FORECASTING COMPUTER SYSTEM RELIABILITY WITH A HANDHELD PROGRAMMABLE CALCULATOR

Equations, code, and user instructions for a calculator program which provides a fast, quantitative approach to understanding, evaluating, and improving computer hardware reliability

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Contemporary computers made up of IC chip elements inherently provide better reliability than their predecessors, built with relays, vacuum tubes, or discrete transistors. Even today, however, non-redundant computer systems rarely enjoy mean-time-between-failures of over 2000 hours. Where there are stringent reliability specifications, essential computer components should be duplicated. Incorporating redundant hardware improves both reliability and maintainability—prerequisites for long, trouble free periods of continuous computer system operation.

Evaluating and enhancing computer system reliability are vitally important in nearly all processing applications. The equations and handheld calculator programs incorporated here provide computer designers with the tools necessary for making accurate reliability decisions. Redundancy must be introduced at the optimum level in the hierarchy of computer system organization in order to achieve maximum reliability at reasonable cost and complexity. The reliability models described ascertain the elements that need to be replicated, and determine the resulting total system reliability. It is assumed that the reader is familiar with handheld programmable calculators.

Programmable Calculators

The handheld programmable calculator offers convenience, portability, and a capability for powerful decision making. Texas Instruments' model TI-59 when used with a companion PC-100A thermal printer also produces alphanumeric messages and labels the results. The Reliability Analysis Program, using the instructions set forth in the Instructions, executes on the TI-59 in only a few seconds running time, with or without printer attached, and provides answers to intricate "what if" questions. It is easier and more revealing to determine computer system reliability interactively on a programmable calculator than to run a FORTRAN batch implementation of the program on a large computer.

Inputs to Reliability Model

To determine overall system reliability, element mean time between failures (MTBF), mean-time-to-repair (MTTR), the configuration, system failure modes, and minimum downtime that constitutes a failure must all be specified. MTBF and MTTR values for individual elements can be computed from trouble report statistics, estimated

TI-59 Reliability Analysis Program

Partition: 479.59

Program Listing:

```
000:  Lbl B′ STO 12 CP ( x=t 016 RCL 12 x Dsz B 06 1 ) INV SBR
019:  Lbl List STO 00 Lbl Prt RCL Ind 00 Op 01 Op 20 RCL Ind 00 Op 02
035:  Op 20 RCL Ind 00 Op 03 Op 20 RCL Ind 00 Op 04 Op 20 Op 05 INV SBR
052:  Lbl ifflg Op 00 RCL 54 Op 01 Op 05 INV SBR Lbl D.MS Op 00 RCL 55
069:  Op 01 RCL 56 Op 02 Op 05 INV SBR Lbl C x⇄t 2 2 STO 00 5 8 GTO Σ+
089:  Lbl A x⇄t 5 7 Lbl Σ+ Fix 2 STO 13 ifflg 0 x̄ Stflg 0 Adv SBR Prt
108:  Lbl x̄ Adv SBR ifflg RCL Ind 13 Op 04 x⇄t Op 06 STO 01 1/x SUM 02
125:  RCL 02 1/x STO 03 SBR D.MS RCL Ind 13 Op 04 RCL 03 Op 06 INV SBR
141:  Lbl B x⇄t SBR ifflg SBR x≥t + RCL 01 = ÷ RCL 01 = 1/x Prd 04 1 −
161:  RCL 04 = ÷ RCL 04 x RCL 03 = x⇄t SBR D.MS Lbl x≥t RCL 58 Op 04
180:  x⇄t Op 06 INV SBR Lbl D x⇄t SBR ifflg SBR π ÷ x⇄t ( x⇄t + RCL 01 )
199:  +/− + 1 = Prd 05 RCL 05 − 1 = +/− STO 04 ÷ RCL 05 x RCL 03 = STO
221:  03 x⇄t SBR D.MS Lbl π RCL 57 Op 04 x⇄t Op 06 INV SBR Lbl A′ Adv
238:  1 4 SBR List Pgm 01 SBR CLR 1 STO 04 STO 05 INV Stflg 0 Fix 0
256:  INV SBR Lbl C′ Adv 3 8 SBR List RCL 04 INV Fix Prt INV SBR Lbl E
272:  Fix 0 STO 06 Adv 2 6 SBR List SBR Prt RCL 06 Prt R/S STO 07 SBR Prt
291:  RCL 07 Prt R/S Fix 2 STO 02 ÷ ( RCL 07 − RCL 06 + STO 08 1 = STO
312:  01 SBR ifflg RCL 02 x⇄t SBR x≥t SBR D.MS RCL 01 x⇄t SBR x≥t R/S
328:  x x⇄t SBR ifflg ( SBR π ÷ RCL 02 ) y^x RCL 08 x RCL 08 B′ x ( RCL 06
350:  − 1 ) B′ ÷ RCL 07 B′ = STO 03 ÷ x⇄t ( x⇄t + RCL 01 = STO 04 SBR
372:  D.MS RCL 03 x⇄t SBR π INV SBR Lbl D′ Adv Stflg 1 STO 09 4 2 SBR List
390:  0 Lbl E′ STO 11 ifflg 1 Dsz 4 6 Adv SBR List RCL 11 Fix 0 Prt SBR
409:  Prt Lbl Dsz Fix 2 RCL 59 Op 04 RCL 09 Op 06 RCL 09 ÷ RCL 03 =
428:  STO 10 +/− INV lnx x RCL 10 y^x RCL 11 ÷ RCL 11 B′ = INV Fix
446:  INV Stflg 1 Prt Inv SBR
```

Prestored Data Registers:

```
14:   3517272413  1424272437  4500133113  2745362436  5100361735  2417360030
20:   3216172700  2000000000  5100003313  3513272717  2700303216  1727002000
26:   5100001331  4500262032  2120310030  3216172720  3132400041  3124373600
32:   3517344124  3517166400  3132400041  3124373600  1342132427  1314271764
38:   7500134213  2427131424  2724374500  6400000000  7500351727  2413142427
44:   2437450024  3100000000  7500333532  1413142427  2437450032  2100000000
50:   2113242741  3517360024  3100000000           0  4131243720  3641143645
56:   3637173020  2030371421  2030373735  2023353664
```

Note: With calculator in learn mode, enter keystrokes found in program listing and store tabulated numbers into registers. Record calculator memory on two mag cards for future use. Reading these cards is a convenient way to reload entire program.

from reliability handbooks, or obtained from vendors. A failure must be defined in terms of equipment and downtime for the application being analyzed. For example, if the back-end computer of a large realtime switching network is not operational for a brief time span, such downtime can be masked by a frontend processor. With batch systems, brief soft failures may be tolerated even more readily.

The first step in determining the reliability of any computer installation is to draw a diagram showing the primary elements joined so as to indicate how their relationship affects system operation. This diagram does not usually represent actual physical connections. Thus, large and complicated configurations can be reduced to three basic reliability dependencies: series, parallel, and K of N (any K elements required to be operational

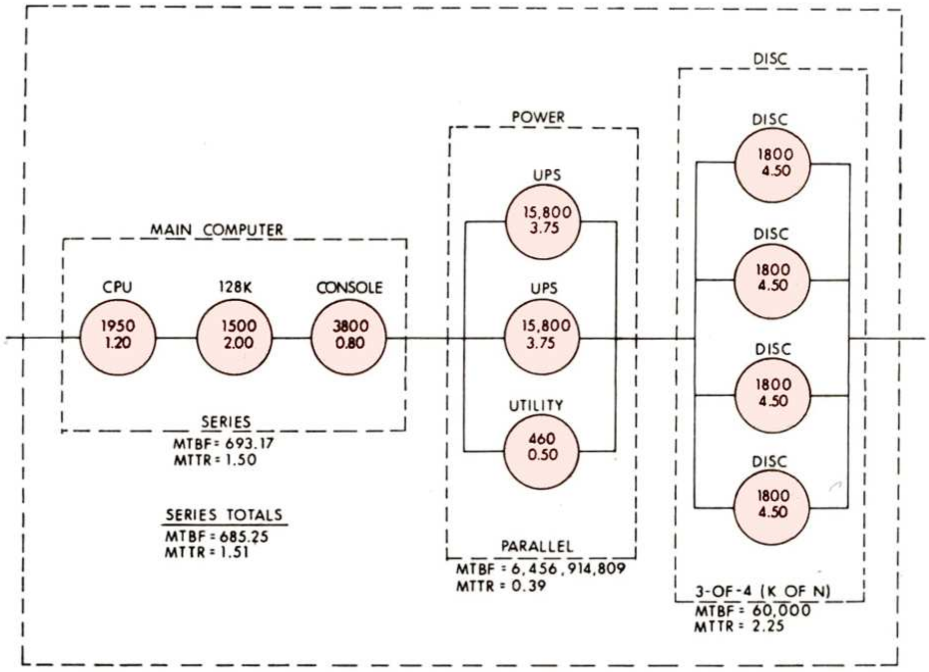

Fig 1 Reliability diagram for computer system configuration. Processor, memory, console, minimum of one power source, and at least three of four available discs are required for operational system. Top number within each circle is element MTBF, bottom number is MTTR

of N elements available). For example, in the reliability diagram shown in Fig 1, the processor (CPU), memory (128k), and operator's console are tied in series; if any one of these three elements fails, the entire computer system goes down. Next, two uninterruptible power supplies (UPS) and utility company lines are connected in parallel, indicating that only one of these three sources is necessary to energize the system. Finally, the disc subsystem is operational if any three of four available discs work. As indicated by the outermost dashed lines in Fig 1, the three subsystems are strung in series to calculate total computer system reliability.

To evaluate this total computer system, calculate dependability in successive stages (initially components, next first-level subsystems, then larger subsystems), building on each set of results until the entire installation is encompassed. Primary elements in Fig 1 are grouped by shorter dashed lines into three first-level subsystems: main computer, power, and disc. Intermediate MTBF and MTTR results for these three subsystems are used to compute the MTBF and MTTR for the total computer installation. Most computer configurations can be translated easily and directly into a hierarchy of series, parallel, and K of N network models.

Series Configurations

The statistical likelihood of series elements in a string being operational is calculated by multiplying together the probability of finding each element working. Thus, the overall long-term availability (A) of a series string of repairable components is the product of their individual availabilities:

$$A_s = A_1 \times A_2 \ldots \times A_n \qquad (1)$$

The reliability analysis calculator program multiplies availabilities and assumes that defective modules in a computer installation are either repairable or replaceable. This algorithm cannot be applied to nonrepairable subsystems because their long-term availabilities are zero.

The series formula for total subsystem MTBF is analogous to the equation for evaluating a circuit of parallel resistors; combined resistance is always less than the value of any one parallel resistor. Similarly, total MTBF in a series reliability diagram is always lower than the MTBF of its least reliable element. The inverse of series subsystem MTBF can be calculated by adding together the inverses of individual element MTBFs:

Instructions for Running Reliability Analysis Program on TI-59 Calculator

Step	Procedure	Enter	Press	Display
S.0	Series Configuration		START (A′)	1.
S.1	Enter MTBF and MTTR for each series element	Element MTBF	MTBF_s (A)	Cumulative subsystem MTBF
S.2	Repeat steps S.1 and S.2 for each series element. Cumulative result is displayed after each entry. After each element is added, you can interrupt loop and compute the following:	Element MTTR	MTTR_s (B)	Cumulative subsystem MTTR
S.3	Availability		A (C′)	Availability
S.4	Reliability	Time period t	R(t) (D′)	Reliability
S.5	Probability of n failures. Reliability must be calculated first.	n	P(n) (E′)	Probability of n failures
SS.0	Series Configuration for calculation of MTBF only.		START (A′)	1.
SS.1	Enter MTBF for each series element. Repeat step SS.1 for each element. Cumulative result is displayed after each entry. After each MTBF is entered, you can compute the following:	Element MTBF	MTBF_s (A)	Cumulative subsystem MTBF
SS.2	Reliability	Time period t	R(t) (D′)	Reliability
SS.3	Probability of n failures. Reliability must be calculated first.	n	P(n) (E′)	Probability of n failures
P.0	Parallel Configuration		START (A′)	1.

$$1/\text{MTBF}_s = 1/\text{MTBF}_1 + 1/\text{MTBF}_2 \ldots + 1/\text{MTBF}_n. \qquad (2)$$

This calculation holds for both repairable and nonrepairable subsystems. The subscript s is used to denote subsystem values.

Another way of explaining Eq (2) is that the total failure rate ($1/\text{MTBF}_s$) is the sum of the failure rates of its series constituent elements. According to this concept, the MTBF of a series configuration, with two identical modules, has half the MTBF of one element. If a third identical component is added, series subsystem MTBF drops to one-third of the element MTBF; if more series modules are included, MTBF_s falls proportionately.

For repairable subsystems, MTBF_s, MTTR_s, and A_s are mutually related by

$$A_s = \text{MTBF}_s/(\text{MTBF}_s + \text{MTTR}_s) \qquad (3)$$

Step	Procedure	Enter	Press	Display
P.1	Enter MTTR and MTBF for each parallel element.	Element MTTR	MTTR$_p$ (C)	Cumulative subsystem MTTR
P.2	Repeat steps P.1 and P.2 for each parallel element. Cumulative result is displayed after each entry. After each element is added, you can interrupt loop and compute the following:	Element MTBF	MTBF$_p$ (D)	Cumulative subsystem MTBF
P.3	Availability		A (C′)	Availability
P.4	Reliability	Time period t	R(t) (D′)	Reliability
P.5	Probability of n failures. Reliability must be calculated first.	n	P(n) (E′)	Probability of n failures
KN.0	K of N Configuration			
KN.1	Input number of elements required	K	K-of-N (E)	
KN.2	Input number of elements available	N	R/S	
KN.3	All elements are identical. Element MTTR and MTBF need to be input only once.	Element MTTR	R/S	Subsystem MTTR
KN.4		Element MTBF	R/S	Subsystem MTBF
KN.5	Availability		A (C′)	Availability
KN.6	Reliability	Time period t	R(t) (D′)	Reliability
KN.7	Probability of n failures. Reliability must be calculated first.	n	P(n) (E′)	Probability of n failures

Note: "Press" column lists describing mnemonic, written on calculator mag program card. Actual calculator keystrokes shown in parentheses

After any two of these parameters have been determined, this equation can be used to solve for the remaining unknown; the equation, solving for MTTR_s in terms of A_s and MTBF_s, is

$$\text{MTTR}_s = \frac{(1 - A_s)}{A_s}\,\text{MTBF}_s \qquad (4)$$

For all series networks, repairable or not, total reliability is the product of the reliabilities of each of the individual elements:

$$R_s = R_1 \times R_2 \ldots \times R_n \qquad (5)$$

The reliability analysis calculator program first solves for MTBF_s, and then uses the following equation:

$$R_s = e - t/\text{MTBF}_s \qquad (6)$$

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RELIABILITY ANALYSIS

* SERIES MODEL -

UNIT-
    1950.00     MTBF
SUBSYSTEM-
    1950.00     MTBF
UNIT-
       1.20     MTTR
SUBSYSTEM-
       1.20     MTTR

UNIT-
    1500.00     MTBF
SUBSYSTEM-
     847.83     MTBF
UNIT-
       2.00     MTTR
SUBSYSTEM-
       1.65     MTTR

UNIT-
    3800.00     MTBF
SUBSYSTEM-
     693.17     MTBF
UNIT-
       0.80     MTTR
SUBSYSTEM-
       1.50     MTTR

^ AVAILABILITY =
 .9978441824

^ RELIABILITY IN
    2080.00     HRS=
 .0497521423
```

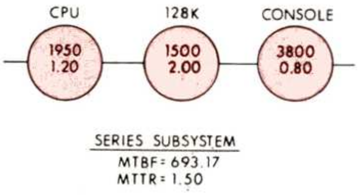

Fig 2 Three primary elements connected in series. Failure of only one element brings down entire subsystem. Series connection is always less reliable than its weakest link, in this case main storage (128k)

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RELIABILITY ANALYSIS

* SERIES MODEL -              ^ PROBABILITY OF
                                        0.
UNIT-                         FAILURES IN
      693.17     MTBF                 2080.00     HRS=
SUBSYSTEM-                     .0480566918
      693.17     MTBF
UNIT-                         ^ PROBABILITY OF
        1.50     MTTR                   1.
SUBSYSTEM-                    FAILURES IN
        1.50     MTTR                 2080.00     HRS=
                               .1458700273
UNIT-
 6456914809.     MTBF         ^ PROBABILITY OF
SUBSYSTEM-                              2.
      693.17     MTBF         FAILURES IN
UNIT-                                 2080.00     HRS=
        0.39     MTTR          .2213850358
SUBSYSTEM-
        1.50     MTTR         ^ PROBABILITY OF
                                        3.
UNIT-                         FAILURES IN
    60000.00     MTBF                 2080.00     HRS=
SUBSYSTEM-                     .2239954522
      685.25     MTBF
UNIT-
        2.25     MTTR
SUBSYSTEM-
        1.51     MTTR

^ AVAILABILITY =
 .9978032837

^ RELIABILITY IN
     2080.00     HRS=
 .0480566918
```

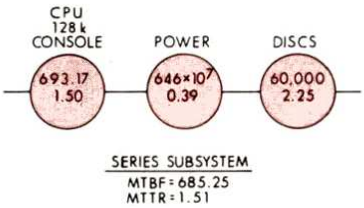

Fig 3 Series model for calculating dependability of entire computer system. Total system dependability is determined by joining first-level subsystems in series. MTBF and MTTR results from Figs 2, 4, and 5 are shown in reliability circles. MTBF for power and disc subsystems is already high, so further improvements in total system dependability would require adding backup processors, memories, and/or consoles

to determine series reliability; however, results are the same as if element reliabilities had been multiplied.

Series Calculations

To compute reliability or failure probability, the MTBF of at least one element needs to be input; however, both MTBF and MTTR are needed for availability. Pressing calculator key C′ displays availability. Reliability, R(t), is the probability of successful uninterrupted operation without any failure(s) over a given time span (t). Calculate reliability by entering the time period, in the same units used for MTBF and MTTR (usually in hours); pressing key D′ displays reliability. Multiply by 100 to convert to percent. The probability of any number of failures occurring during time interval t, P(n), is computed by entering the number of failures, n, and pressing key E′. This failure probability parameter is an expansion of reliability, the probability of zero failures; therefore, an alternate method of calculating reliability is to key in zero followed by E′. The time period over which failure probabilities are calculated is the same period used for reliability. Make certain to calculate, or recalculate, reliability over that desired time span first.

On the TI-59, press the A′ key to initiate either series or parallel calculations. For the series model, enter the MTBF of the first element, then press key A. Enter its MTTR, and press key B. Repeat this sequence for every element in the series connection as follows: A′, MTBF_1, A, MTTR_1, B, MTBF_2, A, MTTR_2, B, . . . , MTBF_n, A, MTTR_n, B. A cumulative MTBF is displayed each time key A is pressed, and a cumulative MTTR is displayed each time B is pressed, so that the effect of adding each series element is immediately apparent. This cyclic entry process can be interrupted after each element is entered to compute availability (key C′), reliability (key D′), or failure probability (key E′) of the series components included thus far (see Instructions, Steps S.0 to S.5).

Sometimes, only failure rates are provided for the individual elements in a series connection. In such cases, the problem becomes one of determining the failure rate, or alternatively the MTBF of the whole series string. Initialize the model by pressing key A′. Since MTBF (h/failure) is the inverse of failure rate (failures/h), enter the failure rate for the first element; then press calculator keys 1/x and A. Repeat this process for each series element. Given element failure rates, the keystroke sequence is: A′, failure rate_1, 1/x, A, failure rate_2 1/x, A, . . . , failure rate_n, 1/x, A. Alternatively, if given element MTBFs, the keystroke sequence is: A′, MTBF_1, A, MTBF_2, A, . . . , MTBF_n, A. The MTBF for the string of accumulated elements will be displayed each time key A is pressed; to convert this MTBF to its equivalent failure rate, press key 1/x. This entry process can also be interrupted to calculate both reliability and failure probability for the string of series elements thus far accumulated (see

Instructions, Steps SS.0 to SS.3). Availability calculations, however, require MTTR and therefore can only be accomplished as described in the preceding paragraph.

Figs 2 and 3 illustrate the use of the calculator program in computing dependability parameters for series configurations. In Fig 2, three primary elements (CPU, 128k memory, and console) are connected in series and their MTBF and MTTR values are entered consecutively. As each element value is entered, subsystem MTBF and MTTR values are immediately calculated. When the CPU is input, it is the first and only element included in the series subsystem; subsystem MTBF and MTTR are the same as that for the CPU. After the 128k memory is also incorporated, subsystem MTBF is reduced to only 847.83 h, and subsystem MTTR equals 1.65 h. MTBF_s falls even lower, to 693.17 h when the console value is added. Series elements can be configured and input in any order, but final results will always be the same. After each element is entered, the computer designer can temporarily suspend further input to solve for availability (key C′), reliability (key D′), and/or failure probabilities (key E′) of the interim subsystem.

Fig 3 indicates the use of the series model in calculating the dependability of an entire computer installation. Three first-level subsystems, whose MTBFs and MTTRs have been calculated in Figs 2, 4, and 5, are the series elements. By studying MTBF values, the experienced computer designer can estimate the final overall MTBF. The first subsystem is the limiting resource because it has the lowest MTBF, 693.17 h; power and disc subsystems have MTBF values many times greater, 6,456,914,809 h and 60,000 h, respectively. The total MTBF of a series connection is less than the MTBF of any of its elements, and the reliability analysis program confirms this by predicting a total MTBF_s for the overall computer installation of 685.25 h.

Fig 3 also shows failure probability calculations. The arbitrarily chosen 2080-h time period could represent either 2080 nonstop hours, 260 8-h days, 130 online 16-h days, etc. The probability of zero failures produces the same result as reliability, 4.81%, because they both have the same definition. According to the printout, the probability of 1 failure occurring in 2080 h is 14.59%, the probability of 2 failures is 22.14%, the probability of 3 failures is 22.40%, etc.

Parallel Configurations

The inherent redundancy of a parallel configuration, where only one element needs to be working, enhances reliability. Parallel networks whose defective elements can be repaired or replaced are the most desirable, and are many times more reliable than a nonrepairable parallel configuration, which ceases to operate when each of its elements has experienced only one failure. Since most computer installations fall into the repairable

```
RELIABILITY ANALYSIS

*  PARALLEL MODEL -

UNIT-
        3.75    MTTR
SUBSYSTEM-
        3.75    MTTR
UNIT-
    15800.00    MTBF
SUBSYSTEM-
    15800.00    MTBF

UNIT-
        3.75    MTTR
SUBSYSTEM-
        1.88    MTTR
UNIT-
    15800.00    MTBF
SUBSYSTEM-
 33301133.17    MTBF

UNIT-
        0.50    MTTR
SUBSYSTEM-
        0.39    MTTR
UNIT-
      460.00    MTBF
SUBSYSTEM-
 6456914809.    MTBF

^ AVAILABILITY =
 .9999999999

^ RELIABILITY IN
     2080.00    HRS=
 .9999996779
```

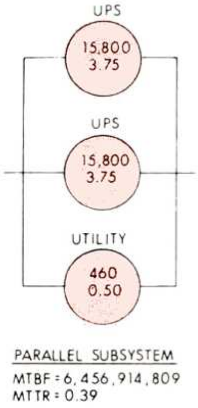

Fig 4 Paralleling elements increases reliability. Parallel subsystem works as long as any one of its constituent elements operates. This computer system is usually energized by one of its redundant uninterruptible power supplies. Should both these power sources fail, bypass transfer switch connects ac bus to normal utility company voltage

```
*  ANY K-OF-N MODEL-
NO. UNITS REQUIRED=
            3.
NO. UNITS AVAILABLE=
            4.
UNIT-
        4.50    MTTR
SUBSYSTEM-
        2.25    MTTR
UNIT-
     1800.00    MTBF
SUBSYSTEM-
    60000.00    MTBF

^ AVAILABILITY =
 .9999625014

^ RELIABILITY IN
     2080.00    HRS=
 .9659273384

^ PROBABILITY OF
            1.
FAILURES IN
     2080.00    HRS=
 .0334854811
```

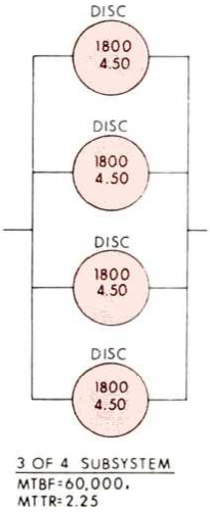

Fig 5 3-of-4 disc subsystem. Working combination of any three discs (K=3) suffices for this four-disc (N=4) subsystem to be operational. The K of N model, K elements required of N elements available, is restricted to situations where all elements have identical MTBF and MTTR values. Reliability diagrams of data storage peripherals should indicate any dependency on specific files as well as on raw hardware storage capacity. If different file is stored on each of four discs, and realtime system needs to access all of these files, proper representation would be either 4-of-4 or series model category, paralleling key modules is a technique commonly used to significantly upgrade system reliability. The strategy for evaluating dependability parameters in repairable systems is to initially determine A_s and MTTR$_s$. Unavailability is defined as percent availability subtracted from 100%. The model calculates the unavailability of each parallel element by subtracting its availability from one:

$$U_1 = 1 - A_1, U_2 = 1 - A_2, \ldots, U_n = 1 - A_n \quad (7)$$

Repairable subsystem unavailability (U_s) is the product of all element unavailabilities. The model multiplies the unavailabilities of the parallel elements together and determines unavailability for the entire parallel subsystem:

$$U_s = U_1 \text{ x } U_2 \ldots \text{ x } U_n \quad (8)$$

Subtracting U_s from 1 reconverts back to parallel subsystem availability:

$$A_s = 1 - U_s \quad (9)$$

Parallel subsystem MTTR$_s$ is calculated with the same equation used in the series configuration for MTBF$_s$; just replace each MTBF that appeared in the equation by its corresponding MTTR:

$$1/\text{MTTR}_s = 1/\text{MTTR}_1 + 1/\text{MTTR}_2 \ldots + 1/\text{MTTR}_n \quad (10)$$

Consider two parallel elements, each with an MTTR of one hour. If one of these elements fails, the parallel subsystem continues to operate. Total parallel outages are unlikely and will occur only if the second module simultaneously fails sometime during the 1-h downtime of the first element. The probability of the second element failing is approximately uniform over this 1-h downtime, and it will occur, on average, after 0.5 h has transpired. Repair of the first unit will, on average, be accomplished in another 0.5 h; therefore, overall parallel subsystem MTTR$_s$ is 0.5 h. Both the equation for MTTR$_s$ and this intuitive line of reasoning lead to the same result. If a third element, with the same MTTR as the other two is added in parallel, only a simultaneous triple failure could bring down the parallel subsystem; in this case, MTTR$_s$ would be 0.333 h.

After solving for A_s and MTTR$_s$, the general equation for availability, with its terms rearranged, is used to determine parallel subsystem MTBF$_s$:

$$\text{MTBF}_s = (\text{MTTR}_s)\ (A_s)/(1 - A_s) \quad (11)$$

The above equations for repairable parallel networks have been implemented in the reliability analysis calculator program. They apply only to systems where repair or replacement of malfunctioning parts is feasible. These equations will not work for nonrepairable configurations because long-term availability is zero and MTTRs are not definable.

However, it is still possible to approximate MTBF$_s$ and R_s for nonrepairable parallel subsystems. Beginning with reliability, the approach is the same as that used for availability. Define unreliability as reliability subtracted from 100%. Calculate the unreliability of each parallel element by subtracting its reliability from 1. Nonrepairable subsystem unreliability is the product of all element unreliabilities. Multiply the unreliabilities of the parallel elements together to compute the unreliability of the entire parallel subsystem. Subtract subsystem unreliability from 100% to reconvert it to parallel subsystem reliability, R_s. The equation for this process is

$$R_s = 1 - [(1 - e^{-t/\text{MTBF}_1}) \text{ x } (1 - e^{-t/\text{MTBF}_2}) \text{ x } \ldots \text{ x } (1 - e^{-t/\text{MTBF}_n})] \quad (12)$$

After R_s is known, the second step is to find parallel subsystem MTBF$_s$. The relationship between MTBF$_s$ and R_s is expressed by Eq (6). Solving for MTBF$_s$ yields

$$\text{MTBF}_s = -t/(\ln R_s) \quad (13)$$

where ln denotes the natural logarithm.

Parallel Calculations

These are handled similarly to the series procedure. The main modification is that an element's MTTR value is entered before its corresponding MTBF. Key in the element MTTR, and then press key C. Next, enter the corresponding MTBF, and press key D. Continue this cycle until all parallel elements have been included, for example, A′, MTTR$_1$, C, MTBF$_1$, D, MTTR$_2$, C, MTBF$_2$, D, . . . , MTTR$_n$, C, MTBF$_n$, D. Each time key C is pressed, the cumulative MTTR for the elements included thus far is displayed. After pressing D, the cumulative MTBF is displayed. As for series connections, the parallel entry process can also be interrupted after each element is input to calculate subsystem availability (key C′), reliability (key D′), or failure probability (key E″) (see Instructions, Steps P.0 to P.5).

For comparison, consider two parallel elements, each of which has an MTBF of 100 h and an MTTR of 1 h. If these elements are repairable, their parallel configuration has an MTBF$_s$ of 5100 h and an $R_s(2080)$ of 66.51%, according to the calculator program. If two nonrepairable elements, each with a 100-h MTBF, are connected in parallel, total subsystem MTBF$_s$ is no more than several hundred h, and reliability, R(2080), is smaller than 1%. MTBF$_s$ can only be approximated for nonrepairable configurations because the precision of calculations is sensitive to the reliability time period chosen. Nevertheless, the difference in reliability levels is dramatic, going from 66.51% probability of no failures in 2080 h to almost certain failure.

Fig 4 shows three power sources in parallel. Running the model indicates the effect of increasing redundancy. Paralleling two UPS supplies, without a static bypass transfer switch, gives an MTBF$_s$ of 33,301,133.17 h, an MTTR of 1.88 h (half the UPS MTTR of 3.75 h), and a five-year reliability $R_s(43800)$ of 99.869%. Adding a static bypass transfer switch to connect the computer to utility power in the event of UPS failures increases the MTBF_s by several orders of magnitude and the reliability $R_s(43800)$ to 99.999%. Even this small 0.130% increase in reliability is significant for a critical application. If computer workload and turnaround time are paramount, the cost of replicated hardware may be justified by comparison to lost operation and revenue should potential downtime occur.

K of N Configurations

A K of N network connection frequently encountered in computer systems is a grouping of N elements, of which K elements must be working. When K is equal to 1, only one of the elements is needed to sustain subsystem operation, the equivalent of a parallel configuration. If all N elements are essential (K equals N), the group is actually a series network. Equations for calculating MTTR_s and MTBF_s for general K of N repairable configurations are

$$\text{MTTR}_s = \frac{\text{MTTR}_1}{N-K+1} \tag{14}$$

$$\text{MTBF}_s = \text{MTBF}_1 \left[\frac{\text{MTBF}_1}{\text{MTTR}_1}\right]^{(N-K)} \times \frac{(N-K)!\quad(K-1)!}{N!} \tag{15}$$

These two equations have been coded into the reliability analysis calculator program. The K of N equations assume that all elements have the same MTBF and MTTR values. General availability Eq (3) is used to solve for A_s in terms of MTTR_s and MTBF_s.

K of N Calculations

The K of N model requires no initialization; follow the keystroke sequence given in the Table, Steps KN.0 to KN.7. Input an integer value for K and press key E; then, enter N and press the run/stop key, R/S. Input element MTTR, press R/S, and the MTTR of the total subsystem is displayed. Similarly, enter element MTBF, press R/S, and the display shows total subsystem MTBF. Subsystem MTBF and MTTR values are automatically stored in registers, so that they will not have to be reentered for further availability (key C′), reliability (key D′), and failure probability (key E′) calculations.

A reliability diagram for a 3-required-of-4 available disc subsystem is depicted in Fig 5. The accompanying printout indicates the order in which input parameters are entered: K, N, element MTTR, and element MTBF. After element MTTR is keyed in, the program halts, displaying a subsystem MTTR_s of 2.25 h in this case. Similarly, when element MTBF is input, the program displays an MTBF_s of 60,000 h. Availability (key C′) and reliability in 2080 h (key D′) are also shown 99.99625014% and 96.59273384% respectively.

K of N dependability parameters, when elements are not alike, are complex and difficult to calculate. There are also configurations, especially in systems having dual sets of data storage devices and redundant files, for which there is no method by which hardware interdependencies can be represented with absolute accuracy on a reliability diagram. In these situations, it is usually best to work up two sets of simple calculations, one optimistic and the other pessimistic. Such a sensitivity analysis provides reliability bounds and indicates whether a more detailed study would prove worthwhile. The bibliography listed in the Appendix explores this subject further. Reliability and MTBF can also be evaluated for K of N subsystems containing elements that cannot be repaired, as also described in the Appendix.*

Summary

This article presents the equations, code, and user instructions for a TI-59 calculator program that can estimate the overall reliability of a computer installation, identify specific subsystems most prone to failure, and show where additional redundancy would help the most.

The configuration design objective is to reach a specified satisfactory level of reliability at minimum cost. Excess reliability wastes dollars while excessive downtime jeopardizes operation and results in costly failures. This software tool predicts computer installation reliability and determines the level and elements at which replication efforts should be directed. The reliability analysis calculator program is easy to use and it addresses issues at a level of detail that makes efficient use of the computer designer's time. It permits interactive analysis with a response time of seconds, which allows the investigation of many prospective reconfigurations. Reliability alternatives, along with their cost tradeoffs, can then be explored for factual appraisal and decision making based on quantitative results.

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*The appendix to this article contains a discussion of types of computer system failures, simplifying modeling assumptions made in deriving reliability equations, a list of pertinent equations, detailed instructions for programming the TI-59 calculator, code for the SR-52 calculator, an illustrative trouble report, and an extensive bibliography. Interested readers may obtain a copy of this appendix by writing to: The Editor, *Computer Design*, 11 Goldsmith St, Littleton, MA 01460.