Eight Calculating Queens

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Terry Smith's article on the eight Queens problem brought back fond memories of the first time I "solved" the problem—more than 20 years ago, on an IBM 650. Just for the fun of it, I resurrected my program and adapted it to a programmable pocket calculator—the Texas Instruments SR-56. Listing 6, the resulting 100 step program, is shown. The running time for the entire set of 92 solutions is around 14 hours.

A word or two about the algorithm underlying this program—the problem becomes quite a bit simpler if it is restated from a geometric to a numeric orientation, namely:

> Determine all 8 digit numbers of the form
> $D_1\ D_2\ D_3\ D_4\ D_5\ D_6\ D_7\ D_8$ –

consisting of all the digits 1 thru 8 such that the absolute value of the difference of any pair of digits D_a and D_b does *not* equal the difference between their relative positions in the number, ie:

$$|D_a - D_b| \neq |a - b|$$

As a matter of further interest, I programmed the same approach on my Signetics 2650 microprocessor. The program, with initialization but excluding display steps, required 63 bytes of memory, and it took a little less than 4 seconds to display all 92 solutions on my screen.

I feel that four further comments on Mr Smith's article are relevant.

1. The number of unique solutions to the Queens problem is 12, not 23 as reported. This becomes more obvious when the solutions are represented as numbers rather than as geometric figures as stated

before. With no convention defined for displaying a "solution," we are free to define the "home position" (1 in Di) as occupying any of the 4 corners of the chessboard. Also, we are further free to measure the digit values Di either horizontally or vertically from the home position, with the digit position i being measured in the other (vertical or horizontal) direction. Thus, each solution is one of a family of eight identical solutions (ie: the first solution, 15863724, leads immediately to the solutions 17582463, 36428571, 42736851, 57263148, 63571428, 82417536, and 84136275). The "sharp" reader will note that the total number of solutions, 92, is not divisible by 8. The reason is that one of the solutions—35281746 in its minimum form—is partially symmetrical and generates itself as a variation.

2. Mr Smith suggests that his next problem will be to substitute "Maharajahs" for Queens, where a Maharajah combines the moves of Queen and Knight. Inspection of the solutions will demonstrate that this is impossible—any solution of the Maharajahs problem must also be a solution to the Queens problem, and each of the solutions to the Queens problem has at least one pair of pieces that are a Knight's move apart.

3. A more interesting variation to the problem is to generalize the problem for an n by n chessboard.

4. In the 1950s, a game manufacturer had a somewhat popular game consisting of an 8 by 8 pegboard together with eight sets of colored pegs, each set a different color and consisting of eight pegs. The object of the game was to place all 64 pegs into the board so that there were no two pegs of the same color in any horizontal, vertical, or diagonal row. If you try to program this problem, you'll find it very difficult to debug. (A clue is that the game manufacturer offered a $1000 prize to anyone who could send in a solution—and he's still in business.)■

Registers

0 Trial Digit
1 Solution
2 Number of digits
3 △
4 Partial solution
5 8 ————
6 Number of solutions

Listing 6.

LOC	CODE	KEY	LOC	CODE	KEY	LOC	CODE	KEY	LOC	CODE	KEY
00	38	*CMs	25	33	STO	50	94	=	75	34	RCL
01	00	0	26	03	3	51	37	*x=t	76	01	1
02	32	x≷t	27	34	RCL	52	06	6	77	64	x
03	34	RCL	28	01	1	53	03	3	78	01	1
04	00	0	29	33	STO	54	28	Ix I	79	00	0
05	35	SUM	30	04	4	55	74	−	80	74	−
06	01	1	31	01	1	56	34	RCL	81	29	*INT
07	92	.	32	35	SUM	57	03	3	82	33	STO
08	01	1	33	03	3	58	94	=	83	00	0
09	30	*PROD	34	34	RCL	59	12	INV	84	94	=
10	01	1	35	04	4	60	37	*x=t	85	33	STO
11	34	RCL	36	37	*x=t	61	03	3	86	01	1
12	02	2	37	00	0	62	01	1	87	22	GTO
13	74	−	38	03	3	63	27	*DSZ	88	06	6
14	08	8	39	64	X	64	02	2	89	03	3
15	33	STO	40	01	1	65	04	4	90	01	1
16	00	0	41	00	0	66	01	1	91	35	SUM
17	94	=	42	74	−	67	12	INV	92	09	9
18	37	*x=t	43	12	INV	68	35	SUM	93	34	RCL
19	09	9	44	29	*INT	69	02	2	94	01	1
20	00	0	45	33	STO	70	34	RCL	95	41	R/S
21	01	1	46	04	4	71	02	2	96	22	GTO
22	35	SUM	47	74	−	72	37	*x=t	97	07	7
23	02	2	48	34	RCL	73	09	9	98	05	5
24	00	0	49	00	0	74	09	9	99	41	R/S

Execution Notes: Load Program, (RST), (R/S), read solution. **Be Patient!!**