

## Computer Program

# A Fast Fourier Transform (FFT) Program for a Programmable Pocket Calculator TI 59

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### Summary

This paper presents an FFT-program written for the use of the programmable pocket calculator of Texas Instruments TI 59. The program consists of the following routines: Input/output; FFT forward and inverse; shuffling and a routine that converts the FFT-coefficients in case that real numbers were put in. The maximal number of input data is 32 complex or 64 real data. Listings of all routines are given and the program handling is described. Peak sharpening and removal of background (PARG) are examples to demonstrate the handling of routines.

### 1. Introduction

The widespread availability of digital computers and the development of an algorithm for a Fast Fourier Transform (FFT) by *Cooley and Tukey* (1965) have led to the FFT becoming a very powerful tool in a large sphere of applications. Several versions of the Cooley-Tukey algorithm have been published, and reviews are given by *Bergland* (1969) and *Singleton* (1969), among others. One of the shortest FFT-subroutines written in FORTRAN known to the author was given by *Ahmed and Rao* (1975). The possibility of high speed convolution and correlation, of digital filtering etc. with the aid of FFT led to applications in image processing and spectroscopy, and in this article, to XRMA with an EDS.

The program described in this paper is written for a programmable pocket calculator (TI 59, Texas Instruments) and is derived from the version of *Ahmed and Rao*. It can be helpful for those who want to become familiar with the principle of FFT or who have

no direct access to a computer but want to test the utility of FFT for a special problem (if the number of data is small enough). It can be used, for example, to test a correction procedure for a single spectral peak when programming a computer is too expensive.

### 2. Program structure

In all routines addressing is absolute to provide a fast execution. The memory of TI 59 is used in the following partitioning:

Memory No 000–239: Program registers 000–239

Memory No 959–240: Data registers 00–89

That means: block 2, 3 and 4 are reserved for data only. They can be read in from 3 sides of magnetic cards (or written to). The program routines are situated in block 1 and must be read in from one side of card. In this way the program can be exchanged optionally without affecting the content of data registers. This is very similar to the “common area” of FORTRAN.

A listing of 4 routines is given in Table 5.

1. Input and output of data (called “ENTER” and “BRING OUT”).  
There are separate routines for real and complex numbers.
2. FFT forward and inverse (called “DFT”).
3. Shuffling of FFT-coefficients (called “ARRANGE”).
4. Conversion of FFT-coefficients if real numbers were entered (called “CONVERT”).

DFT and CONVERT are supported by program 04 of the solid state standard software module “Master 1”. The output routines are written for the use of printer PC-100 A.

**3. Data capacity**

The program allows to transform a number of data which is integer power of 2 (radix 2). The maximum number of data points is 32 complex or 64 real numbers. The data are stored in registers starting at No 20. After DFT the coefficients are returned in the same registers. The registers 00–19 are required to store parameters.

**4. Execution time for N data points**

The execution time of the routines is tabulated in Table 1.

It can be seen that the execution time of DFT's is approximately proportional to  $N \ln N$ , those of shuffling and print proportional to  $N$ , only conversion is slightly less than proportional to  $N$ . It should be kept in mind that a faster print out is achieved by pressing the keys: 20 2nd list.

**5. Preparation of magnetic cards**

When one of the program parts listed above has been keyed in it must be written on a magnetic card.

Attention: Before writing or reading a card make sure that memory partitioning is of the right order! (Press: 2nd Op 16 239.89 must be displayed. If not: press 9 2nd Op 17)

The cards should be marked as shown in Fig. 1.

Naturally the system of marking will be varied to suit one's own convenience, and this arrangement suggested here is only a recommendation and oriented around the hints on the display and the synonyms (Table 2).

Table 1 Execution times

Complex data			
routine	execution time (s) for		
	N = 8	N = 16	N = 32
BRING OUT	25	50	95
DFT forward	70	180	430
ARRANGE	27	60	110
DFT inverse	72	185	448
ARRANGE	20	45	85

Real data			
routine	execution time (s) for		
	N = 16	N = 32	N = 64
BRING OUT	27	51	95
DFT forward	73	180	432
ARRANGE	28	55	115
CONVERT	32	55	107
DFT inverse	73	180	433
ARRANGE	21	40	82
BRING OUT	30	50	95

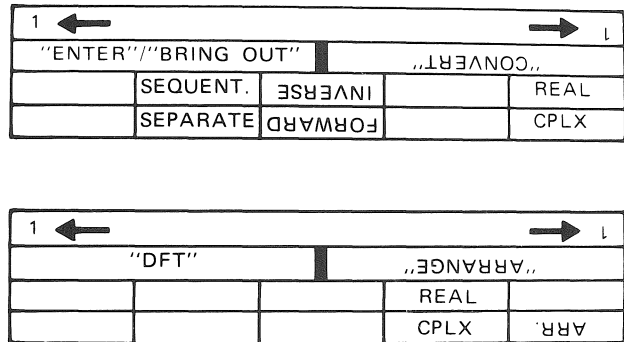


Fig. 1 Marking of magnetic cards

Table 2 Synonyms

Pgm. No.	user defined key	synonym	functions
0.1	E'	ENTER	Input, complex data
0.2	E	ENTER	Input, real data
0.3	B'	BRING OUT	Print out, complex data, real and imag. part separate
0.4	B	BRING OUT	Print out, sequential
1.1	D'	DFT	FFT forward, complex data
1.2	D	DFT	FFT forward, real data
1.3	SBR INV	DFT	FFT inverse
2	A	ARRANGE	Shuffling of FFT coeffs.
3.1	C	CONVERT	Conversion of FFT coeffs. after FFT forward
3.2	C'	CONVERT	Conversion of FFT coeffs. before starting FFT inverse

## 6. Program handling

Load one of the routines just described, paying attention to memory partitioning.

### 0. ENTER (input) and BRING OUT (output) routines

To permit a reliable data input the program consists of 2 parts. The display indicates in every case the index  $i$  of the data to be keyed in. Except when beginning an operation, the display "0" indicates that the operation is finished. An integer number displayed means that the real part of the data should be entered. Similarly, the input of the imaginary part is indicated by displaying "index of data.1".

#### 0.1.1 ENTER N complex data

Program start

Key in: Number of data to put in  
Press: 2nd E  
Display: "0"

Program execution

Key in: Real part of first data  $x_0$   
Press: R/S  
Display: "0.1"  
Key in: Imag. part of first data  $x_0$   
Press: R/S  
Display: "1"  
Key in: Real part of  $x_1$   
Press: R/S  
Display: "1.1"  
Key in: Imag. part of  $x_1$   
etc.

#### 0.1.2 ENTER 2 N real data

Program start

Key in: Half of the number of data to put in  
Press: 2nd E  
Display: "0"

Program execution

Key in:  $x_0$   
Press: R/S  
Display: "0.1"  
Key in:  $x_1$   
Press: R/S  
Display: "1"  
Key in:  $x_2$   
etc.

0.2 ENTER N complex data setting their imaginary parts generally to zero.

Program start

Key in: Number of data to put in  
Press: E  
Display: "0"

Program execution

Key in: Real part of  $x_0$   
Press: R/S  
Display: "1"  
Key in: Real part of  $x_1$   
Press: R/S  
etc.

The BRING OUT routines are written to perform – as far as possible – a comfortable print out. There are 2 types of output:

1. Data are printed in the same sequence as stored in the data registers. The index of data is printed at the right side of the paper. This method may be preferred to print out real data. It can be used for complex data also, but the right side index is no longer suitable.
2. The real and imaginary parts of the data are printed separately.

#### 0.3 Separate print of real and imaginary parts of data

Program start

Press: 2nd B  
Print: "R"  
"x<sub>0</sub>" (real part) "00"  
"x<sub>1</sub>" (real part) "01"  
.  
.  
.  
"x<sub>n</sub>" (real part) "n"  
"I"  
"x<sub>0</sub>" (imag. part) "00"  
"x<sub>1</sub>" (imag. part) "01"  
.  
.  
.  
"x<sub>n</sub>" (imag. part) "n"

#### 0.4 Sequential BRING OUT

Program start

Press: B  
Print: "CPLX"  
"x<sub>0</sub>" "00"  
"x<sub>1</sub>" "01"  
"x<sub>2</sub>" "02"  
etc.

### 1. DFT (FFT)

#### 1.1 DFT forward, complex data

Program start

Press: 2nd D  
Display: "2" indicating that calculation is completed and ARRANGE/A must follow

1.2 DFT forward, real data

Program start  
 Press: D  
 Display: "2" as above

1.3 DFT inverse

Program start  
 Press: SBR INV  
 Display: "2" as above

2. ARRANGE (shuffling)

Program start  
 Press: A  
 Display: "0" indicating the end of operations  
 or: "3.1" indicating that calculation is completed and CONVERT/C must follow

3. CONVERT (only if real data were entered)

3.1 CONVERT the result of DFT to DFT coefficients

Program start  
 Press: C  
 Display: "0" indicating the end of operations

3.2 CONVERT DFT coefficients of real data to prepare inverse DFT

Program start  
 Press: 2nd C  
 Display: "1.3" indicating that calculation is completed and DFT/SRB INV must follow

7. Sequence of operations

*Complex data, forward transform*

ENTER/2nd E; [optionally: BRING OUT/2nd B];  
 DFT/2nd D; ARRANGE/A; BRING OUT/2nd B.

*Complex data, inverse transform*

CONVERT/2nd C; DFT/SRB INV; ARRANGE/A;  
 BRING OUT/2nd B.

Real data can be transformed in two ways:

1. Real data are considered as complex data having imaginary parts equal to zero. In this case data can be entered according to 0.2.
2. Real data are entered in the data registers without regard to the fact that odd numbered registers are provided for the imaginary parts of complex data. This method needs an additional routine to convert the DFT coefficients. Nevertheless execution becomes faster and the maximum number of data points twice as large.

*Real data, forward transform*

ENTER/2nd E; [optionally: BRING OUT/B];  
 DFT/D; ARRANGE/A; CONVERT/C; BRING OUT/2nd B.

*Real data, inverse transform*

CONVERT/2nd C; DFT/SRB INV; ARRANGE/A;  
 BRING OUT/B.

The characters right of the slash are a reference to the key to be pressed.

8. An example of application

Table 3 and Fig. 2 present a part of a spectrum obtained from EDS by adding a pure background of Bremsstrahlung from a carbon target to a pure peak

Table 3 Count rate and current index of a part of an EDS spectrum

CPLX			
3726.	00	4340.	16
3714.	01	4171.	17
3734.	02	4248.	18
3672.	03	4066.	19
3668.	04	3947.	20
3770.	05	3899.	21
3720.	06	3897.	22
3678.	07	3777.	23
3724.	08	3771.	24
3794.	09	3771.	25
3893.	10	3636.	26
3976.	11	3690.	27
4093.	12	3667.	28
3988.	13	3681.	29
4120.	14	3658.	30
4151.	15	3725.	31

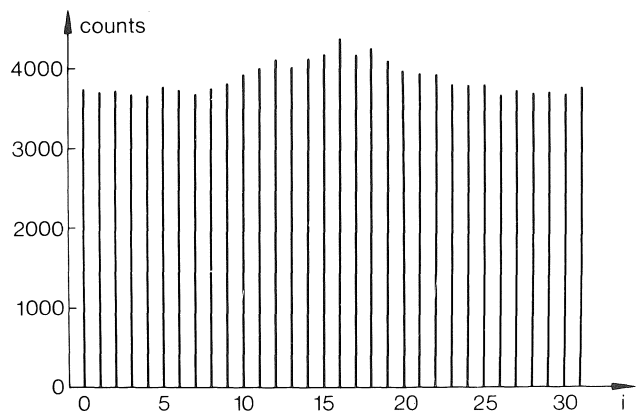


Fig. 2 Count rate and current index of a part of an EDS spectrum

generated by a Co<sub>57</sub>-source. The sum of counts in all channels of the peak (peak integral) is 5,118 counts, the integral of peak plus background numbers 123,365 counts. The energy-scale is replaced by running numbers, the index *i* (*i* = 0, 1, ..., *N*-1). The data are ordered so that the peak maximum is placed at *N*/2 and were entered in the calculator with the aid of routine "ENTER". 16 was keyed in and the program started by pressing "2nd E". Table 3 was printed out with "BRING OUT/B".

Thereafter "DFT" was loaded and executed by key "D" followed by "ARRANGE/A" and "CONVERT/C". The DFT coefficients now resident in the data registers were printed out by "BRING OUT/2nd B" as shown in Table 4. The two sets of numbers – indicated by "R" and "I" respectively – are the cosine- and sine-coefficients of the DFT.

Table 4 FFT coefficients of data points given in Table 3

R		I	
3855.15625	00	0.	00
-121.3611744	01	-3.517942454	01
53.66598451	02	-2.791983743	02
-6.582778884	03	-1.986192532	03
6.67428639	04	-7.712961078	04
-4.012319279	05	14.64237968	05
11.05892292	06	-7.206566881	06
-1.654167289	07	-4.407844716	07
.9375000003	08	-1.65625	08
-1.140785671	09	-4.591211761	09
-2.506645647	10	14.76063188	10
-4.92233112	11	.2923104881	11
4.95071361	12	-4.4629610788	12
1.081214375	13	2.531825377	13
9.156738217	14	1.050215014	14
-14.90765773	15	3.416777606	15
9.96875	16	0.	16

As the sine-coefficients of a symmetric curve are zero, a first step to suppress the non-symmetric part (background) can be to set all sine-coefficients to zero. To get a smoothed backtransform the cosine-coefficient of higher frequencies should be cut off. In this example all cosine-coefficients starting at *i* = 11 were set to zero (to keep the example simple no attempt was made to suppress side lobes). Finally, the higher frequencies of the remaining coefficients should be raised. This was done by multiplying the coefficients, by a factor *f<sub>i</sub>* (*f<sub>i</sub>* = 0, 0.1, 0.2, ..., 1.0 for *i* = 0, 1, ..., 10). This is comparable with high pass filtering in analogue technique. Now the modified coefficients were transformed back by loading "CONVERT" and starting the routine by pressing "2nd C". Next steps: "DFT/SRB INV", "ARRANGE/A" and printing by "BRING OUT/B". The result of this procedure is given in Fig. 3. This method is sometimes used for peak sharpening and to facilitate peak detection.

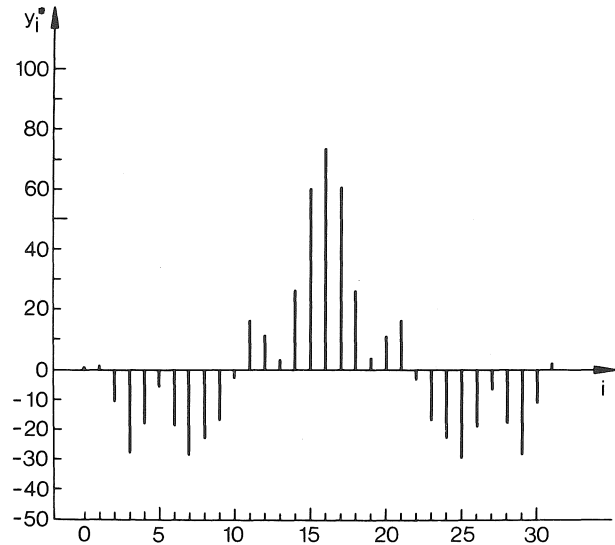


Fig. 3 Spectrum after high pass filtering and smoothing

This procedure however does not perform a quantitative separation of peak and background. To evaluate the peak integral use can be made of PARG (Schwarz 1977). This method of "Peak Analysis and Regeneration of a Gaussian Curve" is based on the following considerations:

A spectral peak should be of Gaussian form. The FFT coefficients of a Gaussian curve are also Gaussian.

If the peak follows the relation:

$$y_i = h e^{-\lambda (x_i - \bar{x})^2} \tag{1}$$

then the cosine-coefficients *a<sub>n</sub>* of DFT are given by:

$$|a_n| = \frac{h}{N} \sqrt{\frac{\pi}{\lambda}} e^{-\frac{n^2 \pi^2}{N^2 \lambda}} \tag{2}$$

It can be shown that the coefficients *a<sub>1</sub>* and *a<sub>2</sub>* are nearly unaffected by the background under convenient conditions (given in this example!). In such a case *a<sub>0</sub>* and the peak integral  $\Sigma y_i$  are easy to calculate:

$$a_0 = |a_1| \left| \frac{a_1}{a_2} \right|^{1/3} ; \quad \Sigma y_i = a_0 N \tag{3}$$

In the example under discussion we obtain:  
*a<sub>0</sub>* = 159.2966564 ;  $\Sigma y_i$  = 5097.5

Compared with the theoretical value the error is about -0.4 %.

Table 5 Program listings

Routines: 0.1 – 0.4 ENTER/BRING OUT

000	76	LBL	060	21	31	120	05	05	180	22	22
001	15	E	061	87	IFF	121	00	0	181	43	RCL
002	71	SBR	062	01	01	122	42	STD	182	02	DF
003	00	00	063	00	00	123	01	01	183	67	EQ
004	22	22	064	74	74	124	86	STF	184	02	EQ
005	43	RCL	065	93	.	125	05	05	185	05	05
006	02	02	066	01	.	126	01	1	186	97	DS2
007	91	R/S	067	44	SUM	127	00	0	187	00	00
008	72	ST*	068	02	02	128	00	0	188	01	01
009	01	01	069	86	STF	129	42	STD	189	62	62
010	02	2	070	01	01	130	33	03	190	25	INV
011	44	SUM	071	61	STD	131	01	1	191	97	DS2
012	01	01	072	00	00	132	42	STD	192	04	04
013	69	DP	073	54	54	133	02	02	193	02	02
014	22	22	074	93	.	134	02	2	194	37	37
015	97	DS2	075	01	1	135	00	0	195	69	DP
016	00	00	076	22	INV	136	44	SUM	196	00	00
017	00	00	077	44	SUM	137	01	01	197	02	2
018	05	05	078	02	02	138	43	RCL	198	04	4
019	00	0	079	69	DP	139	16	16	199	69	DP
020	31	R/S	080	22	22	140	42	STD	200	01	01
021	91	R/S	081	97	DS2	141	00	00	201	01	1
022	47	CHS	082	00	00	142	01	1	202	61	STD
023	42	STD	083	00	00	143	32	XIT	203	01	1
024	10	10	084	51	51	144	43	RCL	204	22	22
025	42	STD	085	00	0	145	11	11	205	87	IFF
026	00	00	086	91	R/S	146	22	INV	206	01	01
027	65	x	087	91	R/S	147	67	EQ	207	02	02
028	02	2	088	76	LBL	148	01	01	208	24	24
029	65	x	089	17	E*	149	53	53	209	01	1
030	42	STD	090	69	DP	150	44	SUM	210	42	STD
031	16	16	091	00	00	151	44	SUM	211	02	02
032	02	2	092	03	3	152	00	0	212	08	8
033	95	=	093	05	5	153	43	RCL	213	32	XIT
034	42	STD	094	69	DP	154	05	05	214	01	1
035	15	15	095	01	01	155	00	0	215	00	0
036	91	R/S	096	91	R/S	156	49	FRD	216	00	0
037	08	8	097	61	STD	157	00	0	217	44	SUM
038	95	=	098	01	01	158	08	8	218	03	03
039	42	STD	099	17	17	159	32	XIT	219	86	STF
040	17	17	100	76	LBL	160	98	ADV	220	01	1
041	02	2	101	12	8	161	69	DP	221	01	01
042	00	0	102	69	DP	162	05	05	222	01	01
043	42	STD	103	00	00	163	43	RCL	223	86	86
044	01	01	104	01	1	164	03	03	224	22	INV
045	92	RTN	105	05	5	165	85	+	225	86	STF
046	76	LBL	106	03	3	166	43	RCL	226	01	01
047	10	E*	107	03	3	167	02	02	227	01	01
048	71	SBR	108	02	2	168	95	=	228	00	0
049	00	00	109	07	7	169	69	DP	229	42	STD
050	22	22	110	04	4	170	04	04	230	02	02
051	97	DS2	111	04	4	171	73	RC+	231	03	03
052	86	STF	112	63	DP	172	01	01	232	05	5
053	01	01	113	00	00	173	69	DP	233	32	XIT
054	43	RCL	114	69	DP	174	06	06	234	61	STD
055	02	02	115	01	01	175	43	RCL	235	01	01
056	69	DP	116	44	SUM	176	05	05	236	00	0
057	72	ST*	117	42	STD	177	44	SUM	237	00	0
058	01	01	118	04	04	178	01	01	238	91	R/S
059	69	DP	119	42	STD	179	69	DP	239	91	R/S

Routines: 1.1 – 1.3 DFT

000	76	LBL	060	01	1	120	36	36	180	01	01
001	15	E	061	54	)	121	69	DP	181	06	6
002	01	1	062	65	x	122	37	37	182	04	4
003	94	+/-	063	43	RCL	123	73	RC+	183	00	0
004	42	STD	064	15	15	124	06	06	184	00	0
005	12	12	065	95	=	125	42	STD	185	01	1
006	42	STD	066	42	STD	126	01	01	186	03	3
007	11	11	067	03	03	127	95	+	187	03	3
008	61	STD	068	39	CHS	128	73	RC+	188	05	5
009	00	00	069	48	EXC	129	07	07	189	03	3
010	27	27	070	03	03	130	22	INV	190	05	5
011	76	LBL	071	38	SIN	131	44	SUM	191	69	DP
012	14	14	072	65	x	132	01	01	192	02	02
013	01	1	073	45	RCL	133	95	=	193	01	1
014	42	STD	074	12	12	134	72	ST*	194	03	3
015	11	11	075	95	=	135	06	06	195	03	3
016	42	STD	076	42	STD	136	38	FGH	196	01	1
017	42	STD	077	04	04	137	04	04	197	02	2
018	12	12	078	43	RCL	138	13	13	198	02	2
019	61	STD	079	10	10	139	72	ST*	199	01	1
020	00	00	080	42	STD	140	07	07	200	07	7
021	27	27	081	08	08	141	69	DP	201	00	0
022	76	LBL	082	53	(	142	27	27	202	00	0
023	22	INV	083	43	RCL	143	32	XIT	203	69	DP
024	01	1	084	08	08	144	72	ST*	204	03	03
025	42	STD	085	75	75	145	07	07	205	06	6
026	12	12	086	43	RCL	146	43	RCL	206	03	3
027	70	RD	087	13	13	147	13	13	207	01	1
028	43	RCL	088	85	+	148	75	=	208	03	3
029	10	10	089	43	RCL	149	01	1	209	00	0
030	42	STD	090	05	05	150	95	=	210	00	0
031	14	14	091	54	)	151	22	INV	211	00	0
032	23	LNK	092	65	x	152	44	SUM	212	00	0
033	55	+	093	02	2	153	08	08	213	00	0
034	02	2	094	85	+	154	97	DS2	214	00	0
035	23	LNK	095	01	1	155	08	08	215	69	DP
036	42	STD	096	09	9	156	00	0	216	00	04
037	42	STD	097	95	=	157	82	82	217	99	DP
038	00	00	098	42	STD	158	97	DS2	218	05	05
039	43	RCL	099	06	06	159	05	05	219	02	2
040	14	14	100	85	+	160	00	00	220	91	R/S
041	43	RCL	101	85	+	161	61	61	221	01	01
042	13	13	102	13	13	162	95	DS2	222	00	0
043	55	+	103	95	=	163	00	00	223	00	0
044	02	2	104	42	STD	164	00	00	224	00	0
045	95	=	105	07	07	165	39	39	225	00	0
046	42	STD	106	73	RC+	166	98	ADV	226	00	0
047	14	14	107	06	06	167	69	DP	227	00	0
048	42	STD	108	42	STD	168	00	00	228	00	0
049	05	05	109	02	02	169	03	3	229	00	0
050	55	=	110	85	+	170	01	01	230	00	0
051	97	DS2	111	01	01	171	01	01	231	00	0
052	07	07	112	07	07	172	07	07	232	00	0
053	35	1X	113	22	INV	173	04	4	233	00	0
054	42	STD	114	44	SUM	174	04	4	234	00	0
055	15	15	115	02	02	175	03	3	235	00	0
056	43	RCL	116	85	+	176	07	07	236	00	0
057	72	ST*	117	72	ST*	177	00	0	237	00	0
058	05	05	118	06	06	178	00	0	238	00	0
059	75	-	119	69	DP	179	69	DP	239	00	0

Routine: 2 ARRANGE

000	76	LBL	060	72	ST*	120	04	04	180	01	1
001	15	E	061	07	07	121	01	1	181	03	3
002	01	1	062	69	DP	122	04	04	182	04	04
003	94	+/-	063	27	27	123	13	13	183	69	DP
004	42	STD	064	73	RC+	124	74	SM+	184	02	02
005	11	11	065	06	06	125	06	06	185	06	06
006	65	x	066	42	STD	126	23	INV	186	03	03
007	76	LBL	067	01	01	127	74	SM+	187	03	3
008	13	C	068	75	-	128	07	07	188	06	6
009	01	1	069	73	RC+	129	69	DP	189	01	1
010	95	=	070	07	07	130	26	26	190	04	4
011	42	STD	071	44	SUM	131	69	DP	191	03	03
012	19	19	072	01	01	132	27	27	192	05	5

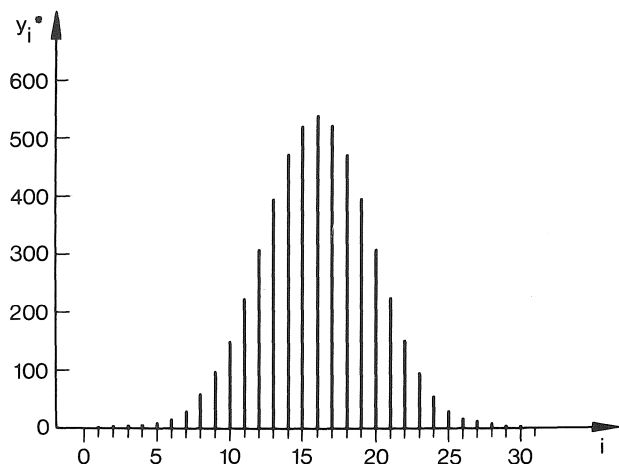


Fig. 4 Gaussian peak regenerated by PG

The total peak can be regenerated using the equation:

$$y_i^* = h e^{-\lambda(i - \bar{i})^2} \quad (i = 0, 1, \dots, N-1) \quad (\bar{i} = N/2) \quad (4)$$

$$\text{where } \lambda = \frac{3\pi^2}{N^2} \frac{1}{\ln \left| \frac{a_1}{a_2} \right|} \quad ; \quad h = a_0 N \sqrt{\frac{\lambda}{\pi}} \quad (5)$$

(see Fig. 4)

Of course the use of FFT does not seem to be very economical if only the first two cosine-coefficients are needed, but this is only intended to serve as an example.

## 9. Final remarks

It would be admirable if the manufacturer of the TI 59 calculator could create a solid state software module having versatile FFT and utility routines. As there would then be much less restriction in respect of the program length, the I/O routines would become more comfortable and the FFT could be accomplished for a mixed radix. This would result not only in quicker and more convenient handling but in a slightly greater number of data registers being available. Reloading of routines (5 in the example above) would be omitted.

## References

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