

Analysis of Polynomial Functions with the TI-59 Calculator

Part 2

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Consider the following polynomial:

$$P(x) = x^4 - 8x^2 + 7$$

For $P(x) = 0$ it is essential to study the characteristic elements, derived polynomials $P'(x)$ and $P''(x)$, and automatically plot the function curve. The procedure is as follows:

1. Read the magnetic card of the main program in groups 1 and 2.
2. Initialize by depressing key A.
3. Enter each of the coefficients with the keys. Start with the coefficient for x^6 by depressing key B each time. A 0 is entered for any term not having a power of x . Thus, you can perform the sequence 0 B, 0 B, 1 B, 0 B, -8 B, 0 B, 7 B.
4. Depress key C.

Depressing key C causes the processing of $P(x)$ to its conclusion with no other intervention.

When reading listing 1, the following are seen successively, separated by program spaces:

- the column of the seven given coefficients or the 0s which replace them
- the group of the lower boundary a and upper boundary b
- the group of interval $(b - a)$ and increment Δx
- the indication of the maximum error

After these appear the following results:

- the group of roots followed by the series 9. 999...? that indicates the end of determination of the roots
- the table of the thirty-nine values of x
- the table of the thirty-nine values of $P(x)$

If it is desired to retain the data for $P(x)$ to plot the function curve later, this is the time to record it in groups 3 and 4.

The procedure for the first derived polynomial is even simpler:

1. Depress key B' once; this causes all the coefficients of $P'(x)$ to be printed one after the other.
2. Depress key C.

The second derived polynomial is obtained in the same manner. The same applies for the derivatives of order n , provided the polynomial remains derivable. Notice that it is useless to reinitialize to change from one polynomial to the next.

Plotting the Function Curve

By convention, hereafter designate the data used in plotting the function curve as listing 2. It can be the table of values of $P(x)$ already recorded or any other that could be substituted for reasons that will be discussed. The plot itself will be designated figure 1.

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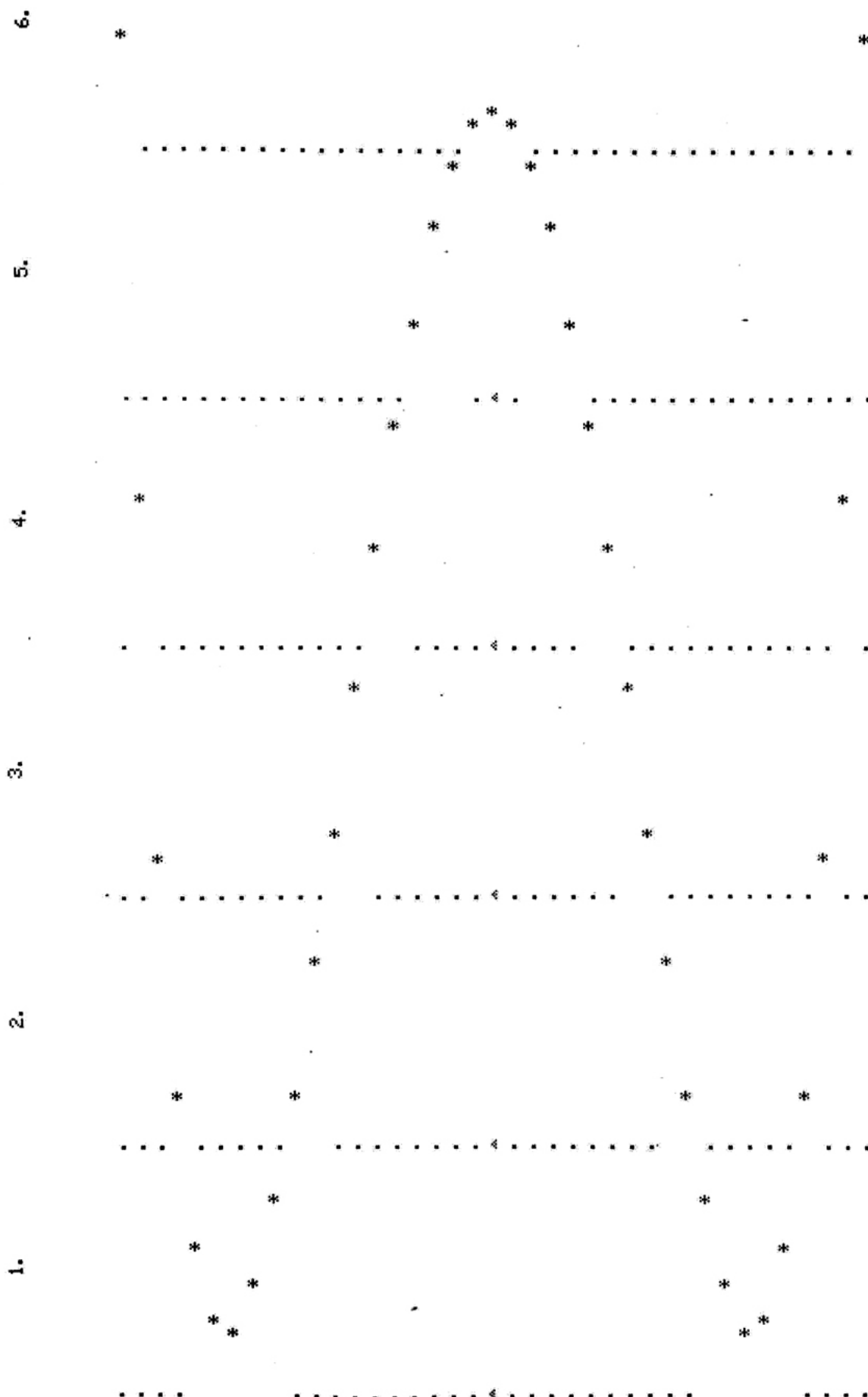


Figure 1: The six segments of output from the TI-59 that define the function curve.

Listing 1: A listing containing the data and error specification for a six-degree polynomial, $P(x)$. Thirty-nine values of x are also printed along with the corresponding values of $P(x)$ calculated for each value of x .

0.	0.	0.
0.	0.	0.
1.	4.	0.
0.	0.	0.
-8.	-16.	3.
0.	0.	0.
7.	0.	-4.
-3.828427125	-3.	-2.154700538
3.828427125	3.	2.154700538
7.656854249	6.	4.309401077
.3828427125	0.3	.2154700538
0.000001	0.000001	0.000001
-2.6457516	-2.000000095	-1.15470057
.9999998053	.00000002725	1.154700295
1.0000000232	2.0000000068	9.9999999 999
2.645751297	9.9999999 999	
9.9999999 999		
-3.637005769	21 -2.85	21 -2.046965511
-3.445584412	22 -2.7	22 -1.939230485
-3.254163056	23 -2.55	23 -1.831495458
-3.0627417	24 -2.4	24 -1.723760431
-2.871320344	25 -2.25	25 -1.616025404
-2.679898987	26 -2.1	26 -1.508290377
-2.488477631	27 -1.95	27 -1.40055535
-2.297056275	28 -1.8	28 -1.292820323
-2.105634919	29 -1.65	29 -1.185085296
-1.914213562	30 -1.5	30 -1.077350269
-1.722792206	31 -1.35	31 -.9696152423
-1.53137085	32 -1.2	32 -.8618802154
-1.339949494	33 -1.05	33 -.7541451884
-1.148528137	34 -0.9	34 -.6464101615
-.9571067812	35 -0.75	35 -.5386751346
-.7656854249	36 -0.6	36 -.4309401077
-.5742640687	37 -0.45	37 -.3232050808
-.3828427125	38 -0.3	38 -.2154700538
-.1914213562	39 -0.15	39 -.1077350269
5. -13	40 -9.2-12	40 -8.9-12
.1914213562	41 0.15	41 .1077350269
.3828427125	42 0.3	42 .2154700538
.5742640687	43 0.45	43 .3232050808
.7656854249	44 0.6	44 .4309401077
.9571067812	45 0.75	45 .5386751346
1.148528137	46 0.9	46 .6464101615
1.339949494	47 1.05	47 .7541451884
1.53137085	48 1.2	48 .8618802154
1.722792206	49 1.35	49 .9696152423
1.914213562	50 1.5	50 1.077350269
2.105634919	51 1.65	51 1.185085296
2.297056275	52 1.8	52 1.292820323
2.488477631	53 1.95	53 1.40055535
2.679898987	54 2.1	54 1.508290377
2.871320344	55 2.25	55 1.616025404
3.0627417	56 2.4	56 1.723760431
3.254163056	57 2.55	57 1.831495458
3.445584412	58 2.7	58 1.939230485
3.637005769	59 2.85	59 2.046965511
76.15249512	21 -11.749125	21 2.856734472
52.96920178	22 -8.883	22 2.427281539
34.42252761	23 -6.381375	23 2.021042278
19.94856125	24 -4.224	24 1.638016689
9.015614845	25 -2.390625	25 1.278204772
1.124224037	26 -0.861	26 .9416065276
-4.192852014	27 0.385125	27 .6282219549
-7.370630645	28 1.368	28 .3380510543
-8.811905689	29 2.107875	29 .0710938257
-8.887247468	30 2.625	30 -.1726497308
-7.935002802	31 2.939625	31 -.3931796153
-6.261295001	32 3.072	32 -.5904958277
-4.140023871	33 3.042375	33 -.7645983681
-1.81286571	34 2.871	34 -.9154872364
.5107266897	35 2.578125	35 -.1043162433
2.653524043	36 2.184	36 -.147623957
4.470520569	37 1.708875	37 -.1228871809
5.848933998	38 1.173	38 -.1286905989
6.708205561	39 0.596625	39 -.1321726497
7.	40 3.68-11	40 -.1333333333
6.708205561	41 -0.596625	41 -.1321726497
5.848933998	42 -1.173	42 -.1286905989
4.470520569	43 -1.708875	43 -.1228871809
2.653524043	44 -2.184	44 -.147623957
.5107266897	45 -2.578125	45 -.1043162433
-1.81286571	46 -2.871	46 -.9154872364
-4.140023871	47 -3.042375	47 -.7645983681
-6.261295001	48 -3.072	48 -.5904958277
-7.935002802	49 -2.939625	49 -.3931796153
-8.887247468	50 -2.625	50 -.1726497308
-8.811905689	51 -2.107875	51 .0710938257
-7.370630645	52 -1.368	52 .3380510543
-4.192852014	53 -.3851250001	53 .6282219549
1.124224037	54 .8609999998	54 .9416065276
9.015614845	55 2.390625	55 1.278204772
19.94856125	56 4.224	56 1.638016689
34.42252761	57 6.381375	57 2.021042278
52.96920178	58 8.883	58 2.427281539
76.15249512	59 11.749125	59 2.856734472

Text continued:

The procedure is as follows in practice:

1. In groups 3 and 4 read listing 2 mentioned above.
2. In groups 1 and 2 read the magnetic card of the program for the function curve.
3. Initialize by depressing key A.
4. With the keys enter the two extremes envisaged for the curve starting with the lower and then each time depressing key B.
5. Depress key C.

Depressing key C initiates the entire process with no other intervention. The six strips obtained are separated by cutting with scissors, and are assembled with glue or adhesive tape. This is the standard automatic procedure, and nothing prevents the operator from applying it in every case using the data collected in listing 1.

However, you may desire to center the reproduction in a smaller field. When examining the table of values of $P(x)$ obtained, it is obvious that, for registers R_{21} thru R_{25} and R_{55} thru R_{59} , small variations in x cause considerable variations in $P(x)$. In other words, the curve ends with parabolic branches. In the same way a photographer takes a close-up of a subject, you can neglect the infinite range and concentrate on useful details.

For this purpose, you must disconnect automatic operation and gain control of the depth of field. This time, the procedure will be as follows:

1. Reread the card of the main program in groups 1 and 2.
2. Initialize by depressing key A.
3. Re-enter the initial coefficients of R_{16} to R_{10} by each time depressing key B as previously indicated.
4. Switch to programming mode LRN, and perform the few modifications required:

- Replace the neutral NOP instructions provided for this purpose at statements 087, 096 and 119 by R/S instructions.
- Replace the initial partition of the interval (a, b) at statements 120 thru 122 by as many NOP instructions.
- Replace all occurrences of ϵ with a deliberately excessive number, for example 999...

5. Return to the calculating mode, and depress key C. The rest of the program will be executed but will stop whenever useful to permit the entry of a data item of your choice:

- Boundary a with the first stop: here, it will be 3 in absolute value but the calculator will recognize it as negative 3.
- Boundary b with the second stop: it will again be 3.
- Partition of the interval (a, b) at the third stop: keep it at 20 on seeing the value of the interval the machine has just printed out after the boundaries.

Naturally, each data entry with the keys is followed by operation of the R/S key to restart the calculation.

What happens now? Without getting involved in a root calculation that is no longer of interest at this point, the

Listing 2: Listing of a sample input of data used to plot the function curve.

0.	-0.9	34	-8.4224	28
0.	-0.75	35	-7.36799375	29
1.	-0.6	36	-5.9375	30
0.	-0.45	37	-4.25849375	31
-8.	-0.3	38	-2.4464	32
0.	-0.15	39	-0.60449375	33
7.	0.	40	1.1761	34
	0.15	41	2.81640625	35
-3.	0.3	42	4.2496	36
3.	0.45	43	5.42100625	37
	0.6	44	6.2881	38
6.	0.75	45	6.82050625	39
0.3	0.9	46	7.	40
	1.05	47	6.82050625	41
9999999.	1.2	48	6.2881	42
	1.35	49	5.42100625	43
	1.5	50	4.2496	44
	1.65	51	2.81640625	45
-2.55	1.8	52	1.1761	46
9.9999999 999	1.95	53	-0.60449375	47
	2.1	54	-2.4464	48
-2.85	2.25	55	-4.25849375	49
-2.7	2.4	56	-5.9375	50
-2.55	2.55	57	-7.36799375	51
-2.4	2.7	58	-8.4224	52
-2.25	2.85	59	-8.96099375	53
-2.1			-8.8319	54
-1.95	21	7.99500625	-7.87109375	55
-1.8	22	1.8241	-5.9024	56
-1.65	23	-2.73749375	-2.73749375	57
-1.5	24	-5.9024	1.8241	58
-1.35	25	-7.87109375	7.99500625	59
-1.2	26	-8.8319		
-1.05	27	-8.96099375		

calculator simply indicates the lowest root in approximate fashion and then rapidly prints out the tables of values of x and $P(x)$ at the assigned values of a and b (see listing 2).

All that remains is to use these values contained in

registers R_{21} to R_{59} for the plot by continuing with the known steps as follows:

6. In groups 1 and 2 read the card of the program for the function curve.
7. Initialize with key A.
8. With the keys punch in -9 B then 8 B to enter the extremes which are obviously appropriate here.
9. Depress key C which delivers the six ideal strips after this mathematical "zooming" as can be seen from looking at the curve in detail (see figure 1).

Above all, the question is one of knowing if this plot is technically satisfactory.

For verification purposes, see if the coordinates of the minimums found by the calculation (± 2 , -9) and the coordinates measured on the plot are consistent. More precisely, determine the abscissa of the minimums with an ordinate of -9 . From the small median triangular sign marking the 0 abscissa on the base line, you can easily count ± 13 intervals each having a value of 0.15, the increment of x . This gives $\pm 13 \times 0.15 = \pm 1.95$. This abscissa is very close to the value calculated (± 2), and it can be said that the plot is extremely accurate.

As for the points of inflection, their ordinate is found to be $-1.888...$ for $P(x)$ evaluated from the roots of $P''(x) = 0$, in other words abscissas of ± 1.154700 . The points of inflection whose abscissa is ± 1.15 in accordance with the calculation fall slightly before the eighth point on the base line at the abscissa ($8 \times 0.15 = 1.20$). This is also very close to the value calculated. These are the points where the curve crosses its tangent.

The zero ordinate can easily be deduced from a simple rule. Given that the difference between minimum and maximum is $9 + 8 = 17$ in absolute value and there are $4 \times 20 + 17 = 97$ elementary intervals between these points, each has a value of 0.175. From this the axis of the abscissas is at $9/0.175 = 51$ intervals from the minimum of the curve.

From the table of $P(x)$ it can be seen that the curve cancels between registers R_{34} and R_{33} , R_{46} and R_{47} , and that this effectively corresponds to the interval 6-7 of the base line. Without providing the precision of a professional plotter, the reproduction obtained is thus of suitable quality given the means employed.

Conclusion

When a procedure is used for a rather long calculation that requires only three keyboard operations:

- initialization by key A
- data entry by key B or B'
- complete execution by key C

the drawbacks of the relative slowness of calculation are considerably reduced.

The TI-59 cannot execute its program with great speed. However, most users can tolerate a delay of a few minutes with no hardship. Some will appreciate the option of allowing users to disable automatic operation to follow their own inspiration. ■