

AD A 096370

NAVAL POSTGRADUATE SCHOOL
Monterey, California



THESIS

DTIC
ELECTED
MAR 16 1981
S D
A

A STUDY OF MULTI-ECHELON
AND MULTI-LOCATION INVENTORY SYSTEM

by

Turgut Büyükkarhan

September 1980

Thesis Advisor:

F. Russell Richards

Approved for public release; distribution unlimited

DTIC FILE COPY

81316081

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE			READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER (6)	2. GOVT ACCESSION NO. AD-A096370	3. RECIPIENT'S CATALOG NUMBER	
4. TITLE (and Subtitle) A Study of Multi-Echelon and Multi-Location Inventory System.		5. TYPE OF REPORT & PERIOD COVERED Master's Thesis; September 1980	6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) Turgut/ G. E. Buyukkarhan	8. CONTRACT OR GRANT NUMBER(s)		
9. PERFORMING ORGANIZATION NAME AND ADDRESS Naval Postgraduate School Monterey, California 93940	10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS		
11. CONTROLLING OFFICE NAME AND ADDRESS Naval Postgraduate School Monterey, California 93940	12. REPORT DATE September 1980		
13. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office) Naval Postgraduate School Monterey, California 93940	14. SECURITY CLASS. (of this report) Unclassified		
15. DECLASSIFICATION/DOWNGRADING SCHEDULE (12) - 150			
16. DISTRIBUTION STATEMENT (of this Report) Approved for public release; distribution unlimited			
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)			
18. SUPPLEMENTARY NOTES			
19. KEY WORDS (Continue on reverse side if necessary and identify by block number) Multi-echelon, multi-location inventory system, costs, back orders.			
20. ABSTRACT (Continue on reverse side if necessary and identify by block number) The reorder point and reorder quantity for a multi-echelon inventory system consisting of two levels were determined through the use of a mathematical model and a computer simulation was used to verify the results. The main echelon supported two lower echelon stock points and reordered using a continuous review inventory policy. The two			

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE/When Data Entered:

lower echelon stock points operated under periodic review policies.

The measure of effectiveness used was to minimize total system costs subject to a constraint on the maximum number of back orders per year.

The results from the mathematical model were used as input values for the simulation and measures of effectiveness were compared. An alternative procedure was proposed and simulated; and the results of the three products were compared.

DD Form 1473
1 Jan 73
S/N 0102-014-6601

2 UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE/When Data Entered:

Approved for public release; distribution unlimited

A Study of Multi-Echelon
And Multi-Location Inventory System

by

Turgut Büyükkarhan
Lieutenant, Turkish Navy
B.S., Naval Postgraduate School, 1980

Submitted in partial fulfillment of the
requirements for the degree of

MASTER OF SCIENCE IN OPERATIONS RESEARCH

from the
NAVAL POSTGRADUATE SCHOOL
September 1980

Author

Büyükkarhan

Approved by:

J.R. Richards

Thesis Advisor

Ellen F. Roland

Second Reader

Neal T. Marshall

Chairman, Department of Operations Research

L. M. Woodward

Dean of Information and Policy Sciences

ABSTRACT

The reorder point and reorder quantity for a multi-echelon inventory system consisting of two levels were determined through the use of a mathematical model and a computer simulation was used to verify the results.

The main echelon supported two lower echelon stock points and reordered using a continuous review inventory policy. The two lower echelon stock points operated under periodic review policies.

The measure of effectiveness used was to minimize total system costs subject to a constraint on the maximum number of back orders per year.

The results from the mathematical model were used as input values for the simulation and measures of effectiveness were compared. An alternative procedure was proposed and simulated; and the results of the three products were compared.

TABLE OF CONTENTS

I.	INTRODUCTION-----	8
II.	A PROBABILISTIC MATHEMATICAL MODEL FOR MULTI-ECHELON INVENTORY SYSTEM-----	10
	A. DESCRIPTION OF MODEL-----	10
	B. OBJECTIVE OF MODEL-----	10
	C. ASSUMPTIONS OF PROBABILISTIC MATHEMATICAL MODEL-----	11
	1. Main System-----	11
	2. System One-----	13
	3. System Two-----	15
	D. ANALYTICAL SOLUTION TO THE MODEL-----	15
	1. Main System-----	16
	2. System One and System Two-----	19
	a. Ordering and Reviewing Costs-----	19
	b. Inventory Carrying Cost-----	19
	c. Stockout Costs-----	20
	3. Objective Function and Constraints-----	22
	4. Optimum Values of Operating Variables-----	23
	E. EXAMPLE PROBLEM-----	26
	1. Additional Assumptions-----	26
	2. Input Values-----	27
	a. Main System-----	28
	b. System One-----	28
	c. System Two-----	28
	3. Solution Procedure and Results-----	29
III.	COMPUTER SIMULATION FOR MULTI-ECHELON, MULTI-LOCATION SINGLE ITEM INVENTORY SYSTEMS-----	36
	A. DESCRIPTION OF THE SIMULATION MODEL-----	36
	1. Main System-----	36
	2. System One and System Two-----	38

B.	HOW TO USE THE PROGRAM-----	38
C.	RUNS-----	39
	1. Starting Conditions-----	39
	2. Results of Runs-----	40
	a. Main System-----	40
	b. System One-----	40
	c. System Two-----	41
	(1) Total Results-----	41
	(a) Main System-----	41
	(b) System One-----	42
	(c) System Two-----	42
	(2) Total Results-----	43
	3. Comparison of Both Starting Conditions-----	43
IV.	COMPARISON OF ANALYTICAL AND COMPUTER SIMULATION RESULTS-----	44
V.	AN ALTERNATIVE SOLUTION-----	49
	A. DESCRIPTION OF ALTERNATIVE PROCEDURE-----	49
	B. COMPUTER SIMULATION RESULTS FOR EARLY WARNING POLICY-----	50
	1. Main System Results-----	51
	2. System One Results-----	51
	3. System Two Results-----	51
	C. COMPARISON OF EARLY WARNING POLICY SIMULATION RESULTS WITH ANALYTICAL AND FIRST SIMULATION RESULTS-----	51
VI.	CONCLUSION-----	53
	APPENDIX A - Flowcharts of First Simulation Model-----	56
	APPENDIX B - Flowcharts of Early Warning Simulation Model-----	68
	APPENDIX C - Simulation Program; Versatec Plotter-----	80
	APPENDIX D - Simulation Program; No Versatec Plotter Output-----	101
	APPENDIX E - Early Warning Simulation Program-----	120

APPENDIX F - Versatec Output of the Program in Appendix C-	138
APPENDIX G - TI-59 Calculator Program for Analytical Solutions for Periodic Review Systems-----	141
APPENDIX H - Variable Definitions for Simulation Programs-	146
BIBLIOGRAPHY-----	148
INITIAL DISTRIBUTION LIST-----	149

I. INTRODUCTION

Despite the fact that all military supply systems and many large civilian corporations have multi-echelon and multi-location inventory systems, few multi-echelon inventory models are currently being used. The policies that have been used are the single echelon and single location inventory policies that attempt to minimize the total variable cost of a single location ignoring total system cost. The difficulties of optimizing a multi-echelon and multi-location inventory system are most likely due to the complexity of demand and the interdependency among the units in different echelons.

A mathematical model for a multi-echelon and multi-location inventory system is developed in Chapter II. The objective of this model is to minimize the total annual variable costs subject to a maximum allowable number of back orders per year for the entire system. This model consists of one first echelon location operating under a continuous review policy and two second echelon systems which operate under periodic review policies. An example is presented to obtain numerical results.

In Chapter III, a computer model is developed to simulate the multi-echelon, multi-location system. This simulation model was used to check the results obtained from

the mathematical model and to demonstrate the interaction between echelons. Graphical plots are generated which show the inventory position of each entity in the model.

In Chapter IV, comparisons are made of the simulation results and the results obtained from the mathematical model. The comparisons suggest some modifications to the operating policy.

The system was simulated again using the modified operating policy and substantial improvements in the effectiveness of the multi-echelon system were observed. These results are described in Chapter V.

Chapter VI summarizes the results of the research and concludes with some suggestions for additional research.

III. A PROBABILISTIC MATHEMATICAL MODEL FOR MULTI-ECHELON INVENTORY SYSTEM

A. DESCRIPTION OF MODEL

In this model there are three systems. One of these systems represents the highest echelon and is called the Main System. The other two, at separate locations, are the lower echelon and are called System One and System Two, respectively. Each system carries its own inventory, and receives random demands. System One and System Two are dependent upon the Main System but independent of each other. System One and System Two can only be resupplied by the Main System. In other words, they cannot order from any other suppliers. The Main System replenishes stocks by placing orders to external suppliers.

B. OBJECTIVE OF MODEL

The objective of this model is to minimize the total expected annual variable costs of three systems subject to a specified expected number of total back orders per year. In fact, this is the same as minimization of total yearly variable costs subject to a specified minimum level of customer satisfaction.

Based on these objectives, decision variables for each system will be calculated to achieve optimum levels for the entire multi-echelon supply system.

C. ASSUMPTIONS OF THE PROBABILISTIC MATHEMATICAL MODEL

1. Main System

It is assumed that the Main System under consideration consists of a single installation which utilizes transaction reporting. This system is governed by a (Q, r) type policy with back orders.

The assumptions in addition to the continuous review assumption are:

- a. The cost of operating the information processing system is independent of Q (reorder quantity) and r (reorder point).
- b. The unit cost C of the item is a constant independent of Q .
- c. The back-order cost is constant (Π), per unit back ordered regardless of the length of time the back-orders exist.
- d. There is never more than a single order outstanding. This assumption implies that when the reorder point is reached, there are no orders outstanding; therefore, the inventory position is equal to the net inventory. Thus, the reorder point will be the same regardless of whether it is based on the inventory position or net inventory.
- e. Procurement lead times are independent and identically distributed (i.i.d) random variables with a gamma distribution.

- f. All variables are treated as continuous.
- g. The demands are Poisson distributed with the mean number of demands per year a constant λ_m .
- h. The reorder point, r , which is based on the inventory position is positive.

With a back orders constraint, it is infeasible to wait until back orders exist before placing an order. Because of this and assumption (d) there will be no back orders outstanding at the reorder point. As was discussed in assumption (d), at the reorder point, the inventory position is equal to the on-hand inventory.

For this model, any one of the three inventory levels on hand, net, or inventory position can be used to define the reorder point; and the reorder point has the same value for any one of them. It should also be noted that to use the on-hand level it must be assumed that after an order arrives, it is sufficient to fill all back orders and raise the on-hand inventory level above the reorder point. If this ever failed to happen, the reorder point would never be reached again and the system would continue to accumulate back orders indefinitely.

When the reorder point is thought of in terms of the inventory position of the system, then assumption (d) guarantees that the on-hand inventory will always exceed the reorder point when an order arrives; otherwise, it would not be possible to have only a single order outstanding.

2. System One

This system, which is at the lower echelon, is governed by a periodic review policy. The operating doctrine is the most widely used type of periodic review which is the order up to R policy. All demands which occur when the system is out of stock are back ordered.

For this periodic review system the time between reviews will be denoted by T, and at each review time a sufficient quantity is ordered to bring the inventory position of the system up to a level, R, regardless of the amount of on-hand inventory. This policy dictates that at review times even if the inventory position is R-1, only one item must be ordered to bring the inventory position up to R, ignoring the high cost of placing the order. Despite its appearance of being illogical, this assumption simplifies the formulation of the system and is very unlikely to occur on high demand items. When an item experiences zero demand in a review period, there is no need for order because the inventory position is already at R.

The other assumptions are:

- a. The cost, J, of making a review is independent of the variables R and T.
- b. The unit cost, C, of the item is constant and independent of the quantity ordered.

81316081

c. Back orders are incurred only in very small quantities. This implies that when an order arrives, it is almost always sufficient to meet any outstanding back orders.

d. The backorder cost is constant, Π , per unit back ordered regardless of the length of time the back order exists.

e. Procurement lead times are i.i.d random variables with a gamma distribution.

f. Orders are received in the same sequence in which they were placed. It should be noted that for (Q, r) models, the two assumptions that orders were received in the sequence placed and that lead times are i.i.d random variables could not both hold rigorously, since there exists a positive probability that two successive orders could be separated by an arbitrarily short time interval. In this model, orders can never be more closely spaced than by an interval of length T . If T is large enough, it is possible, provided that there is a sufficiently small range of variation in the lead time, that both assumptions hold simultaneously.

g. The demands are Poisson distributed with mean, λ .

h. All variables are treated as continuous.

3. System Two

This system is identical to, but independent of, System One.

D. ANALYTICAL SOLUTION TO THE MODEL

As was mentioned previously, the objective of the model is to minimize total system costs subject to a constraint on the number of back orders per year.

The determination of various annual costs of each system may be made independently and placed into a common cost formula. Instead of doing this independently, it is better to find a main system cost expression, System One cost expression and System Two cost expression, and then to add them to each other to determine the total cost formulation and then use this formulation as the objective function of total system. The systems are actually tied together through the constraint on the number of back orders.

For the following notation, the subscript m will indicate that the variables being subscripted belong to the Main System where 1 and 2 indicate System One and System Two, respectively.

The costs that are of interest in this model for each system are the cost of placing an order, the cost of carrying inventory and back order costs.

1. Main System

As was discussed previously, the Main System has a transactions reporting policy or (Q, r) model.

In a continuous review system, a period is defined as the length of time between the receipt of two successive procurements. This time period is a random variable. Because the procurement lead times are random variables, the number of demands for a fixed time are random variables; and the number of items demanded per demand are also random variables. The review period is also a random variable.

In the following material, we itemize the costs.

a. Procurement Cost

λ_m = Expected number of demands per year

Q_m = Order quantity

$\frac{\lambda_m}{Q_m}$ = Average number of procurements per year

A_m = Procurement cost per cycle

$\frac{\lambda_m}{Q_m} A_m$ = Procurement cost per year

b. Inventory Carrying Costs

Because of randomness, it is possible to accumulate a large number of back orders at the end of a cycle. To prevent this from occurring, it is advisable to provide a safety stock to buffer the system from excessive numbers

of back orders. The safety stock is the expected amount of stock on hand when an order arrives. The actual amount of stock on hand when a shipment arrives is clearly random.

S_m = Mean value of on-hand stock when an order arrives

After an order arrives, the expected on-hand inventory increases to $Q+S$ and is reduced to a value of S on the average just before the next order arrives. Therefore, the average on-hand inventory per cycle is:

$$\frac{(Q_m + S_m)}{2} + \frac{S_m}{2} = \frac{Q}{2} + S_m$$

To write S_m in terms of the reorder point, r_m , let us first assume that the lead time τ_m is fixed. Let

$\xi_{\tau}(x; r_m) = r_m - x$ be the net inventory at the time an order arrives,

where x

is the number of units demanded in lead time τ_m . Then,

$$S_m = E_{\tau_m} [\text{Net inventory}] = \int_0^{\infty} \xi_{\tau_m}(x; r_m) f(x; \tau_m) dx$$

where $f(x; \tau_m)$ = Density function of demand in time τ_m

$$S_m = \int_0^{\infty} (r_m - x) f(x; \tau_m) dx = r_m \int_0^{\infty} f(x; \tau_m) dx - \int_0^{\infty} x f(x; \tau_m) dx$$

$$S_m = r_m - \mu_m \quad \text{where } \mu_m = \text{Expected lead time demand.}$$

$$\text{Then } \frac{Q_m}{2} + S_m = \frac{Q_m}{2} + r_m - \mu_m$$

I = Inventory carrying charge

C = Cost of an item

$$\text{Total holding cost/year} = IC (r_m - \mu_m + \frac{Q_m}{2})$$

c. Stockout Costs

$$\text{Let us define } n_{\tau_m}(x; r_m) = \begin{cases} 0 & \text{if } x - r_m < 0 \\ x - r_m & \text{if } x - r_m \geq 0 \end{cases}$$

where $n_{\tau_m}(x; r_m)$ is the number of back orders per cycle.

If $\bar{n}_m(r_m)$ = expected number of back orders per cycle, then

$$\bar{n}_m(r_m) = \int_0^\infty n_{\tau_m}(x; r_m) h(x) dx = \int_0^\infty (x - r_m) h(x) dx$$

$$= \int_{r_m}^\infty x h(x) dx - r_m H(r_m) \text{ where } H(r_m) = P[X > r_m]$$

and $h(x)$ = marginal distribution of leadtime demand.

Therefore, the expected number of back orders/year =

$$\frac{\lambda_m}{Q_m} [r_m \int_{r_m}^\infty x h(x) dx - r_m H(r_m)]$$

and the expected cost of back orders/year =

$$\frac{\Pi_m \lambda_m}{Q_m} [r_m \int_{r_m}^\infty x h(x) dx - r_m H(r_m)]$$

All the terms in the average annual variable cost K_m have now been found:

$$K_m = \frac{\lambda_m}{Q_m} A_m + IC \left[\frac{Q_m}{2} + r_m - \mu_m \right] + \frac{\pi \lambda_m}{Q_m} \left[r_m \int_{r_m}^{\infty} x h(x) dx - r_m H(r_m) \right]$$

2. System One and System Two

It was stated in the assumptions that System One and also System Two both follow a periodic review policy with an order up to R stockage policy. Since System One and System Two are identical to each other; the equations will be derived only for System One.

For convenience, a period is assumed to be the time between the receipt of two successive orders rather than between the placement of two successive orders. Costs are described as follows:

a. Ordering and Reviewing Costs

J = Reviewing cost/cycle

A = Ordering cost/cycle

$$\text{Ordering and reviewing cost/year} = \frac{J_1 + A_1}{T_1}$$

where T_1 is the period length defined in units of years.

b. Inventory Carrying Cost

The expected net inventory just prior to the arrival of an order is $R_1 - \mu_1 - \lambda_1 T_1$, where μ_1 = mean demand during lead time.

The mean rate of demand remains constant over time and the expected demand per period must be the expected

amount ordered, i.e., $\lambda_1 T_1$. If the expected net inventory immediately after the arrival of a procurement is $R_1 - \mu_1$, it is therefore $R_1 - \mu_1 - \lambda_1 T_1$ just prior to the arrival of a procurement.

The expected unit years of storage incurred per period is

$$T_1 \left[\frac{1}{2} (R_1 - \mu_1) + \frac{1}{2} (R_1 - \mu_1 - \lambda_1 T_1) \right] = T_1 [R_1 - \mu_1 - \frac{\lambda_1 T_1}{2}]$$

and average inventory carrying cost/year = $IC [R_1 - \mu_1 - \frac{\lambda_1 T_1}{2}]$.

c. Stockout Costs

First we assume the case where the procurement lead time is constant τ . An order placed at time t will arrive in the system at time $t + \tau$, and the next procurement will arrive in the system at time $t + \tau + T_1$. After the order is placed at time t , the inventory position of the system is R_1 . It is necessary to compute the expected number of back orders occurring between $t + \tau$ and $t + \tau + T_1$. A back order will occur in this period under assumption c if and only if the demand in the time period $\tau + T_1$ exceeds R_1 . Assumption c also assures that after the arrival of the order placed at time t , there will be no remaining back orders, and therefore they must all occur between times $t + \tau$ and $t + \tau + T_1$. Consequently the expected number of back orders incurred per period is

$$R_1 \int_{R_1}^{\infty} (x - R_1) f(x; \tau + T_1) dx \text{ where}$$

$f(x; \tau + T_1)$ = Demand distribution during time $\tau + T_1$.

When lead time is random with density $g(\tau_1)$

with τ_{\min} and τ_{\max} being lower and upper limits respectively and τ_1 and τ_2 , the lead times for the orders placed at times t and $t + T_1$, respectively, the expected number of back orders incurred per period is:

$$\begin{aligned} & \int_{\tau_{\min}}^{\tau_{\max}} \int_{\tau_{\min}}^{\tau_{\max}} R_1 \int_{R_1}^{\infty} (x - R_1) f(x; \tau_2 + T_1) g(\tau_2) g(\tau_1) dx d\tau_2 d\tau_1 \\ &= R_1 \int_{R_1}^{\infty} (x - R_1) \hat{h}(x; T_1) dx \end{aligned}$$

where $\hat{h}(x; T_1) = \int_{\tau_{\min}}^{\tau_{\max}} f(x; \tau_2 + T_1) g(\tau_2) d\tau_2$ which is the demand distribution during time $\tau_2 + T_1$ when lead time is a random variable with density function $g(\tau_2)$. The average number of back orders incurred per year is;

$$E_1(R_1, T_1) = \frac{1}{T_1} R_1 \int_{R_1}^{\infty} (x - R_1) \hat{h}(x; T_1) dx \text{ and the average}$$

back order cost per year equals to $\Pi_1 E_1(R_1, T_1)$ and

$\bar{n}_1(r) = T_1 E_1(R_1, T_2)$ is the expected number of back orders per period.

Finally, the annual variable cost of System One
is:

$$K_1 = \frac{L_1}{T_1} + IC [R_1 - \mu_1 - \frac{\lambda_1 T_1}{2}] + \Pi_1 E_1 (R_1, T_1) \text{ and}$$

likewise the annual variable cost of System Two is:

$$K_2 = \frac{L_2}{T_2} + IC [R_2 - \mu_2 - \frac{\lambda_2 T_2}{2}] + \Pi_2 E_2 (R_2, T_2)$$

where $L_i = J_i + A_i$.

3. Objective Function and Constraints

The objective function of the model consists of the total annual variable costs of each system. Therefore;

$K = K_m + K_1 + K_2$ which is equal to the minimization
of;

$$\begin{aligned} K = & \frac{\lambda_m}{Q_m} A_m + IC [\frac{Q_m}{2} + r_m - \mu_m] + \frac{\Pi_m \lambda_m}{Q_m} \bar{\eta}_m (r) + \frac{L_1}{T_1} \\ & + IC [R_1 - \mu_1 - \frac{\lambda_1 T_1}{2}] + \frac{\Pi_1}{T_1} \bar{\eta}_1 (r) + \frac{L_2}{T_2} + IC [R_2 - \mu_2 - \frac{\lambda_2 T_2}{2}] \\ & + \frac{\Pi_2}{T_2} \bar{\eta}_2 (r) \end{aligned}$$

subject to $\frac{\lambda_m}{Q_m} \bar{\eta}_m (r) + \frac{1}{T_1} \bar{\eta}_1 (r) + \frac{1}{T_2} \bar{\eta}_2 (r) \leq b$

where b is the specified total maximum number of back orders per year for the entire system.

The problem at hand is to calculate the optimum values of Q_m , r_m , R_1 , T_1 , R_2 , T_2 . Since the objective function and the constraint are non-linear functions of the

decision variables, we solve the problem using the Lagrange multiplier approach.

4. Optimum Values of Operating Variables

After including the Lagrange multiplier in the formulation, the new objective function becomes:

Minimize

$$\begin{aligned} L = & \frac{\pi_m}{Q_m} A_m + IC\left[\frac{Q_m}{2} + r_m - \mu_m\right] + \frac{\Pi_m \lambda_m}{Q_m} \bar{\eta}_m(r) + \frac{L_1}{T_1} \\ & + IC[R_1 - \mu_1 - \frac{\lambda_1 T_1}{2}] + \frac{\Pi_1}{T_1} \bar{\eta}_1(r) + \frac{L_2}{T_2} + IC[R_2 - \mu_2 - \frac{\lambda_2 T_2}{2}] \\ & + \frac{\Pi_2}{T_2} \bar{\eta}_2(r) - \theta\left[\left(\frac{\lambda_m}{Q_m} \bar{\eta}_m(r) + \frac{1}{T_1} \bar{\eta}_1(r) + \frac{1}{T_2} \bar{\eta}_2(r)\right) - b\right] \end{aligned}$$

where θ is the Lagrange multiplier. It is through this Lagrange multiplier that the three systems are linked together mathematically.

The optimum values of the unknown variables can be found by taking the derivatives of the objective function with respect to Q_m , R_m , R_1 , T_1 , R_2 , T_2 , θ ; equating them to zero; solving the equations simultaneously; and ensuring the Kuhn-Tucker conditions are satisfied.

The derivatives were taken with respect to Q_m , r_m , R_1 , R_2 , θ . A different procedure was utilized to find T_1 and T_2 .

The derivatives:

$$L_{Q_m} = \frac{\partial L}{\partial Q_m} = - \frac{\lambda_m A_m}{Q_m^2} + \frac{IC}{2} - \frac{\lambda_m}{Q_m^2} \bar{n}_m(r) (\Pi_m - \theta) = 0$$

$$L_{r_m} = \frac{\partial L}{\partial r_m} = IC - \frac{\lambda_m}{Q_m} \hat{H}_m(r) (\Pi_m - \theta) = 0$$

$$L_{R_1} = \frac{\partial L}{\partial R_1} = IC - \frac{1}{T_1} H_1(R_1, T_1) (\Pi_1 - \theta) = 0$$

$$L_{R_2} = \frac{\partial L}{\partial R_2} = IC - \frac{1}{T_2} H_2(R_2, T_2) (\Pi_2 - \theta) = 0$$

$$L_\theta = \frac{\partial L}{\partial \theta} = [(\frac{\lambda_m}{Q_m} \bar{n}_m(r) + \frac{1}{T_1} \bar{n}_1(r) + \frac{1}{T_2} \bar{n}_2(r)) - b] = 0$$

The Kuhn-Tucker conditions:

$$L_{Q_m} \geq 0 \quad Q_m \cdot L_{Q_m} = 0 \quad Q_m \geq 0 \quad g \leq 0$$

$$L_{r_m} \geq 0 \quad r_m \cdot L_{r_m} = 0 \quad r_m \geq 0 \quad \theta \cdot g = 0$$

$$L_{R_1} \geq 0 \quad R_1 \cdot L_{R_1} = 0 \quad R_1 \geq 0 \quad \theta \leq 0$$

$$L_{R_2} \geq 0 \quad R_2 \cdot L_{R_2} = 0 \quad R_2 \geq 0$$

where $g = \frac{\lambda_m}{Q_m} \bar{n}_m(r) + \frac{1}{T_1} \bar{n}_1(r) + \frac{1}{T_2} \bar{n}_2(r) - b$

Therefore the following Kuhn-Tucker conditions must be satisfied:

$$-\frac{\lambda_m A_m}{Q_m^2} + \frac{IC}{2} - \frac{\lambda_m}{Q_m} \bar{\eta}_m(r) (\Pi_m - \theta) \geq 0$$

$$IC - \frac{\lambda_m}{Q_m} \hat{H}_m(r) (\Pi_m - \theta) \geq 0$$

$$IC - \frac{1}{T_1} \hat{H}_1(R_1, T_1) (\Pi_1 - \theta) \geq 0$$

$$IC - \frac{1}{T_2} \hat{H}_2(R_2, T_2) (\Pi_2 - \theta) \geq 0$$

$$-\frac{\lambda_m A_m}{Q_m} + \frac{IC Q_m}{2} - \frac{\lambda_m}{Q_m} \bar{\eta}_m(r) (\Pi_m - \theta) = 0$$

$$IC r_m - \frac{\lambda_m r_m}{Q_m} \hat{H}_m(r) (\Pi_m - \theta) = 0$$

$$IC R_1 - \frac{R_1}{T_1} \hat{H}_1(R_1, T_1) (\Pi_1 - \theta) = 0$$

$$IC R_2 - \frac{R_2}{T_2} \hat{H}_2(R_2, T_2) (\Pi_2 - \theta) = 0$$

$$Q_m \geq 0$$

$$r_m \geq 0$$

$$R_1 \geq 0$$

$$R_2 \geq 0$$

$$\frac{\lambda_m}{Q_m} \bar{\eta}_m(r) + \frac{1}{T_1} \bar{\eta}_1(r) + \frac{1}{T_2} \bar{\eta}_2(r) - b \leq 0$$

$$\theta [\frac{\lambda_m}{Q_m} \bar{\eta}_m(r) + \frac{1}{T_1} \bar{\eta}_1(r) + \frac{1}{T_2} \bar{\eta}_2(r) - b] = 0$$

$$\theta \leq 0$$

From these equations and conditions the optimum values of the unknown variables can be simplified to the following form and the exact values can be found by solving these equations simultaneously:

$$\hat{H}_m(r) = \frac{IC Q_m}{(\Pi_m - \theta) \lambda_m}$$

$$Q_m = \frac{\sqrt{2\lambda_m [A_m + \bar{\tau}_m (\Pi_m - \theta)]}}{IC}$$

$$\hat{H}_1(R_1, T_1) = \frac{IC T_1}{(\Pi_1 - \theta)}$$

$$\hat{H}_2(R_2, T_2) = \frac{IC T_2}{(\Pi_2 - \theta)}$$

The procedure to find T_1 and T_2 and subsequently R_1 and R_2 is through iteration and trial and error. This is not unreasonable. In realistic cases other considerations not modelled here usually dictate the length of a period. In most cases the systems control a large number of different items and the same period length is used for each item. Thus, the period length is usually some convenient calendar or financial period, such as a month or a quarter. The procedure we use is shown in the following example.

E. EXAMPLE PROBLEM

1. Additional Assumptions

For this example it is assumed that each system has Poisson arrivals independent of each other. The customers

demand only one item at a time. System One and System Two are resupplied only from the main system and the main system is resupplied from outside suppliers. To be more realistic it is also assumed that the procurement lead times have a gamma distribution with different mean and variances.

The λ 's will be assumed daily demand or arrival rate per day rather than yearly values.

2. Input Values

Before providing the input values, some clarifications must be provided regarding the demand at the main system. When it is stated that $\lambda_m = 4/\text{day}$, $\lambda_1 = 3/\text{day}$ and $\lambda_2 = 1/\text{day}$. These represent the mean number of demands which arrive each day directly at the respective systems. However, since all demands at Systems One and Two eventually must filter up to the main system, the cumulative demand at the main system has an expected value of $\lambda_1 + \lambda_2 + \lambda_m = 8$. The main system supports not only Systems One and Two, but also has its own customer demands. The model assumes that the demand at the main system is the superposition of the direct demands at the main system and Systems One and Two. However, since the actual demands placed on the main system in the lower echelon are batched, the demand variability is much greater than would be expected by the superposition process. We will evaluate the seriousness of our assumption about the demand process at the main system with the simulation model described in the next chapter.

In this example the input values for each system
are;

a. Main System

λ_m = 8/day (considering the individual customer
arrivals and the remainder being Systems
One and Two average demands)

C = \$50/item

I = 0.23

Π_m = \$5

A_m = \$500

b. System One

λ_1 = 3/day

C = \$50/item

I = 0.23

Π_1 = \$5

A_1 = \$100

J_1 = \$100

c. System Two

λ_2 = 1/day

C = \$50/item

I = 0.23

Π_2 = \$5

A_2 = \$100

J_2 = \$100

The total system is expected not to exceed
 $b = 175$ total number of back orders at the end of a year.

3. Solution Procedure and Results

It was assumed that for each system, procurement lead times are gamma distributed random variables.

The gamma distribution is defined as:

$$g(t) = \frac{1}{\Gamma(\alpha)} \frac{t^{\alpha-1}}{\beta^\alpha} e^{-\frac{t}{\beta}} \quad \text{with}$$

$$\mu = \alpha \cdot \beta \text{ and}$$

$$\text{Var} = \alpha \cdot \beta^2$$

We need the lead-time demand distribution for the case in which lead times are gamma distributed and the process generating demands is Poisson. This is derived below.

The Poisson distribution is

$$f(x) = \frac{(\lambda t)^x e^{-\lambda t}}{x!} \quad \text{and the demand distribution}$$

during lead time is:

$$\begin{aligned} f(x; \tau) &= \int_0^\infty f(x) g(t) dt \\ &= \int_0^\infty \frac{(\lambda t)^x e^{-\lambda t}}{x!} \frac{t^{\alpha-1}}{\Gamma(\alpha)} \frac{e^{-t/\beta}}{\beta^\alpha} dt \\ &= \frac{\lambda^x}{\Gamma(\alpha) \beta^\alpha x!} \int_0^\infty t^{\alpha-1 + x} e^{-(\frac{1}{\beta} + \lambda)t} dt \end{aligned}$$

$$= \frac{\lambda^x}{\Gamma(\alpha) \beta^\alpha x!} \cdot \frac{\Gamma(\alpha+x)}{(\frac{1}{\beta} + \lambda)^{\alpha+x}} \int_0^\infty \frac{(\frac{1}{\beta} + \lambda)^{\alpha+x} t^{\alpha-1+x} e^{-(\frac{1}{\beta} + \lambda)t}}{\Gamma(\alpha+x)} dt$$

$$\int_0^\infty \frac{(\frac{1}{\beta} + \lambda)^{\alpha+x} t^{\alpha-1+x} e^{-(\frac{1}{\beta} + \lambda)t}}{\Gamma(\alpha+x)} dt = 1 \quad \text{so}$$

$$f(x; \tau) = \frac{\lambda^x}{\Gamma(\alpha) \beta^\alpha x!} \cdot \frac{\Gamma(\alpha+x)}{(\frac{1}{\beta} + \lambda)^{\alpha+x}}$$

$$= \frac{\Gamma(\alpha+x)}{\Gamma(\alpha) x!} \left[\frac{\lambda}{(\frac{1}{\beta} + \lambda)} \right]^x \left[\frac{\frac{1}{\beta}}{\frac{1}{\beta} + \lambda} \right]^\alpha$$

$$= \binom{\alpha+x-1}{x} \left[\frac{\lambda}{\frac{1}{\beta} + \lambda} \right]^x \left[\frac{\frac{1}{\beta}}{\frac{1}{\beta} + \lambda} \right]^\alpha$$

$$= \binom{\alpha+x-1}{x} (1-\rho)^x \rho^\alpha$$

$$\mu = \frac{\alpha(1-\rho)}{\rho} = \frac{\alpha(1 - \frac{1/\beta}{1/\beta + \lambda})}{\frac{1/\beta}{1/\beta + \lambda}} = \lambda \alpha \beta$$

$$\text{Var} = \frac{\alpha(1-\rho)}{\rho^2} = \frac{\alpha(1 - \frac{1/\beta}{1/\beta + \lambda})}{\frac{1/\beta}{1/\beta + \lambda}} = \lambda \alpha \beta (1 + \lambda \beta)$$

In fact this is a Negative Binomial distribution

with

$$\rho = \frac{1/\beta}{1/\beta + \lambda} .$$

The parameters α and β for each system are:

$$\alpha_m = 12.8 \quad \alpha_1 = 5.43 \quad \alpha_2 = 2.5$$

$$\beta_m = 3.125 \quad \beta_1 = 4.6 \quad \beta_2 = 6.$$

The main system has a Negative Binomial distributed lead time demand with parameters:

$$\mu_m = 8 \cdot (12.8) (3.125) = 320$$

$$var_m = 8 \cdot (12.8) (3.125) [1+8 \cdot (3.125)] = 8320$$

System One has the parameters:

$$\mu_1 = 3 \cdot (5.43) \cdot (4.6) = 74.934$$

$$var_1 = 3 \cdot (5.43) (4.6) [1+3(4.6)] = 1109.0232$$

System Two has the parameters:

$$\mu_2 = 1 \cdot (2.5) \cdot 6 = 15$$

$$var_2 = 1 \cdot (2.5) \cdot 6 [1+1(6)] = 105$$

Because the negative binomial is computationally intractable, we use the normal approximation for the calculation of lead time demand probabilities. The normal distributions are assumed to have the same mean and variance as the negative binomial distributions they replace.

To solve this example problem, it is necessary to make an initial estimate for the Lagrange multiplier λ .

which must be less than or equal to zero. After the initial estimate if the total number of back orders per year exceeds b, then the absolute value of θ should be increased gradually until the number of back orders converges to b.

For this problem $\theta = -1.5$ works very well. For the main system the formulas are:

$$Q_m = \sqrt{2\lambda_m [A_m + \bar{\eta}_m(r)(H_m - \theta)] / IC}$$

$$H_m(r) = \frac{Q_m IC}{(H_m - \theta)\lambda_m} = P [X > r_m]$$

Q_m is calculated to be 550 and

$$H_m(r) = P [X > r_m] = \frac{550 (0.23) \cdot 50}{(5+1.5) 2920} = 0.3332455216$$

From the inverse standard normal distribution TI-59 calculator program (Appendix G), the reorder level at the main system is found to be:

$$r_m = \sigma_m \cdot (0.4305333395) + \mu_m = 359.2706827$$

$$\bar{\eta}_m(r) = (\mu_m - r_m) H_m(r) + \sigma_m \varphi\left(\frac{r_m - \mu_m}{\sigma_m}\right)$$

where φ is the functional value of standard normal distribution at 0.4305333395

$\bar{\eta}_m(r) = 20.08139334$. Using this value in the Q_m formula yields

$$Q_m = \sqrt{2.2920 [500 + 20.08139334 (5 + 1.5)] / (0.23) \cdot 50} = 568.84$$

which is not equal to the first estimated value. If Q_m is taken 569 then:

$$H_m(r) = \frac{569 (0.23) \cdot 50}{(5 + 1.5) 2920} = 0.3447576396$$

$$r_m = \sigma_m (0.399071201) + \mu_m = 356.4008941$$

$$\bar{\eta}_m(r) = 21.05458646$$

Using this value in the Q_m formula yields

$$Q_m = \sqrt{2.2920 [500 + 21.05438646 (5 + 1.5)] / (0.23) \cdot 50} = 568.69$$

which is very close to the initial Q_m value. A summary of the results for the main system follows:

$$Q_m = 569$$

$$S_m = 36.400891$$

$$\bar{\eta}_m(r) = 21.05438646/\text{period}$$

$$\bar{\eta}_m(r) = 108.047115/\text{year}$$

$$K_m = \$6796.50/\text{year}$$

To solve for the optimum values for Systems One and Two a different procedure is followed. The total annual cost of Systems One and Two is a convex function of the period length T . For different values of T there are different total annual

cost values. The minimum of these values is the optimum total annual variable cost and the corresponding T and R values are the optimum operating values. A TI-59 program was written to perform the line search for the best value of T. The program is found in Appendix G. User information and the features of the program are also in the same appendix. This program evaluates the R value, safety stock, back orders per period, back-orders per year, annual reviewing and ordering cost, annual holding cost, annual back order cost, and finally, the total annual cost.

Using this program the optimum values for Systems One and Two are:

$$R_1 = 306.809389$$

$$T_1 = 2.39 \text{ months (72.6958 days)}$$

$$S_1 = 13.78788901$$

$$\bar{n}_1(r) = 8.670289086/\text{period}$$

$$\bar{n}_1(r) = 43.53283223$$

$$K_1 = \$2634.41/\text{year}$$

$$R_2 = 134.9807003$$

$$T_2 = 4.07 \text{ months (123.7958333 days)}$$

$$S_2 = 0$$

$$\bar{n}_2(2) = 8.13480099/\text{period}$$

$$\bar{n}_2(r) = 23.98467122/\text{year}$$

$$K_2 = \$1377.56/\text{year}$$

The total back orders per year are:

$$\begin{aligned}\bar{n}_m(r) + \bar{n}_1(r) + n_2(r) &= 108.047 + 43.532 + 23.984 \\ &= 175.56\end{aligned}$$

which is the same as $b = 175.56$.

The total annual variable cost is:

$$K = K_m + K_1 + K_2 = 6796.50 + 2634.41 + 1377.56$$

$$K = \$10808.47.$$

III. COMPUTER SIMULATION FOR MULTI-ECHELON MULTI-LOCATION SINGLE ITEM INVENTORY SYSTEM

To check the analytical results, a computer simulation was written which uses the same operating assumptions and input parameters that were made for the analytical model. This model, however, simulates the real world more accurately since some of the simplifying assumptions required to obtain analytical results were not necessary in the simulation. Also, the demands placed at the main system from the lower echelon systems were batched as in the real world.

A. DESCRIPTION OF THE SIMULATION MODEL

As in the analytical case, there are three systems in the simulation model; the main system, System One and System Two. The flow charts of this program are in Appendix A.

1. Main System

This system uses a continuous review policy. When the stock level reaches the reorder point, it orders the amount Q . The decision variable for reordering is the inventory position. When an order is placed, the inventory position increases by an amount of Q . If the new inventory position is less than the reorder point, the system places an additional order for another amount of size Q . Then the

inventory position increases with one more Q. This continues until the inventory position exceeds r. The order policy is thus (nQ, r) . N is the smallest integer that will make the inventory position higher than the reorder point.

There are independent customer arrivals to the main system and also group demands that are placed by System One and System Two when those systems replenished their stocks at the end of their periods.

The number of units demanded per requisition is one. The number of units demanded for resupply to the lower echelon systems is random depending on the demands at the lower echelon systems and the parameters R_1 , R_2 . The program is capable of allowing geometrically distributed quantities demanded per requisition. Back orders at the main system are satisfied by filling the back orders to individual customers first followed by filling any back orders due to System One and System Two on a first-come, first-served basis.

When a demand occurs from the lower echelon systems for which there is not sufficient on-hand inventory at the main system, the maximum amount is filled and the rest is put into the back order queue. As soon as a shipment arrives at the main system, the back orders are filled. The times between the customer arrivals are independent of each other and exponentially distributed. The lead times are also independent of each other and gamma distributed.

2. System One and System Two

Both systems operate identically. The only difference might be in the values of the system parameters. They have the same periodic review policy which is: order up to R at each review time. The program is also capable of making a decision at each review time regarding the placement of an order if the inventory position is less than or equal to a threshold value, is an (r,R) policy. If r is taken to be $R-1$, then the system orders up to R at each review time even if there is only one demand in a period.

The times between customer arrivals are independent of each other and exponentially distributed. The number of units demanded per requisition is one. The lead times are independent of each other and gamma distributed.

B. HOW TO USE THE PROGRAM

Two simulations are given in Appendix C and Appendix D. The programs differ primarily in the type of output that is generated. The program given in Appendix C produces a Versatec plot output showing the inventory position for the main system and the lower echelon systems. A simulated period of up to 4 or 5 years can be run with a Class K job.

On the Versatec output for each system, there are two plots: one plot has a small triangle at each point representing the inventory position. The second plot represents the net inventory. The first figure shows the main system

values, the second figure shows the System One values and the third figure shows System Two values all plotted versus time.

The second program can be used to simulate operations over arbitrarily long periods of time. There are no dimension restrictions. This program gives the net inventory at the end of each period for all the systems; total demand in the periods of both System One and System Two; demand during a lead time for the main system; average on-hand inventory; and average number of items short per day for each system.

The input required by the programs is described by the variable definition list given in Appendix H. The programs use the IMSL subroutines for random number generation.

C. RUNS

1. Starting Conditions

There were two different starting conditions entered for the main system. In the first case the inventory position and net inventory were set initially to equal the order quantity. In the second case, the inventory position and the net inventory were set equal to the reorder point plus the order quantity. For Systems One and Two the inventory positions and net inventories were set initially at R_i .

2. Results of Runs

In all runs the length of time was 10 years. The average yearly results were obtained by dividing the results for the 10 year period by 10.

Using the policy parameters determined by the mathematical model and the first set of starting conditions the following simulated results were obtained.

a. Main System

The number of orders in 10 years = 51.

The number of back orders in 10 years = 3026.

The average on-hand inventory over 10 years =
313.026.

The average safety stock when an order arrives =
56.54 units.

The average annual costs are:

Ordering costs: \$2,550.00

Holding costs: 3,599.80

Back Order Costs: 1,513.00

Total Cost \$7,662.80

b. System One

Number of back orders in 10 years = 587.

The average on-hand inventory over 10 years =
114.482 units.

The average annual costs are:

Ordering and Reviewing Costs:	\$1,004.18
Holding Costs:	1,316.54
Back Orders Costs:	<u>293.50</u>
Total Costs	\$2,614.22

c. System Two

Number of back orders in 10 years = 287.

The average on-hand inventory over 10 years =
57.618 units.

The average annual costs are:

Ordering and Reviewing Costs:	\$ 589.68
Holding Costs:	662.61
Back Orders Cost:	<u>143.50</u>
Total Costs	\$1,395.79

(1) Total results

The total number of back orders for the
whole system per year is $302.6 + 58.7 + 28.7 = 390$.

The total annual variable cost for the
whole system is $\$7,662.80 + \$2,614.22 + \$1,395.79 = \$11,672.81$.

The model was run again with the second set
of starting conditions and the same set of policy parameter
values. The simulation results are summarized below:

(a) Main System

The number of orders in 10 years = 50.

The number of back orders in 10 years =
1665.

The average on-hand inventory over
10 years = 346.937.

The average safety stock when an order
arrives = 77.46.

The average annual costs are;

Ordering Costs: \$2,500.00

Holding Costs: 3,989.78

Back Order Costs: 832.50

Total Costs: \$7,322.28

(b) System One

Number of back orders in 10 years = 582.

The average on-hand inventory over
10 years = 118.696 units.

The average annual costs are;

Ordering and Reviewing Costs: \$1,004.18

Holding Costs: 1,365.00

Back Orders Costs: 291.00

Total Costs: \$2,660.18

(c) System Two

Number of back orders in 10 years: 314.

The average on-hand inventory over 10 years =
57.401 units.

The average annual costs are;

Ordering and Reviewing Costs:	\$ 589.68
Holding Costs:	660.11
Back Orders Costs:	<u>157.00</u>
Total Costs:	\$1,406.79

(2) Total Results

The total number of back orders for whole system per year is $166.5 + 58.2 + 31.4 = 256$.

The total annual variable cost for whole system is $\$7,322.28 + \$2,660.18 + \$1,406.79 = \$11,389.25$.

3. Comparison of Both Starting Conditions

It is obvious that if both results are compared there is a considerable decrement on annual back orders but not a great difference in annual variable costs.

$$\frac{390-256}{390} = 0.3436$$

$$\frac{11672.81 - 11389.25}{11672.81} = 0.02429$$

IV. COMPARISON OF ANALYTICAL AND COMPUTER SIMULATION RESULTS

By comparing the analytical results with the simulation results, it is possible to evaluate the analytical model. If the measures of effectiveness predicted by the analytical model are reasonably close to those generated by the simulation, there is support for the analytical results. In the table below we compare the main measures of effectiveness: back orders and costs.

	<u>Total Back Orders Per Year</u>	<u>Total Variable Costs Per Year</u>
Analytical result	175.56	\$10,808.17
Simulation result (For both starting conditions)	390.00	\$11,672.81 or
	256.1	\$11,389.25

This table shows that the average yearly back orders are considerably higher than what is estimated by the mathematical solution. In fact, the total number of back orders generated in the simulation does not even meet the constraint. Since the solutions produced by the simulation model, especially with respect to the number of back orders do not agree with the mathematical model, this suggests that some of the assumptions made in deriving the mathematical results are not reasonable. The major differences between the simulation results and the analytical results are found in the measures

of effectiveness at the main system. The results for the lower echelon track reasonably closely.

The costs are also a little higher than what is expected but much closer percentage wise.

As an explanation for the large differences observed in the number of back orders at the main system with the simulation and analytical models, let us reexamine the assumptions we made that affect back orders. Back orders results from inadequate amounts of safety stock to protect the inventory system against excessively large numbers of demands in a lead time. The safety stock is manipulated by control of the reorder point r . After the reorder point is hit and an order placed, the system is totally at the mercy of the demands that occur during the lead time. If the variance of lead time demand is underestimated, then large stockouts will occur, even if the mean lead time demand is estimated accurately. The lead time demand is affected by two random quantities: (1) the demand distribution and (2) the lead time distribution.

For purposes of making the mathematics tractable, we assumed in our analytical model that demands at the main system flowed in at a smooth continuous rate λ which was taken to be the sum of the rates of demand incurred directly at the main system λ_m and those which occurred at the lower echelon systems λ_1 and λ_2 . The lead times were assumed to

be normally distributed with mean equal to $\lambda\tau$ and variance
= $\lambda\tau(1-\lambda\beta)$ where τ is the mean value of the lead time.

Because of the relatively high demand rates used in the example runs, the normal assumption should be justified by the Central Limit Theorem. However, as is illustrated by the Versatec plot output (See Appendix F) the demands at the main system are far from smooth. What happens is the demands which arrive directly at the main system cause the inventory position to drop off smoothly. However, when the replenishment orders from the lower echelon are received, large drops occur in the inventory position of the main system. Recall that the lower echelon systems order in batches once each period from the main system. If the main system simply tries to average out demands (as assumed by the analytical model), it will sometimes have very large amounts of excess stocks when shipments arrive and sometimes very large numbers of back orders. The high variance in lead time demand caused by the irregular demand actually seen at the main system causes the problem. Since all demands eventually flow through the main system and this is assumed by our analytical model, the problem cannot lie in the value used for the mean lead time demand.

Let us explore further what happens when reorders are triggered at the main system. Demands directly at the main system eat away smoothly at the inventory position. Then a

very large quantity, say X , is demanded by one of the lower echelon systems. There is a very good chance that the large order placed by the lower echelon system will trigger a reorder by the main system. However, if the demand causes a large overshoot of the reorder point, the main system may have much less stock to live off of until the shipment arrives. For example, suppose the inventory position is $IP = 347$, the reorder point is $r = 300$, and System One places its resupply order for 200 units. An order will be placed by the main system but instead of having 300 units to keep it going until the order is received, it will have only 147 units. It is clear that if 300 is the amount of stock needed to provide reasonable protection against demands in a lead time, large numbers of back orders would be expected. Effectively, in the example above, the reorder level was not $r = 300$ but $r = 147$ and the safety stock negative. The impact of this surge in demand caused by the batching of demands received from the lower echelon systems is to reduce the "effective" reorder point from the value r to a value $r' < r$. Because the actual demand distribution witnessed b, the main system is difficult to describe mathematically, the actual value of r' cannot be determined analytically. However, it is clearly less than r and may be much more so.

In the next chapter we describe a policy modification for operation of the main system that was suggested by the

observations above. The modification attempts to allow the main system to anticipate the surge of demands that will be received by the lower echelon systems.

V. AN ALTERNATIVE SOLUTION

A. DESCRIPTION OF ALTERNATIVE PROCEDURE

Due to the reasons mentioned in Chapter IV, the number of back orders found by simulation were much higher than the number of back orders predicted by the mathematical model. The question is how to run the multi-echelon system so that the large number of back orders seen earlier can be reduced.

Obviously, what needs to be done is to reduce the impact of the very large demands that occur when the lower echelon systems place their resupply orders. With modern day communication and data systems, it would be feasible to allow the main system to "see" every demand that occurs anywhere in the system. If the main system is given the visibility, it will be able to take action to get the stock on its shelves in anticipation of the large demands from the lower echelon systems. The main system can do this if it uses as its reorder point the pseudo inventory position which is like the inventory position except that it decreases only when direct customer demands are encountered at any of the three systems. The pseudo inventory position is unaffected by the batch replenishment demands placed by the lower echelon systems. For example, if a customer requests

a unit from System One, the pseudo inventory position at the main system decreases by one. However, if System One places an order for 200 units at the main system, the pseudo inventory position does not change.

This new policy is referred to in this thesis as the "early warning policy." Clearly, the pseudo inventory position will always reach the reorder point before the inventory position. Therefore, orders will always be placed earlier and consequently, the number of back orders should decrease. The price paid will be in terms of extra holding costs. The results of a simulation using the early warning policy should be more nearly like those of the mathematical model since, in effect, the mathematical model makes the assumption of early warning. Implicitly, assumption of demand which is the superposition of the direct demands occurring at the main system and Systems One and Two is equivalent to the "early warning assumptions."

In the next section, results are given of a simulation of the multi-echelon system with early warning. The flowchart of the simulation model is given in Appendix B. The actual FORTRAN computer program is given in Appendix E.

B. COMPUTER SIMULATION RESULTS FOR EARLY WARNING POLICY

The same starting conditions and input parameters used in the previous simulation run were utilized here. The results are summarized as follows:

1. Main System Results

The number of orders in 10 years = 50.

Total number of back orders in 10 years = 126.

The average on-hand inventory = 488.093.

Average safety stock = 210.96.

The costs are:

Ordering costs	=	\$2,500.00
Holding costs	=	5,613.00
Back order costs	=	63.00
Total costs		\$8,176.07

2. System One Results

Total number of back orders in 10 years = 665.

The average on-hand inventory over 10 years = 117.846.

The costs are:

Ordering and reviewing costs	=	\$1,006.18
Holding Costs	=	1,355.23
Backorder costs	=	332.50
Total costs		\$2,691.91

3. System Two Results

Total number of back orders in 10 years = 312.

The average on-hand inventory over 10 years = 58.552.

The costs are:

Ordering and reviewing costs	=	\$ 589.68
Holding costs	=	673.35
Back orders costs	=	156.00
Total costs		\$1,419.03

C. COMPARISON OF EARLY WARNING POLICY SIMULATION RESULTS WITH ANALYTICAL AND FIRST SIMULATION RESULTS

The comparison will be done with respect to total yearly back orders and total variable system costs (considering all three system entities).

<u>Entire System</u>		
	<u>Number of Back Orders Per Year</u>	<u>Total Cost</u>
Analytical Result	175.56	\$10,808.47
First Simulation Results	256.1	11,389.25
Early Warning Simulation Results	110.3	12,287.01

Since the effects of implementing the early warning policy will be observed primarily at the main system, we also produce the results obtained for the main system individually.

<u>Main System</u>		
	<u>Numbers of Back Orders Per Year</u>	<u>Total Cost</u>
Analytical Result	108.05	\$6,796.50
First Simulation Result	166.50	7,322.28
Early Warning Simulation Results	12.6	8,176.07

This table shows the differences better than the first one. The number of back orders decreases 88.3 percent, simultaneously as the costs increase about 10.44 percent. The reason for the higher cost is because the increment in safety stock increases the carrying costs.

VI. CONCLUSIONS

The differences between the results of the mathematical and the simulation model can be explained largely as a result of the assumptions made about the demand process. In the main system it was assumed that the demand was smooth (the superposition of three Poisson processes), but in the simulation model the actual demand at the main system was as it would be in actual practice. There were batches of demand placed by System One and System Two in addition to the individual customer demands directly at the main system. These demand batches in fact increased the variance in lead time demand beyond that modelled. This explains why many more back orders were generated in the simulation model than what was predicted by the mathematical model.

The simulation model developed in this thesis is useful for making comparisons and examining the effects of policy changes or parameter changes. Moreover, it is one of the best ways to check the reasonableness of all of the simplifying assumptions made in order to obtain analytical solutions.

The mathematical results described in this thesis do not adequately determine the reorder point. The predicted number of stockouts is much less than the simulated numbers. As explained earlier, this is probably due to the assumptions

made in the model about the variance of lead time demand. Further study is needed to determine what could be done in the analytical model to better approximate what happens in actual practice.

The early warning policy discussed in this thesis did provide for great reductions in the number of stockouts system wide. Since stockouts are probably the most important consideration in military supply systems, the early warning policy is recommended, even though the holding costs are larger. Additional study is required to see if the reorder level in the early warning policy can be reduced substantially from the value determined by the mathematical model. Preliminary evidence is that the reorder level can be reduced significantly (25 percent or so) without generating excessively many back orders if the early warning policy is used.

Our simulation models allow us to view the effect of changes but they cannot be used to optimize the values of the policy parameters. For that objective, additional work in the mathematical modelling area is required.

In this thesis we have tried to model a multi-echelon inventory system analytically, by linking together the individual echelons and locations through a single objective function and a constraint on backorders system wide. We, knowingly, were making various simplifying assumptions to facilitate the derivation of solutions. As reported above,

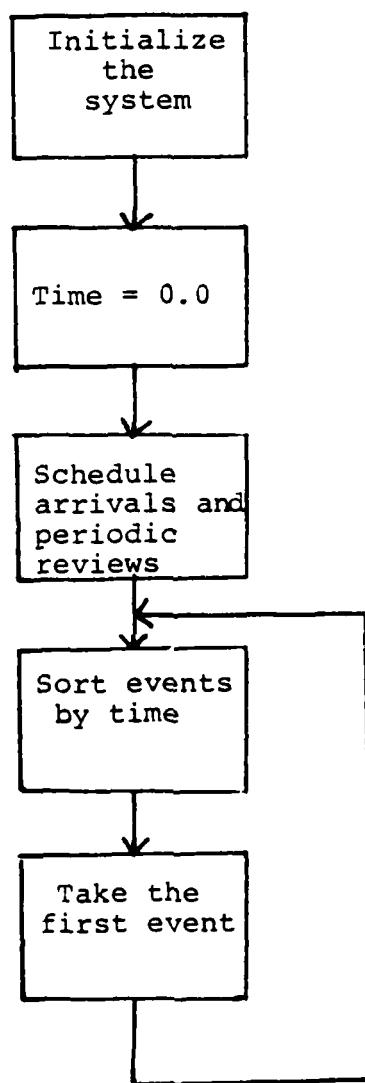
the resulting solution for the reorder level at the main system led to many more back orders than predicted. The other solutions; R_1 , R_2 , Q , T_1 , and T_2 appear to be satisfactory.

In order to model adequately the multi-echelon system, it will probably be necessary to build into the determination of the reorder lead at the main system the values of the parameters R_1 and R_2 at the lower echelon systems. This will be the only way to accurately describe the actual demand process that is observed at the main system. We recommend future work in this area.

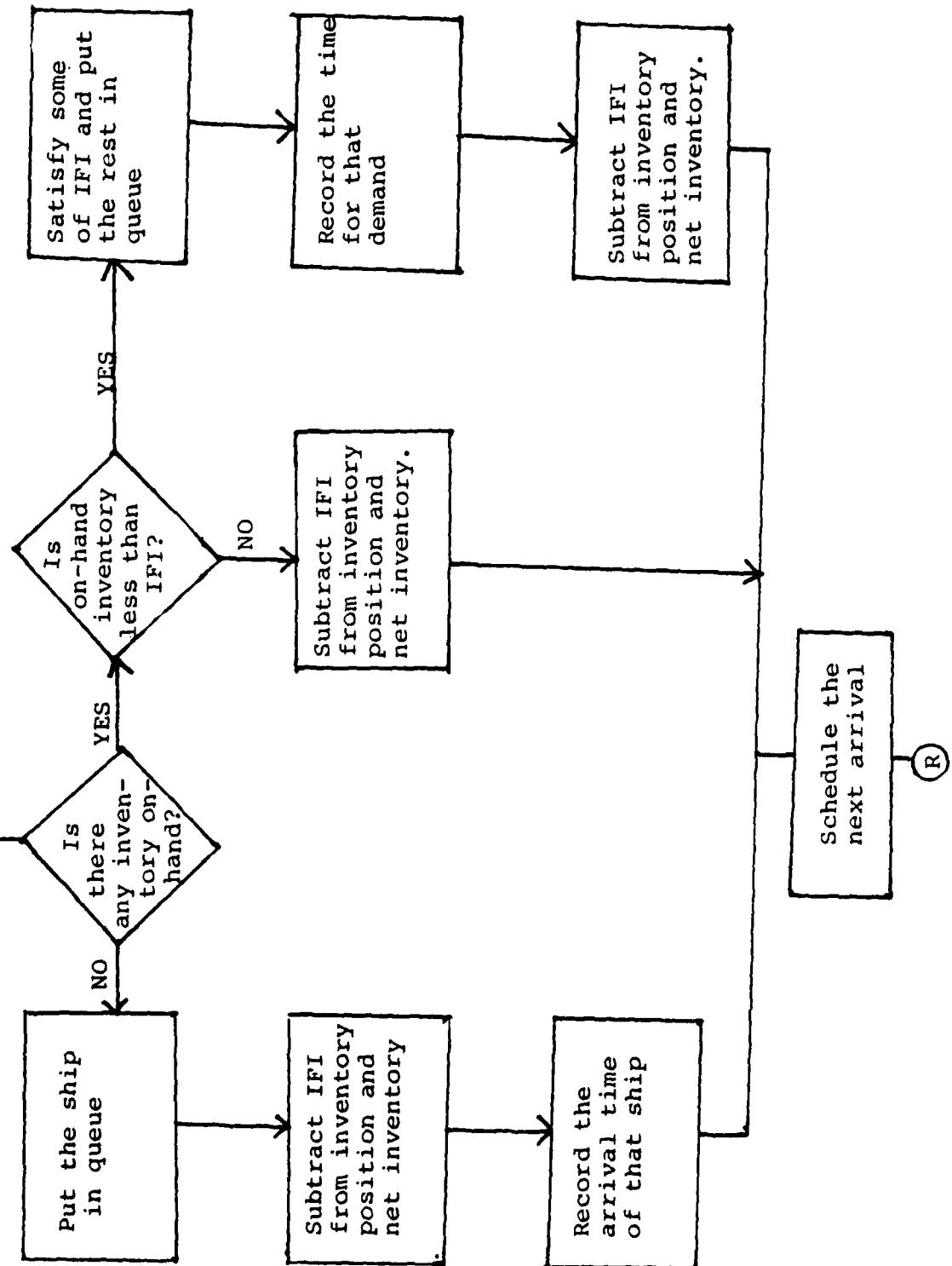
APPENDIX A
FLOWCHARTS OF FIRST SIMULATION MODEL

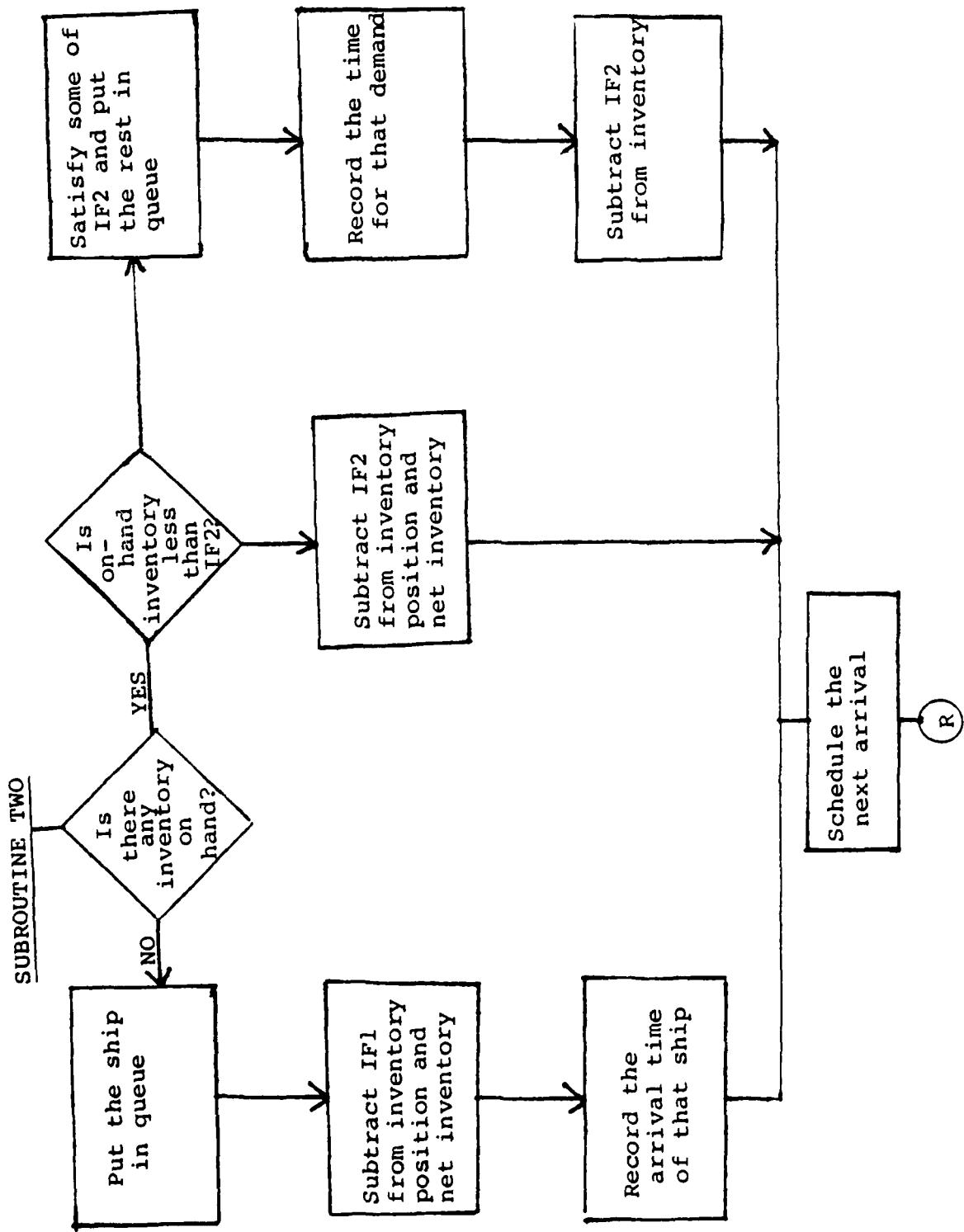
Subroutine One = Ship arrivals to System One
Subroutine Two = Ship arrivals to System Two
Subroutine Three = Ship arrivals to Main System
Subroutine Four = Periodic review of System One
Subroutine Five = Periodic review of System Two
Subroutine Six = Shipment arrival to System One
Subroutine Seven = Shipment arrival to System Two
Subroutine Eight = Shipment arrival to Main System

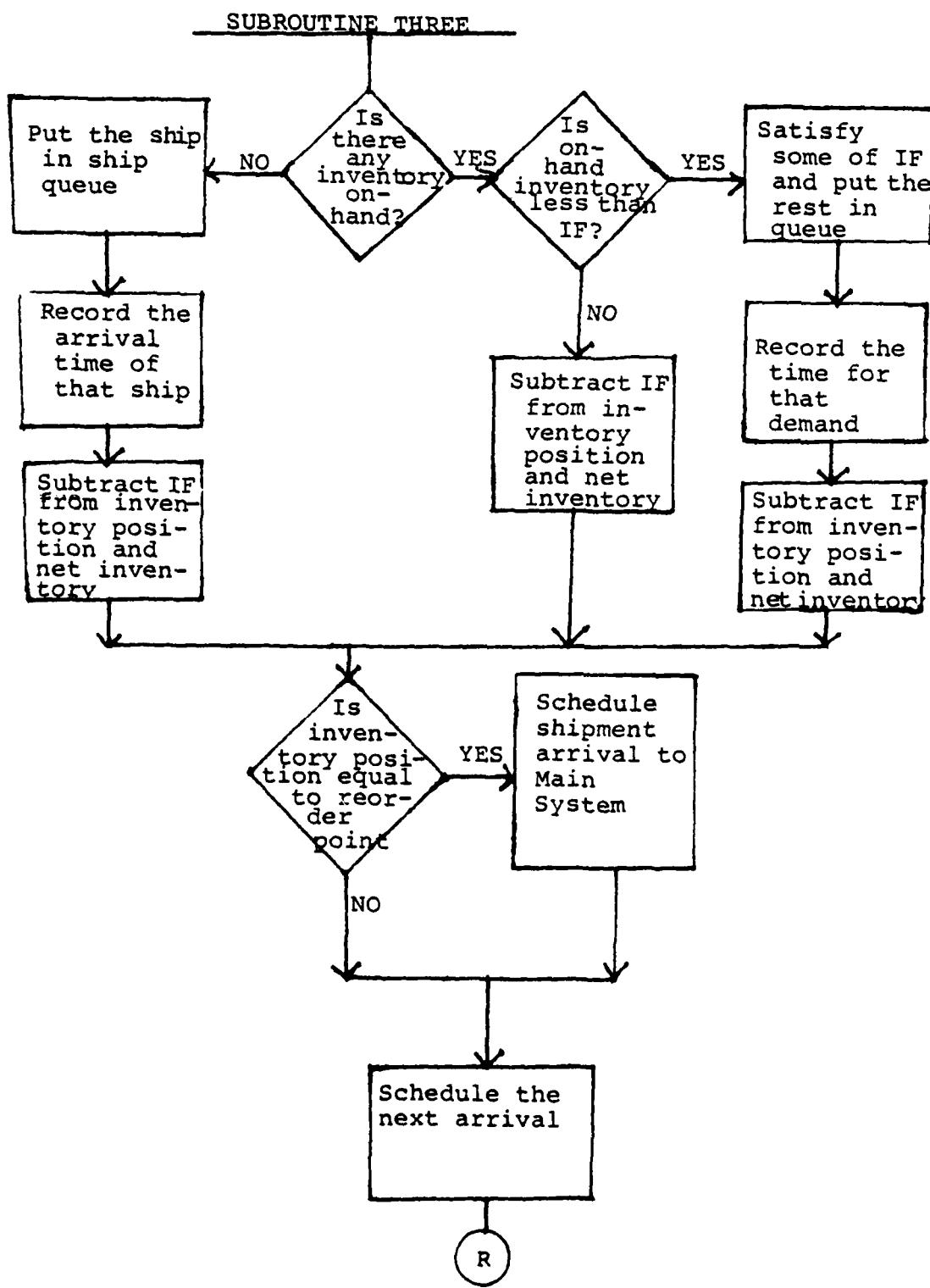
MAIN PROGRAM



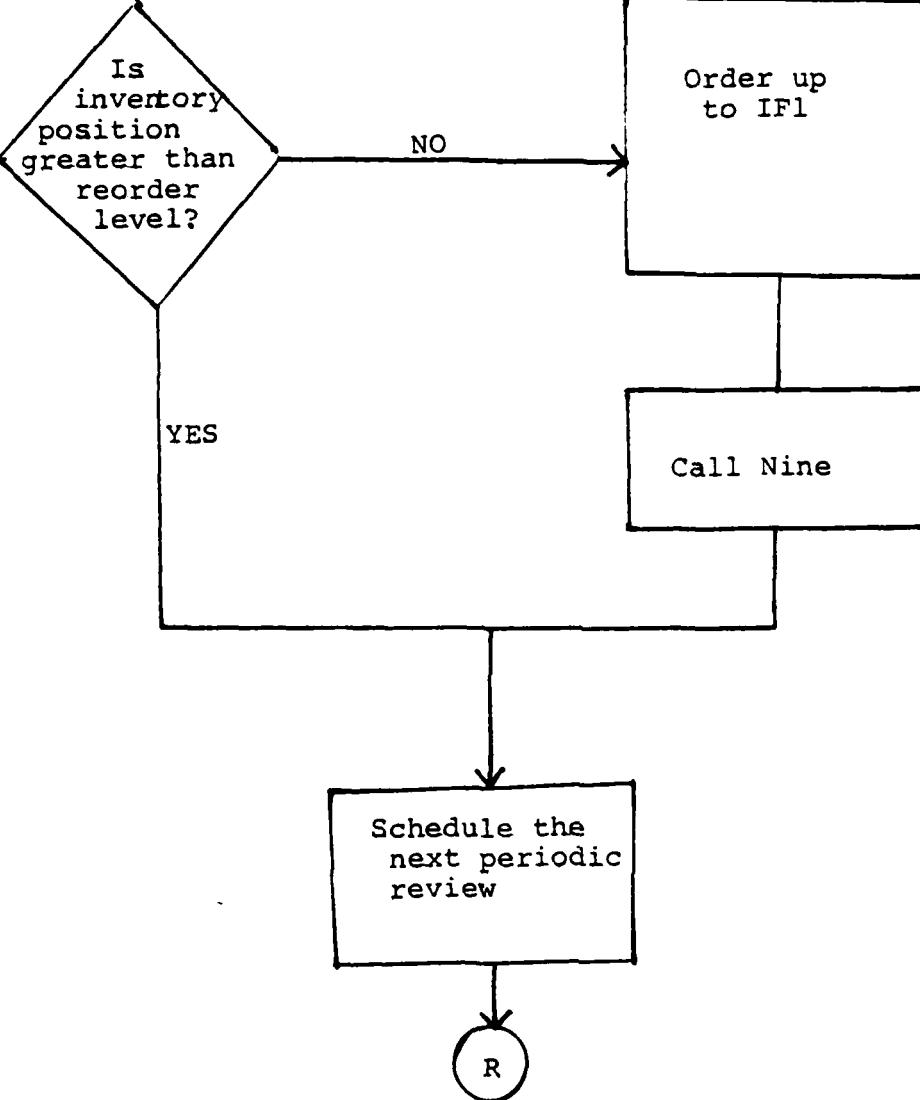
SUBROUTINE ONE

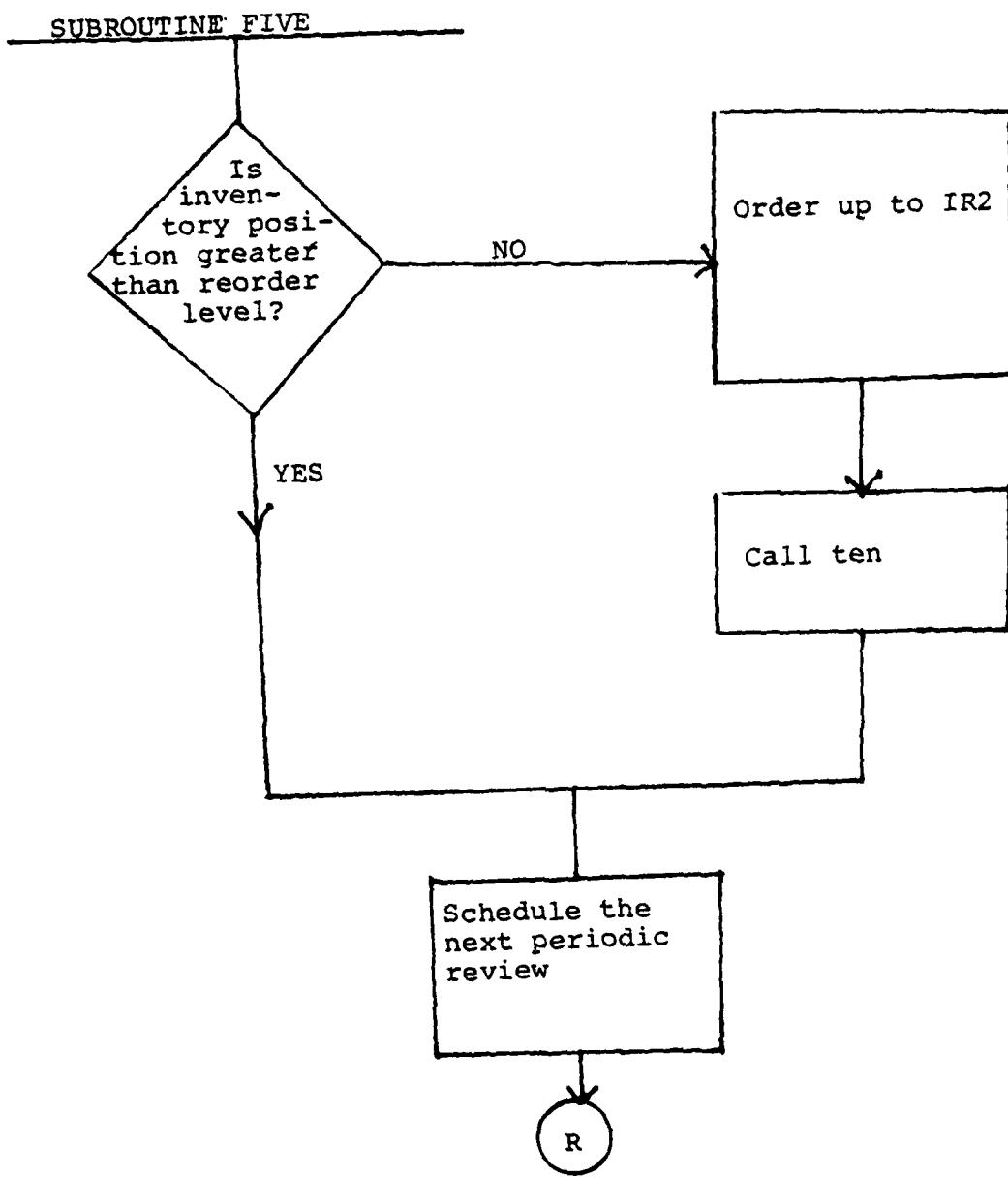


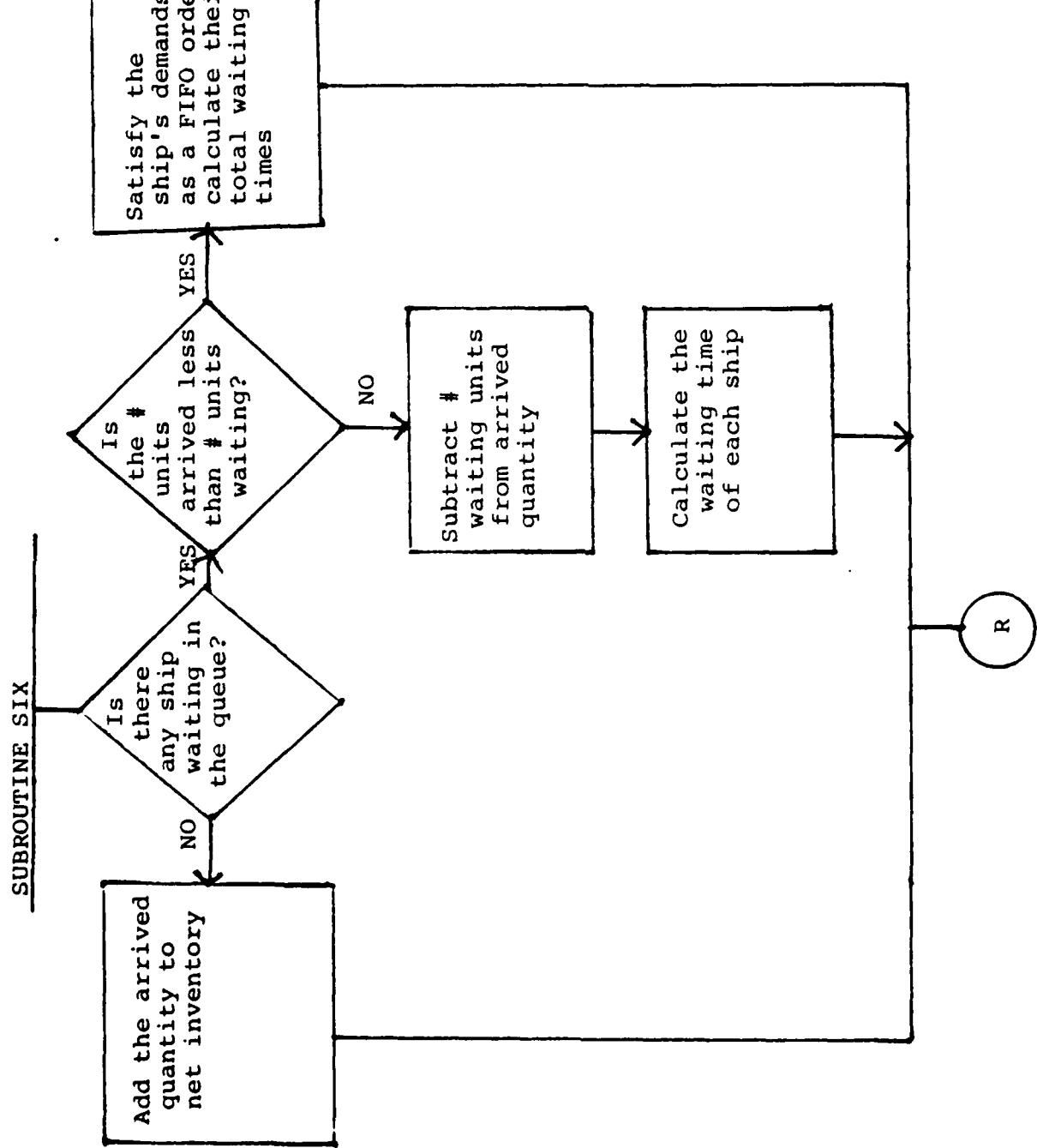


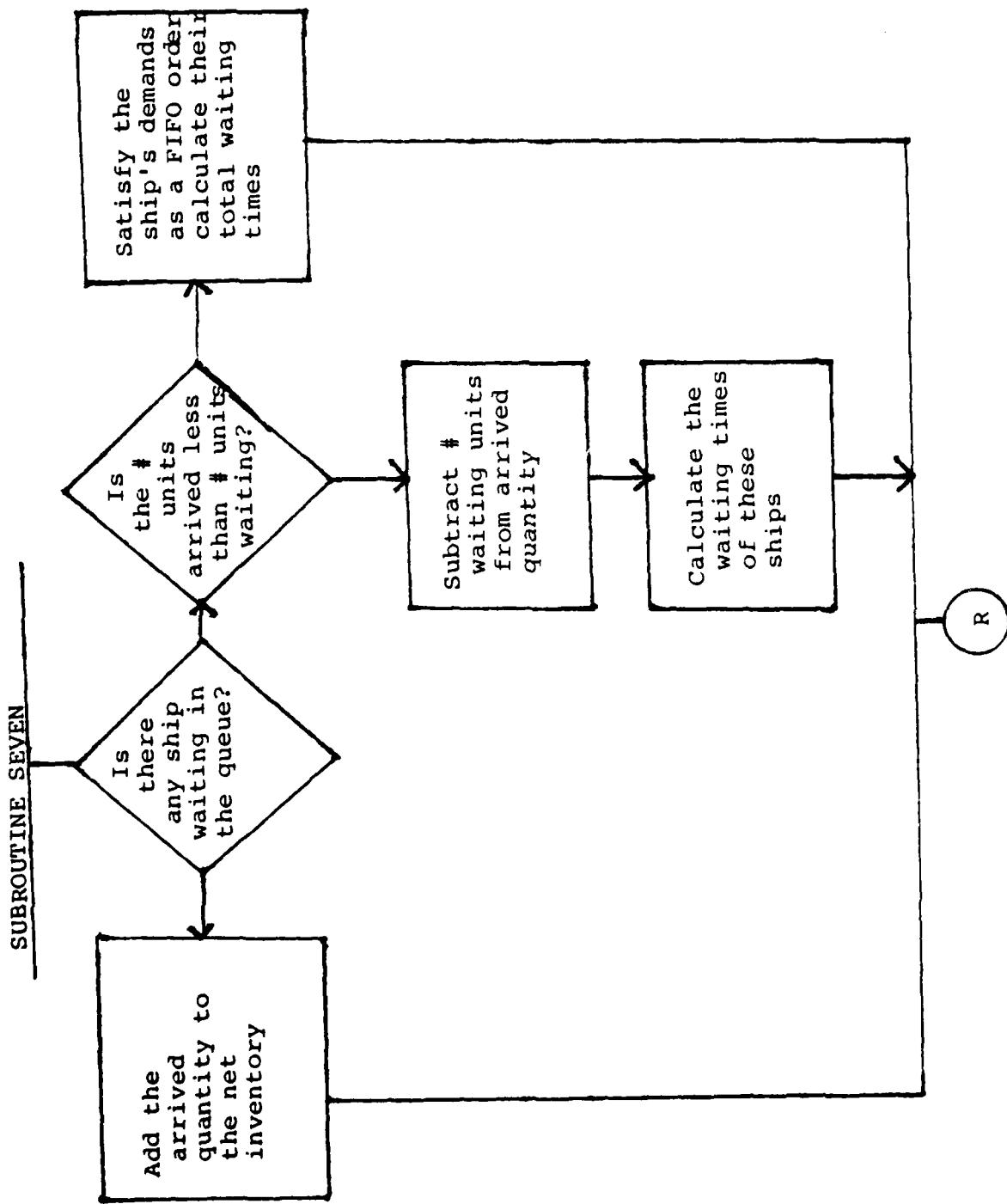


SUBROUTINE FOUR

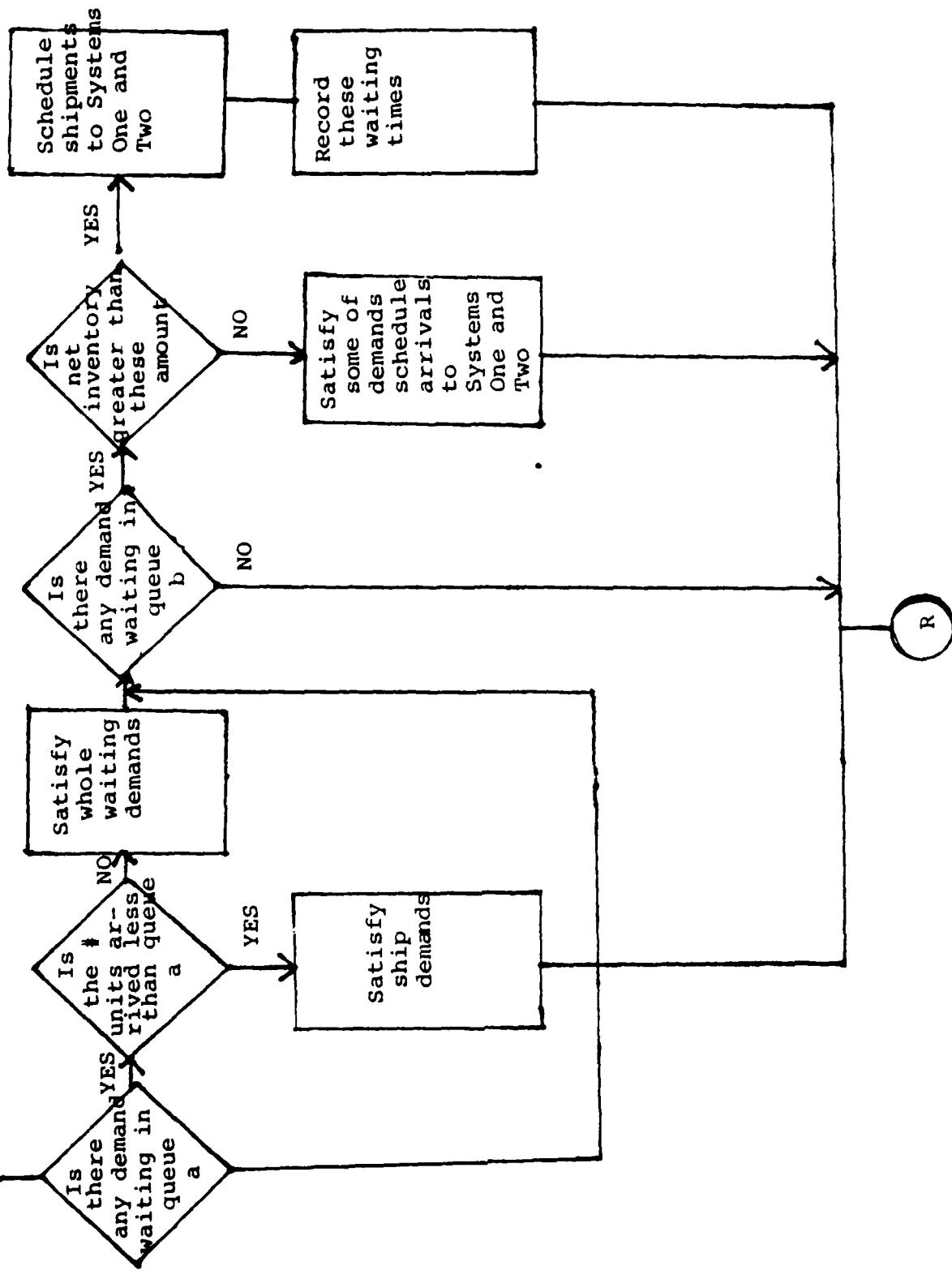


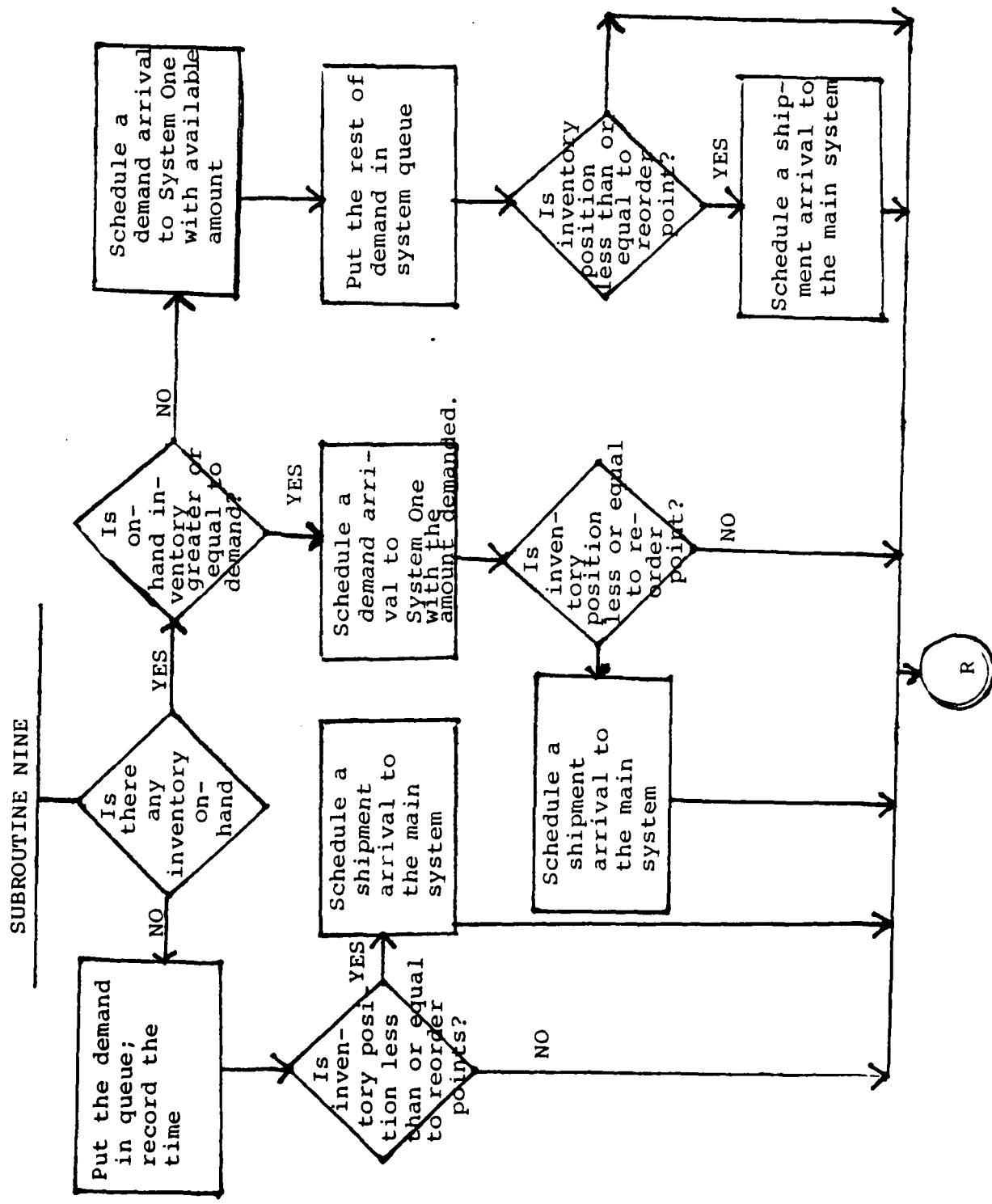


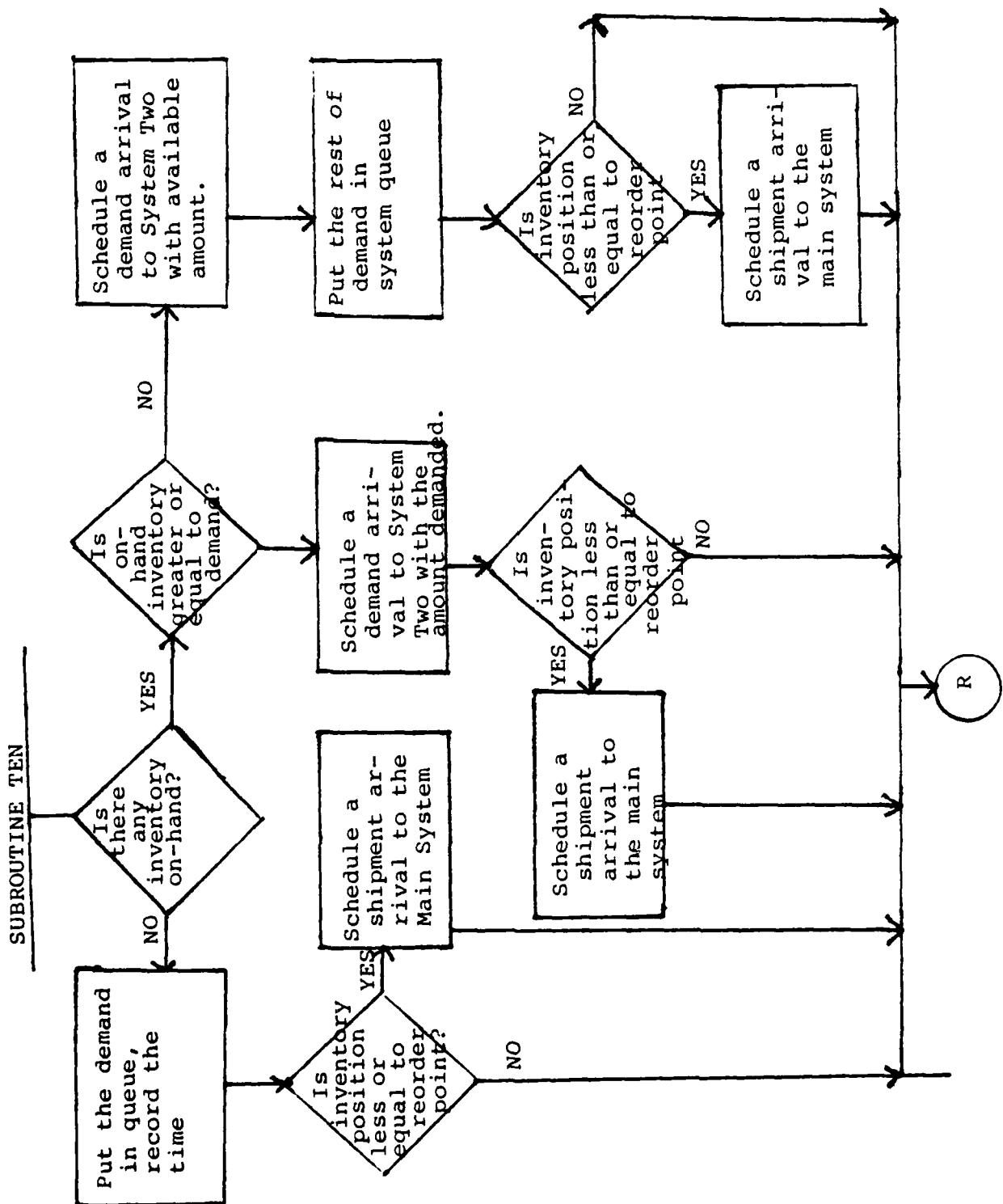




SUBROUTINE EIGHT







APPENDIX B

Flowcharts of Early Warning Simulation Model

Subroutine One = Ship arrivals to System One

Subroutine Two = Ship arrivals to System Two

Subroutine Three = Ship arrivals to the Main System

Subroutine Four = Periodic review of System One

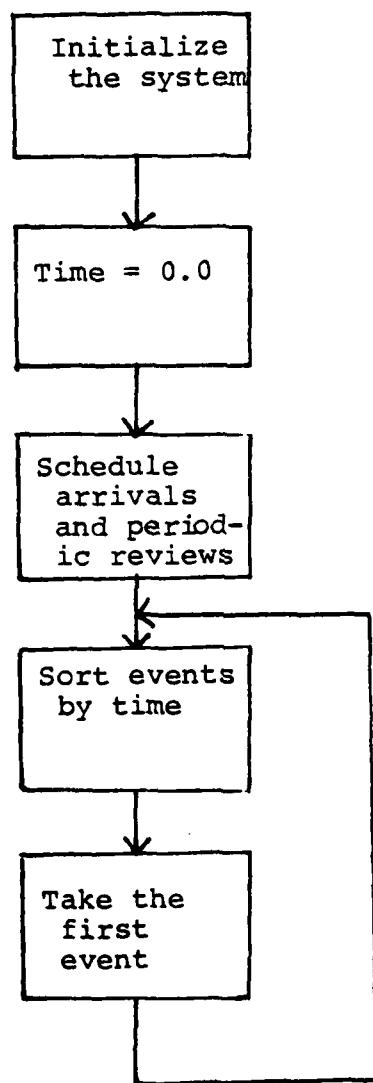
Subroutine Five = Periodic review of System Two

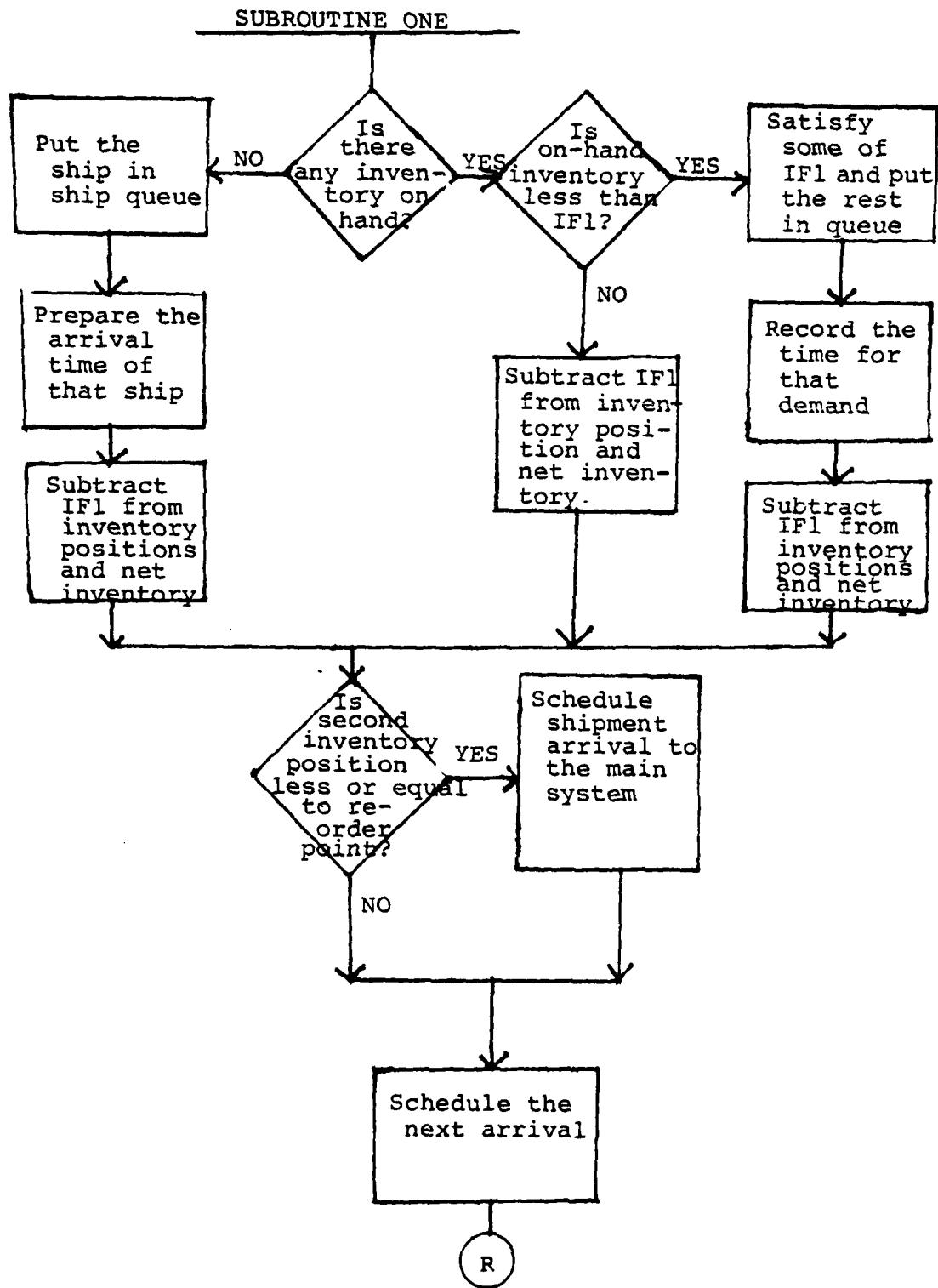
Subroutine Six = Shipment arrival to System One

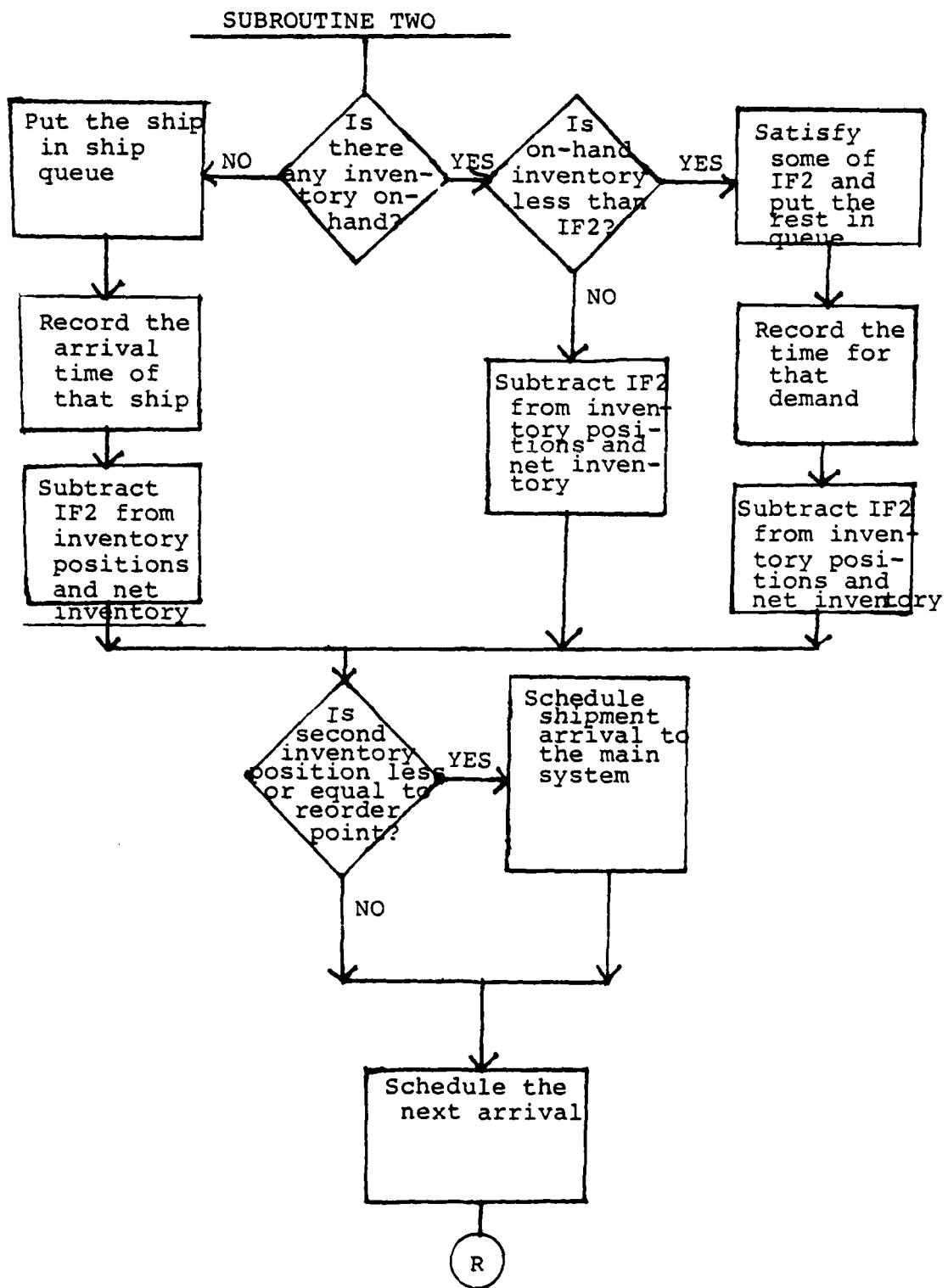
Subroutine Seven = Shipment arrival to System Two

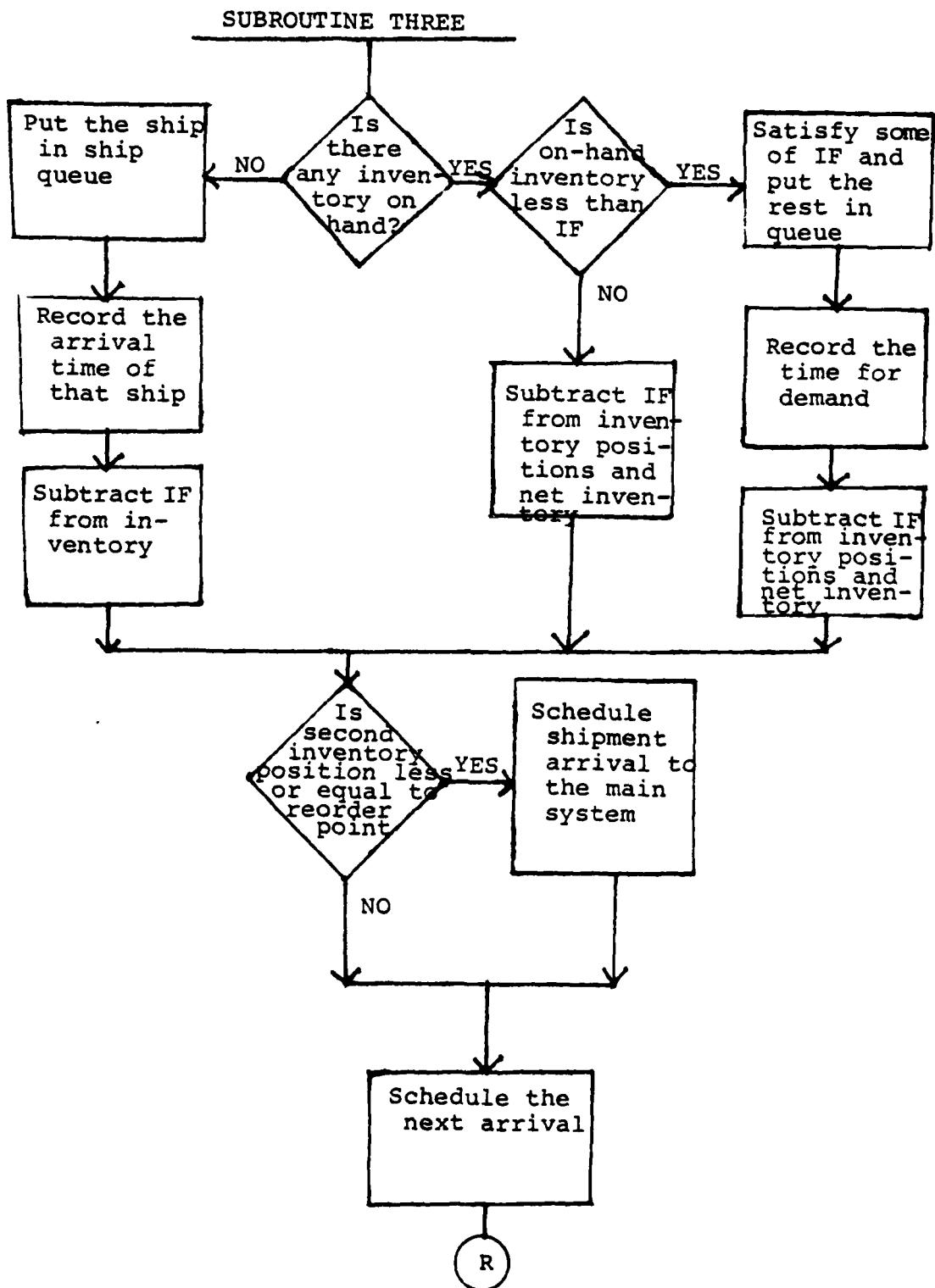
Subroutine Eight = Shipment arrival to the Main System

MAIN PROGRAM

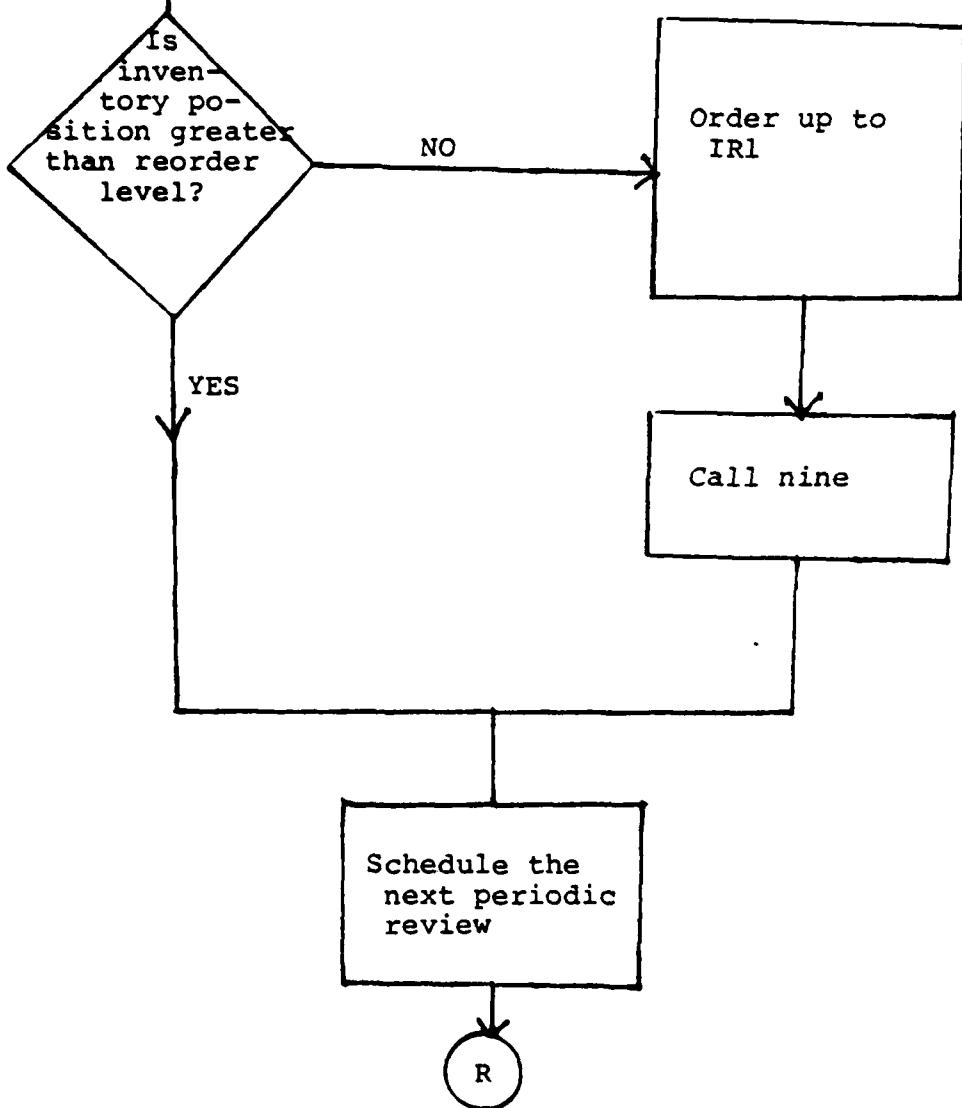




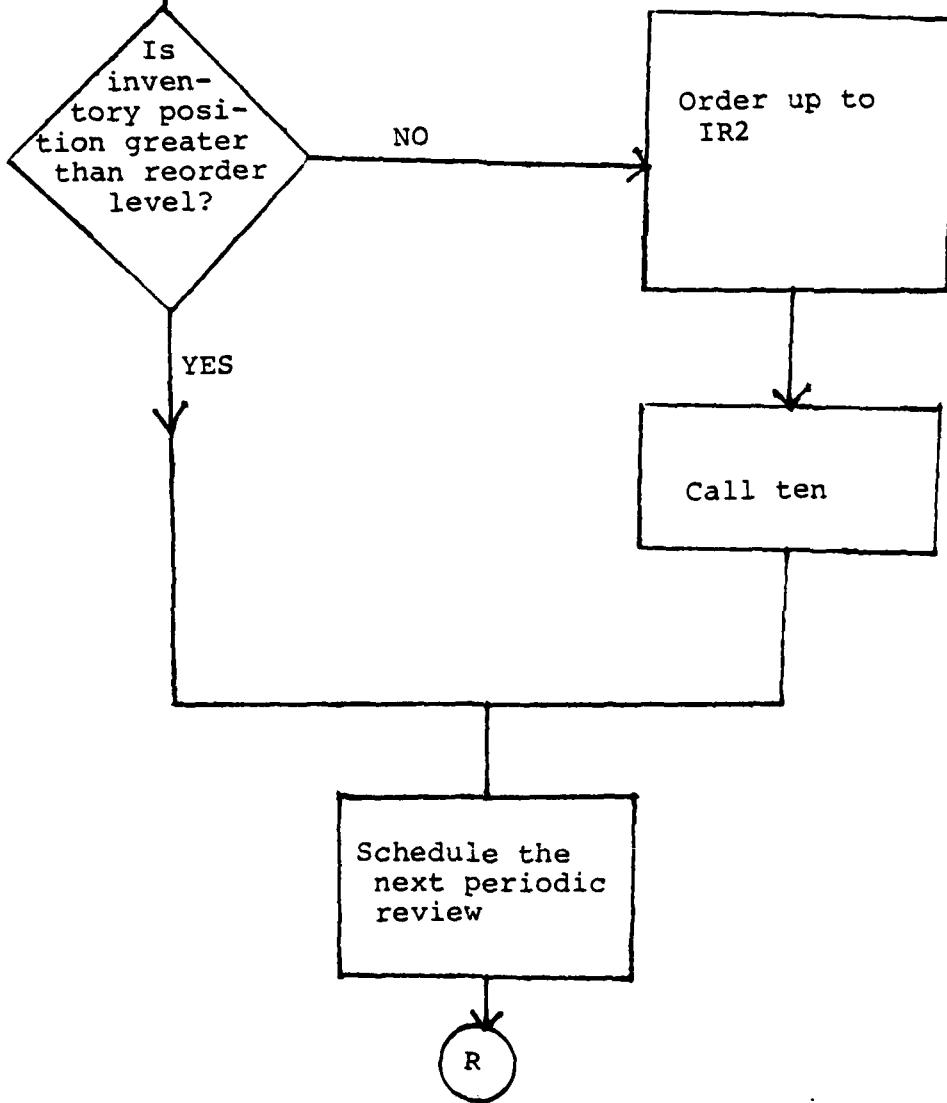


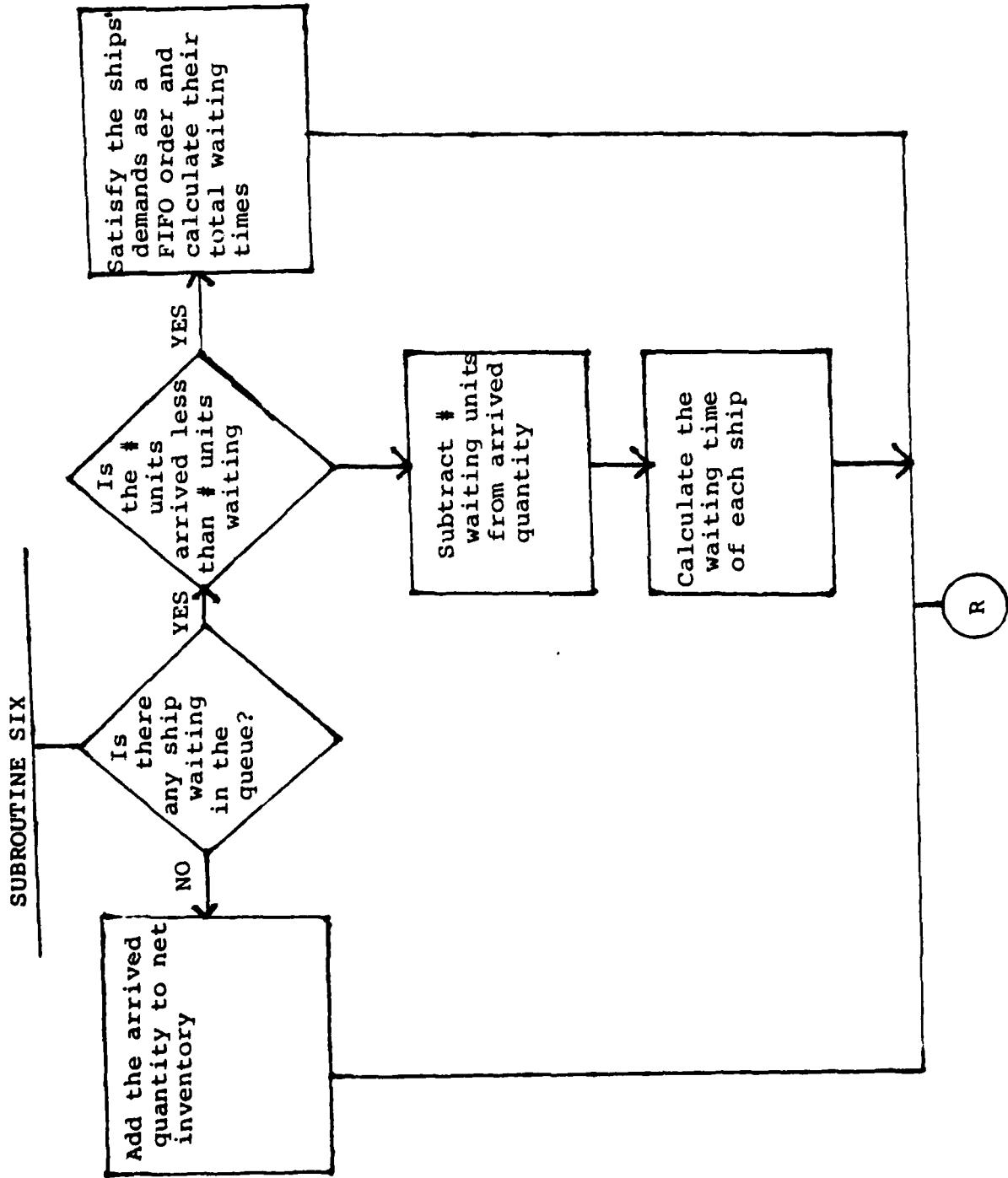


SUBROUTINE FOUR

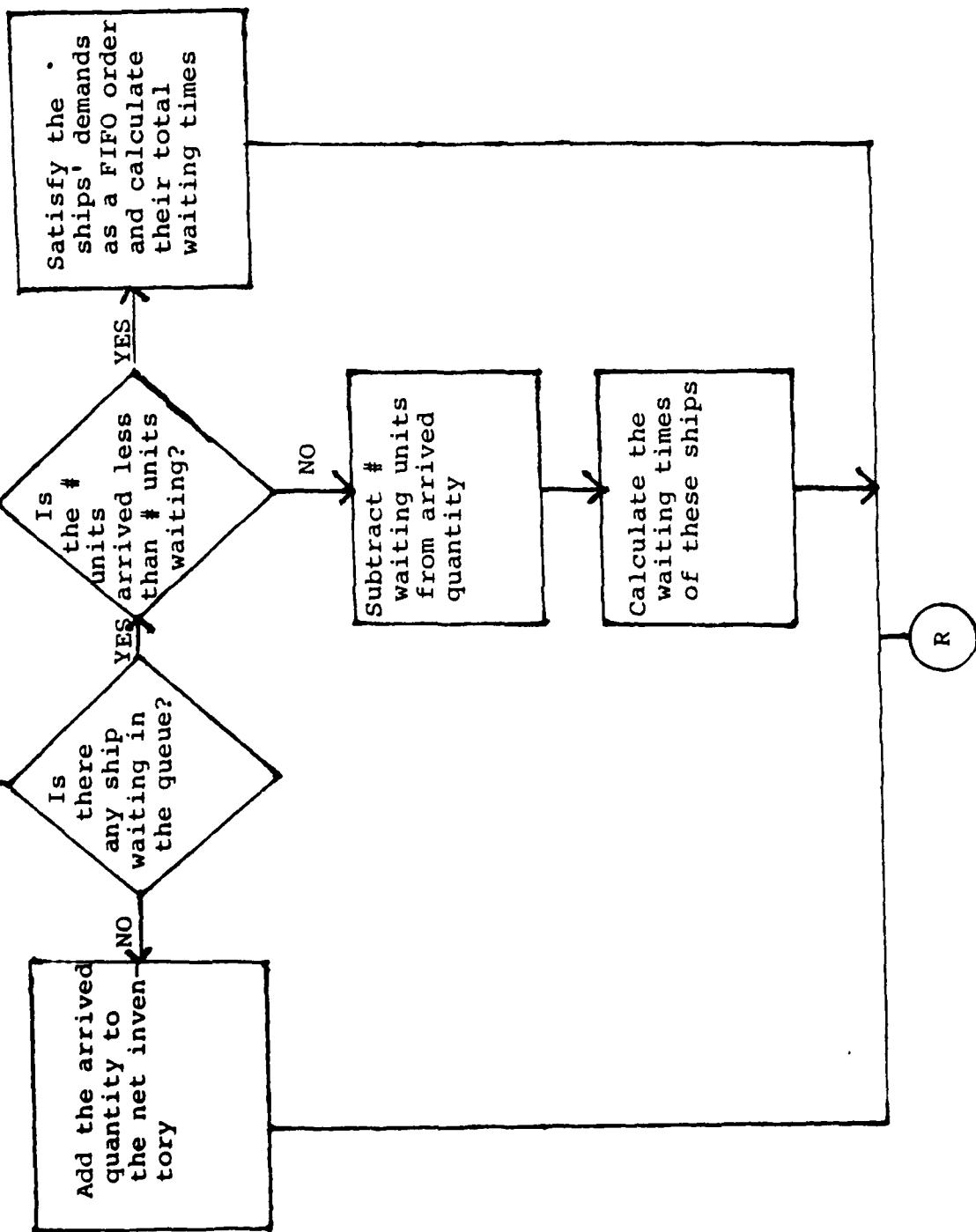


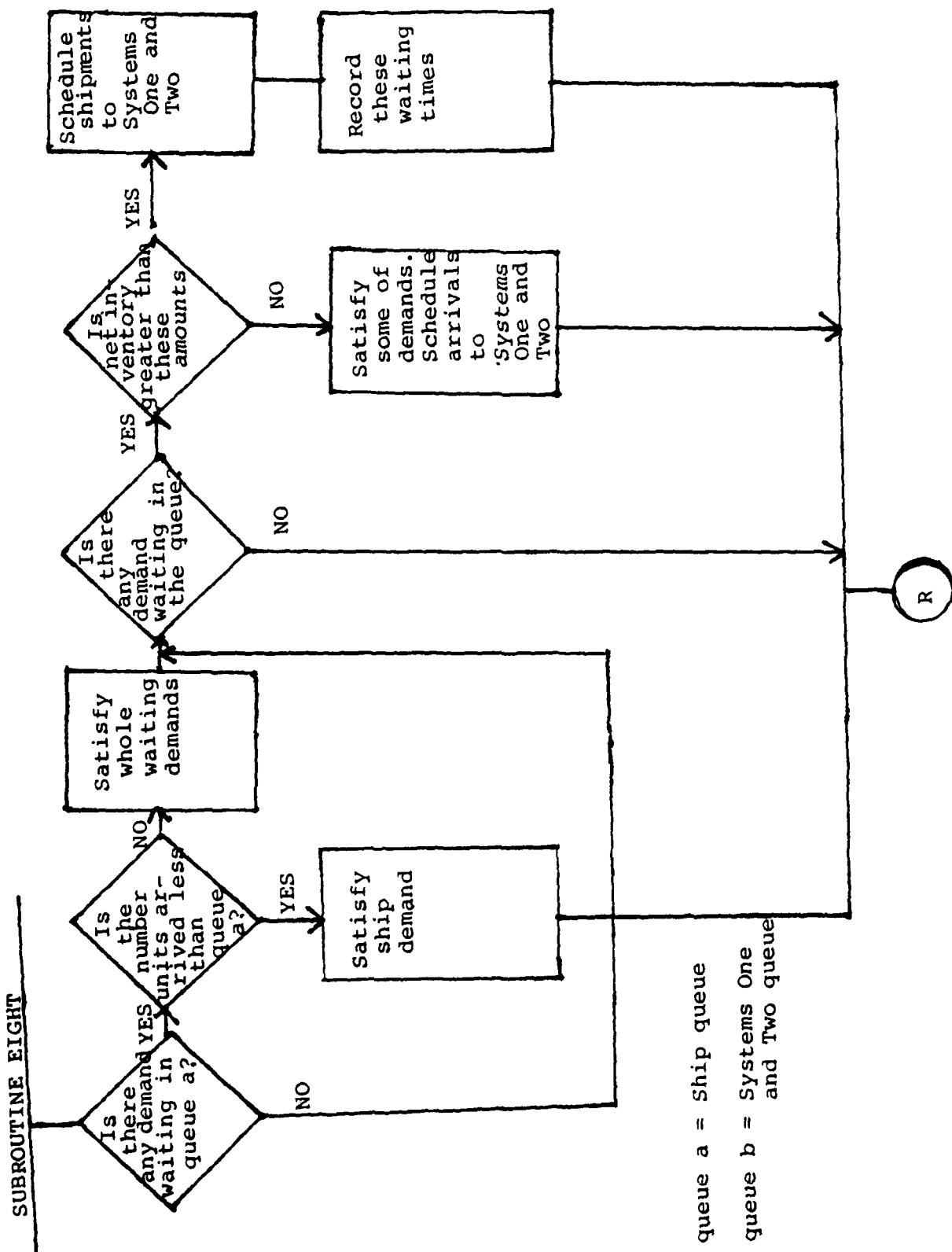
SUBROUTINE FIVE





SUBROUTINE SEVEN

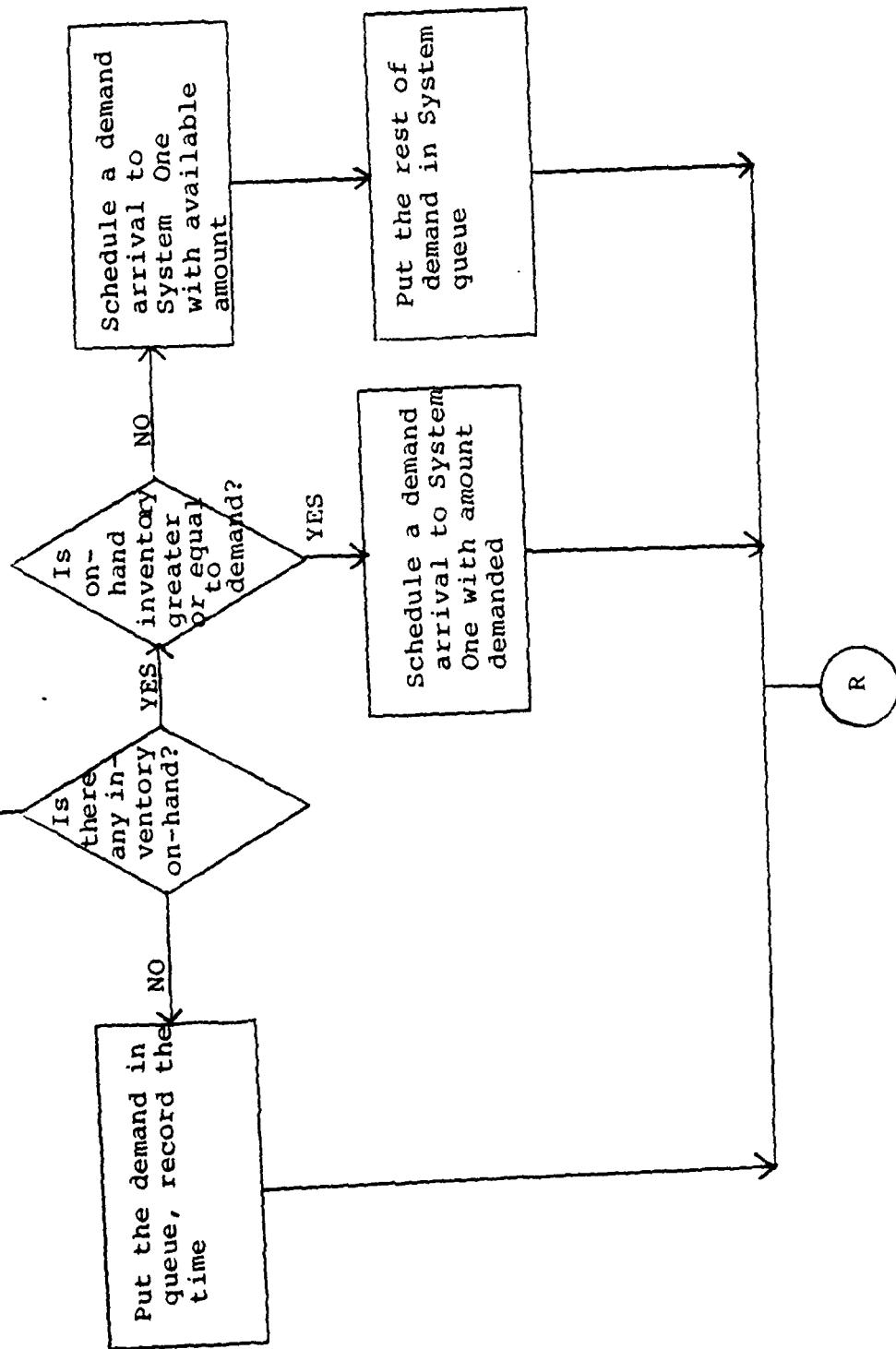


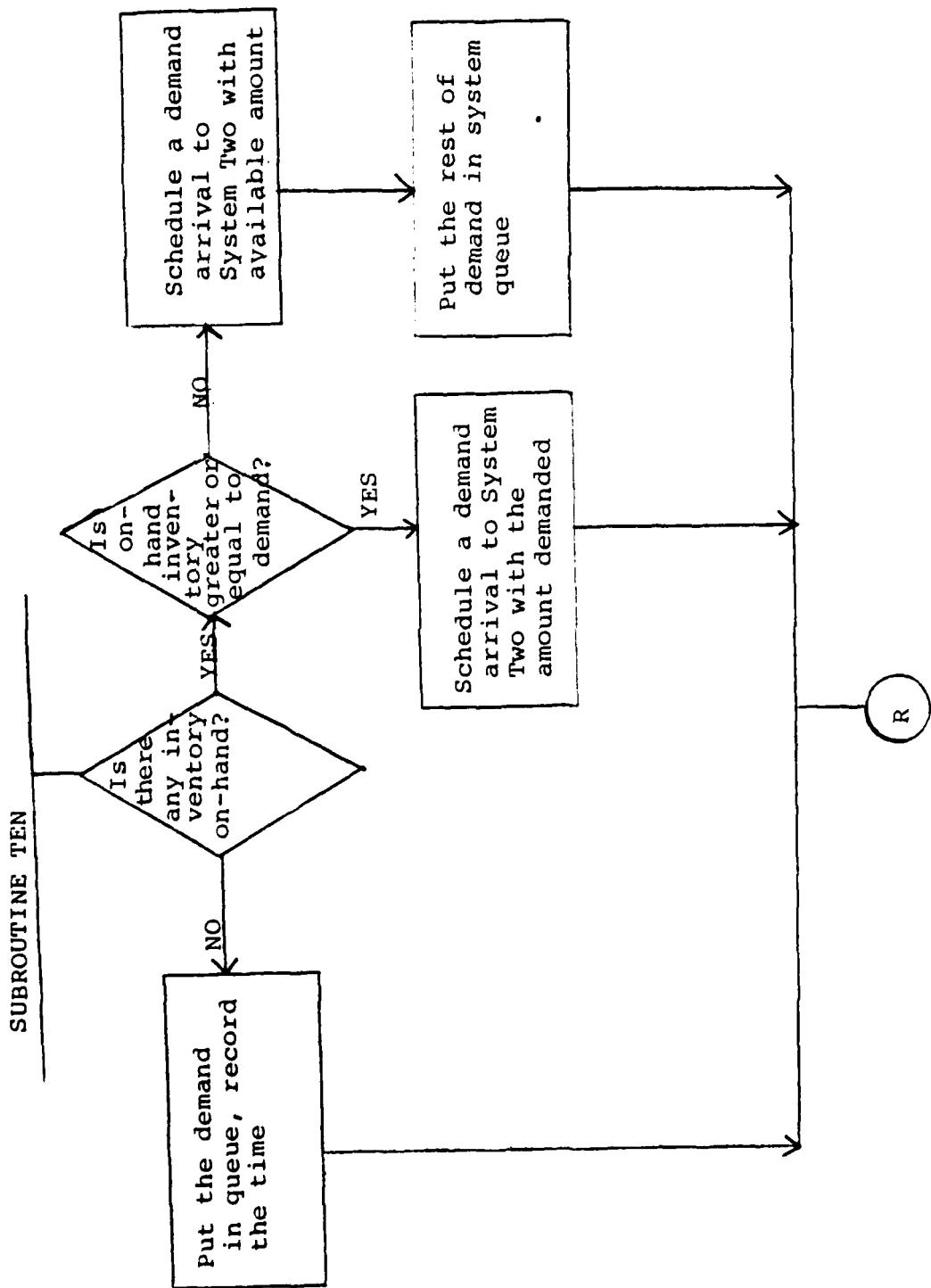


queue a = ship queue

queue b = Systems One
and Two queue

SUBROUTINE NINE





APPENDIX C

Simulation Program; Versatec Plotter

```

REAL *8 DSEED1,DSEED2,DSEED3,DSEED4,DSEED5,DSEED6,DSEED7,DSEED8,DSE
1 ED9
1 DIMENSION EVENT(900),IM(900),QU1(900),QU2(900),QU3(900)
1 QS(900),IG1(900),IG2(900),IG3(900),IA1(900),IA2(900),IA
1 1900)2150),Z1(50),Z2(50),X(400),X1(400),X2(400),Y(400),
1 Y2(400),Y1(400),Y2(400),Y1(400),W1(50),W2(50),WS(50),WK(50),IK1(1),I
1 K2(1),IK3(1),S1(1),S2(1),S3(1)
1 NP=4000
1 NS=900
1 NH=300
1 DSEED01=123456.000
1 DSEED02=247658.000
1 DSEED03=365274.000
1 DSEED04=258732.000
1 DSEED05=541863.000
1 DSEED06=433215.000
1 DSEED07=651563.000
1 DSEED08=265418.000
1 DSEED09=167519.000
1 I=0
1 J=0
1 K=0
1 L=0
1 K1=1
1 K2=1
1 K3=1
1 TW1=0.0
1 TW2=0.0
1 TW3=0.0
1 TW4=0.0
1 TW5=0.0
1 TOH1=0.0
1 TOH2=0.0
1 DO_1 N=1,50
1 EVENT(N)=9999.
1 EVENT(N)=0
1 QU1(N)=0.0
1 QU2(N)=0.0
1 QU3(N)=0.0
1 IG1(N)=0
1 IG2(N)=0
1 IG3(N)=0
1 IA1(N)=0
1 IA2(N)=0
1 CONTINUE
1 DO_2 N=1,10

```

```

IM(N)=0
IS(N)=0.0
IG(N)=0
CONTINUE
2
IF=0
IF1=0
IF2=0
ID=0
ID1=0
ID2=0
100 READ(5,100) IR, IS
      FORMAT(5,100)
      READ(5,100) IR1, IS1, T1
110 FORMAT(5,100) F10, 0
      READ(5,100) F10, 0
120 FORMAT(5,100) IS2, T2
      READ(5,100) IS2, T2
130 FORMAT(5,100) A, B
      READ(5,100) A, B
      READ(5,100) B
      READ(5,130) A1, B1
      READ(5,130) A2, B2
      READ(5,140) X1
      READ(5,140) X2
      READ(5,140) X3
140 FORMAT(5,140)
      READ(5,140) P1
      READ(5,150) P2
      READ(5,150) P3
150 FORMAT(5,150)
      IQ=IR+IS
      IQ1=IR1
      IQ2=IR2
      XL=(YEAR/365.)*33.
      TIME=0.0
      IP=IR+IS
      IP1=IR1
      IP2=IR2
      X(K1)=IQ
      Y(K1)=IP
      Y(X(K1))=TIME
      Y1(K2)=IQ1
      Y1(K2)=IP1
      Y1(K2)=TIME
      X2(K3)=IQ2
      Y2(K3)=IP2
      Y2(K3)=TIME
      CALL G3EXN (DSEED1,XM1,!;S1)
      CALL GGEOT (DSEED7,1,P1,WZ,IK1)

```

```

1 IF1=1
2 DO 3 KI=1,NM
3   I1=KI
4   IF(EVENT(I1).EQ.99999.) GO TO 4
5   CONTINUE
6   IF(EVENT(I1)=TIME+S1(1)
7     EVENT(I1)=TIME+S2(1)
8     CALL GGEVN (DSEED2,XM21,S2)
9     CALL GGEDT (DSEED8,1,P2,WS,IK2)
10    DO 11 KI=1,NM
11    I1=KI
12    IF(EVENT(I1).EQ.99999.) GO TO 6
13    CONTINUE
14    IF(EVENT(I1)=TIME+S3(1)
15      EVENT(I1)=TIME+S3(1)
16      DO 17 KI=1,NM
17      I1=KI
18      IF(EVENT(I1).EQ.99999.) GO TO 8
19      CONTINUE
20      IF(EVENT(I1)=TIME+S3(1)
21        EVENT(I1)=3
22        DO 23 KI=1,NM
23        I1=KI
24        IF(EVENT(I1).EQ.99999.) GO TO 10
25        CONTINUE
26        IF(EVENT(I1)=TIME+T1
27          EVENT(I1)=4
28          DO 29 KI=1,NM
29          I1=KI
30          IF(EVENT(I1).EQ.99999.) GO TO 12
31          CONTINUE
32          IF(EVENT(I1)=TIME+T2
33            EVENT(I1)=5
34            DO 35 KI=1,NM
35            I1=KI
36            IF(EVENT(I1).EQ.99999.) GO TO 14
37            CONTINUE
38            IF(EVENT(I1)=YEAR
39              EVENT(I1)=9
40              GO TO 26
41              IF(EVENT(IN)=99999.
42                IN=1
43                IF(EVENT(IN)=0
44                  DO 45 KI=2,NM

```



```

CALL PLOTG(Y1,X1,K2,1,1.0,'TIME',4,'INVENTORY QUANTITY',18,0,0,0,
100.0,0.0,X1,Y2,V1,K2,2,1.52,'TIME',4,'INVENTORY QUANTITY',18,0,0,0
1 CALL PLOTG(X1,Y1,K2,2,1.52,'TIME',4,'INVENTORY QUANTITY',18,0,0,0
1 CALL PLOTG(X1,Y1,K2,2,1.52,'TIME',4,'INVENTORY QUANTITY',18,0,0,0
1 CALL PLOTG(X1,Y1,K2,2,1.52,'TIME',4,'INVENTORY QUANTITY',18,0,0,0
1 CALL PLOTG(X1,Y1,K2,2,1.52,'TIME',4,'INVENTORY QUANTITY',18,0,0,0
1 CALL PLOT(X1,Y1,K2,2,1.52,'TIME',4,'INVENTORY QUANTITY',18,0,0,0
1 CALL PLOT(0.0,0.0,999)

STOP
DEBUG SUBCHK
END
SUBROUTINE ONE (I,TIME,EVENT,IEVENT,QU1,IQ1,IG1,IP1,IF1,XM1,P1,DSE
1 EDL,DSEED7,K2,NP,X1,Y1,V1,NM,NS,TOMH)
1 REAL*8 DSEED7,DSEED1,DSEED7
1 DIMENSION EVENT(NS),QU1(NS),IG1(NS),X1(NS),Y1(NS),V1(NS)
1 SS=Y1(K2)
K2=K2+1
X1(K2)=IQ1
Y1(K2)=IP1
YF(IQ1,G=0) GO TO 1
I=I+1
IC1=IQ1-IF1
IP1=IP1-IF1
QU1(I)=TIME
IG1(I)=IF1
K2=K2+1
X1(K2)=IQ1
Y1(K2)=TIME
GO TO 3
1 WW=0.0
WW=(TIME-SS)*X1(K2)
TOMH=TOMH+WW
IF(IQ1LTIF1) GO TO 2
IC1=IQ1-IF1
IP1=IP1-IF1
K2=K2+1
X1(K2)=IQ1
Y1(K2)=IP1
Y1(K2)=TIME
GO TO 3
2 I=I+1
IC1=IQ1-IF1
IP1=IP1-IF1
QU1(I)=TIME

```

```

1 G1(1)=IF1-IQ1
K2=K2+1
X1(K2)=IQ1
Y1(K2)=IP1
Y1(K2)=TIME
3 CALL GGEXN (DSEED1,XM1,I,S1)
CALL GGEO (DSEED7,I,PI,WZ,IK1)
IF1=1
DO 4 K1=1,NM
  I=K1
  IF(EVENT(I).EQ.99999.) GO TO 5
  CONTINUE
  4 WRITE(6,100)
  100 FORMAT(10F10.0)
  EVENT(I)=TIME+SI(I)
  IEVENT(I)=1
  RETURN
  DEBUG SUBCHK
END
SUBROUTINE TWO (J,TIME,EVENT,IEVENT,QU2,IG2,IQ2,IP2,IF2,XM2,P2,DSE
1 EDE2,DSEED8,K3,YP1,X2,Y2,V2,NS,TOH2)
  REAL*8 DSEED8
  DIMENSION EVENT(NS),IEVENT(NS),QU2(NS),X2(NS),P2(NS)
  1 NS=50
  1 K2(1),S2(1)
  SS=Y2(K3)
  K3=K3+1
  X2(K3)=IQ2
  Y2(K3)=IP2
  Y2(K3)=TIME
  1 IF(IQ2.GT.0) GO TO 1
  J=J+1
  1 IC2=IQ2-IF2
  1 IP2=IP2-IF2
  QU2(J)=TIME
  1 G2(J)=IF2
  K3=K3+1
  X2(K3)=IQ2
  Y2(K3)=IP2
  Y2(K3)=TIME
  GO TO 3
  1
  1 WW=0.0
  1 WW=(T14E-SS)*X2(K3)
  T0H2=TOH2+WW
  1 F(IQ2-LT*IF2) GO TO 2
  1 G2=IQ2-IF2
  1 P2=IP2-IF2
  K3=K3+1
  X2(K3)=IQ2

```

```

Y2(K3)=IP2
GO TO 3
2   J=J+1
    IQ2=IP2-IF2
    IQ2=IP2-J=TIME
    IQ2=IP2-IF2
    K3=K3+1
    X2(K3)=IP2
    Y2(K3)=TIME
    CALL GGEVN {DSEED2,XM21,S2}
    CALL GGEOT {DSEED8,1,P2,WS,IK2}
    IF2=1
DO 4  K1=1,NM
  IF(EVENT(11).EQ.99999.1 GO TO 5
  CONTINUE
  WRITE(6,100)
  FORMAT(11,EVENT(LIST) IS FULL)
100 5 EVENT(11)=TIME+S2(1)
  RETURN(11)=2
  RETURN
  DEBUG SUBCHK
END
SUBROUTINE THREE (L,BTIME,EVENT,QU3,IG3,IP,IF,XM3,P3,DSEE
1 D3,DSEED6,DSEED9,A,B,K1,NP,X,Y,V,NM,NS,IA,IR,IS,OH}
REAL*B,DSEED3,DSEED6,DSEED9
DIMENSION EVENT(NS),EVENT(NS),NS,WK(50),S3(1),R(1)
LSS=Y(K1)
K1=K1+1
X(K1)=IP
Y(K1)=TIME
IF(IP.GT.0) GO TO 1
L=L+1
IP=IP-1
IQ=IP-1
QU3(L)=TIME
IG3(L)=IF
K1=K1+1
X(K1)=IP
Y(K1)=TIME
IF(WW.GT.0) GO TO 3
WW=0.0
1

```

```

WW=(TIME-SS)*X(K1)
TOH=TOH+WW
IF(IQ.LT.IF) GO TO 2
IQ=IQ-IF
IP=IP-IF
K1=K1+1
X(K1)=IQ
Y(K1)=IP
Y(K1)=TIME
GO TO 3
2 L=L+1
Q(3)(L)=TIME
I(3)=IF-IQ
IP=IP-IF
K1=K1+1
X(K1)=IP
Y(K1)=TIME
Y(F(IP,GT,S)) GO TO 6
3 CALL GGAMS(DSEED6,A,B,L,Z,R)
DO 4 KI=1,NM
I=KI
IF(EVENT(III).EQ.99999.) GO TO 5
CONTINUE
4 WRITE(6,100)
100 FORMAT(1X,LIST IS FULL)
5 EVENT(II)=TIME+R(II)
EVENT(II)=8
ML=0
70 ML=ML+1
IP=IP+IR
IF(IP.LT.IS) GO TO 70
IA(II)=ML*IR
K1=K1+1
X(K1)=IQ
Y(K1)=IP
Y(K1)=TIME
Y(CALL GGEXN(DSEED3,XM3,L,S3))
6 CALL GGEO(T,DSEED9,I,P3,WK,IK3)
IF(I=1,7,KI=1,NM
I=KI
IF(EVENT(III).EQ.99999.) GO TO 8
7 CONTINUE
8 WRITE(6,100)
EVENT(II)=TIME+S3(1)
EVENT(II)=3

```

```

RETURN
DEBUG SUBCHK
END
SUBROUTINE FOUR (K1,IQ,IP1,IP2,TIME,EVENT,IEVENT,IM,IG,QS,IA,IA1,IR
1 IS1,IS1TOH)
1 IS1*8 DSEED4 DSEED6
DIMENSION EVENT(NS) IM(NS) IA1(NS),QS(NS),IG(NS),IA(NS)
1 X(NP),Y(NP),V(NP),X1(NP),Y1(NP),V1(NP),V2(NP)
1 ID=0
1 IF(IP1.GT.IS1) GO TO 1
K2=K2+1
X1(K2)=IQ1
Y1(K2)=IP1
ID1=IR1-IP1
IP1=IR1
K2=K2+1
X1(K2)=IQ1
Y1(K2)=TIME
CALL NINE (K1,IQ,IDL,TIME,EVENT,IEVENT,IM,IG,QS,IA,IA1,IR,IS,IP,A,B
1 ID0,2,K1=1,NM
1 II=KI
1 IF(EVENT(II).EQ.99999.) GO TO 3
2 CONTINUE
2 WRITE(6,100)
100 FORMAT(1F10.0)
3 EVENT(II)=TIME+1
EVENT(II)=4
RETURN
DEBUG SUBCHK
END
SUBROUTINE FIVE (K1,IQ,IP2,TIME,EVENT,IEVENT,IM,IG,QS,IA,IA2,IR
1 IS2,IS2TOH)
1 IS2*8 DSEED5 DSEED6
DIMENSION EVENT(NS) IM(NS) IA2(NS),QS(NS),IG(NS),IA(NS)
1 X(NP),Y(NP),V(NP),X2(NP),Y2(NP),V2(NP)
1 ID2=0
1 IF(IP2.GT.IS2) GO TO 1
K3=K3+1
X2(K3)=IQ2
Y2(K3)=IP2
Y2(K3)=TIME
ID2=IR2-IP2
IP2=IR2

```

```

K3=K3+1
X2(K3)=IP2
Y2(K3)=TIME
CALL TENSEEDS(IQ1, ID2, TIME, EVENT, IEVENT, IMIC, QS, IA, IA2, IR, IS, IP, A, B,
 1 A2, B2, K1=1, NM
 1 DO 2 K1=1, NM
 1 IF(EVENT(III).EQ.99999.) GO TO 3
 2 CONTINUE
 2 WRITE(6, 100)
 100 FORMAT(1F10.0)
 3 EVENT(III)=TIME+T2
 3 EVENT(III)=5
 3 RETURN
 3 DEBUG SUBCHK
END
SUBROUTINE SIX (I, IJ, TIME, IAI, IQ1, QUIL, IGL1, TW1, IPL1, K2, NP, X1, V1, Y1, N
 1 NS)
 1 DIMENSION IAI(NS), IGL1(NS), QUIL(NS), X1(NS), V1(NP), Y1(NP)
 1 K2=K2+1
 1 X1(K2)=IQ1
 1 Y1(K2)=IPL1
 1 Y1(K2)=TIME
 1 IQ1=IQ1+IAI(IJ)
 1 K2=K2+1
 1 X1(K2)=IQ1
 1 Y1(K2)=IPL1
 1 Y1(K2)=TIME
 1 ITD=0
 1 IF(I.EQ.0) GO TO 8
 1 DO 1 N=1, 1
 1 ITD=ITD+IG1(N)
 1 CONTINUE
 1 IF((IAI(IJ)).LT.ITD) GO TO 3
 1 DO 2 N=1, 1
 1 W=0.
 1 W=(TIME-QUIL(N))*IG1(N)
 1 QUIL(N)=0.
 1 TW1=TW1+W
 1 IG1(N)=0
 2 CONTINUE
 2 IAI(IJ)=0
 2 I=0
 2 GO TO 8
 3 DO 5 N=1, 1
 3 IF((IAI(IJ)).GE.IG1(N)) GO TO 4

```

```

1 IF(IA1(I,J)) EQ.0. GO TO 6
2 IF(I(N)=IG1(N)-IA1(I,J)*IA1(I,J))
3 TW1=TW1+W
4 IA1(I,J)=IA1(I,J)-IG1(N)
5 QU1(N)=0.
6 DO 7 N=1,1
7 IF(QU1(N).EQ.0.) GO TO 7
8 M=M+1
9 QU1(M)=QU1(N)
10 IG1(M)=IG1(N)
11 CONTINUE
12 I=M
13 RETURN
14 DEBUG SUBCHK
15 END
16 SUBROUTINE SEVEN (J,IJ,TIME,IA2,IQ2,QU2,IG2,TW2,IP2,K3,NP,X2,V2,Y2
17 ,NH,NS)
18 DIMENSION IA2(NS),IG2(NS),QU2(NS),X2(NS),V2(NS),Y2(NS)
19 K3=K3+1
20 X2(K3)=IQ2
21 Y2(K3)=IP2
22 Y2(K3)=TIME
23 IQ2=IQ2+IA2(IJ)
24 X2(K3)=IQ2
25 Y2(K3)=IP2
26 Y2(K3)=TIME
27 ITD=0
28 IF(J.EQ.0) GO TO 8
29 DO 1 N=1,J
30 ITD=ITD+IG2(N)
31 CONTINUE
32 IF(IA2(IJ).LT.ITD) GO TO 3
33 DO 2 N=1,J
34 ITD=ITD+IG2(N)
35 IF(IA2(IJ).LT.ITD) GO TO 3
36 W=0.
37 QU2(N)=0.
38 TW2=TW2+W
39 IG2(N)=0
40 CONTINUE
41 IA2(IJ)=0

```

```

J=0
GO TO 8
3 DO 5 N=1,J
1F(I,A2(I,J)) .GE. IG2(N) ) GO TO 4
1F(I,A2(I,J)) .EQ. 0 ) GO TO 6
IG2(N)=IG2(N)-IA2(I,J)
H=(TIME-QU2(N))*IA2(I,J)
TH2=TW2+H
GO TO 6
GIA2(I,J)=IA2(I,J)-IG2(N)
H=(TIME-QU2(N))*IG2(N)
TH2=TW2+H
QU2(N)=0.
CONTINUE
5 J=0
DO 7 N=1,J
1F(QU2(N).EQ.0.) GO TO 7
J=J+1
QU2(J)=QU2(N)
IG2(J)=IG2(N)
CONTINUE
7 J=JJ
8 RETURN
DEBUG SUBCHK
END
SUBROUTINE EIGHT(L,K,IJ,TIME,EVENT,IEVENT,IQ,IA1,IA2,IG,QS,IM,
1Q3,1G3,A,B,AL,BL,A2,B2,TW3,TW4,TW5,DSEED4,DSEED5,K1,NP,X,Y,V,NM,N
1REAL*B DSEED4*DSEED5
DIMENSION EVENT(NS),IA(NS),IA1(NS),IA2(NS),X(NS),Z1(50),Y(NS),V(NS),R1(NS),
1I(NS),QU3(NS),IG3(NS),R2(NS)
11 ITD=ITD+IG3(N)

```

```

1 CONTINUE
  IF(IA(IJ).LT.ITD) GO TO 3
  DO 2 N=1,L
    W=0.0
    W=(TIME-QU3(N))*IG3(N)
    QU3(N)=W
    TW3=TW3+W
    IG3(N)=0
2 CONTINUE
  IA(IJ)=IA(IJ)-ITD
  L=0
  GO TO 8
3 DO 5 N=1,L
  W=0.0
  IF((IA(IJ).GE.IG3(N)).GO TO 4
  IF((IA(IJ).EQ.0.0).GO TO 6
  IG3(N)=IG3(N)-IA(IJ)
  W=(TIME-QU3(N))*IA(IJ)
  TW3=TW3+W
  GO TO 6
4 W=(TIME-QU3(N))-IG3(N)
  QU3(N)=0.
  IG3(N)=0
5 CONTINUE
6 DO 7 N=L,L
  IF(QU3(N).EQ.0.0) GO TO 7
  LL=LL+1
  QU3(LL)=QU3(N)
  IG3(LL)=IG3(N)
7 CONTINUE
  L=LL
  GO TO 31
8 IF(K.EQ.0) GO TO 31
  ITD=0
  DO 9 N=1,K
    ITD=ITD+IG(N)
9 CONTINUE
  IF((IA(IJ).LT.ITD) GO TO 16
  IAA=0
  DO 10 N=1,K
    IF((IM(N).NE.1)) GO TO 10
    W=0.0
    W=(TIME-QS(N))*IG(N)
    TW4=TW4+W
    IAA=IAA+IG(N)

```

```

QS(N)=0.
IM(N)=0
10 CONTINUE
    IF(IAA.EQ.0) GO TO 32
    DO 14 KI=1,NM
        I=KI
        IF(EVENT(I).EQ.99999.) GO TO 12
        CONTINUE
        WRITE(6,100)
100 FORMAT(1 EVENT LIST IS FULL')
112 CALL GGAMS(DSEED4,A1,B1,I,Z1,R1)
EVENT(I)=TIME+R1(I)
IAA(I)=IAA
14 IAB=0
32 DO 13 N=1,K
    IF(IM(N).NE.2) GO TO 13
    W=0
    W=(T TIME-QS(N))*IG(N)
    TW5=TW5+W
    IAB=IAB+IG(N)
    QS(N)=0.
    IM(N)=0
13 CONTINUE
    IF(IAB.EQ.0) GO TO 31
    DO 14 KI=1,NM
        I=KI
        IF(EVENT(I).EQ.99999.) GO TO 15
        CONTINUE
14 WRITE(6,100)
15 CALL GGAMS(DSEED5,A2,B2,I,Z2,R2)
EVENT(I)=TIME+R2(I)
IA2(I)=IAB
GO TO 31
16 IAA=0
    IAB=0
    DO 20 N=1,K
        IF(IM(N).NE.1) GO TO 17
        IF(IAA(IJ).GE.IG(N)) GO TO 18
        IG(N)=IG(N)-IA(IJ)
        W=(T TIME-QS(N))*IA(IJ)
        TW4=TW4+W
        IAA=IAA+IA(IJ)
        GO TO 21
17 IF(IM(N).NE.2) GO TO 20

```

```

18 IF(IA(I,J) .GE. IG(N)) GO TO 19
    IG(N)=IG(N)-IA(I,J)
    W=(TIME-QS(N))*IA(I,J)
    TW5=TW5+W
    IA(IAB+IA(I,J))
    GO TO 21
19 W=0.
    IAA=IAA+IG(N)
    IA(I,J)=IA(I,J)-IG(N)
    QS(N)=0.
    IG(N)=0.
    IN(N)=0
    GO TO 20
20 W=0.
    IAB=IAB+IG(N)
    IA(I,J)=IA(I,J)-IG(N)
    TW5=TW5+W
    QS(N)=0.
    IG(N)=0
    CONTINUE
21 IF(IAA.EQ.0.) GO TO 24
    DO 22 KI=1,NM
    II=KI
    IF(EVENT(II).EQ.99999.) GO TO 23
22 CONTINUE
    WRITE(6,100)
23 CALL GGAMS(DSEED4,A1,B1,I,Z1,R1)
    EVENT(II)=TIME+R1(II)
    I=EVENT(II)=6
    IA(II)=IAA
    IF(IAB.EQ.0.) GO TO 27
    DO 25 KI=1,NM
    II=KI
    IF(EVENT(II).EQ.99999.) GO TO 26
25 CONTINUE
    WRITE(6,100)
26 CALL GGAMS(DSEED5,A2,B2,I,Z2,R2)
    EVENT(II)=7
    IA2(II)=IAB
    NN=0
27 DO 30 N=1,K
    IF(QS(N).EQ.0.) GO TO 30
    NN=NN+1

```

```

IF( IM(N).NE.1) GO TO 28
IM(NN)=1
GO TO 29
28 IM(NN)=2
QS(NN)=QS(N)
IG(NN)=IG(N)
30 CONTINUE
K=NN
31 RETURN
END

SUBROUTINE NINE(K,IQ,IDL,TIME,EVENT,LEVENT,IM,IG,QS,IA,IA1,IR,IS,
1 IP,A,B,AL,B1,DSEED4,DSEED6,K1,NP,X,Y,V,NM,NS,TOH)
REAL*8 DSEED4,DSEED6
DIMENTION EVENT(NS),LEVENT(NS),IM(NS),IA1(NS),QS(NS),IG(NS)
1 Z(50),Z1(50),X(NP),Y(NP),V(NP),R1(I),R2(I)
SS=Y(K1)
K1=K1+1
X(K1)=IQ
Y(K1)=IP
IF(IQ.LE.0) GO TO 11
WW=0.0
TOH=TIME-E-SS)*X(K1)
TOH=TOH+WW
IF(IQ.GE.IDL) GO TO 6
CALL GGAMS(DSEED4,AL,B1,1,Z1,R1)
DO 1 KI=1,NM
1 I=KI
IF(EVENT(I).EQ.99999.) GO TO 2
1 CONTINUE
1 WRITE(6,100)
100 FORMAT(1I10)
1 EVENT(I)=TIME+R1(I)
1 IA1(I)=IQ
K=K+1
QS(K)=TIME
IG(K)=ID1-IQ
1 H(K)=1
1 Q=IQ-1D1
1 P=IP-1D1
K1=K1+1
X(K1)=IP
Y(K1)=TIME
1 IF(IP.GT.IS) GO TO 14
DO 3 KI=1,NM

```

```

II=KI
IF(EVENT(II).EQ.99999.) GO TO 4
3 CONTINUE(6,100)
4 CALL GGAMS(DSEED6,A,B,L,Z,R)
EVENT(II)=8
ML=0
70 ML=ML+1
IP=IP+IR
IF(IP.LT.IS) GO TO 70
IA(II)=ML*IR
KI=KI+1
X(KI)=IQ
Y(KI)=TIME
GO TO 14
6 CALL GGAMS(DSEED4,A1,B1,L,Z1,R1)
DO 7 KI=1,NM
II=KI
IF(EVENT(II).EQ.99999.) GO TO 8
7 CONTINUE(6,100)
8 WRITE(6,100)
EVENT(II)=TIME+R1(1)
IA(II)=ID1
IQ=IP-ID1
IP=IP-ID1
KI=KI+1
X(KI)=IQ
Y(KI)=TIME
IF(IP.GT.IS) GO TO 14
DO 9 KI=1,NM
II=KI
IF(EVENT(II).EQ.99999.) GO TO 10
9 CONTINUE(6,100)
WRITE(6,100)
10 CALL GGAMS(DSEED6,A,B,L,Z,R)
EVENT(II)=8
ML=0
71 ML=ML+1
IP=IP+IR
IF(IP.LT.IS) GO TO 71
IA(II)=ML*IR
KI=KI+1
X(KI)=IQ

```

```

Y(K1)=TIME
GO TO 14
11 K=K+1
QS(K)=TIME
IG(K)=ID1
IQ=ID1
IP=ID1
K1=K1+1
X(K1)=IQ
Y(K1)=TIME
IF(IP.GT.IS) GO TO 14
DO 12 K1=1,NM
11 =K1
IF(EVENT(II).EQ.99999.) GO TO 13
12 CONTINUE
WRITE(6,100)
13 CALL GSNS(DSEED6,A,B,L,Z,R)
EVENT(II)=TIME+R(1)
EVENT(II)=8
ML=0
72 IP=IP+IR
IF(IP.LT.IS) GO TO 72
1A(K1)=ML*IR
K1=K1+1
X(K1)=IQ
Y(K1)=TIME
14 RETURN
DEBUG SUBCHK
SUBROUTINE TEN(K1,IQ,ID2,TIME,IEVENT,IM,IG,QS,IA,IA2,IR,IS,I
P,IA,B,A2,DSEED5,DSEED6,K1,NP,X,Y,V,NM,NS,TOH)
REAL*8 DSEED5,DSEED6
DIMENSION EVENT(NS),IEVENT(NS),IM(NS),IA2(NS),QS(NS),IG(NS),IA(NS)
I,Z(50),Z2(50),X(NP),Y(NP),V(NP),R(1),R2(1)
SS=Y(K1)
K1=K1+1
X(K1)=IQ
Y(K1)=IP
IF(IQ.LE.0) GO TO 11
WW=TIME-SS)*X(K1)
TOH=TOH+WW

```

```

1 IF((IQ.GE.ID2)) GO TO 6
2 CALL GGAMS (DSEED5,A2,B2,1,22,R2)
3 DO 1 KI=1,NM
4 IF(EVENT(III).EQ.99999.) GO TO 2
5 CONTINUE
6 WRITE(6,100)
7 FORMAT(1,EVENT(LIST IS FULL'))
8 EVENT(III)=TIME+R2(1)
9 EVENT(II)=IQ
10 IA2(II)=IQ
11 K=K+1
12 GS(K)=TIME
13 GK(K)=ID2-IQ
14 IQ=IQ-ID2
15 IP=IP-ID2
16 K1=K1+1
17 X(K1)=IQ
18 Y(K1)=TIME
19 IF(IP.GT.IS) GO TO 14
20 DO 3 KI=i,NM
21 IF(EVENT(III).EQ.99999.) GO TO 14
22 CONTINUE
23 WRITE(6,100)
24 CALL GGAMS (DSEED6,A,B,1,Z,R1)
25 EVENT(III)=TIME+R1(1)
26 IML=0
27 MLL=ML+1
28 IP=IP+IR
29 IF((IP.LT.IS)) GO TO 70
30 K1=K1+1
31 X(K1)=ML*IR
32 Y(K1)=IP
33 VTO 14
34 CALL GGAMS (DSEED5,A2,B2,1,22,R2)
35 DO 7 KI=1,NM
36 IF(EVENT(III).EQ.99999.) GO TO 8
37 CONTINUE
38 WRITE(6,100)
39 EVENT(III)=TIME+R2(1)
40 EVENT(III)=7

```

```

1 A2(II)=ID2
1 Q=IQ-ID2
1 P=IP-ID2
1 K1=K1+1
1 X(K1)=IQ
1 Y(K1)=TIME
1 F(IP.GT.IS) GO TO 14
DO 9 K1=1,NM
1 I=K1
1 IF(EVENT(II).EQ.9999.) GO TO 10
9 CONTINUE
9 WRITE(6,100)
10 CALL GGAMS(DSEED6,A,B,1,Z,R)
10 EVENT(II)=TIME+R(1)
10 EVENT(II)=8
ML=0
71 ML=ML+1
1 P=IP+IR
1 F(IP+LT.IS) GO TO 71
1 A(II)=ML*IR
1 K1=K1+1
1 X(K1)=IQ
1 Y(K1)=IP
1 V(K1)=TIME
1 GO TO 14
1 K=K+1
1 QS(K)=TIME
1 G(K)=ID2
1 H(K)=2
1 I=IQ-ID2
1 P=IP-ID2
1 K1=K1+1
1 X(K1)=IP
1 Y(K1)=TIME
1 F(IP.GT.IS) GO TO 14
DO 12 K1=1,NM
1 I=K1
1 IF(EVENT(II).EQ.9999.) GO TO 13
12 CONTINUE
12 WRITE(6,100)
13 CALL GGAMS(DSEED6,A,B,1,Z,R)
13 EVENT(II)=TIME+R(1)
13 EVENT(II)=8
ML=0
72 ML=ML+1
1 P=IP+IR

```

```
IF(IPI=LI) GO TO 72  
IA(LI)=ML*IR  
KI=KI+1  
X(KI)=IQ  
V(KI)=IP  
Y(KI)=TIME  
14 RETURN SUBCHK  
DEBUG  
END
```

APPENDIX D

Simulation Program; No Versatec Plotter Output

```

REAL*8 DSEED1,DSEED2,DSEED3,DSEED4,DSEED5,DSEED6,DSEED7,DSEED8,DSEED9
1 DIMENSION EVENT(900),IM(900),IS(900),I3(900),I1(900),QU1(900),QU2(900),
1 Q(900),IG1(900),IG2(900),IG3(900),IA1(900),IA2(900),IA3(900),IA4(900),
1 IZ1(50),IZ2(50),WK(50),WS(50),WZ(50),IK1(1),IK2(1),IK3(1),IK4(1),
1 IS1(1),IS2(1),IS3(1),KA(10),AK(10)
1 NS=900
1 NH=30
DSEED1=123456.000
DSEED2=247658.000
DSEED3=365274.000
DSEED4=258732.000
DSEED5=541863.000
DSEED6=433215.000
DSEED7=651563.000
DSEED8=265418.000
DSEED9=167519.000
1=0
J=0
K=0
L=0
SS1=0.0
SS2=0.0
TH1=0.0
TH2=0.0
TH3=0.0
TH4=0.0
TH5=0.0
TOH1=0.0
TOH2=0.0
DO 1 N=1,50
EVENT(N)=99.999.
1 EVENT(N)=0
QU1(N)=0.0
QU2(N)=0.0
QU3(N)=0.0
IG1(N)=0
IG2(N)=0
IG3(N)=0
IA1(N)=0
IA2(N)=0
IA3(N)=0
IA4(N)=0
CONTINUE
1 DO 2 N=1,10
1 IM(N)=0
1 IS(N)=0

```

```

1 G(N)=0
2 K(N)=0
3 CNTINJE
4 IF=0
5 IF1=0
6 IF2=0
7 ID=0
8 ID1=0
9 ID2=0
10 READ(5,100) IR,IS
110 FORMAT(5,110) IR1,IS1,T1
110 FORMAT(5,110) F10,0
110 READ(5,120) IR2,IS2,T2
120 FORMAT(5,120) YEAR
120 FORMAT(5,130) A,B
130 FORMAT(5,12F4,0)
130 READ(5,130) A1,B1
130 READ(5,130) A2,B2
130 READ(5,140) XM1
130 READ(5,140) XM2
130 READ(5,140) XM3
140 FORMAT(5,F4,0) P1
140 READ(5,150) P2
140 READ(5,150) P3
150 FORMAT(F6,0)
150 IQ=IR+IS
150 IQ1=IR1
150 IQ2=IR2
150 Q1=100000
150 Q1=100000
150 Q2=100000
150 D1=100000
150 D2=100000
150 XL=(YEAR/365.)**48.
150 TIME=0.0
150 IP=IR+IS
150 IP1=IR1
150 IP2=IR2
150 CALL GGEXN (DSEED1,XM1,1,S1)
150 CALL GGEGT (DSEED7,1,P1,WZ,IK1)
150 IF1=1
150 DO 3 K1=1,NM
150 II=KI
150 IF(EVENT(III).EQ.99999.1 GO TO 4

```

```

3 CONTINUE(II)=TIME+S1(1)
4 IF(EVENT(II)=1
CALL GGEVN (DSEED2,XM2,1,S2)
CALL GGEOT (DSEED8,1,P2,WS,IK2)
DO 5 K1 =1,NM
      IF(K1
IF(EVENT(II).EQ.99999.1 GO TO 6
5 CONTINUE(II)=TIME+S2(1)
6 IF(EVENT(II)=2
CALL GGEVN (DSEED3,XM3,1,S3)
CALL GGEOT (DSEED9,1,P3,WK,IK3)
IF=1
DO 7 K1 =1,NM
      IF(K1
IF(EVENT(II).EQ.99999.1 GO TO 8
7 CONTINUE(II)=TIME+S3(1)
8 IF(EVENT(II)=3
DO 9 K1 =1,NM
      IF(K1
IF(EVENT(II).EQ.99999.1 GO TO 10
9 CONTINUE(II)=TIME+T1
10 IF(EVENT(II)=4
DO 11 K1 =1,NM
      IF(K1
IF(EVENT(II).EQ.99999.1 GO TO 12
11 CONTINUE(II)=TIME+T2
12 IF(EVENT(II)=5
DO 13 K1 =1,NM
      IF(K1
IF(EVENT(II).EQ.99999.1 GO TO 14
13 CONTINUE(II)=YEAR
14 IF(EVENT(II)=9
GO TO 26
15 EVENT(IN)=99999.
16 EVENT(IN)=0
26 IN=1
      DO 16 K1=2,NM
      IF(EVENT(IN).GT.EVENT(K1)) IN=K1
CONTINUE(II)=TIME-EVENT(IN)
      IL=EVENT(IN)

```

```

1 J=IN (17 18 19 20 21 22 23 24 25) !!
2 CALL ONE (I TIME,EVENT,QU1,IG1,IQ1,IP1,IF1,XM1,P1,DSEED1,DSTURO1460
3 IED7,NM,NS,TOH1,SSI,IQQ1)
4 GO TO 15
5 CALL TWO (J TIME,EVENT,QU2,IG2,IQ2,IP2,IF2,XM2,P2,DSEED2,DSTURO1470
6 IED7,NM,NS,TOH2,SSI,IQQ2)
7 CALL THREE (L TIME,EVENT,QU3,IG3,IQ3,IP3,IF3,XM3,P3,DSEED3,DS
8 IED6,DSEED9,A,B,NM,NS,IA,IR,IS,TOH,SS,KA,AK,IQQ),ITURO1480
9 CALL FOUR (K,IQ,IQ1,IP1,TIME,EVENT,IEVENT,IM,IG,QS,IA,IA1,IR,IR1,ITURO1490
10 IP,A,B,A1,B1,DSEED4,DSEED6,NM,T2,NS,IS1,TOH,SS,KA,AK,IQQ,IQ1,NDTURO1500
11 ) GO TO 15
12 CALL FIVE (K,IQ,IQ2,IP2,TIME,EVENT,IEVENT,IM,IG,QS,IA,IA2,IR,IR2,ITURO1510
13 IP,A,B,A2,B2,DSEED5,DSEED6,NM,T2,NS,IS2,TOH,SS,KA,AK,IQQ,IQ2,NDTURO1520
14 ) GO TO 15
15 CALL SIX (I,J,TIME,IA1,IQ1,QU1,IG1,TW1,IP1,NM,NS) TUR01530
16 GO TO 15
17 CALL SEVEN (J,I,J,TIME,IA2,IQ2,QU2,IG2,TW2,IP2,NM,NS) TUR01540
18 GO TO 15
19 CALL EIGHT (L,K,I,J,TIME,EVENT,IEVENT,IQ,IA,IA1,IA2,IG,2S,IM,IG,3,IG,TURO1550
20 13,A,B,A1,B1,A2,B2,TW3,TW4,TW5,DSEED5,NM,NS,IP,KA,AK,IQQ) TUR01560
21 14,TW1,IYEAR TUR01570
22 15,TW1,IYEAR TUR01580
23 16,TW1,IYEAR TUR01590
24 17,TW1,IYEAR TUR01600
25 18,TW1,IYEAR TUR01610
26 19,TW1,IYEAR TUR01620
27 20,TW1,IYEAR TUR01630
28 21,TW1,IYEAR TUR01640
29 22,TW1,IYEAR TUR01650
30 23,TW1,IYEAR TUR01660
31 24,TW1,IYEAR TUR01670
32 25,TW1,IYEAR TUR01680
33 26,TW1,IYEAR TUR01690
34 27,TW1,IYEAR TUR01700
35 28,TW1,IYEAR TUR01710
36 29,TW1,IYEAR TUR01720
37 30,TW1,IYEAR TUR01730
38 31,TW1,IYEAR TUR01740
39 32,TW1,IYEAR TUR01750
40 33,TW1,IYEAR TUR01760
41 34,TW1,IYEAR TUR01770
42 35,TW1,IYEAR TUR01780
43 36,TW1,IYEAR TUR01790
44 37,TW1,IYEAR TUR01800
45 38,TW1,IYEAR TUR01810
46 39,TW1,IYEAR TUR01820
47 40,TW1,IYEAR TUR01830
48 41,TW1,IYEAR TUR01840
49 STOP DEBUG SUBCHK TUR01850
50 END
51 SUBROUTINE ONE (I TIME,EVENT,QU1,IG1,IQ1,IP1,IF1,XM1,P1,DSEED1,DSTURO1860
52 IED1,DSEED7,NM,NS,TOH1,SSI,IQQ1)
53 REAL*8 DSEED1,DSEED7
54 DIMENSION EVENT(NS),IEVENT(NS),QU1(NS),IG1(NS),WL(50),IK1(11),SI(11)
55 IF(IQ1.GT.0) GO TO 1
56 I=I+1

```

```

1 IQ1=IQ1-IF1
1 IP1=IP1-IF1
1 QD1=IQ1-TIME
1 Q1(1)=IQ1-IF1
GO TO 3
1 WW=0.0
1 TOH=TOH1+WW
1 FFL=IQ1-LT*IF1) GO TO 2
1 IQ1=IQ1-IF1
1 IP1=IP1-IF1
1 QD1=IQQ1-IF1
GO TO 3
1 =I+1
1 IQ1=IQ1-IF1
1 IP1=IP1-IF1
1 QD1=IQQ1-IF1
1 Q1(1)=TIME
1 IQ1(I)=IF1-IQ1(DSEEED1,XM1,I,S1)
3 CALL CGEXN (DSEEED7,1,PI,WZ,IK1)
1 IF1=1
DO 4 KI=1,NM
1 ZXI
1 IF(EVENT(111).EQ.99999.) GO TO 5
4 CONTINUE
1 WRITE(6,100)
100 FORMAT(111,111)
100 EVENT(I,I)=TIME+SI(I)
100 SI=TIME
RETURN
DEBUG SUBCHK
END
SUBROUTINE TWO (J,TIME,EVENT,IEVENT,QU2,IG2,IQ2,IP2,IF2,XM2,P2,DSE
1 ED2,DSEEED8,NM,NS,TOH2,SS2,IQQ2)
1 REAL *8 DSEEED2*DSEEED8
1 DIMENSION EVENT(NS),IEVENT(NS),QU2(NS),IG2(NS),WS(NS),IK2(1),S2(1)
1 F(IQ2,GT,0),S3 TO 1
1 =J+1
1 Q2=IQ2-IF2
1 P2=IP2-IF2
1 QD2=IQQ2-IF2
1 G2(J)=IF2
GO TO 3
1 WW=0.0

```

```

WW=(TIME-SS)*IQ2
TOH=TOH+WW
1Q2=1Q2-LT*IF2
1P2=1P2-IF2
1QQ2=1QQ2-IF2
GO TO 3
2 J=J+1
1Q2=1Q2-IF2
1P2=1P2-IF2
1QQ2=1QQ2-IF2
TIME=TIME
1G2(J)=IF2-IQ2
CALL GGEOT(DSEED2,XM2,I,WS,IK2)
3 IF2=1
DO 4 KI=1,NM
I=KI
IF(EVENT(II).EQ.99999.) GO TO 5
CONTINUE
4 WRITE(6,100)
100 FORMAT(1X,'EVENT LIST IS FULL')
5 EVENT(1)=TIME+S2(I)
EVENT(II)=2
SS2=TIME
RETURN DEBUG SUBCHK
END
SUBROUTINE THREE (L,TIME,EVENT,IEVENT,QU3,IG3,IQ,IP,IF,XM3,P3,DSEET)
JD31DSEED6*DSEED9
REAL*8 DSEED3,DSEED9
DIMENSION EVENT(NS),IEVENT(NS),QU3(NS),IG3(NS),IA(NS),WK(50)
1 IK3(1) S3(1) R(1) KA(10) AK(10)
1 IF(IQ,LT,0) GO TO 1
L=L+1
IP=IP-IF
IQ=IQ-IF
1QQ=1QQ-IF
Q3(L)=TIME
1G3(L)=IF
GO TO 3
1 WW=0.0
WW=(TIME-SS)*IQ
TOH=TOH+WW
1F(IQ,LT,IF) GO TO 2
1Q=1Q-IF
1QQ=1QQ-IF
1P=1P-IF

```

```

      GO TO 3
 2   L=L+1
     Q=U3(L)=TIME
     I=Q-IF
     IP=IP-IF
     ICQ=I*QQ-IF
     IF(IP.GT.I) GO TO 6
     CALL GGAMS(DSEED6,A,B,L,Z,R)
     DO 4 KI=1,NM
     IF(KI=EVENT(II).EQ.99999.) GO TO 5
     CONTINUE
 4   WRITE(6,100)
 100  FORMAT(1I10)
 5   EVENT(II)=TIME+R(1)
     EVENT(II)=8
     ML=0
 70   ML=ML+1
     IP=IP+IR
     IF(IP.LT.IS) GO TO 70
     IA(II)=ML*IR
     DO 60 KT=1,10
     LS=KT
     IF(AK(LS).EQ.0.0) GO TO 61
 60   CONTINUE
 61   KA(LS)=IQ
     AK(II)=EVENT(II)
     CALL GGEXN(DSEED3,XM31,S3)
     CALL GEOT(DSEED9,1,P3,WK,IK3)
     IF=1
     DO 7 KI=1,NM
     I=KI
     IF(EVENT(II).EQ.99999.) GO TO 8
     CONTINUE
 7   WRITE(6,100)
 8   EVENT(II)=TIME+S3(1)
     IS=TIME
     RETURN
     DEBUG SUBCHK
END
SUBROUTINE FOUR (K,IQ,IQ1,IPL,TIME,EVENT,IEVENT,IM,IGQS,IA,IA1,IR,
 1,IR1,IS1,IP,A,B,A1,B1,DSEED4,DSEED6,NM,T1,NS,IS1,TC,H,SS,K,A,K,IQ,Q,
 1,IQ1,ND1)
REAL*8 DSEED4,DSEED6
DIMENSION EVENT(NS),IEVENT(NS),IM(NS),QS(NS),IG(NS),IA(NS)
 1,KA(10),AK(10)

```

```

ND1=ND11-IQQ1
ND11=0
WRITE(6,250) ND1
250 FORMAT(6,250) ND1
FORMAT(6,250) ND1
AMOUNT OF DEMAND IN A PERIOD OF 1=*,110)
ID1=0
IF(IP1.GT.IS1) GO TO 1
ID1=IR1-IP1
IP1=IR1
CALL NINE(K,IQ1,IDL,TIME,EVENT,IEVENT,IM,IG,QS,IA,IA1,IR,IS,IP,A,B)
IP1=B1*DSEED4,DSEED6,NM,NS,TOH,SS,KA,AK,IQQ1
1 DO 2 KI=1,NM
   I=KI
   IF(EVENT(III).EQ.99999.) GO TO 3
2 CONTINUE
   WRITE(6,100)
100 FORMAT(6,100)
   EVENT(III)=TIME+T1
3 EVENT(III)=4
   ND11=IQQ1
   RETURN
DEBUG SUBCHK
END
SUBROUTINE FIVE(K,IQ1,IQ2,IP2,TIME,EVENT,IEVENT,IM,IG,QS,IA,IA2,IR,
1 IR2,IS,IP,A,B,A2,B2,DSEED5,DSEED6,NM,T2,NS,IS2,TOH,SS,KA,AK,IQQ1
1,IQQ2,ND2)
2 DSEED5,DSEED6
REAL,*8
DIMENSION EVENT(NS),IEVENT(NS),IM(NS),QS(NS),IG(NS),IA(NS)
1 KA(10),AK(10)
ND2=ND22-IQQ2
ND2=0
WRITE(6,250) ND2
250 FORMAT(6,250) ND2
AMOUNT OF DEMAND IN A PERIOD OF SYSTEM 2=*,110)
ID2=0
IF(IP2.GT.IS2) GO TO 1
ID2=IR2-IP2
IP2=IR2
CALL TEN(K,IQ1,IDL2,TIME,EVENT,IEVENT,IM,IG,QS,IA,IA2,IR,IS,IP,A,B,
1 A2,B2,DSEED5,DSEED6,NM,NS,TOH,SS,KA,AK,IQQ1
1 DO 2 KI=1,NM
   I=KI
   IF(EVENT(III).EQ.99999.) GO TO 3
2 CONTINUE
   WRITE(6,100)
100 FORMAT(6,100)
   EVENT(III)=TIME+T2
3 EVENT(III)=5
   ND22=IQQ2
   RETURN

```

```

DEBUG SUBCHK
END
SUBROUTINE SIX (I,J,IJ,TIME,IAI(NS),IGI(NS),QUI(NS),IP1,NM,NS)
DIMENSION IAI(NS),IGI(NS),QUI(NS)
WRITE(6,50) IJ
50 FORMAT ('/; EXPECTED NET INVENTORY OF SYSTEM 1 AT THE END OF CYCLE',TUR03900
1=I6)
1 IJ=IQ1+IA1(IJ)
1 ITD=0
1 IF(I.EQ.0) GO TO 8
DO 1 N=1,I
1 ITD=ITD+IG1(N)
1 CONTINUE
1 IF(IA1(IJ).LT.ITD) GO TO 3
DO 2 N=1,I
W=0
W=(TIME-QUI(N))*IG1(N)
QUI(N)=0
IG1(N)=0
2 CONTINUE
2 IA1(IJ)=0
I=0
3 GO TO 8
3 DO 5 N=1,I
IF(IA1(IJ).GE.IG1(N)) GO TO 4
IF(IA1(IJ).EQ.0) GO TO 6
IG1(N)=IG1(N)-IA1(IJ)
TW1=TW1+W
GO TO 6
4 IA1(IJ)=IA1(IJ)-IG1(N)
W=(TIME-QUI(N))*IG1(N)
QUI(N)=0.
IG1(N)=0
5 CONTINUE
5 MM=0
6 DO 7 N=1,I
IF(QUI(N).EQ.0.) GO TO 7
MM=MM+1
QUI(MM)=QUI(N)
IG1(MM)=IG1(N)
7 CONTINUE
7 I=MM
8 RETURN
8 DEBUG SUBCHK

```

```

END
SUBROUTINE SEVEN (J, TIME, IA2(NS), IG2(NS), QU2(NS) Q2, QU2, IG2, TW2, IP2, NM, NS)
DIMENSION IA2(NS), IG2(NS), QU2(NS)
WRITE(6,50) IQ2
50 FORMAT ('/ ', EXPECTED NET INVENTORY OF SYSTEM 2 AT THE END OF CYCLE')
      1=16
      IQ2=IA2+IA2(IJ)
      ITD=0
      IF(J.EQ.0) GO TO 8
      DO 1 N=1,J
      ITD=ITD+IG2(N)
      1 CONTINUE
      IF(IA2(IJ).LT.ITD) GO TO 3
      DO 2 N=1,J
      W=0
      W=(TIME-QU2(N))*IG2(N)
      QU2(N)=0
      TW2=TW2+W
      2 CONTINUE
      J=0
      3 DO 5 N=1,J
      W=0
      IF(IA2(IJ).GE.IG2(N)) GO TO 4
      IF(IA2(IJ).EQ.0) GO TO 6
      IG2(N)=IG2(N)-IA2(IJ)
      W=(TIME-QU2(N))*IA2(IJ)
      TW2=TW2+W
      4 IA2(IJ)=IA2(IJ)-IG2(N)
      W=(TIME-QU2(N))*IG2(N)
      QU2(N)=0
      5 CONTINUE
      J=0
      6 DO 7 N=1,J
      IF(QU2(N).EQ.0.) GO TO 7
      JJ=JJ+1
      QU2(JJ)=QU2(N)
      IG2(JJ)=IG2(N)
      7 CONTINUE
      J=JJ
      8 RETURN
      DEBUG SUBCHK
      END

```

```

SUBROUTINE EIGHT (L,K,I,J,TIME,EVENT,IEVENT,IQ,IA1,IA2,IG,QS,IM,TURO4810
1Q,QU3,IG3,A,B,AL,B1,A2,B2,IW3,TW4,TW5,DSEED4,DSEED5,DSEED6,DSEED7,
1REAL*8 DSEED4*DSEED5
DIMENSION EVENT(NS),IEVENT(NS),IA1(NS),IA2(NS),IG(NS),QS(NS),
1I,IM(NS),QU3(NS),IG3(NS),Z1(50),Z2(50),RI(10),KA(10),AK(10),
1I,I0=0
JZ=0
JZ=1Q
DO 60 KT=1,10
L5=KT
IF(IAK(L5).EQ.TIME) GO TO 61
60 CONTINUE
61 ND=KA(L5)-1QQ
KA(L5)=0
AK(L5)=0.
WRITE(6,250) ND,TIME
250 FORMAT ('//',1X,'DEMAND DURING LEAD TIME=',I10,' TIME=' ,F15.5)
1Q=IQ+IA(I,J)
WRITE(6,50) JZ,TIME
50 FORMAT ('//',1X,'EXPECTED NET INVENTORY OF MAIN SYSTEM AT THE END OF CY
1CLE=,16.2X,IQ=,16.2X,TIME=',F10.4)
1F(L,Eq,0) GO TO 8
DO 1 N=1,L
ITD=ITD+IG3(N)
1 CONTINUE
1F(IA(I,J),LT,ITD) GO TO 3
DO 2 N=1,L
W=0.
W=(TIME-QU3(N))*IG3(N)
QU3(N)=0.
TW3=TW3+W
IG3(N)=0
2 CONTINUE
1A(I,J)=IA(I,J)-ITD
L=0
GO TO 8
3 DO 5 N=1,L
W=0.
IF(IA(I,J).GE.IG3(N)) GO TO 6
1F(IA(I,J),EQ,0) GO TO 6
IG3(N)=IG3(N)-IA(I,J)
W=(TIME-QU3(N))*IA(I,J)
TW3=TW3+W
GO TO 6
4 IA(I,J)=IA(I,J)-IG3(N)
W=(TIME-QU3(N))*IG3(N)

```

```

      QU3(N)=0.
      5   IG3(N)=0
      CONTINUE
      6   DO 7 N=1,L.EQ.0.) GO TO 7
      L=LL
      LL=LL+1
      QU3(LL)=QU3(N)
      IG3(LL)=IG3(N)
      CONTINUE
      7
      L=LL
      GO TO 31
      8   IF(K.EQ.0) GO TO 31
      ITD=0
      DO 9 N=1,K
      9   CONTINUE
      IF(IA(IJ).LT.ITD) GO TO 16
      IAA=0
      DO 10 N=1,K
      10  IF(IM(N).NE.1) GO TO 10
      W=0.
      W=(TIME-QS(N))*IG(N)
      TW4=TW4+W
      IAA=IAA+IG(N)
      QS(N)=0.
      IM(N)=0
      10 CONTINUE
      11  IAA.EQ.0) GO TO 32
      DO 11 KI=1,NM
      11  IF(EVENT(III).EQ.99999.) GO TO 12
      11 CONTINUE
      WRITE(6,100)
      100 FORMAT(1000
      CALL GAMS(DSEED4,A1,B1,I,ZI,R1)
      EVENT(III)=TIME+R1(I)
      EVENT(III)=6
      IAA(II)=IAA
      32  IAB=0
      DO 13 N=1,K
      13  IF(IM(N).NE.2) GO TO 13
      W=0.
      W=(TIME-QS(N))*IG(N)
      TW5=TW5+W
      IAB=IAB+IG(N)
      QS(N)=0.

```

```

IG(N)=0          TUR05770
IM(N)=0          TUR05780
CONTINUE EQ:0 GO TO 31 TUR05790
IF(IAB.EQ.0) GO TO 100 TUR05800
DO 14 K=1,NM TUR05810
I=K I=K
IF(EVENT(III).EQ.9999.) GO TO 15 TUR05820
CONTINUE TUR05830
14 WRITE(6,100) TUR05840
15 CALL GGAMS(DSEED$1A2,B2,1,L2,R2) TUR05850
EVENT(III)=7 TUR05860
IA2(II)=IAB TUR05870
TUR05880
TUR05890
TUR05900
TUR05910
TUR05920
TUR05930
TUR05940
TUR05950
TUR05960
TUR05970
TUR05980
TUR06000
TUR06010
TUR06020
TUR06030
TUR06040
TUR06050
TUR06060
TUR06070
TUR06080
TUR06090
TUR06100
TUR06110
TUR06120
TUR06130
TUR06140
TUR06150
TUR06160
TUR06170
TUR06180
TUR06190
TUR06200
TUR06210
TUR06220
TUR06230
TUR06240

13
14
15
16
17
18
19

```

13 IG(N)=0
 IM(N)=0
 CONTINUE EQ:0 GO TO 31
 IF(IAB.EQ.0) GO TO 100
 DO 14 K=1,NM
 I=K I=K
 IF(EVENT(III).EQ.9999.) GO TO 15
 CONTINUE
 14 WRITE(6,100)
 15 CALL GGAMS(DSEED\$1A2,B2,1,L2,R2)
 EVENT(III)=7
 IA2(II)=IAB
 GO TO 31
 16 IAA=0
 IAB=0
 DO 20 N=1,K
 IF(IA(II)*GE;IG(N)) GO TO 17
 IG(N)=IG(N)-IA(II)
 W=(TIME-QS(N))*IA(II)
 TW4=TW4+W
 IAA=IA+IA(II)
 GO TO 21
 IF(IA(II)*NE;2) GO TO 20
 IG(N)=IG(N)-IA(II)
 W=(TIME-QS(N))*IA(II)
 TW5=TW5+W
 IAB=IAB+IA(II)
 GO TO 21
 17 W=0.
 IAA=IAA+IG(N)
 IA(II)=IA(II)-IG(N)
 W=(TIME-QS(N))*IG(N)
 TW4=TW4+W
 QS(N)=0.
 IG(N)=0.
 IM(N)=0
 GO TO 20
 18 W=0.
 IAB=IAB+IG(N)
 IA(II)=IA(II)-IG(N)
 W=(TIME-QS(N))*IG(N)
 TW5=TW5+W
 QS(N)=0.
 IG(N)=0.
 IM(N)=0

```

20 CONTINUE
21 IF(IAA.EQ.0) GO TO 24
22 DO 22 K=I,NM
23 IF(EVENT(III).EQ.99999.) GO TO 23
24 WRITE(6,100)
25 CALL GGAMS(DSEED4,A1,B1,I,Z1,R1)
26 EVENT(III)=TIME+R1(I)
27 IF(EVENT(III)=IAA) GO TO 27
28 DO 25 K=I,NM
29 IF(EVENT(III).EQ.0) GO TO 27
30 CONTINUE
31 IF(EVENT(III).EQ.99999.) GO TO 26
32 WRITE(6,100)
33 CALL GGAMS(DSEED5,A2,B2,I,Z2,R2)
34 EVENT(III)=TIME+R2(I)
35 IF(EVENT(III)=IAB) GO TO 35
36 DO 30 N=1,K
37 IF(QS(N).NE.0) GO TO 30
38 IM>NN+1
39 NN=NN+1
40 IF(IM>NN).NE.1) GO TO 30
41 IM>NN=1
42 GO TO 29
43 IM>NN=2
44 QS>NN)=QS(N)
45 IG>NN)=IG(N)
46 CONTINUE
47 K>NN
48 RETURN
49 DEBUG SUBCHK
50 END
51 SUBROUTINE MIVE(K,IQ,IDL,TIME,EVENT,IM,IG,QS,IA,IR,IS,TURO6610
52 ,IP,A,B,AL,B1,DSEED4,DSEED6,NM,NS,TOH,SS,KA,AK,QQ)
53 REAL*B,DSEED4,DSEED6
54 DIMENSION EVENT(NS),IM(NS),QS(NS),IA(NS),IA(NS)
55 IZ(50),Z1(50),R(1),R(1),KA(10),KA(10)
56 ID=0
57 IF(IQ.LE.0) GO TO 11
58 WW=0.0
59 WW=(TIME-SS)*IQ
60 TOH=TOH+WW
61 IF(IQ.GE.ID) GO TO 6
62 CALL GGAMS(DSEED4,A1,B1,I,Z1,R1)

```

```

DO 1 KI=1,NM
1 I=KI
1 IF(EVENT(III).EQ.99999.) GO TO 2
1 CONTINUE
1 WRITE(6,100)
100 FORMAT(1X,EVENT(LIST) IS FULL.)
1 EVENT(III)=6
1 AI(III)=IQ
1 K=K+1
1 QS(K)=TIME
1 G(K)=ID1-IQ
1 N(K)=1
1 Q=IQ-101
1 P=IP-ID1
1 QQ=IP-Q-ID1
1 F(IP+3,T,IS) GO TO 14
DO 3 KI=1,NM
3 I=KI
3 IF(EVENT(III).EQ.99999.) GO TO 4
3 CONTINUE
3 WRITE(6,100)
4 CALL GGAMS(DSEED6,A,B,L,Z,R)
4 EVENT(III)=TIME+R(1)
4 EVENT(III)=8
ML=0
70 ML=ML+1
70 IP=IP+IR
70 IF(IP>LI*IS) GO TO 70
70 IA(II)=ML*IR
70 DO 60 KI=1,10
70 L5=KI
70 IF(AK(L5).EQ.0.) GO TO 61
60 CONTINUE
61 KA(L5)=IQ
61 AK(L5)=EVENT(III)
61 GO TO 14
6 CALL GGAMS(DSEED4,A1,B1,L,Z1,R1)
6 DO 7 KI=1,NM
7 I=KI
7 IF(EVENT(III).EQ.99999.) GO TO 8
7 CONTINUE
7 WRITE(6,100)
8 EVENT(III)=TIME+R1(1)
8 EVENT(III)=6
8 AI(III)=ID1
8 Q=IQ-101
8 P=IP-ID1

```

```

100=IQ-Q-1D1
101=IP+LT*IS) 30 TO 14
11-KI
11-EVENT(III).EQ.99999.) GO TO 10
9  CONTINUE
  WRITE(6,100)
10  CALL CGAMS(DSEED6,A,B,1,Z,R)
    EVENT(III)=TIME+R(1)
    EVENT(III)=8
    ML=0
    ML=ML+1
    IP=IP+IR
    IF(IP+LT*IS) GO TO 71
    IA(II)=ML*IR
    DO 63 KT=1,10
    L5=KT
    IF(AK(L5)).EQ.0.0) GO TO 64
    CONTINUE
    AK(L5)=IQQ
    63 K=K+1
    AK(L5)=EVENT(III)
    GO TO 14
    64 QS(K)=TIME
    IC(K)=ID1
    IM(K)=1
    IQ=IQ-ID1
    IP=IP-ID1
    IQ=IQ-ID1
    IF(IP.GT.IS) GO TO 14
    DO 12 KT=1,NM
    12 IF(EVENT(III).EQ.99999.) GO TO 13
    CONTINUE
    13 WRITE(6,100)
    CALL CGAMS(DSEED6,A,B,1,Z,R)
    EVENT(III)=TIME+R(1)
    EVENT(III)=8
    ML=0
    ML=ML+1
    IP=IP+IR
    IF(IP+LT*IS) GO TO 72
    IA(II)=ML*IR
    SS=TIME
    RETURN
    DEBUG SUBCHK
    SUBROUTINE TEN (K,IQ,ID2,TIME,EVENT,IEVENT,IM,IG,QS,IA,IA2,IR,IS,IT)
    TUR07210
    TUR07220
    TUR07230
    TUR07240
    TUR07250
    TUR07260
    TUR07270
    TUR07280
    TUR07290
    TUR07300
    TUR07310
    TUR07320
    TUR07330
    TUR07340
    TUR07350
    TUR07360
    TUR07370
    TUR07380
    TUR07390
    TUR07400
    TUR07410
    TUR07420
    TUR07430
    TUR07440
    TUR07450
    TUR07460
    TUR07470
    TUR07480
    TUR07490
    TUR07500
    TUR07510
    TUR07520
    TUR07530
    TUR07540
    TUR07550
    TUR07560
    TUR07570
    TUR07580
    TUR07590
    TUR07600
    TUR07610
    TUR07620
    TUR07630
    TUR07640
    TUR07650
    TUR07660
    TUR07670
    TUR07680

```

```

1 P,A,B,A2,B2,DSEED5,DSEED6,NM,NS,TOH,SS,KA,AK,IQQ)
2 REAL*8 DSEED5,DSEED6
3 DIMENSION EVENT(NS),IEVENT(NS),IM(NS),IA2(NS),QS(NS),IG(NS),IA(NS)
4 IZ(50),Z2(50),R(10),R2(10),KA(10),AK(10)
5 ID=0
6 IF(IQ.LE.0) GO TO 11
7 W=0.0
8 W=(T1*E-SS)*IQ
9 TOH=TOH+WW
10 IF(IQ.GE.ID2) GO TO 6
11 CALL GGAMS(DSEED5,A2,B2,1,Z2,R2)
12 DO 1 KI=1,NM
13 I=KI
14 IF(EVENT(I)).EQ.99999.) GO TO 2
15 CONTINUE
16 WRITE(6,100)
17 FORMAT(1X,EVENT(I),IS FULL)
18 EVENT(I)=TIME+R2(1)
19 I=EVENT(I)=7
20 IA2(I)=IQ
21 K=K+1
22 QS(K)=TIME
23 IG(K)=ID2-IQ
24 IM(K)=2
25 IQ=IQ-ID2
26 IP=IP-ID2
27 IQQ=IQQ-ID2
28 IF(IP.GT.IS) GO TO 14
29 DO 3 KI=1,NM
30 I=KI
31 IF(EVENT(I)).EQ.99999.) GO TO 4
32 CONTINUE
33 WRITE(6,100)
34 CALL GGAMS(DSEED6,A,B,1,Z,R)
35 EVENT(I)=TIME+R(1)
36 ML=0
37 ML=ML+1
38 IP=IP+IR
39 IF(IP.LT.IS) GO TO 70
40 IA(I)=ML*IR
41 DO 60 KT=1,10
42 LS=KT
43 IF(AK(LS).EQ.0.0) GO TO 61
44 CONTINUE
45 KA(LS)=IQQ
46 AK(LS)=EVENT(III)
47 GO TO 14

```

```

6 CALL GGAMS (DSEED5,A2,B2,1,22,R2)
DO 7 KI=1,NM
    I=KI
    IF(EVENT(II).EQ.99999.) GO TO 8
    7 CONTINUE
    8 WRITE(6,100)
    EVENT(II)=TIME+R2(1)
    IA2(II)=ID2
    IQ=IQ-ID2
    IP=IP-ID2
    IQ=IQ-ID2
    IF(IP.GT.IS) GO TO 14
    IQ=KI
    DO 9 KI=1,NM
        IF(EVENT(II).EQ.99999.) GO TO 10
        9 CONTINUE
        10 WRITE(6,100)
        CALL GGAMS (DSEED6,A,B,L,Z,R)
        EVENT(II)=TIME+R2(1)
        IEVENT(II)=8
        ML=0
        71 ML=ML+1
        IP=IP+IR
        IF(IP.LT.IS) GO TO 71
        IA(II)=ML*IR
        DO 63 KT=1,10
        L5=KT
        IF(AK(L5).EQ.0.0) GO TO 64
        63 CONTINUE
        KA(L5)=IQ
        AK(L5)=EVENT(II)
        GO TO 14
        64
        K=K+1
        QS(K)=TIME
        IG(K)=ID2
        IM(K)=2
        IQ=IQ-ID2
        IQ=IQ-ID2
        IP=IP-ID2
        IF(IP.GT.IS) GO TO 14
        DO 12 KI=1,NM
            I=KI
            IF(EVENT(II).EQ.99999.) GO TO 13
            12 CONTINUE
            13 WRITE(6,100)
            CALL GGAMS (DSEED6,A,B,L,Z,R)
            EVENT(II)=TIME+R2(1)

```

```
1 EVENT(III)=8
ML=0
ML=ML+1
IP=IP+IR
IF(IP.LT.IS) GO TO 72
IA(II)=ML+IR
IS=TIME
RETURN
DEBUG SUBCHK
END
```

```
TUR08650
TUR08660
TUR08670
TUR08680
TUR08690
TUR08700
TUR08710
TUR08720
TUR08730
TUR08740
```

APPENDIX E

Early Warning Simulation Program

```

REAL *8 DSEED1,DSEED2,DSEED3,DSEED4,DSEED5,DSEED6,DSEED7,DSEED8,DSEED9
DIMENSION EVENT(900),IM(900),QU1(900),QU2(900),QU3(900)
1)QS(900) 1GI(900) 1G3(900) 1I3(900) 1AI(900) 1A2(900)
1900) 12(50) 121(50) 122(50) W{50},WS(50),WZ(50),
1)S1(1),S2(1),S3(1),KA(10),AK(10)
1)NH=300
DSEED1=123456.000
DSEED2=247658.000
DSEED3=365274.000
DSEED4=258732.000
DSEED5=541863.000
DSEED6=433215.000
DSEED7=651563.000
DSEED8=265418.000
DSEED9=167519.000
I=0
J=0
K=0
L=0
SS=0.0
SS1=0.0
SS2=0.0
TW1=0.0
TW2=0.0
TW3=0.0
TW4=0.0
TW5=0.0
TOH1=0.0
TOH2=0.0
DO 1 N=1,50
EVENT(Y)=99999.
1)QU1(N)=0.0
1)QU2(N)=0.0
1)QU3(N)=0.0
G1=G2=G3=0.0
IA1(N)=0
IA2(N)=0
IA(N)=0
CONTINUE
1)OC2(N)=1.0
1)OM(N)=0
1)OS(N)=0.0

```

```

1 G(N)=0          TUR00490
1 K(N)=0          TUR00500
1 A(K,N)=0.0      TUR00510
1 CONTINUE        TUR00520
1 F=0             TUR00530
1 F1=0            TUR00540
1 F2=0            TUR00550
1 D=0             TUR00560
1 D1=0            TUR00570
1 D2=0            TUR00580
1 READ(5,100) IR, IS    TUR00590
100 FORMAT(2110) IR1,IS1,T1
100 READ(5,110) IR1,IS1,T1
110 FORMAT(2110) F10.0
110 READ(5,110) F10.0
110 READ(5,110) IR2,IS2,T2
110 READ(5,110) IR2,IS2,T2
120 FORMAT(5,120) YEAR
120 READ(5,120) YEAR
130 FORMAT(5,130) A1,B1
130 READ(5,130) A1,B1
130 FORMAT(5,130) A2,B2
130 READ(5,130) A2,B2
130 FORMAT(5,140) XX1
130 READ(5,140) XX1
130 FORMAT(5,140) XX2
130 READ(5,140) XX2
130 FORMAT(5,140) XX3
130 READ(5,140) XX3
140 FORMAT(F4.0) P1
140 READ(5,150) P1
140 FORMAT(F4.0) P2
140 READ(5,150) P2
140 FORMAT(F4.0) P3
140 READ(5,150) P3
150 FORMAT(F6.0)
150 IQ=IR+IS
150 TIME=0.0
150 IP=IR+IS
150 IP=IR+IS
150 IP=IR1
150 IP=IR1
150 IP2=IR2
150 CALL GGEXN (DSEED1,XM1,1,S1)
150 CALL GGEXN (DSEED7,1,P1,WZ,K1)
150 IF1=1
150 DO 3 KI=1,NM
150 I=KI

```

```

1 IF(EVENT(11).EQ.99999.) GO TO 4
2 CONTINUE
3 EVENT(11)=TIME+S1(1)
4 EVENT(11)=1
CALL GGEXN (DSEED2,XM2,1,S2)
CALL GGEO (DSEED8,1,P2,WS,IK2)
5 IF2=1
DO 5 KI =1,NM
5 IF(KI=KI) EQ. 99999.1 GO TO 6
CONTINUE
6 EVENT(11)=TIME+S2(1)
EVENT(11)=2
CALL GGEXN (DSEED3,XM3,1,S3)
CALL GGEO (DSEED9,1,P3,WK,IK3)
IF=1
DO 7 KI =1,NM
7 IF(EVENT(11).EQ. 99999.1 GO TO 8
CONTINUE
8 EVENT(11)=TIME+S3(1)
EVENT(11)=3
DO 9 KI =1,NM
9 IF(KI=KI) EQ. 99999.1 GO TO 10
CONTINUE
10 EVENT(11)=TIME+S3(1)
EVENT(11)=4
DO 11 KI =1,NM
11 IF(KI=KI) EQ. 99999.1 GO TO 12
CONTINUE
12 EVENT(11)=TIME+T1
EVENT(11)=5
DO 13 KI =1,NM
13 IF(KI=KI) EQ. 99999.1 GO TO 14
CONTINUE
14 EVENT(11)=YEAR
GO TO 26
15 EVENT(11)=99999.
16 EVENT(IN)=0
IN=1
DO 16 KI =2,NM
16 IF(EVENT(IN).GT.EVENT(KI)) IN=KI
CONTINUE
TIME=EVENT(IN)

```

```

IL=IEVENT( IN)
IJ=IN
GO TO 11
11 CALL ONE (17 18 19 20 21 22 23 24 25) IL
17 CALL ONE (11 12 13 14 15 16 17 18 19 20 21, DSEED1, DS
18 CALL TWO (J TIME EVENT QU2, IQ2, IP2, IF2, XM2, P2, DSEED2, DS
19 CALL THREE (L TIME, EVENT, IEVENT, QU3, IG3, IQ, IP, XM3, P3, DSEED3, DS
20 CALL FOUR (K, IA1, BI1, DSEED4, DSEED6, NM, T1, NS, IS1, TOH, SS, KA, AK, IQQ, I
21 CALL FIVE (K, IA2, BI2, DSEED5, DSEED6, NM, T2, NS, IS2, TOH, SS, KA, AK, IQQ, I
122) GO TO 15
22 CALL SIX (I, J, TIME, IA1, IQ1, QU1, IG1, TW1, IP1, NM, NS)
23 CALL SEVEN (J, I, TIME, IA2, IQ2, QU2, IG2, TW2, IP2, NM, NS)
24 CALL EIGHT (L, K, I, J, TIME, EVENT, IEVENT, IQ, IA1, IA2, IG, QS, IM, QU3, IG
13, A, B, A1, B1, A2, B2, TW3, TW4, TW5, DSEED4, DSEED5, NM, NS, IP, KA, AK, IQQ, I
25 GO TO 15
25 TW1=TW1/YEAR
TW2=TW2/YEAR
TW3=TW3/YEAR
TW4=TW4/YEAR
TW5=TW5/YEAR
200 1 FORMAT (6, 2, 00), TW1, TW2, TW3, TW4, TW5, F10.4, * F10.4, * TW3=*, F10.4, * TW4=*, F10.4, *
1 TOH=TOH/YEAR
TOH1=TOH1/YEAR
TOH2=TOH2/YEAR
WRITE (6, 2, 01), TOH1, TOH2, F10.3, * TOH1=*, F10.3, * TOH2=*, F10.3, *
201 FORMAT (*, TOH=*, F10.3, * TOH1=*, F10.3, * TOH2=*, F10.3)
STOP
DEBUG SUBCHK
END
SUBROUTINE ONE (L TIME, EVENT, QU1, IG1, IQ1, IP1, XM1, P1, DS
1 ED1, DSEED7, NM, VS, TOH1, SS1, IQQ1, IPP, IS, A, B, IR, DSEED6, IA, KA, AK, IQQ, TUR01880
1 REAL *8 DSEED1, DSEED7, DSEED6
DIMENSION EVENT(NS), EVENT(NS), QU1(NS), WZ(50), IK1(1), S1(1) TUR01890
1, Z(50), R(1), IA(NS), KA(10), AK(10) TUR01900
1 TUR01910
1 TUR01920

```

```

1 IPP=IPP-IFI
1 IQQ=IQQ-IFI
1 FC(IPP,GT,IS) GO TO 6
CALL GGAMS (DSEED6,A,B,L,Z,R)
DO 7 KI=1,NM
    I=KI
    IF(EVENT(I,I)=TIME+R(1))
        WRITE(16,100)
        EVENT(I,I)=TIME+R(1)
    END
    ML=0
    ML=ML+1
    IPP=IPP+IR
    IF(IPP.LT.IS) GO TO 70
    IA(I,I)=ML*IR
    DO 60 KT=1,10
        L5=KT
        IF(AK(L5).EQ.0.0) GO TO 61
        CONTINUE
        60 KA(L5)=IQQ
        AK(IQ1.GT.0) GO TO 61
        61 IQ1=IQQ-EVENT(I,I)
        IF(IQ1.GT.0) GO TO 1
        I=I+1
        IQ1=IP1-IFI
        IQD1=IQ1-IFI
        QUIT(I,I)=TIME
        IGC1(I,I)=IFI
        GO TO 3
        3  B=0.0
        B=B+T14E-SS11*IQ1
        T0H1=TDH1+WW
        IF1=IUT(IF1) GO TO 2
        I=I+1
        IP1=IP1-IFI
        IQD1=IQ1-IFI
        QUIT(I,I)=IFI
        IGC1(I,I)=IFI
        GO TO 3
        2  I=I+1
        IP1=IP1-IFI
        IQD1=IQ1-IFI
        QUIT(I,I)=IFI
        IGC1(I,I)=IFI
        CALL G2EXN (DSFTN,1,1,P1,W1,K1)
        CALL G2EF0T (DSFTN,1,1,P1,W1,K1)
        DO 4 KI=1,NM

```

```

1 I=K1
1 IF(IEVENT(111).EQ.99999.) GO TO 5
4 CONTINUE
4 WRITE(6,100)
105 FORMAT(111,EVENT LIST IS FULL')
1 IEVENT(111)=1
1 SSI=TIME
1 RETURN
1 DEBUG SUBCHK
END
SUBROUTINE TWO (J, TIME, EVENT, IEVENT, IQ2, QU2, IC2, XM2, IP2, DS, SET)
1 ED2, DSSEED8, NM, VS2, OH2, SS2, IQ2, IPP, IS, A, B, IR, DSSEED6, IA, KA, IQJ
1 REAL*8 DSSEED2, DSSEED8, DSSEED6
1 DIMENSION NS, EVENT(NS), IEVENT(NS), QU2(NS), WS(50), IK2(11), S2(11)
1 Z(50) R(11), IA(NS), KA(10), AK(10)
1 IPP=IPP-IF2
1 IQ2=IQ2-IF2
1 F(IPP, GT, IS) GO TO 6
1 CALL GSAMS(DSEED6,A,B,I,Z,R)
DO 7 KI=1,NM
1 I=KI
1 IF(IEVENT(111).EQ.99999.) GO TO 8
7 CONTINUE
7 WRITE(6,100)
8 EVENT(111)=TIME+R(11)
8 IEVENT(111)=8
ML=0
70 ML=ML+1
1 IPP=IPP+IR
1 IF((IPP*LT*IS) GO TO 70
1 IA(111)=ML*IR
DO 60 KT=1,10
L5=KT
1 IF(IAK(L5).EQ.0.0) GO TO 61
60 CONTINUE
61 KA(L5)=IQQ
1 AK(L5)=EVENT(111)
6 AF(IQ2, GT, 0) GO TO 1
1 J=J+1
1 IC2=IQ2-IF2
1 IP2=IP2-IF2
1 IQ2=IQ2-IF2
1 Q2(J)=TIME
1 G2(J)=IF2
GO TO 3
1 WW=0
1 WW=(T14E-SS2)*IQ2

```

```

1 OH2=TOH2+WW
1 F(1Q2-LT)F2) GO TO 2
1 C2=1Q2-IF2
1 P2=1P2-IF2
1 QQ2=1QQ2-IF2
GO TO 3
2 J=J+1
1 Q2=1Q2-IF2
1 P2=1P2-IF2
1 QQ2=1QQ2-IF2
1 Q2(J)=IF2-
1 Q2(GGEXN(DSEED2,XM2,L,S2)
3 CALL GGEDT(DSEED8,I,P2,WS,IK2)
1 F2=1
DO 4 KI=1,NM
1 I=KI
1 IF(EVENT(I)).EQ.99999.1 GO TO 5
1 CONINJE
4 WRITE(6,100)
100 FORMAT(1,EVENT(LIST IS FULL))
5 EVENT(I)=TIME+S2(I)
1 EVENT(I)=2
1 SS2=TIME
RETURN
DEBUG SUBCHK
END
SUBROUTINE THREE (L,TIME,EVENT,JU3,IG3,IQ,IP,IF,XM3,P3,DSEEET
1D3,DSEED6,DSEED9,A,B,NM,NS,IA,IR,IS,TOH,SS,KA,AK,IPQ,IPP)
1REAL*8 DSEED3,DSEED6,DSEED9
DIMENSION EVENT(NS),IG3(NS),QU3(NS),Z(50),IA(NS),WK(50)
1 IK3(1),S3(1),R(1),KA(1),AK(1)
1 IP=IPP-IF
1 IF(IQ.JT.0) GO TO 1
L=L+IP-IF
1 IQ=IQ-1
1 QQ=1Q2-IF
1 Q3(L)=TIME
1 G3(L)=IF
GO TO 3
1 WW=0
1 WW=(TIME-SS)*IQ
1 TCH=TOH+WW
1 IF(IQ.LT.IF) GO TO 2
1 IQ=IQ-1
1 GO=IP-IF
1 IP=IP-IF

```

```

      GO TO 3
  2 L=L+1
      Q=3(L)=TIME
      I=3(L)=IF-IQ
      I=I-Q-1
      IP=IP-1F
      IQ=IQ-1F
      IF(IP,GT,1,S) 30 TO 6
      CALL GGAM(S,DSEED6,A,B,L,Z,R)
      DO 4 KI=1,NM
      I=KI
      IF(EVENT(II).EQ.99999.) GO TO 5
      CONTINUE
  4 WRITE(6,100)
      FORMAT(6,EVENT(LIST IS FULL'))
  100 FORMAT(II)=TIME+R(1)
  105 EVENT(II)=8
      EVENT(II)=0
      ML=ML+1
  70 IPP=IPP+IR
      IF(IPP,LT,IS) GO TO 70
      IA(II)=ML*IR
      DO 60 KT=1,10
      LS=KT
      IF(AK(LS).EQ.0.0) GO TO 61
      CONTINUE
  61 KA(LS)=IQ
      AK(LS)=EVENT(II)
      IPP=IR+IS
      CALL GGEXN(DSEED3,XM31,S3)
      CALL GGEDT(DSEED9,1,P3,WK,K3)
      IF=1
      DO 7 KI=1,NM
      I=KI
      IF(EVENT(II).EQ.99999.) GO TO 8
      CONTINUE
  7 WRITE(6,100)
  8 EVENT(II)=TIME+S3(1)
      SS=TIME
      RETURN
      DEBUG SUBCHK
END
SUBROUTINE FOUR(K,IQ,IQL,IP,TIME,EVENT,IEVENT,IM,IG,QS,IA,IA1,IR
  IRI,IS,IP,A,B,AL,BL,DSEED4,DSEED6,NM,TI,NS,ISI,IOH,SS,KA,AK,IQQ,
  IQ1,ND1)
REAL*8 DSEED4,DSEED6
DIMENSION EVENT(NSI),IEVENT(NSI),IM(NSI),QS(NSI),IG(NSI),IA(NSI),TUR
  O3840

```

```

1 KA(10) AK(10)
1 ND1=ND1 1-IQQ1
ND1=0
250 WRITE (6,250) ND1
FORMAT (/,250) ND1
1 IF (IP1 GT IS1) GO TO 1
1 ID1=IR1-IP1
ID1=IR1
1 IP1=IR1
CALL NINE (K,IQ2,TIME,EVENT,IEVENT,IM,IQS,IA,IA1,IR,IS,IP,A,B)
1 AI,B1,DSEED6,NM,NS,TOH,SS,KA,AK,IQQ1
1 DO 2 KI=1,NM
1 I1=KI
1 IF (EVENT(I1).EQ. 99999.) GO TO 3
2 CONTINUE
2 WRITE (6,100)
100 FORMAT (/, EVENT LIST IS FULL')
3 EVENT(I1)=TIME+T1
1 EVENT(I1)=4
1 ND1=IQQ1
RETURN
DEBUG SUBCHK
END
SUBROUTINE FIVE (K,IQ2,IP2,TIME,EVENT,IEVENT,IM,IQS,IA,IA2,IQQ1,IR,
1 IR2,IS1,IP,A,B,A2,B2,DSEED5,DSEED6,NM,T2,NS,IS2,TOH,SS,KA,AK,IQQ1,
1 IQ2,ND2)
1 Q2,ND2
1 Q2*DSEED5*DSEED6
REAL*8 DSEED5,DSEED6
DIMENSION EVENT(NS),IEVENT(NS),IM(NS),QS(NS),IG(NS),IA(NS)
1 KA(10) AK(10)
1 ND2=ND2-1
ND2=0
250 WRITE (6,250) ND2
FORMAT (/,ND2
1 IF (IP2 GT IS2) GO TO 1
1 ID2=IR2-IP2
1 P2=IR2
CALL TEN (K,IQ2,IP2,TIME,EVENT,IEVENT,IM,IQS,IA,IA2,IR,IS,IP,A,B)
1 A2,B2,DSEED5,DSEED6,NM,NS,TOH,SS,KA,AK,IQQ1
1 DO 2 KI=1,NM
1 I1=KI
1 IF (EVENT(I1).EQ. 99999.) GO TO 3
2 CONTINUE
2 WRITE (6,100)
100 FORMAT (/, EVENT LIST IS FULL')
3 EVENT(I1)=TIME+T2
1 EVENT(I1)=5
ND2=IQQ2

```

```

RETURN
DEBUG SUBCHK
END
SUBROUTINE SIX(I,J,TIME,IAI,IGI,QUI,IGL,NS)
DIMENSION IAI(NS),IGI(NS),QUI(NS)
WRITE(6,50) IAI
50 FORMAT(1X, EXPECTED NET INVENTORY OF SYSTEM 1 AT THE END OF CYCLE
      1=I, QUI=IAI(I,J)
      ITD=0
      IF(I.EQ.0) GO TO 8
      DO 1 N=1,1
      ITD=ITD+IGI(N)
      1 CONTINUE
      IF((IAI(I,J).LT.ITD) GO TO 3
      DO 2 N=1,I
      W=0
      W=(TIME-QUI(N))*IGI(N)
      QUI(N)=0.
      TWI=TWI+W
      IGI(N)=0
      2 CONTINUE
      2 IAI(I,J)=0
      I=0
      GO TO 8
      3 DO 5 N=1,1
      IF((IAI(I,J).GE.IGI(N)) GO TO 4
      IF((IAI(I,J).EQ.0) GO TO 6
      IGI(N)=IGI(N)-IAI(I,J)
      W=(TIME-QUI(N))*IAI(I,J)
      TWI=TWI+W
      GO TO 6
      4 IAI(I,J)=IAI(I,J)-IGI(N)
      W=(TIME-QUI(N))*IGI(N)
      QUI(N)=0.
      IGI(N)=0
      5 CONTINUE
      5 MM=0
      DO 7 N=1,1
      IF(QUI(N).EQ.0.) GO TO 7
      MM=MM+1
      QUI(MM)=QUI(N)
      IGI(MM)=IGI(N)
      7 CONTINUE
      7 I=MM
      8 RETURN

```

```

DEBUG SUBCHK
END
SUBROUTINE SEVEN (J1,J2,TIME,IA2(N),IG2(N),QU2(N),NM,NS)
DIMENSION IA2(NS),IG2(NS),QU2(NS)
WRITE(6,50) IQ2
50 FORMAT ('/ . EXPECTED NET INVENTORY OF SYSTEM 2 AT THE END OF CYCLE')
      ITD=0
      IQ2=IA2(IJ)
      IF(J.EQ.0) GO TO 8
      DO 1 N=1,J
      ITD=ITD+IG2(N)
      1 CONTINUE
      IF((IA2(IJ).LT.ITD) GO TO 3
      DU 2 N=1,J
      H=0
      W=(TIME-QU2(N))*IG2(N)
      QU2(N)=0
      TW2=TW2+W
      2 CONTINUE
      2 IA2(IJ)=0
      J=0
      3 DO 5 N=1,J
      H=0
      IF(IA2(IJ).GE.IG2(N)) GO TO 4
      IF(IA2(IJ).EQ.0) GO TO 6
      IG2(N)=IG2(N)-IA2(IJ)
      W=(TIME-QU2(N))*IA2(IJ)
      TW2=TW2+W
      GO TO 6
      4 IA2(IJ)=IA2(IJ)-IG2(N)
      W=(TIME-QU2(N))*IG2(N)
      QU2(N)=0
      IG2(N)=0
      5 CONTINUE
      5 JJJ=0
      DO 7 N=1,J.EQ.0.1 GO TO 7
      JJJ=JJ+1
      QU2(JJJ)=QU2(N)
      IG2(JJJ)=IG2(N)
      7 CONTINUE
      7 J=J
      8 RETURN
      DEBUG SUBCHK

```

```

END SUBROUTINE EIGHT (L,K,I,J,TIME,EVENT,IEVENT,IQ,IA1,IA2,IG,QS,IM
1 QU3,1G3,A,B,A1,B1,A2,B2,TW3,TW4,TW5,DSEED4,DSEED5,NM,NS,IP,KA,AK,
1 IQ)
1 REAL*8 DSEED4,DSEED5
1 DIMENSION EVENT(NS),IEVENT(NS),IA1(NS),IA2(NS),IG(NS),
1 I1,I2,I3,I4,I5,I6,I7,I8,I9,I10,I11,I12,I13,I14,I15,I16,I17,I18,I19,I20
1 I1D=0
1 J2=0
1 JZ=1Q
1 DO 60 KT=1,10
1 L5=KT
1 IF(IAK(L5).EQ.TIME) GO TO 61
1 60 CONTINUE
1 ND=KA(L5)-1QQ
1 KA(L5)=0
1 AK(L5)=0
1 WRITE(6,250) ND,TIME,Demand LEAD TIME=*,I10,* TIME=*,F15.51
1 250 FORMAT(1Q+1A(1J)
1 IQ=IQ+IA(L5)
1 WRITE(6,50) JZ,IQ,TIME
1 50 FORMAT(1Q*,JZ,IQ,TIME
1 ICLE=16,2X,EXPected NET INVENTORY OF MAIN SYSTEM AT THE END OF CY
1 TIME=*,I16,2X,TIME=*,F10.4)
1 IF(L.EQ.0) GO 10 8
1 DO 1 N=1,L
1 LTD=LTD+IG3(N)
1 CONTINUE
1 IF((IA(IJ).LT.IRD)) GO TO 3
1 3 DO 2 N=1,L
1 W=0
1 QU3=TIME-QU3(N)*IG3(N)
1 QU3(N)=0
1 TW3=TW3+W
1 IG3(N)=0
1 2 CONTINUE
1 IA(IJ)=IA(IJ)-LTD
1 L=0
1 GO TO 8
1 3 DO 5 N=1,L
1 W=0
1 IF(IA(IJ)*GE*IG3(N)) GO TO 4
1 IF(IA(IJ)*EQ.0) GO TO 6
1 IG3(N)=IG3(N)-IA(IJ)
1 W=(TIME-QU3(N))*IA(IJ)
1 TW3=TW3+W
1 4 GO TO 5
1 IA(IJ)=IA(IJ)-IG3(N)
1 W=(TIME-QU3(N))*IG3(N)

```

```

 $W_3 = T W_3 + W$ 
 1  $Q_{33}(N) = 0.$ 
 1  $I_{G3}(N) = 0.$ 
 5 CONTINUE
 6 DO 7  $N=1^L$ .EQ.0.) GO TO 7
    L=LL
    IF( $Q_{33}(N) = 0.$ ) GO TO 31
     $Q_{33}(LL+1) = Q_{33}(N)$ 
     $I_{G3}(LL) = I_{G3}(N)$ 
 7 CONTINUE
 8 GO TO 31
 1 TD=0
 1 TD=TD+IG(N)
 9 CONTINUE
 9 IF( $I_{IA}(I,J) \cdot LT.IID$ ) GO TO 16
 1 AA=0
 10 DO 9  $N=1^K$ 
    IF( $IIM(N).NE.1$ ) GO TO 10
     $W=0.$ 
     $W=(TIME-QS(N))*IG(N)$ 
     $T W_4 = T W_4 + W$ 
     $I_{AA}=I_{AA}+IG(N)$ 
    QS(N)=0.
    IG(N)=0.
    IM(N)=0
 10 CONTINUE
 10 IF( $I_{AA} \cdot EQ.0$ ) GO TO 32
    DO 11  $K=1,NM$ 
     $I=KI$ 
    IF(EVENT(III).EQ.99999.) GO TO 12
    11 CONTINUE
    11 WRITE(6,100)
    100 FORMAT(11 EVENT LIST IS FULL')
    12 CALL GGAMS(DSEEED4A1,B1,I,Z1,R1)
    EVENT(III)=TIME+R1(1)
    EVENT(III)=6
    IAI(1)=IAA
 12 IAB=0
 12 DO 13  $N=1^K$ 
    IF( $IIM(N).NE.2$ ) GO TO 13
     $W=0.$ 
     $W=(TIME-QS(N))*IG(N)$ 
    IAB=IAB+IG(N)

```

```

QS(N)=0.
IAA=0
CONTINUE
IF(IAB.EQ.0) GO TO 31
DO 14 K=1,NM
 14 CONTINUE
  IF(EVENT(111).EQ.99999.) GO TO 15
  WRITE(6,100)
  CALL GCAMS(DSEED5,A2,B2,L,Z2,R2)
  EVENT(111)=7
  IA2(111)=IAB
  GO TO 31
 16 IAB=0
  DO 20 N=1,K
    IF(IM(N)*NE*1.) GO TO 17
    IF(IA(IJ)*GE*IG(N)) GO TO 18
    IG(N)=IG(N)-IA(IJ)*IA(IJ)
    W=(TIME-QS(N))*IA(IJ)
    TW4=TW4+W
    IAA=IAA+IA(IJ)
    GO TO 21
  17 IF(IM(N)*NE*2.) GO TO 20
    IF(IG(N)=IG(N)-IA(IJ))
    W=(TIME-QS(N))*IA(IJ)
    TW5=TW5+W
    IAB=IAB+IA(IJ)
    GO TO 21
  18 W=0.
    IAA=IAA+IG(N)
    IA(IJ)=IA(IJ)-IG(N)
    W=(TIME-QS(N))*IG(N)
    TW4=TW4+W
    QS(N)=0.
    IG(N)=0.
    IM(N)=0
    GO TO 20
  19 W=0.
    IAB=IAB+IG(N)
    IA(IJ)=IA(IJ)-IG(N)
    W=(TIME-QS(N))*IG(N)
    TW5=TW5+W
    QS(N)=0.
    IG(N)=0

```

```

20 IM(N)=0
21 IF(IAA.EQ.0.) GO TO 24
22 IF(K1=K1) CONTINUE
23 WRITE(6,100)
24 CALL GGAMS(DSEED4,A1,B1,L,ZL,R1)
25 EVENT(L)=IAA
26 EVENT(L)=IAB
27 DO 25 K1=1,NM
28 IF(EVENT(L).EQ.99999.) GO TO 26
29 WRITE(6,100)
30 CALL GGAMS(DSEED5,A2,B2,L,ZZ,R2)
31 EVENT(L)=IAB
32 EVENT(L)=IAB
33 NN=0
34 DO 30 N=1,K
35 IF(QS(N).EQ.0.) GO TO 30
36 IF(IM(N).NE.1) GO TO 28
37 IM(NN)=1
38 GO TO 29
39 QS(NN)=QS(N)
40 IG(NN)=IG(N)
41 CONTINUE
42 K=NN
43 RETURN
44 DEBUG SUBCHK
45 END
46 SUBROUTINE NINE(K,IQIDL,TIME,EVENT,IM,IG,QS,IA,IA1,IR,IS,
47 1IP,A,B,A1,B1,DSEED4,DSEED6,NM,NS,TOH,SS,KA,AK,IQ)
48 REAL*8 DSEED4,DSEED6
49 DIMENSION EVENT(NS),IM(NS),QS(NS),IG(NS),IA(NS)
50 IZ(50),ZI(50),R(1),R(1),KA(10),AK(10)
51 ID=0
52 IF(IQ.LE.0) GO TO 11
53 WW=0.0
54 WW=(1.14E-SS)*IQ
55 TOH=TOH+WW
56 IF(IQ.GE.ID1) GO TO 6

```

```

CALL GGAMS (DSEED4,A1,B1,I,ZI,R1)
DO 1 KI=1,NM
1 I=KI
1 IF(EVENT(III).EQ.99999.) GO TO 2
1 CONTINUE
1 WRITE(6,100)
100 FORMAT(1F10.0)
100 EVENT(III)=TIME+R1(1)
1 EVENT(III)=6
1 AI(1)=IQ
1 GS(K)=TIME
1 G(K)=IDI-IQ
1 M(K)=1
1 IQ=IQ-ID1
1 IP=IP-ID1
1 IF(IP.GT.IS) GO TO 14
DO 60 KT=1,10
1 S=KT
1 IF(AK(L5).EQ.0.) GO TO 61
60 CONTINUE
61 KA(L5)=IQ
1 AK(L5)=EVENT(III)
1 IP=IR+IS
60 TO 14
6 CALL GGAMS (DSEED4,A1,B1,I,ZI,R1)
DO 7 KI=1,NM
1 I=KI
1 IF(EVENT(III).EQ.0.) GO TO 8
1 CONTINUE
1 WRITE(6,100)
1 EVENT(III)=TIME+R1(1)
1 EVENT(III)=6
1 AI(1)=ID1
1 IQ=IQ-ID1
1 IP=IP-ID1
1 IF(IP.GT.IS) GO TO 14
DO 63 KT=1,10
1 S=KT
1 IF(AK(L5).EQ.0.0) GO TO 64
63 CONTINUE
64 KA(L5)=IQ
1 AK(L5)=EVENT(III)
1 IP=IR+IS
60 TO 14
11 K=K+1
1 GS(K)=TIME
1 G(K)=IDI

```

```

IM(K)=1
IQ=IQ-ID1
IP=IP-ID1
SS=TIME
14 RETURN SUBCHK
DEND
SUBROUTINE TEN (K, IQ, ID2, TIME, EVENT, IM, IG, QS, IA, IA2, IR, IS, ITURO7690
  IP, A, B, A2, B2, DSEED5, DSEED6, NM, NS, TOH, SS, KA, AK, IQQ)
REAL*8 DSEED5, DSEED6
DIMENSION EVENT(NS), IM(NS), IA2(NS), QS(NS), IG(NS), IA(NS)
1 Z(50), Z2(50), R(1), R2(1), KA(10), AK(10)
ID=0
IF(IQ.LE.0) GO TO 11
WW=0.0
WW=(TIME-SS)*IQ
TOH=TOH+WW
IF(IQ.GE.1D2) GO TO 6
CALL GGAMS (DSEED5, A2, B2, 1, Z2, R2)
DO 1 KI=1,NM
1 IF(EVENT(KI).EQ.99999.) GO TO 2
1 CONTINUE
1 WRITE(6,100)
100 FORMAT(1F100)
EVENT(1)=TIME+R2(1)
EVENT(1)=7
IA2(1)=IQ
K=K+1
QS(K)=TIME
IG(K)=ID2-IQ
IM(K)=2
IP=IP-ID2
1 IF(IP.GT.IS) GO TO 14
DO 60 KT=1,10
L5=KT
IF(AK(L5).EQ.0.0) GO TO 61
60 CONTINUE
61 KA(L5)=IQQ
AK(L5)=EVENT(1)
IP=IP+IS
GO TO 14
6 CALL GGAMS (DSEED5, A2, B2, 1, Z2, R2)
DO 7 KI=1,NM
7 IF(EVENT(KI).EQ.99999.) GO TO 8
7 CONTINUE

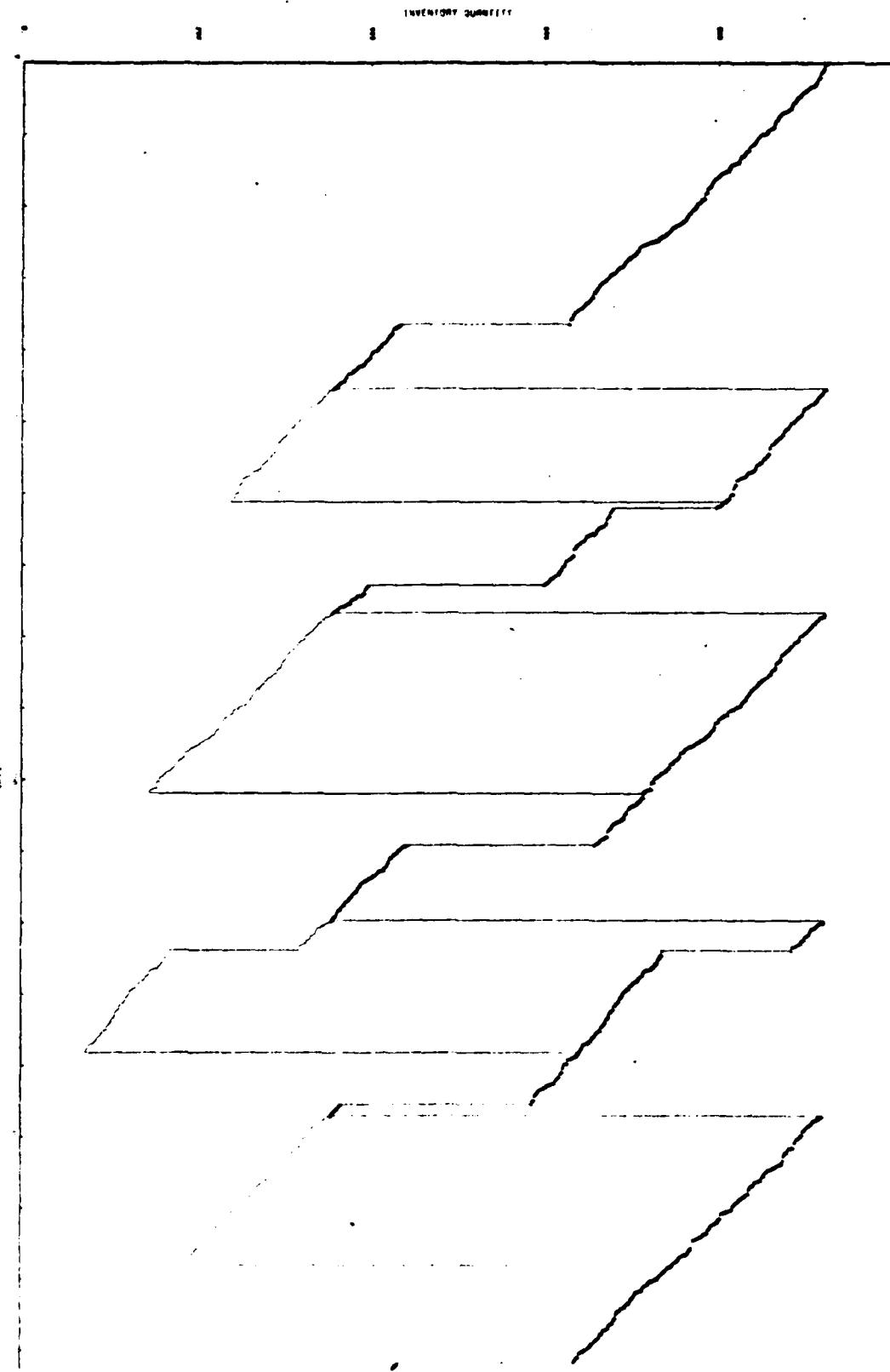
```

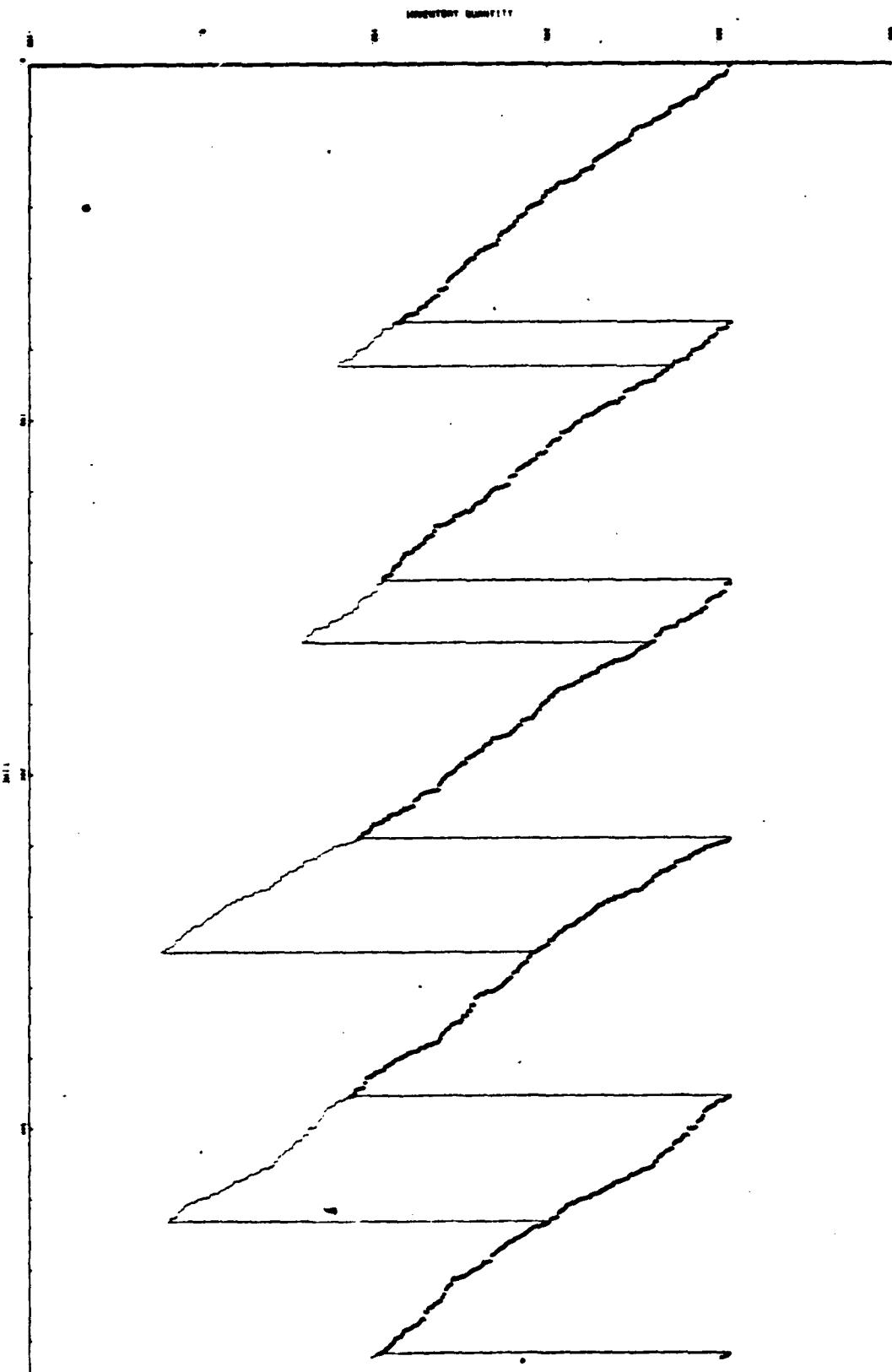
```

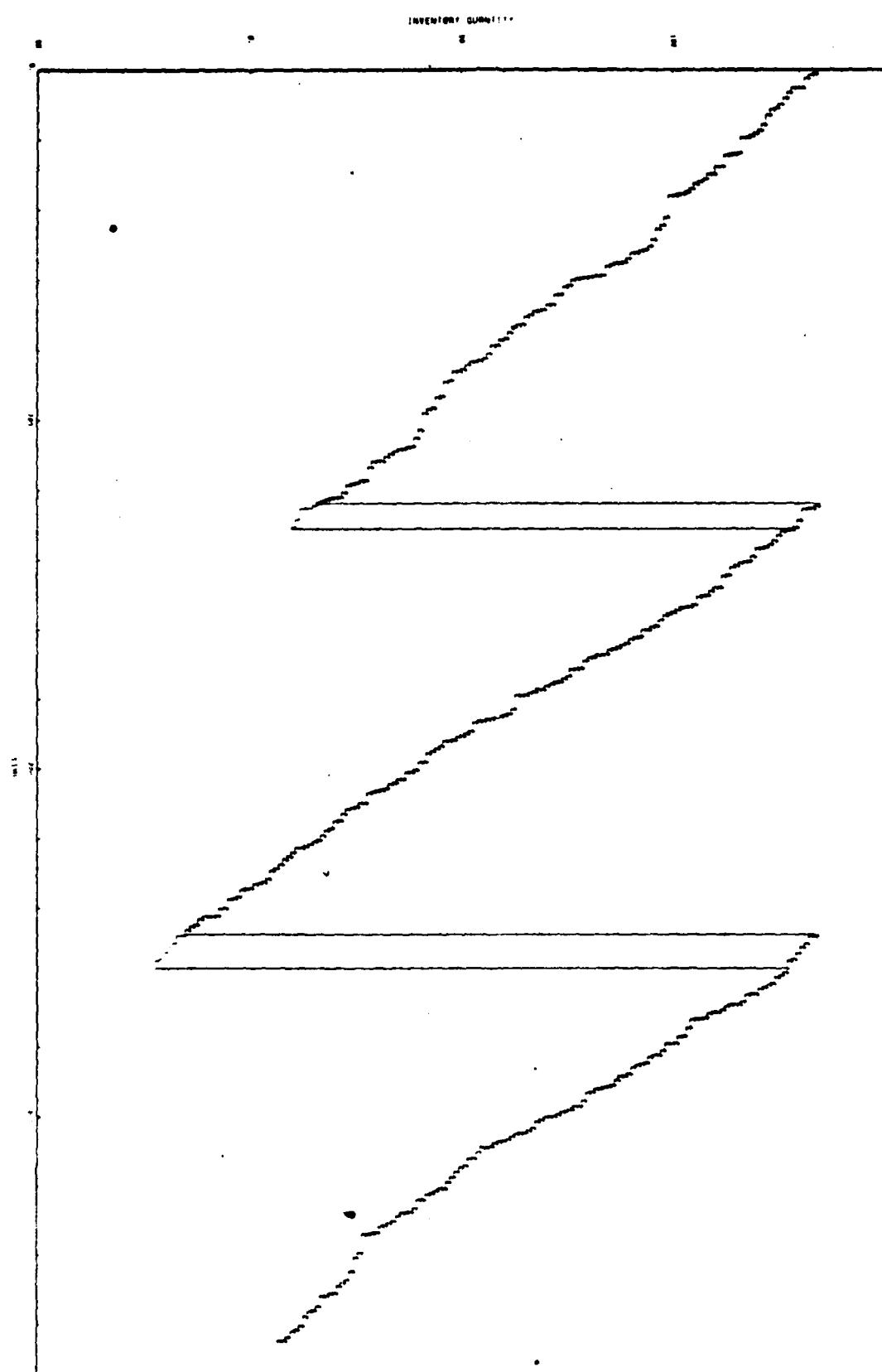
      WRITE(61,100)
100   EVENT(111)=TIME+R2(11)
      IEVENT(111)=7
      IA2(111)=102
      IQ=IQ-ID2
      IP=IP-ID2
      IF(IP.GT.IS) GO TO 14
      DO 63 KT=1,10
      LS=KT
      IF(AK(LS).EQ.0.0) GO TO 14
      63  CONTINUE
      AK(LS)=199
      AK(LS)=EVENT(111)
      IP=IP+IS
      GO TO 14
      64  K=K+1
      QS(K)=TIME
      LG(K)=ID2
      IM(K)=2
      IQ=IQ-ID2
      IP=IP-ID2
      SS=TIME
      14  RETURN DEBUG SUBCHK
      END

```

APPENDIX F
Versatec Output of the Program in Appendix C







APPENDIX G

TI-59 Calculator Program for Analytical Solutions for Periodic Review Systems

This program was written for probabilistic periodic review inventory models having gamma distributed lead times and Poisson arrivals and uses. Normal distribution for demand during lead time instead of Negative Binomial.

Input Requirements

The following variables should be stored in the registers shown before the variables.

STO 01 = λ (arrival rate per day)

STO 02 = C

STO 03 = I

STO 04 = Π

STO 05 = J

STO 06 = A

STO 07 = $\alpha-1$

STO 08 = β

This program also requires the TI-59 applied statistics module.

Enter	Press	Display
T	A	R
	B	S
C		$n(r)/\text{period}$
D		$n(r)/\text{year}$
E		Annual review and order cost
A'		Annual inventory carrying cost
B'		Annual shortage cost
C'		Total annual variable cost

312 43 RCL
313 02 02
314 65 X
315 43 RCL
316 03 03
317 54)
318 42 STO
319 55 55
320 91 R/S
321 76 LBL
322 17 B'
323 53 C
324 43 RCL
325 53 53
326 65 X
327 43 RCL
328 04 04
329 54)
330 42 STO
331 56 56
332 91 R/S
333 76 LBL
334 18 C'
335 53 C
336 43 RCL
337 54 54
338 85 +
339 43 RCL
340 55 55
341 85 +
342 43 RCL
343 56 56
344 54)
345 42 STO
346 57 57
347 91 R/S

APPENDIX H

Variable Definitions for Simulation Programs

IQ = Net inventory
IP = Inventory position
IPP = Second inventory position for early warning system
IR = Order quantity
IS = Reorder level
ID = Number of items demanded by systems
IF = Number of items demanded per demand by ships
QU = Ship queue
QS = Lower echelon's group demand queue in the main system
IG = Amount of demand for each demand waiting in the queue
IA = Amount of shipment arrived
IM = Index. If it is equal to 1, that means that the demand waiting in the main system queue to be filled belongs to System One.
TW = Total waiting time
TOH = Total average on-hand inventory
T = Length of a period
EVENT = This indicates the subroutines
IEVENT = This indicates time of subroutines scheduled
X = Net inventory variable for Versatec plotter
V = Inventory position variable for Versatec plotter
Y = Time for Versatec plotter

WK = Work space for geometric random variable
WS = Work space for geometric random variable
WZ = Work space for geometric random variable
S = Exponential random number
IK = Geometric random number
WW = Increment
I = Indicates the number of ships waiting in the ship queue at System One
J = Indicates the number of ships waiting in the ship queue at System Two
L = Indicates the number of ships waiting in the ship queue at the main system
K = Indicates the number of demand batches waiting in the group demand queue at the main system
SS = Time indicator
IQQ = Counter for ship arrivals to indicate the number of items demanded per period
ML = Multiplier for the number of batches of demand to be ordered from outside supplier
KA = Indicates the last change on net inventory of the main system in order to get the number of items demanded in a lead time
AK = Time of last change
A = Scale parameter for a gamma distribution
B = Shape parameter for a gamma distribution
XM = Ship arrival rate per day
P = Probability of success for geometric distribution

BIBLIOGRAPHY

Hadley, G., and Whitin, T.M., Analysis of Inventory Systems,
Prentice-Hall, 1963.

INITIAL DISTRIBUTION LIST

	No. Copies
1. Defense Technical Information Center Cameron Station Alexandria, Virginia 22314	2
2. Library, Code 0142 Naval Postgraduate School Monterey, California 93940	2
3. Department Chairman, Code 55 Department of Operations Research Naval Postgraduate School Monterey, California 93940	1
4. Asso. Prof. F. Russell Richards, Code 55Rh Department of Operations Research Naval Postgraduate School Monterey, California 93940	4
5. LT Turgut Büyükkarhan Cennet Mah. Hurriyet Cadde Yildiz Apt. No: 64 A Blok Daire 12 Küçükçekmece/Istanbul TURKEY	1