% larger, $P < .08$. This indicates that it is for very small values of the circular coverage function that the approximate formula has a higher chance of being off by a significant percentage.

In conclusion, the programmable formula for the circular coverage function, together with figures 1 and 2, enables one to evaluate the function with a maximum error on the order of .0002 for $0 \leq R' \leq 4.5$ and $0 \leq r' \leq 5$.

TABLE 1
TI-59 PROGRAM FOR CIRCULAR COVERAGE OF A
CIRCULAR NORMAL DISTRIBUTION^a

000	42	STD	033	54)	065	43	RCL	097	04	4
001	08	08	034	33	X ²	066	04	04	098	85	+
002	55	+	035	95	=	067	75	-	099	43	RCL
003	43	RCL	036	42	STD	068	01	1	100	04	04
004	10	10	037	06	06	069	95	=	101	33	X ²
005	95	=	038	75	-	070	77	GE	102	54)
006	42	STD	039	01	1	071	87	IFF	103	95	=
007	04	04	040	85	+	072	43	RCL	104	91	R/S
008	43	RCL	041	53	(073	04	04	105	81	RST
009	09	09	042	43	RCL	074	33	X ²	106	76	LBL
010	55	+	043	04	04	075	85	+	107	87	IFF
011	43	RCL	044	33	X ²	076	04	4	108	43	RCL
012	10	10	045	55	+	077	95	=	109	07	07
013	95	=	046	53	(078	35	1/X	110	13	C
014	42	STD	047	02	2	079	65	X	111	94	+/-
015	05	05	048	85	+	080	02	2	112	85	+
016	33	X ²	049	43	RCL	081	65	X	113	01	1
017	65	X	050	05	05	082	43	RCL	114	95	=
018	02	2	051	33	X ²	083	05	05	115	91	R/S
019	85	+	052	54)	084	33	X ²	116	81	RST
020	02	2	053	54)	085	95	=	117	76	LBL
021	95	=	054	22	INV	086	94	+/-	118	13	C
022	65	X	055	45	YX	087	22	INV	119	76	LBL
023	02	2	056	03	3	088	23	LN _X	120	11	A
024	55	+	057	95	=	089	65	X	121	36	PGM
025	09	9	058	55	+	090	02	2	122	14	14
026	55	+	059	43	RCL	091	65	X	123	11	A
027	53	(060	06	06	092	43	RCL	124	36	PGM
028	02	2	061	34	FX	093	04	04	125	14	14
029	85	+	062	95	=	094	33	X ²	126	12	0
030	42	STD	063	42	STD	095	55	+	127	92	RTN
031	05	05	064	07	07	096	53	(
032	33	X ²									

^aThe offset r is stored in register 09, the radius R of the covering circle in 08, and the standard deviation σ in 10. After r and σ are stored, keying in R and pressing R/S will initiate the program. Occasionally the display will flash due to underflow in the calculation, but the displayed value will be correct. The master library module must be inserted.

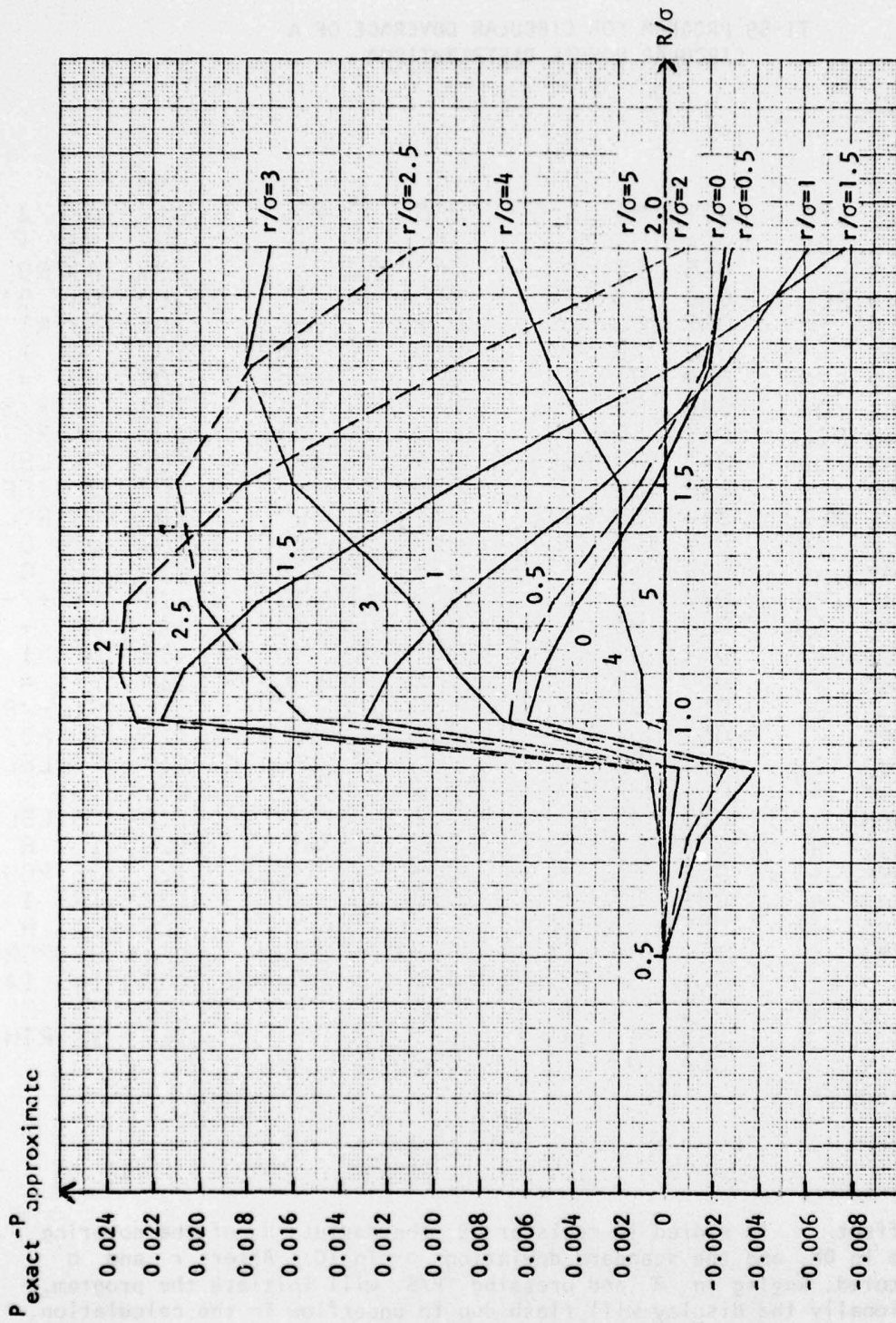


FIG. 1: ERROR MADE IN USING APPROXIMATION TO CIRCULAR COVERAGE FUNCTION FOR CIRCULAR NORMAL DISTRIBUTION ($0 \leq R/\sigma \leq 2$)^a

^a r is offset, R is radius of covering circle, and σ is standard deviation

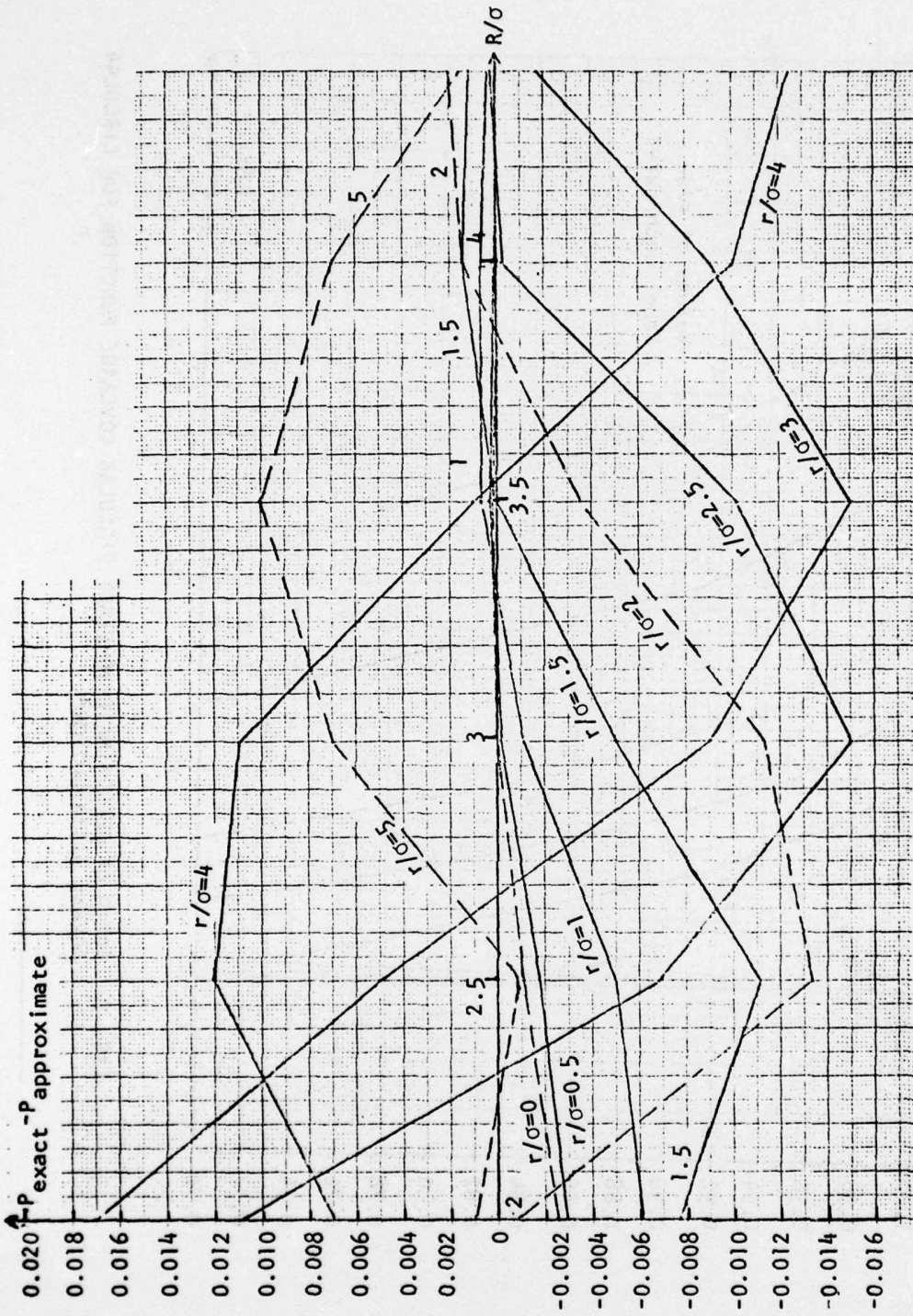


FIG. 2: ERROR MADE IN USING APPROXIMATION TO CIRCULAR COVERAGE FUNCTION FOR CIRCULAR NORMAL DISTRIBUTION ($2 \leq R/\sigma \leq 4.5$)^a

^a r is offset, R is radius of covering circle, and σ is standard deviation

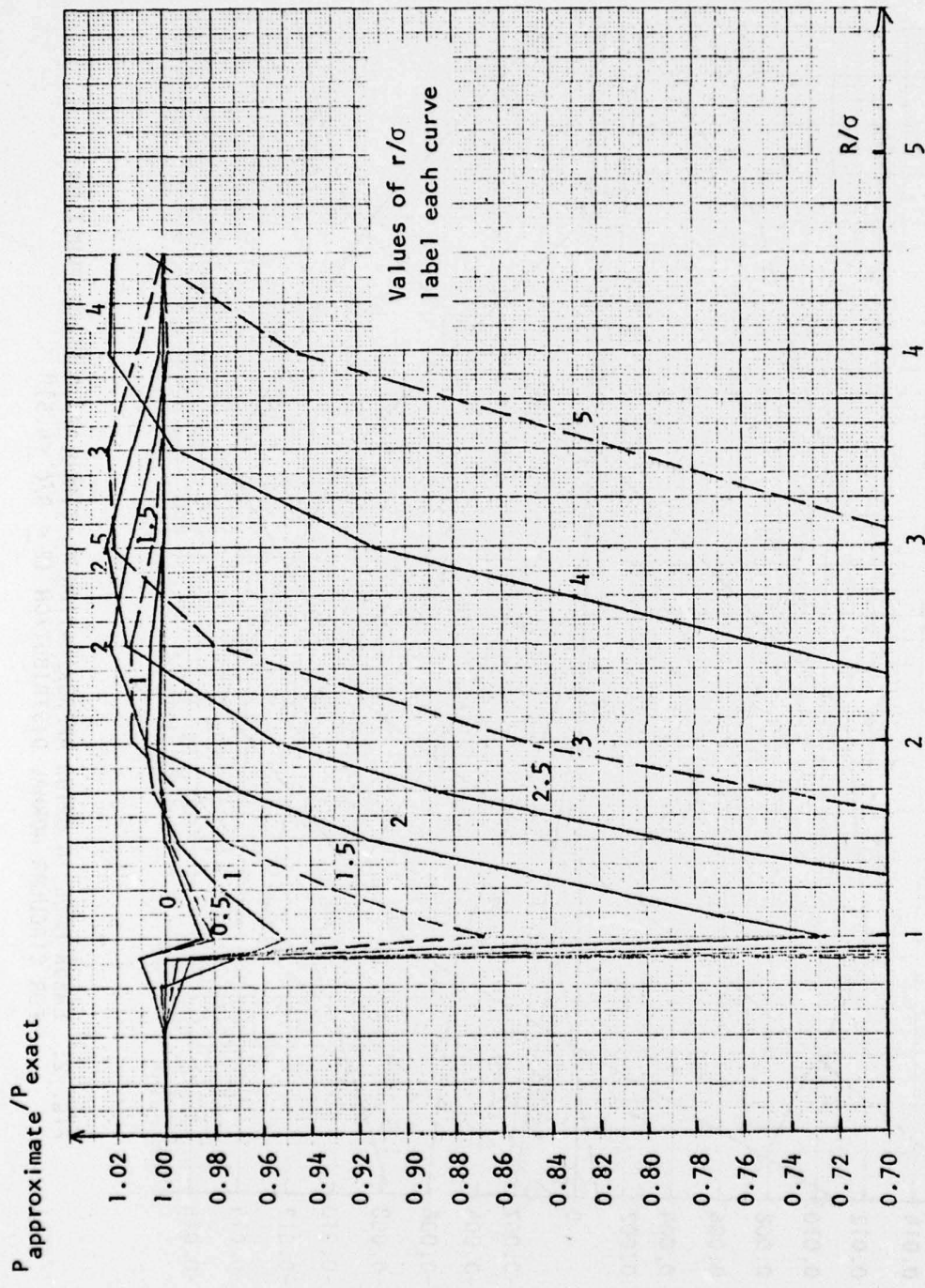


FIG. 3: RATIO OF APPROXIMATE-TO-EXACT CIRCULAR COVERAGE FUNCTION FOR CIRCULAR NORMAL DISTRIBUTION^a

^a r is offset, R is radius of covering circle, σ is standard deviation.

TABLE 2

COMPARISON OF EXACT AND APPROXIMATE VALUES OF THE CIRCULAR COVERAGE FUNCTION OF A CIRCULAR NORMAL DISTRIBUTION^a

r/σ	R/σ	P_{exact}	$P_{\text{approx.}}$	$\frac{P_{\text{approx.}}}{P_{\text{exact}}}$	$P_{\text{exact}} - P_{\text{approx.}}$	$100 \left(\frac{P_{\text{exact}} - P_{\text{approx.}}}{P_{\text{exact}}} \right)$
0	.25	.0308	.0308	1.000	0	0
	.5	.1175	.1176	1.001	-.0001	-.1
	.75	.2452	.2466	1.006	-.0014	-.6
	.9	.3330	.3368	1.011	-.0038	-1.1
	1	.3935	.3876	.985	.0059	1.5
	1.1	.4539	.4485	.988	.0054	1.2
	1.25	.5422	.5384	.993	.0038	.7
	1.5	.6754	.6749	.999	.0005	.1
	1.75	.7837	.7856	1.002	-.0019	-.2
	2	.8647	.8672	1.003	-.0025	-.3
	2.5	.9561	.9572	1.001	-.0011	-.1
	3	.9889	.9889	1.000	0	0
	3.5	.9978	.9976	1.000	.0002	0
	4	.9997	.9996	1.000	.0001	0
	4.5	1.0000	.9999	1.000	.0001	0
.5	.25	.0272	.0272	1.000	0	0
	.5	.1045	.1046	1.001	-.0001	-.1
	.75	.2200	.2210	1.005	-.0010	0.4
	.9	.3009	.3035	1.009	-.0026	-.9
	1	.3573	.3506	.981	.0067	1.9
	1.1	.4144	.4080	.985	.0064	1.5
	1.25	.4991	.4943	.990	.0048	1.0
	1.5	.6309	.6298	.998	.0011	.2
	1.75	.7430	.7447	1.002	-.0017	-.2
	2	.8309	.8338	1.003	-.0029	-.4
	2.5	.9383	.9401	1.002	-.0018	-.2
	3	.9822	.9823	1.000	-.0001	0
	3.5	.9959	.9956	1.000	.0003	0
	4	.9993	.9991	1.000	.0002	0
	4.5	.9999	.9998	1.000	.0001	0

^a r is offset, R is radius of coverage circle, and σ is standard deviation.

TABLE 2
(continued)

r/σ	R/σ	P_{exact}	$P_{\text{approx.}}$	$\frac{P_{\text{approx.}}}{P_{\text{exact}}}$	$P_{\text{exact}} - P_{\text{approx.}}$	$100 \left(\frac{P_{\text{exact}} - P_{\text{approx.}}}{P_{\text{exact}}} \right)$	
1	.25	.0188	.0188	1.000	0	0	
	.5	.0735	.0735	1.000	0	0	
	.75	.1589	.1591	1.001	-.0002	-.1	
	.9	.2217	.2222	1.002	-.0005	-.2	
	1	.2671	.2542	.952	.0129	4.8	
	1.1	.3146	.3027	.962	.0119	3.8	
	1.25	.3883	.3792	.977	.0091	2.3	
	1.5	.5120	.5093	.995	.0027	.5	
	1.75	.6286	.6316	1.005	-.0030	-.5	
	2	.7310	.7371	1.008	-.0061	-.8	
	2.5	.8792	.8842	1.006	-.0050	-.6	
	3	.9563	.9574	1.001	-.0011	-.1	
	3.5	.9874	.9867	.999	.0007	.1	
	4	.9971	.9964	.999	.0007	0	
	4.5	.9995	.9992	1.000	.0003	0	
	1.5	.25	.0102	.0102	1.000	0	0
		.5	.0408	.0408	1.000	0	0
		.75	.0922	.0920	.998	.0002	.3
		.9	.1328	.1321	.995	.0007	.5
		1	.1638	.1422	.868	.0216	13.2
1.1		.1976	.1769	.895	.0207	10.5	
1.25		.2533	.2357	.931	.0176	6.9	
1.5		.3563	.3473	.975	.0090	2.5	
1.75		.4665	.4669	1.001	-.0004	-.1	
.2		.5763	.5840	1.013	-.0077	-1.3	
2.5		.7677	.7787	1.014	-.0110	-1.4	
3		.8962	.9011	1.005	-.0049	-.5	
3.5		.9626	.9624	1.000	.0002	0	
4		.9892	.9877	.998	.0015	.2	
4.5		.9975	.9965	.999	.0010	.1	

TABLE 2
(continued)

r/σ	R/σ	P_{exact}	$P_{\text{approx.}}$	$\frac{P_{\text{approx.}}}{P_{\text{exact}}}$	$P_{\text{exact}} - P_{\text{approx.}}$	$100 \left(\frac{P_{\text{exact}} - P_{\text{approx.}}}{P_{\text{exact}}} \right)$
2	.25	.0043	.0043	1.000	0	0
	.5	.0179	.0179	1.000	0	0
	.75	.0429	.0427	.995	.0002	.5
	.9	.0645	.0638	.989	.0007	1.0
	1	.0819	.0592	.723	.0227	27.7
	1.1	.1019	.0784	.769	.0235	23.1
	1.25	.1371	.1139	.831	.0232	16.9
	1.5	.2092	.1911	.913	.0181	8.7
	1.75	.2970	.2878	.969	.0092	3.1
	2	.3965	.3973	1.002	-.0008	-.2
	2.5	.6059	.6193	1.022	-.0134	-2.2
	3	.7856	.7970	1.015	-.0114	-1.5
	3.5	.9047	.9082	1.004	-.0035	-.4
	4	.9659	.9645	.999	.0014	.1
	4.5	.9902	.9881	.998	.0021	.2
2.5	.25	.0014	.0014	1.000	0	0
	.5	.0062	.0062	1.000	0	0
	.75	.0160	.0159	.994	.0001	.5
	.9	.0252	.0250	.992	.0002	.6
	1	.0332	.0177	.533	.0155	46.7
	1.1	.0428	.0253	.591	.0175	40.9
	1.25	.0609	.0410	.673	.0199	32.7
	1.5	.1021	.0811	.794	.0210	20.6
	1.75	.1589	.1411	.888	.0178	11.2
	2	.2321	.2213	.953	.0108	4.7
	2.5	.4184	.4252	1.016	-.0068	-1.6
	3	.6230	.6379	1.024	-.0149	-2.4
	3.5	.7963	.8065	1.013	-.0102	-1.3
	4	.9100	.9120	1.002	-.0020	-.2
	4.5	.9679	.9657	.998	.0022	.2

TABLE 2
(continued)

r/σ	R/σ	P_{exact}	$P_{\text{approx.}}$	$\frac{P_{\text{approx.}}}{P_{\text{exact}}}$	$P_{\text{exact}} - P_{\text{approx.}}$	$100 \left(\frac{P_{\text{exact}} - P_{\text{approx.}}}{P_{\text{exact}}} \right)$
3	.25	.000	.000	1.000	0	0
	.5	.002	.002	1.000	0	0
	.75	.005	.005	1.000	0	0
	.9	.008	.008	1.000	0	0
	1	.011	.004	.364	.007	66.5
	1.1	.015	.006	.400	.009	61.5
	1.25	.022	.011	.500	.011	50.0
	1.5	.041	.025	.610	.016	37.9
	1.75	.071	.053	.746	.018	25.8
	2	.113	.096	.850	.017	14.6
	2.5	.246	.241	.980	.005	2.0
	3	.433	.442	1.021	-.009	-2.1
	3.5	.634	.649	1.024	-.015	-2.3
	4	.803	.812	1.011	-.009	-1.1
	4.5	.914	.914	1.000	0	0
4	.25	.000	.000	1.000	0	0
	.5	.000	.000	1.000	0	0
	.75	.000	.000	1.000	0	0
	.9	.000	.000	1.000	0	0
	1	.001	.000	.000	.001	100.0
	1.1	.001	.000	.000	.001	100.0
	1.25	.002	.000	.000	.002	100.0
	1.5	.003	.001	.333	.002	66.7
	1.75	.008	.003	.375	.005	56.3
	2	.015	.008	.533	.007	49.6
	2.5	.048	.036	.750	.012	24.0
	3	.126	.115	.913	.011	8.4
	3.5	.263	.262	.996	.001	.2
	4	.450	.460	1.022	-.010	-2.3
	4.5	.648	.661	1.020	-.013	-2.0

TABLE 2
(continued)

r/σ	R/σ	P_{exact}	$P_{\text{approx.}}$	$\frac{P_{\text{approx.}}}{P_{\text{exact}}}$	$P_{\text{exact}} - P_{\text{approx.}}$	$100 \left(\frac{P_{\text{exact}} - P_{\text{approx.}}}{P_{\text{exact}}} \right)$
5	.25	.000	.000	1.000	0	0
	.5	.000	.000	1.000	0	0
	.75	.000	.000	1.000	0	0
	.9	.000	.000	1.000	0	0
	1	.000	.000	1.000	0	0
	1.1	.000	.000	1.000	0	0
	1.25	.000	.000	1.000	0	0
	1.5	.000	.000	1.000	0	0
	1.75	.000	.000	1.000	0	0
	2	.001	.000	.000	.001	100.0
	2.5	.004	.002	.500	.002	50.0
	3	.017	.010	.588	.007	38.5
	3.5	.053	.043	.811	.010	18.3
	4	.133	.126	.947	.007	5.3
	4.5	.272	.274	1.007	-.002	-.8

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