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HANDBOOK FOR THE ANALYSIS OF ENGAGEMENTS WITH MOBILE TARGETS

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Handbook

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This document presents a methodology and data for estimating the probability of a mobile target being present as a function of the time after observation. Handbook data are presented for specific cases and in a generalized form. Coding for performing the calculations on a TI-59 calculator is also included.

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I. INTRODUCTION AND SUMMARY

Current targeting manuals such as the *Staff Officer's Field Manual, Nuclear Weapons Employment Doctrine and Procedures*, FM-101-31-1 (Ref. 1), do not include a means of analyzing the engagement of targets that move. This is reflected by the fact that no parameters which represent motion (for instance, speed or dwell time) are used in the manual's methodology. By implication, all targets are fixed targets.

This document describes a methodology and provides the data required for a realistic analysis of a mobile target engagement. For this report a mobile target is defined both as a target that moves nearly continuously (such as a tank company) and as one that moves only occasionally (such as an artillery battery or command post). Specifically, this manual provides a means of estimating the probability that a target is still present at an observed location as a function of time from the observation where the time the target stopped is unknown. With this methodology targets can then be evaluated not only on the basis of expected fractional coverage as in the manuals, but also on the basis of whether there is an adequate likelihood that they will still be present when a weapon arrives.

Results are summarized in Figure 1. This chart shows the probability of a target being present at an observed location as a function of the expected target dwell time (τ) and the acquisition/engagement time (t). Its use is best illustrated by an example. Let us assume an expected target dwell time (τ) of 12 hours and that the time (t) necessary to acquire and process the target information, to communicate it to required elements, to make decisions, to plan and prepare weapon use,

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1. *Staff Officer's Field Manual, Nuclear Weapons Employment Doctrine and Procedures*, Department of the Army, FM-101-31-1, March 1977.

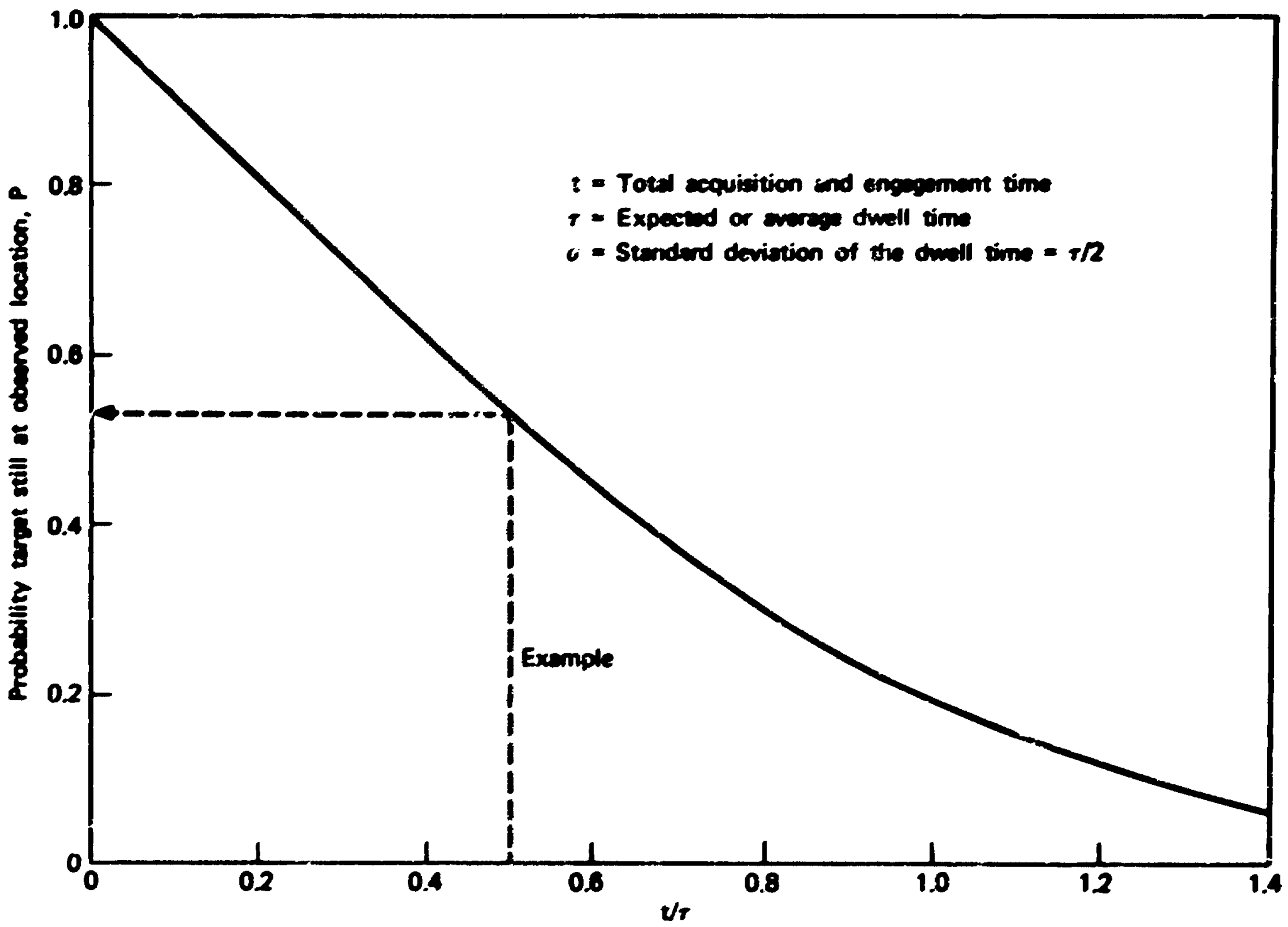

Figure 1. Probability of target presence.

and to employ the weapon is 6 hours. The ratio of t to τ is therefore 0.5. The resultant expected probability is about 0.53. Thus, there is slightly better than a 50/50 chance of the target still being present when the weapon actually arrives for the example.

Figure 1 is based upon a particular assumption concerning the dwell time statistics,[*] but it is a representative curve with general applicability. The remainder of this report describes the methodology in detail and presents results for other cases in greater detail.

Specifically, the sections of this report present a brief description of the methodology (with a detailed development of the equations in an appendix); tabular data for target analysis of specific cases; an expanded generalized curve; and sample cases. Appendix A presents the derivation of the equations while Appendix B details a TI-59 calculator code for calculating target presence probabilities.

---

[*] The standard deviation of the expected dwell time is assumed to be one-half of the expected dwell time. Section IV of this report presents generalized data that permit evaluation of other values for standard deviation of the dwell time.

II. METHODOLOGY

In this section the basic methodology for the evaluation of mobile targets is described. Appendix A presents a detailed development of the equations. The problem addressed by the methodology is:

> What is the probability of a target being at an observed location at some time (Δt) later? Factors to be considered include how long the target can be expected to stay fixed, when did it stop relative to the observation, and how much time is required to respond (i.e., to place a weapon on the target).

The methodology begins with the assumption that any target can be modeled as having a characteristic average dwell time (τ) at a given location. The value of τ is, of course, strongly dependent upon the particular scenario and situation but, in any case, the value can be estimated. The dwell time has some expected deviation (σ) about τ; i.e., not all targets of the given type in that situation would move precisely at time τ.

A reasonable assumption is that the actual distribution of dwell times for a given target type and situation will be Gaussian, i.e., the bell-shaped curve as shown in Figure 2.* The curve is symmetrical about the mean dwell time τ with the degree of spread of the curve determined by the standard deviation σ. The physical meaning of the curve is that the most probable move time is τ; however, some units move earlier and some move later. If the spread (σ) is large, then fewer units move at τ and more move earlier and later than τ.

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*It does not appear that the actual mathematical form of the dwell time distribution is a critical assumption. The results are most sensitive to the parameter τ regardless of the distribution model.

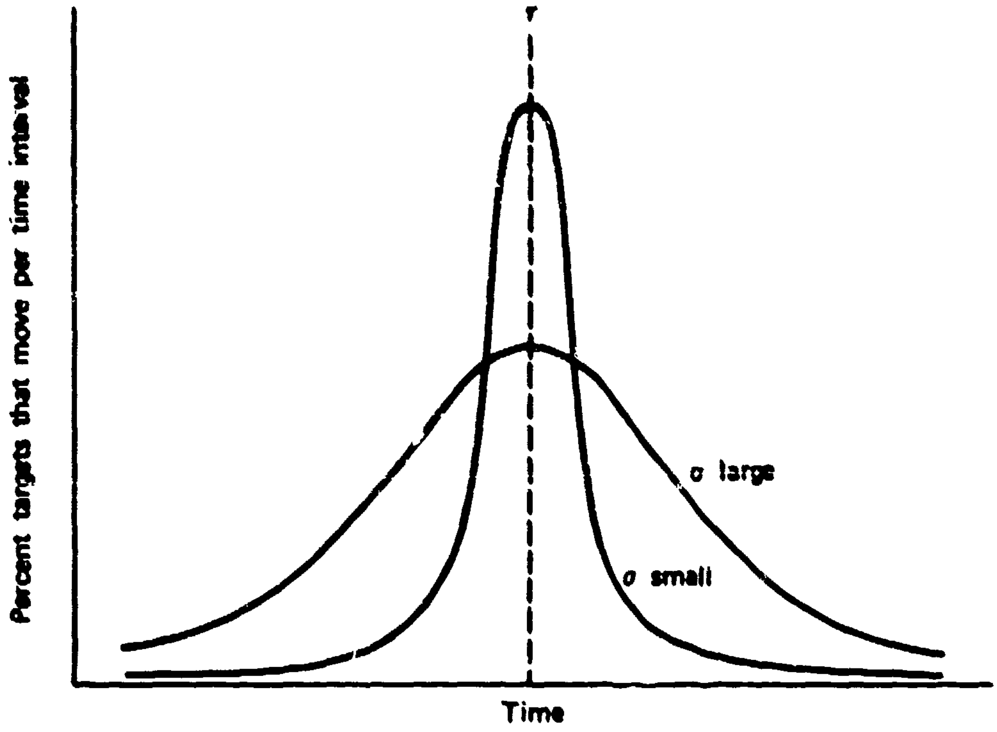

Figure 2. Target dwell time distribution model.

Selection of a dwell time model (τ and σ) permits calculation of the probability of a target being present as a function of time. The probability that a target moves <u>before</u> a given time is equal to the area under the dwell time distribution curve up to that time. This probably is expressed mathematically as

$$P_M = \int_o^t f(t)dt$$

where f(t) $\equiv$ Gaussian distribution function.

The probability of a target being present (P_p) at a given time is one minus the probability that it has moved before that time:

$$P_p = 1 - P_M$$

These manipulations result in a curve as illustrated in Figure 3.

Not knowing at what time the detection occurred relative to the time the target actually stopped complicates the problem. The mathematics of this complication are addressed in detail in Appendix A. The final result, i.e., the probability that the target will still be present at its original position at a time t after it was detected, is

$$P\ (t) = \frac{\sigma\sqrt{\frac{2}{\pi}}\ e^{-\ (t-\tau)\ ^2/2\sigma^2} - (t-\tau)\ \left\{1 - \operatorname{erf}\left[\frac{(t-\tau)}{\sigma\sqrt{2}}\right]\right\}}{\sigma\sqrt{\frac{2}{\pi}}\ e^{-\tau\ ^2/2\sigma^2} + \tau\left[1 + \operatorname{erf}\left(\frac{\tau}{\sigma\sqrt{2}}\right)\right]}$$

This equation, when normalized to present the probability in terms of the ratio t/τ and with the assumption that $\sigma = \tau/2$, results in the curve presented in Figure 1. Note the difference between Figures 1 (stop time not known) and Figure 3

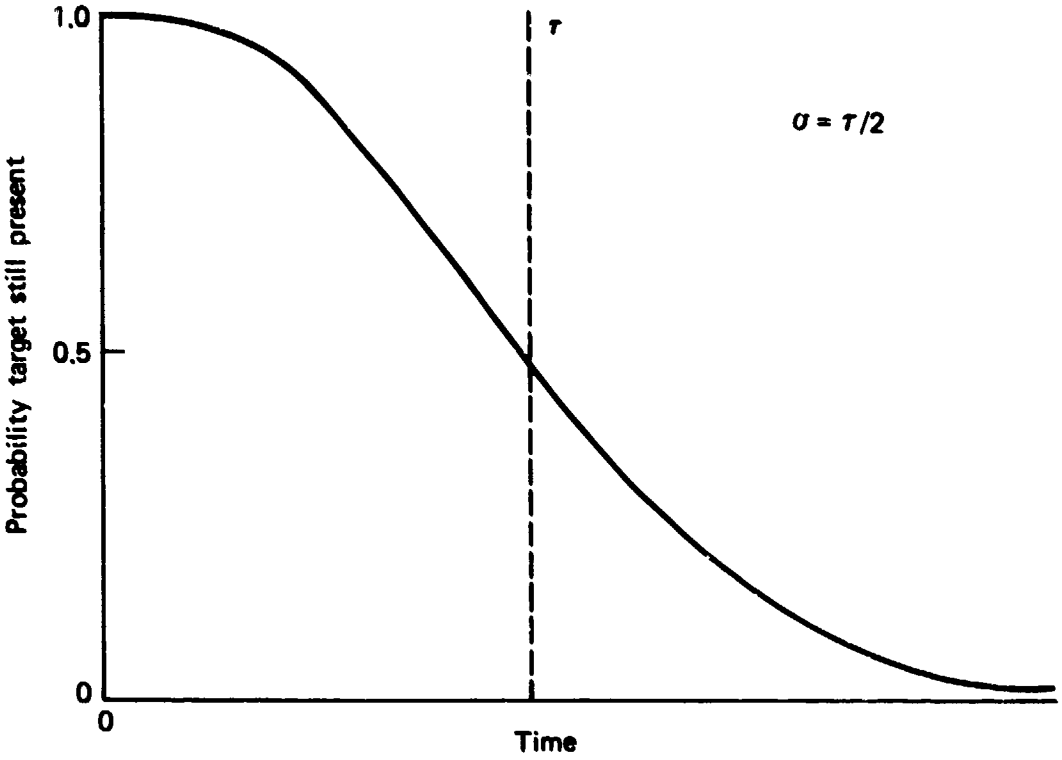

Figure 3. Probability target is still present with the time the target stops known.

(stop time known). For $t = \tau$, the former gives a probability of the target still being present of less than 20 percent while the latter, as expected, yields a probability of 50 percent. Alternatively, for a desired probability (e.g., 50 percent) t must be about 50 percent of τ with the stop time unknown or about equal to τ with the stop time known.

The remainder of this report will treat only the case where the stop time is unknown. This is probably the most realistic assumption when the demands on the target acquisition system in wartime are considered, especially with limited resources and degraded capabilities due to enemy actions.

In summary, the aforementioned methodology provides a means of estimating the probability of target presence as a function of time. The presence probability alone could be used as a criterion for selecting targets or it could be combined with the expected fractional coverage for a given weapon calculated from the field manuals as follows:

$$F' = F \times P(t)$$

where F' = revised expected coverage

F = expected coverage using weapon W assuming static target (FM-101-31 or AP 550)

$P(t)$ = probability target at observed location.

Figure 4 shows an example result using this approach. The target is assumed to remain in one place an average time (τ) of 4 hours with $\sigma = 1$ hour. The static fraction coverages are 0.29 and 0.83 for 600 m and 200-m target location errors, respectively. The figure illustrates that if the desired expected coverage is 0.30, then it is unachievable with a target location error of 600 m. If the target were acquired with a system providing 200-m accuracy, then up to 2.6 hours can elapse between the observation and the attack execution while maintaining an <u>expected</u> coverage of 0.30. Note that in any

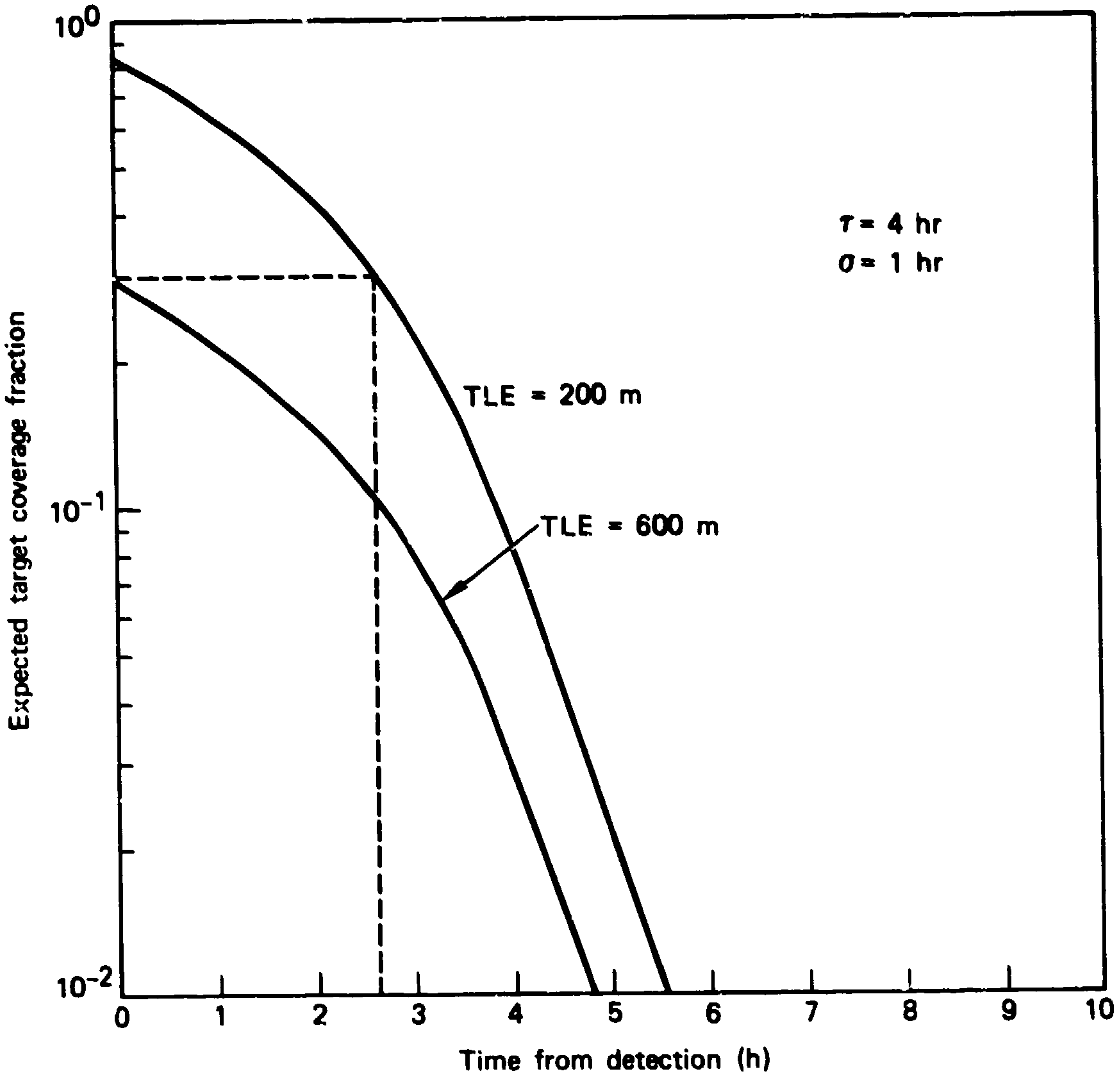

Figure 4. Effect of target movement on expected coverage (target stop time unknown).

individual case, a target will receive either 0.83 coverage or zero. Over a large number of targets the coverage to the targets considered together will be 0.30.

III. TABULAR DATA FOR TARGET ANALYSIS

Data are presented in this section for the estimation of target presence probabilities for selected values of the dwell time (τ) and standard deviation of the dwell time (σ) as noted in Table 1. Results for these cases are presented in Tables 2 through 23. All results assume the time the target stopped is unknown. See Section IV if other values of σ are desired.

TABLE 1. SPECIFIC VALUES OF τ AND σ FOR WHICH TABLES GENERATED

Case	Average dwell time, τ (h)	Standard deviation, σ (h)	
		Set A ($\sigma = \tau/3$)	Set B ($\sigma = \tau$)
1	0.1	0.033	0.1
2	0.2	0.067	0.2
3	0.5	0.167	0.5
4	1	0.333	1
5	2	0.667	2
6	4	1.333	4
7	6	2	6
8	12	4	12
9	24	8	24
10	48	12	48
11	96	32	96

TABLE 2. PROBABILITY OF TARGET PRESENCE, CASE 1A

```
AVERAGE RESIDENCE TIME(HOURS)              = .1
DEVIATION OF AVERAGE RESIDENCE TIME(HOURS)= .03333


TIME(HOURS)  TIME(MIN)  PROB TARGET PRESENT
  0.000         0.0              1.000
   .005          .3              0.950
   .010          .6              0.900
   .015          .9              0.850
   .020         1.2              0.801
   .025         1.5              0.751
   .030         1.8              0.702
   .035         2.1              0.653
   .040         2.4              0.605
   .045         2.7              0.557
   .050         3.0              0.510
   .055         3.3              0.464
   .060         3.6              0.419
   .065         3.9              0.375
   .070         4.2              0.333
   .075         4.5              0.294
   .080         4.8              0.256
   .085         5.1              0.221
   .090         5.4              0.189
   .095         5.7              0.159
   .100         6.0              0.133
   .105         6.3              0.109
   .110         6.6              0.089
   .115         6.9              0.071
   .120         7.2              0.056
   .125         7.5              0.044
   .130         7.8              0.033
```

TABLE 3. PROBABILITY OF TARGET PRESENCE, CASE 1B

```
AVERAGE RESIDENCE TIME(HOURS)                   = .1
DEVIATION OF AVERAGE RESIDENCE TIME(HOURS)      = .1


 TIME(HOURS)   TIME(MIN)   PROB TARGET PRESENT
   0.000          0.0               1.000
    .005           .3               0.961
    .010           .6               0.923
    .015           .9               0.886
    .020          1.2               0.849
    .025          1.5               0.813
    .030          1.8               0.778
    .035          2.1               0.743
    .040          2.4               0.710
    .045          2.7               0.676
    .050          3.0               0.644
    .055          3.3               0.613
    .060          3.6               0.582
    .065          3.9               0.552
    .070          4.2               0.523
    .075          4.5               0.495
    .080          4.8               0.468
    .085          5.1               0.442
    .090          5.4               0.416
    .095          5.7               0.392
    .100          6.0               0.368
    .105          6.3               0.346
    .110          6.6               0.324
    .115          6.9               0.303
    .120          7.2               0.283
    .125          7.5               0.264
    .130          7.8               0.246
    .135          8.1               0.229
    .140          8.4               0.213
    .145          8.7               0.197
    .150          9.0               0.183
    .155          9.3               0.169
    .160          9.6               0.156
    .165          9.9               0.143
    .170         10.2               0.132
    .175         10.5               0.121
    .180         10.8               0.111
    .185         11.1               0.102
    .190         11.4               0.093
    .195         11.7               0.085
    .200         12.0               0.077
    .205         12.3               0.070
    .210         12.6               0.063
    .215         12.9               0.057
    .220         13.2               0.052
    .225         13.5               0.047
    .230         13.8               0.042
    .235         14.1               0.038
    .240         14.4               0.034
    .245         14.7               0.030
```

TABLE 4. PROBABILITY OF TARGET PRESENCE, CASE 2A

```
AVERAGE RESIDENCE TIME(HOURS)              = .2
DEVIATION OF AVERAGE RESIDENCE TIME(HOURS)= .06667


 TIME(HOURS)  TIME(MIN)  PROB TARGET PRESENT
   0.000        0.0          1.000
    .010         .6          0.950
    .020        1.2          0.900
    .030        1.8          0.850
    .040        2.4          0.801
    .050        3.0          0.751
    .060        3.6          0.702
    .070        4.2          0.653
    .080        4.8          0.605
    .090        5.4          0.557
    .100        6.0          0.510
    .110        6.6          0.464
    .120        7.2          0.419
    .130        7.8          0.375
    .140        8.4          0.333
    .150        9.0          0.294
    .160        9.6          0.256
    .170       10.2          0.221
    .180       10.8          0.189
    .190       11.4          0.159
    .200       12.0          0.133
    .210       12.6          0.109
    .220       13.2          0.089
    .230       13.8          0.071
    .240       14.4          0.056
    .250       15.0          0.044
    .260       15.6          0.033
```

TABLE 5. PROBABILITY OF TARGET PRESENCE, CASE 2B

AVERAGE RESIDENCE TIME(HOURS) = .2
DEVIATION OF AVERAGE RESIDENCE TIME(HOURS) = .2

TIME(HOURS)	TIME(MIN)	PROB TARGET PRESENT
0.000	0.0	1.000
.010	.6	0.961
.020	1.2	0.923
.030	1.8	0.886
.040	2.4	0.849
.050	3.0	0.813
.060	3.6	0.778
.070	4.2	0.743
.080	4.8	0.710
.090	5.4	0.676
.100	6.0	0.644
.110	6.6	0.613
.120	7.2	0.582
.130	7.8	0.552
.140	8.4	0.523
.150	9.0	0.495
.160	9.6	0.468
.170	10.2	0.442
.180	10.8	0.416
.190	11.4	0.392
.200	12.0	0.368
.210	12.6	0.346
.220	13.2	0.324
.230	13.8	0.303
.240	14.4	0.283
.250	15.0	0.264
.260	15.6	0.246
.270	16.2	0.229
.280	16.8	0.213
.290	17.4	0.197
.300	18.0	0.183
.310	18.6	0.169
.320	19.2	0.156
.330	19.8	0.143
.340	20.4	0.132
.350	21.0	0.121
.360	21.6	0.111
.370	22.2	0.102
.380	22.8	0.093
.390	23.4	0.085
.400	24.0	0.077
.410	24.6	0.070
.420	25.2	0.063
.430	25.8	0.057
.440	26.4	0.052
.450	27.0	0.047
.460	27.6	0.042
.470	28.2	0.038
.480	28.8	0.034
.490	29.4	0.030

TABLE 6. PROBABILITY OF TARGET PRESENCE, CASE 3A

```
AVERAGE RESIDENCE TIME(HOURS)                = .5
DEVIATION OF AVERAGE RESIDENCE TIME(HOURS)= .16667


 TIME(HOURS)   TIME(MIN)   PROB TARGET PRESENT
   0.000          0.0            1.000
    .025          1.5            0.950
    .050          3.0            0.900
    .075          4.5            0.850
    .100          6.0            0.801
    .125          7.5            0.751
    .150          9.0            0.702
    .175         10.5            0.653
    .200         12.0            0.605
    .225         13.5            0.557
    .250         15.0            0.510
    .275         16.5            0.464
    .300         18.0            0.419
    .325         19.5            0.375
    .350         21.0            0.333
    .375         22.5            0.294
    .400         24.0            0.256
    .425         25.5            0.221
    .450         27.0            0.189
    .475         28.5            0.159
    .500         30.0            0.133
    .525         31.5            0.109
    .550         33.0            0.089
    .575         34.5            0.071
    .600         36.0            0.056
    .625         37.5            0.044
    .650         39.0            0.033
```

TABLE 7. PROBABILITY OF TARGET PRESENCE, CASE 3B

AVERAGE RESIDENCE TIME(HOURS) = .5
DEVIATION OF AVERAGE RESIDENCE TIME(HOURS) = .5

TIME(HOURS)	TIME(MIN)	PROB TARGET PRESENT
0.000	0.	1.000
.025	2.	0.961
.050	3.	0.923
.075	5.	0.886
.100	6.	0.849
.125	8.	0.813
.150	9.	0.778
.175	11.	0.743
.200	12.	0.710
.225	14.	0.676
.250	15.	0.644
.275	17.	0.613
.300	18.	0.582
.325	20.	0.552
.350	21.	0.523
.375	23.	0.495
.400	24.	0.468
.425	26.	0.442
.450	27.	0.416
.475	29.	0.392
.500	30.	0.368
.525	32.	0.346
.550	33.	0.324
.575	35.	0.303
.600	36.	0.283
.625	38.	0.264
.650	39.	0.246
.675	41.	0.229
.700	42.	0.213
.725	44.	0.197
.750	45.	0.183
.775	47.	0.169
.800	48.	0.156
.825	50.	0.143
.850	51.	0.132
.875	53.	0.121
.900	54.	0.111
.925	56.	0.102
.950	57.	0.093
.975	59.	0.085
1.000	60.	0.077
1.025	62.	0.070
1.050	63.	0.063
1.075	65.	0.057
1.100	66.	0.052
1.125	68.	0.047
1.150	69.	0.042
1.175	71.	0.038
1.200	72.	0.034
1.225	74.	0.030

TABLE 8. PROBABILITY OF TARGET PRESENCE, CASE 4A

```
AVERAGE RESIDENCE TIME(HOURS)             = 1
DEVIATION OF AVERAGE RESIDENCE TIME HOURS)= .33333


 TIME(HOURS)  TIME(MIN)  PROB TARGET PRESENT
   0.000         0.            1.000
    .050         3.            0.950
    .100         6.            0.900
    .150         9.            0.850
    .200        12.            0.801
    .250        15.            0.751
    .300        18.            0.702
    .350        21.            0.653
    .400        24.            0.605
    .450        27.            0.557
    .500        30.            0.510
    .550        33.            0.464
    .600        36.            0.419
    .650        39.            0.375
    .700        42.            0.333
    .750        45.            0.294
    .800        48.            0.256
    .850        51.            0.221
    .900        54.            0.189
    .950        57.            0.159
   1.000        60.            0.133
   1.050        63.            0.109
   1.100        66.            0.089
   1.150        69.            0.071
   1.200        72.            0.056
   1.250        75.            0.044
   1.300        78.            0.033
```

TABLE 9. PROBABILITY OF TARGET PRESENCE, CASE 4B

```
AVERAGE RESIDENCE TIME(HOURS)                = 1
DEVIATION OF AVERAGE RESIDENCE TIME(HOURS)= 1


 TIME(HOURS)   TIME(MIN)   PROB TARGET PRESENT
   0.000          0.                1.000
    .050          3.                0.961
    .100          6.                0.923
    .150          9.                0.886
    .200         12.                0.849
    .250         15.                0.813
    .300         18.                0.778
    .350         21.                0.743
    .400         24.                0.710
    .450         27.                0.676
    .500         30.                0.644
    .550         33.                0.613
    .600         36.                0.582
    .650         39.                0.552
    .700         42.                0.523
    .750         45.                0.495
    .800         48.                0.468
    .850         51.                0.442
    .900         54.                0.416
    .950         57.                0.392
   1.000         60.                0.368
   1.050         63.                0.346
   1.100         66.                0.324
   1.150         69.                0.303
   1.200         72.                0.283
   1.250         75.                0.264
   1.300         78.                0.246
   1.350         81.                0.229
   1.400         84.                0.213
   1.450         87.                0.197
   1.500         90.                0.183
   1.550         93.                0.169
   1.600         96.                0.156
   1.650         99.                0.143
   1.700        102.                0.132
   1.750        105.                0.121
   1.800        108.                0.111
   1.850        111.                0.102
   1.900        114.                0.093
   1.950        117.                0.085
   2.000        120.                0.077
   2.050        123.                0.070
   2.100        126.                0.063
   2.150        129.                0.057
   2.200        132.                0.052
   2.250        135.                0.047
   2.300        138.                0.042
   2.350        141.                0.038
   2.400        144.                0.034
   2.450        147.                0.030
```

TABLE 10. PROBABILITY OF TARGET PRESENCE, CASE 5A

```
AVERAGE RESIDENCE TIME(HOURS)              = 2
DEVIATION OF AVERAGE RESIDENCE TIME(HOURS)= .66667


 TIME(HOURS)   TIME(MIN)   PROB TARGET PRESENT
   0.000          0.               1.000
    .100          6.               0.950
    .200         12.               0.900
    .300         18.               0.850
    .400         24.               0.801
    .500         30.               0.751
    .600         36.               0.702
    .700         42.               0.653
    .800         48.               0.605
    .900         54.               0.557
   1.000         60.               0.510
   1.100         66.               0.464
   1.200         72.               0.419
   1.300         78.               0.375
   1.400         84.               0.333
   1.500         90.               0.294
   1.600         96.               0.256
   1.700        102.               0.221
   1.800        108.               0.189
   1.900        114.               0.159
   2.000        120.               0.133
   2.100        126.               0.109
   2.200        132.               0.089
   2.300        138.               0.071
   2.400        144.               0.056
   2.500        150.               0.044
   2.600        156.               0.033
```

TABLE 11. PROBABILITY OF TARGET PRESENCE, CASE 5B

AVERAGE RESIDENCE TIME(HOURS) = 2
DEVIATION OF AVERAGE RESIDENCE TIME(HOURS)= 2

TIME(HOURS)	TIME(MIN)	PROB TARGET PRESENT
0.000	0.	1.000
.100	6.	0.961
.200	12.	0.923
.300	18.	0.886
.400	24.	0.849
.500	30.	0.813
.600	36.	0.778
.700	42.	0.743
.800	48.	0.710
.900	54.	0.676
1.000	60.	0.644
1.100	66.	0.613
1.200	72.	0.582
1.300	78.	0.552
1.400	84.	0.523
1.500	90.	0.495
1.600	96.	0.469
1.700	102.	0.442
1.800	108.	0.416
1.900	114.	0.392
2.000	120.	0.368
2.100	126.	0.346
2.200	132.	0.324
2.300	138.	0.303
2.400	144.	0.283
2.500	150.	0.264
2.600	156.	0.246
2.700	162.	0.229
2.800	168.	0.213
2.900	174.	0.197
3.000	180.	0.183
3.100	186.	0.169
3.200	192.	0.156
3.300	198.	0.143
3.400	204.	0.132
3.500	210.	0.121
3.600	216.	0.111
3.700	222.	0.102
3.800	228.	0.093
3.900	234.	0.085
4.000	240.	0.077
4.100	246.	0.070
4.200	252.	0.063
4.300	258.	0.057
4.400	264.	0.052
4.500	270.	0.047
4.600	276.	0.042
4.700	282.	0.038
4.800	288.	0.034
4.900	294.	0.030

TABLE 12. PROBABILITY OF TARGET PRESENCE, CASE 6A

```
AVERAGE RESIDENCE TIME(HOURS)            = 4
DEVIATION OF AVERAGE RESIDENCE TIME(HOURS)= 1.333


 TIME(HOURS)  TIME(MIN)  PROB TARGET PRESENT
   0.000         0.              1.000
    .200        12.              0.950
    .400        24.              0.900
    .600        36.              0.850
    .800        48.              0.801
   1.000        60.              0.751
   1.200        72.              0.702
   1.400        84.              0.653
   1.600        96.              0.605
   1.800       108.              0.557
   2.000       120.              0.510
   2.200       132.              0.464
   2.400       144.              0.419
   2.600       156.              0.375
   2.800       168.              0.333
   3.000       180.              0.294
   3.200       192.              0.256
   3.400       204.              0.221
   3.600       216.              0.189
   3.800       228.              0.159
   4.000       240.              0.133
   4.200       252.              0.109
   4.400       264.              0.089
   4.600       276.              0.071
   4.800       288.              0.056
   5.000       300.              0.044
   5.200       312.              0.033
```

TABLE 13. PROBABILITY OF TARGET PRESENCE, CASE 6B

AVERAGE RESIDENCE TIME(HOURS) = 4
DEVIATION OF AVERAGE RESIDENCE TIME(HOURS)= 4

TIME(HOURS)	TIME(MIN)	PROB TARGET PRESENT
0.000	0.	1.000
.200	12.	0.961
.400	24.	0.923
.600	36.	0.886
.800	48.	0.849
1.000	60.	0.813
1.200	72.	0.778
1.400	84.	0.743
1.600	96.	0.710
1.800	108.	0.676
2.000	120.	0.644
2.200	132.	0.613
2.400	144.	0.582
2.600	156.	0.552
2.800	168.	0.523
3.000	180.	0.495
3.200	192.	0.468
3.400	204.	0.442
3.600	216.	0.416
3.800	228.	0.392
4.000	240.	0.368
4.200	252.	0.346
4.400	264.	0.324
4.600	276.	0.303
4.800	288.	0.283
5.000	300.	0.264
5.200	312.	0.246
5.400	324.	0.229
5.600	336.	0.213
5.800	348.	0.197
6.000	360.	0.183
6.200	372.	0.169
6.400	384.	0.156
6.600	396.	0.143
6.800	408.	0.132
7.000	420.	0.121
7.200	432.	0.111
7.400	444.	0.102
7.600	456.	0.093
7.800	468.	0.085
8.000	480.	0.077
8.200	492.	0.070
8.400	504.	0.063
8.600	516.	0.057
8.800	528.	0.052
9.000	540.	0.047
9.200	552.	0.042
9.400	564.	0.038
9.600	576.	0.034
9.800	588.	0.030

TABLE 14. PROBABILITY OF TARGET PRESENCE, CASE 7A

AVERAGE RESIDENCE TIME(HOURS) = 6
DEVIATION OF AVERAGE RESIDENCE TIME(HOURS)= 2

TIME(HOURS)	TIME(MIN)	PROB TARGET PRESENT
0.000	0.	1.000
.200	12.	0.967
.400	24.	0.933
.600	36.	0.900
.800	48.	0.867
1.000	60.	0.834
1.200	72.	0.801
1.400	84.	0.768
1.600	96.	0.735
1.800	108.	0.702
2.000	120.	0.669
2.200	132.	0.637
2.400	144.	0.605
2.600	156.	0.573
2.800	168.	0.541
3.000	180.	0.510
3.200	192.	0.479
3.400	204.	0.448
3.600	216.	0.419
3.800	228.	0.389
4.000	240.	0.361
4.200	252.	0.333
4.400	264.	0.307
4.600	276.	0.281
4.800	288.	0.256
5.000	300.	0.233
5.200	312.	0.210
5.400	324.	0.189
5.600	336.	0.169
5.800	348.	0.150
6.000	360.	0.133
6.200	372.	0.117
6.400	384.	0.102
6.600	396.	0.089
6.800	408.	0.077
7.000	420.	0.066
7.200	432.	0.056
7.400	444.	0.048
7.600	456.	0.040
7.800	468.	0.033

TABLE 15. PROBABILITY OF TARGET PRESENCE, CASE 7B

```
AVERAGE RESIDENCE TIME(HOURS)                  = 6
DEVIATION OF AVERAGE RESIDENCE TIME(HOURS) = 6


 TIME(HOURS)   TIME(MIN)   PROB TARGET PRESENT
   0.000          0.              1.000
    .200         12.              0.974
    .400         24.              0.949
    .600         36.              0.923
    .800         48.              0.899
   1.000         60.              0.874
   1.200         72.              0.849
   1.400         84.              0.825
   1.600         96.              0.802
   1.800        108.              0.778
   2.000        120.              0.755
   2.200        132.              0.732
   2.400        144.              0.710
   2.600        156.              0.687
   2.800        168.              0.666
   3.000        180.              0.644
   3.200        192.              0.623
   3.400        204.              0.602
   3.600        216.              0.582
   3.800        228.              0.562
   4.000        240.              0.542
   4.200        252.              0.523
   4.400        264.              0.504
   4.600        276.              0.486
   4.800        288.              0.469
   5.000        300.              0.450
   5.200        312.              0.433
   5.400        324.              0.416
   5.600        336.              0.400
   5.800        348.              0.384
   6.000        360.              0.368
   6.200        372.              0.353
   6.400        384.              0.338
   6.600        396.              0.324
   6.800        408.              0.310
   7.000        420.              0.296
   7.200        432.              0.283
   7.400        444.              0.271
   7.600        456.              0.258
   7.800        468.              0.246
   8.000        480.              0.235
   8.200        492.              0.224
   8.400        504.              0.213
   8.600        516.              0.202
   8.800        528.              0.192
   9.000        540.              0.183
   9.200        552.              0.173
   9.400        564.              0.164
   9.600        576.              0.156
   9.800        588.              0.147
```

TABLE 16. PROBABILITY OF TARGET PRESENCE, CASE 8A

```
AVERAGE RESIDENCE TIME(HOURS)              = 12
DEVIATION OF AVERAGE RESIDENCE TIME(HOURS)= 4


 TIME(HOURS)  TIME(MIN)  PROB TARGET PRESENT
   0.000         0.              1.000
    .500        30.              0.958
   1.000        60.              0.917
   1.500        90.              0.875
   2.000       120.              0.834
   2.500       150.              0.793
   3.000       180.              0.751
   3.500       210.              0.710
   4.000       240.              0.669
   4.500       270.              0.629
   5.000       300.              0.589
   5.500       330.              0.549
   6.000       360.              0.510
   6.500       390.              0.471
   7.000       420.              0.433
   7.500       450.              0.397
   8.000       480.              0.361
   8.500       510.              0.327
   9.000       540.              0.294
   9.500       570.              0.262
  10.000       600.              0.233
  10.500       630.              0.205
  11.000       660.              0.179
  11.500       690.              0.155
  12.000       720.              0.133
  12.500       750.              0.113
  13.006       789.              0.095
  13.500       810.              0.080
  14.000       840.              0.066
  14.500       870.              0.054
  15.000       900.              0.044
  15.500       930.              0.035
```

TABLE 17. PROBABILITY OF TARGET PRESENCE, CASE 8B

AVERAGE RESIDENCE TIME(HOURS) = 12
DEVIATION OF AVERAGE RESIDENCE TIME(HOURS)= 12

TIME(HOURS)	TIME(MIN)	PROB TARGET PRESENT
0.000	0.	1.000
.500	30.	0.968
1.000	60.	0.936
1.500	90.	0.905
2.000	120.	0.874
2.500	150.	0.843
3.000	180.	0.813
3.500	210.	0.784
4.000	240.	0.755
4.500	270.	0.726
5.000	300.	0.698
5.500	330.	0.671
6.000	360.	0.644
6.500	390.	0.618
7.000	420.	0.592
7.500	450.	0.567
8.000	480.	0.542
8.500	510.	0.518
9.000	540.	0.495
9.500	570.	0.472
10.000	600.	0.450
10.500	630.	0.429
11.000	660.	0.408
11.500	690.	0.388
12.000	720.	0.368
12.500	750.	0.349
13.000	780.	0.331
13.500	810.	0.313
14.000	840.	0.296
14.500	870.	0.280
15.000	900.	0.264
15.500	930.	0.249
16.000	960.	0.235
16.500	990.	0.221
17.000	1020.	0.207
17.500	1050.	0.195
18.000	1080.	0.183
18.500	1110.	0.171
19.000	1140.	0.160
19.500	1170.	0.149
20.000	1200.	0.139
20.500	1230.	0.130
21.000	1260.	0.121
21.500	1290.	0.113
22.000	1320.	0.105
22.500	1350.	0.097
23.000	1380.	0.090
23.500	1410.	0.083
24.000	1440.	0.077
24.500	1470.	0.071

TABLE 18. PROBABILITY OF TARGET PRESENCE, CASE 9A

```
AVERAGE RESIDENCE TIME(HOURS)              = 24
DEVIATION OF AVERAGE RESIDENCE TIME(HOURS)= 8


 TIME(HOURS)  TIME(MIN)  PROB TARGET PRESENT
   0.000         0.            1.000
   1.000        60.            0.958
   2.000       120.            0.917
   3.000       180.            0.875
   4.000       240.            0.834
   5.000       300.            0.793
   6.000       360.            0.751
   7.000       420.            0.710
   8.000       480.            0.669
   9.000       540.            0.629
  10.000       600.            0.589
  11.000       660.            0.549
  12.000       720.            0.510
  13.000       780.            0.471
  14.000       840.            0.433
  15.000       900.            0.397
  16.000       960.            0.361
  17.000      1020.            0.327
  18.000      1080.            0.294
  19.000      1140.            0.262
  20.000      1200.            0.233
  21.000      1260.            0.205
  22.000      1320.            0.179
  23.000      1380.            0.155
  24.000      1440.            0.133
  25.000      1500.            0.113
  26.000      1560.            0.095
  27.000      1620.            0.080
  28.000      1680.            0.066
  29.000      1740.            0.054
  30.000      1800.            0.044
  31.000      1860.            0.035
```

TABLE 19. PROBABILITY OF TARGET PRESENCE, CASE 9B

```
AVERAGE RESIDENCE TIME(HOURS)               = 24
DEVIATION OF AVERAGE RESIDENCE TIME(HOURS) = 24


TIME(HOURS)  TIME(MIN)  PROB TARGET PRESENT
  0.000         0.                1.000
  1.000        60.                0.968
  2.000       120.                0.936
  3.000       180.                0.905
  4.000       240.                0.874
  5.000       300.                0.843
  6.000       360.                0.813
  7.000       420.                0.784
  8.000       480.                0.755
  9.000       540.                0.726
 10.000       600.                0.698
 11.000       660.                0.671
 12.000       720.                0.644
 13.000       780.                0.618
 14.000       840.                0.592
 15.000       900.                0.567
 16.000       960.                0.542
 17.000      1020.                0.518
 18.000      1080.                0.495
 19.000      1140.                0.472
 20.000      1200.                0.450
 21.000      1260                 0.429
 22.000      1320.                0.408
 23.000      1380.                0.388
 24.000      1440.                0.368
 25.000      1500.                0.349
 26.000      1560.                0.331
 27.000      1620.                0.313
 28.000      1680.                0.296
 29.000      1740.                0.280
 30.000      1800.                0.264
 31.000      1860.                0.249
 32.000      1920.                0.235
 33.000      1980.                0.221
 34.000      2040.                0.207
 35.000      2100.                0.195
 36.000      2160.                0.183
 37.000      2220.                0.171
 38.000      2280.                0.160
 39.000      2340.                0.149
 40.000      2400.                0.139
 41.000      2460.                0.130
 42.000      2520.                0.121
 43.000      2580.                0.113
 44.000      2640.                0.105
 45.000      2700.                0.097
 46.000      2760.                0.090
 47.000      2820.                0.083
 48.000      2880.                0.077
 49.000      2940.                0.071
```

TABLE 20. PROBABILITY OF TARGET PRESENCE, CASE 10A

AVERAGE RESIDENCE TIME(HOURS) = 48
DEVIATION OF AVERAGE RESIDENCE TIME(HOURS)= 12

TIME(HOURS)	TIME(MIN)	PROB TARGET PRESENT
0.000	0.	1.000
2.000	120.	0.958
4.000	240.	0.917
6.000	360.	0.875
8.000	480.	0.833
10.000	600.	0.792
12.000	720.	0.750
14.000	840.	0.709
16.000	960.	0.667
18.000	1080.	0.625
20.000	1200.	0.584
22.000	1320.	0.543
24.000	1440.	0.502
26.000	1560.	0.462
28.000	1680.	0.422
30.000	1800.	0.382
32.000	1920.	0.344
34.000	2040.	0.307
36.000	2160.	0.271
38.000	2280.	0.237
40.000	2400.	0.204
42.000	2520.	0.174
44.000	2640.	0.147
46.000	2760.	0.122
48.000	2880.	0.100
50.000	3000.	0.080
52.000	3120.	0.064
54.000	3240.	0.049
56.000	3360.	0.038

TABLE 21. PROBABILITY OF TARGET PRESENCE, CASE 10B

AVERAGE RESIDENCE TIME(HOURS) = 48
DEVIATION OF AVERAGE RESIDENCE TIME(HOURS)= 48

TIME(HOURS)	TIME(MIN)	PROB TARGET PRESENT
0.000	0.	1.000
2.000	120.	0.969
4.000	240.	0.936
6.000	360.	0.905
8.000	480.	0.874
10.000	600.	0.843
12.000	720.	0.813
14.000	840.	0.784
16.000	960.	0.755
18.000	1080.	0.726
20.000	1200.	0.698
22.000	1320.	0.671
24.000	1440.	0.644
26.000	1560.	0.618
28.000	1680.	0.592
30.000	1800.	0.567
32.000	1920.	0.542
34.000	2040.	0.518
36.000	2160.	0.495
38.000	2280.	0.472
40.000	2400.	0.450
42.000	2520.	0.429
44.000	2640.	0.408
46.000	2760.	0.388
48.000	2880.	0.368
50.000	3000.	0.349
52.000	3120.	0.331
54.000	3240.	0.313
56.000	3360.	0.296
58.000	3480.	0.280
60.000	3600.	0.264
62.000	3720.	0.249
64.000	3840.	0.235
66.000	3960.	0.221
68.000	4080.	0.207
70.000	4200.	0.195
72.000	4320.	0.183
74.000	4440.	0.171
76.000	4560.	0.160
78.000	4680.	0.149
80.000	4800.	0.139
82.000	4920.	0.130
84.000	5040.	0.121
86.000	5160.	0.113
88.000	5280.	0.105
90.000	5400.	0.097
92.000	5520.	0.090
94.000	5640.	0.083
96.000	5760.	0.077
98.000	5880.	0.071

TABLE 22. PROBABILITY OF TARGET PRESENCE, CASE 11A

```
AVERAGE RESIDENCE TIME(HOURS)              = 96
DEVIATION OF AVERAGE RESIDENCE TIME(HOURS)= 32


 TIME(HOURS)   TIME(MIN)  PROB TARGET PRESENT
   0.000          0.              1.000
   5.000        300.              0.948
  10.000        600.              0.896
  15.000        900.              0.844
  20.000       1200.              0.793
  25.000       1500.              0.741
  30.000       1800.              0.690
  35.000       2100.              0.639
  40.000       2400.              0.589
  45.000       2700.              0.539
  50.000       3000.              0.490
  55.000       3300.              0.443
  60.000       3600.              0.397
  65.000       3900.              0.352
  70.000       4200.              0.310
  75.000       4500.              0.270
  80.000       4800.              0.233
  85.000       5100.              0.198
  90.000       5400.              0.167
  95.000       5700.              0.138
 100.000       6000.              0.113
 105.000       6300.              0.091
 110.000       6600.              0.073
 115.000       6900.              0.057
 120.000       7200.              0.044
 125.000       7500.              0.033
```

TABLE 23. PROBABILITY OF TARGET PRESENCE, CASE 11B

```
AVERAGE RESIDENCE TIME(HOURS)               = 96
DEVIATION OF AVERAGE RESIDENCE TIME(HOURS)= 96


 TIME(HOURS)  TIME(MIN)  PROB TARGET PRESENT
    0.000         0.              1.000
    5.000       300.              0.960
   10.000       600.              0.920
   15.000       900.              0.882
   20.000      1200.              0.843
   25.000      1500.              0.806
   30.000      1800.              0.769
   35.000      2100.              0.733
   40.000      2400.              0.698
   45.000      2700.              0.664
   50.000      3000.              0.631
   55.000      3300.              0.598
   60.000      3600.              0.567
   65.000      3900.              0.536
   70.000      4200.              0.507
   75.000      4500.              0.478
   80.000      4800.              0.450
   85.000      5100.              0.424
   90.000      5400.              0.398
   95.000      5700.              0.373
  100.000      6000.              0.349
  105.000      6300.              0.327
  110.000      6600.              0.305
  115.000      6900.              0.284
  120.000      7200.              0.264
  125.000      7500.              0.246
  130.000      7800.              0.228
  135.000      8100.              0.211
  140.000      8400.              0.195
  145.000      8700.              0.180
  150.000      9000.              0.165
  155.000      9300.              0.152
  160.000      9600.              0.139
  165.000      9900.              0.128
  170.000     10200.              0.117
  175.000     10500.              0.107
  180.000     10800.              0.097
  185.000     11100.              0.088
  190.000     11400.              0.080
  195.000     11700.              0.072
  200.000     12000.              0.065
  205.000     12300.              0.059
  210.000     12600.              0.053
  215.000     12900.              0.048
  220.000     13200.              0.043
  225.000     13500.              0.038
  230.000     13800.              0.034
  235.000     14100.              0.030
```

IV. GENERALIZED DATA FOR TARGET ANALYSIS

Figure 5 presents the generalized curve for the probability of a target being present as a function of time from detection. The probability is presented as a function of two parameters, τ/σ and t/τ, where

τ = average target dwell time

σ = standard deviation of the dwell time

t = time.

This curve is applicable to all values of τ and σ and may be used for those cases for which the tabular data in Section III are inadequate. Examples illustrating its use are presented in Section V. Coding for calculating these data on the TI-59 is presented in Appendix B.

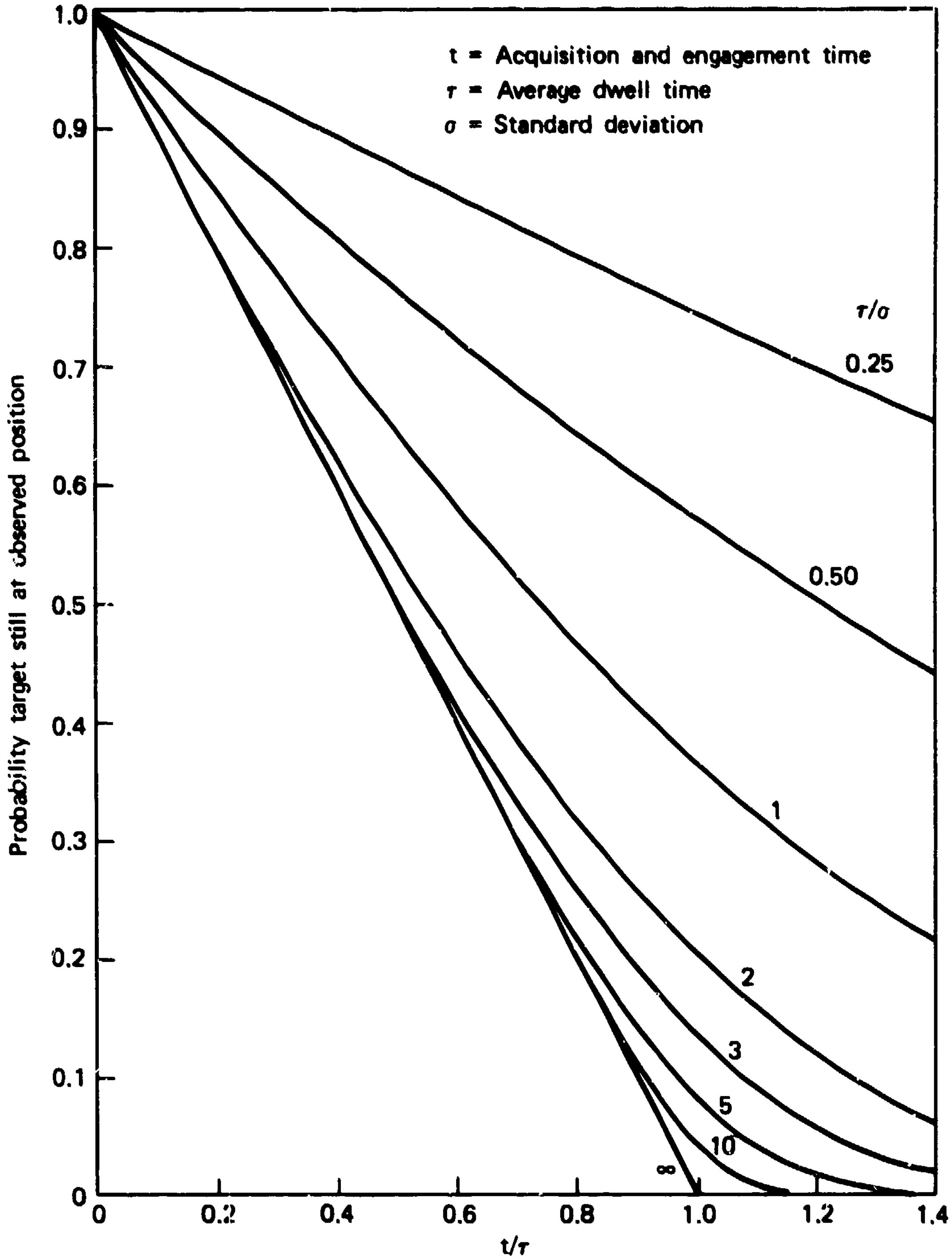

Figure 5. Generalized curve for estimating probability target is still at observed location.

V. SAMPLE CASES

Case 1

Compute the probability that a target is still at an observed position 3 hours after the observation if the target is assumed to have an average dwell time (τ) of 4 hours with a deviation (σ) of 1.33 hours.

Solution: Table 12, p. 26 applicable.
Answer is P = 0.294.

Case 2

For the above case, what is the revised expected fractional coverage with a 1-KT weapon with a CEP of 140 m, a 200-m target location error, and a 200-m target radius?

Solution: The damage radius is about 660 m with a criterion of latent lethality. FM-101-31 estimates the static fractional coverage to be 0.98. The revised expected fractional coverage is 0.95 x 0.294 = 0.29.

Case 3

For a target represented by a dwell time (τ) of 6 hours and a standard deviation (σ) of 6 hours, what is the allowable response time for a desired probability of target presence of 0.30?

Solution: Table 15, p. 29 is applicable.
By interpolation, T = 6.9 hours.

Case 4

What is the probability that a target is present after 3 hours if it is represented by the parameters τ = 3.2 hours and σ = 1.5?

Solution: Generalized curve, p. 31.

$$\frac{\tau}{\sigma} = \frac{3.2}{1.5} = 2.13$$

$$\frac{t}{\tau} = \frac{3.0}{3.2} = 0.94$$

By interpolation, P = 0.22.

Case 5

For the values of τ and σ assumed in Case 4, what is the allowable response time if the desired expected coverage is 0.50?

Solution: From the generalized curve with:

$$\frac{\tau}{\sigma} = 2.13, \quad P = 0.5$$

$$\frac{t}{\tau} = 0.54$$

$$t = (0.54)(3.2) = 1.73 \text{ hours.}$$

APPENDIX A. EQUATIONS FOR TARGET PERMANENCE

We begin by assuming that the probability of the target leaving (i.e., beginning to move from) its original position between t and t + dt is

$$P_1(t)dt = \frac{1}{C\sigma\sqrt{2\pi}} e^{-(t-\tau)^2/2\sigma^2} dt$$

where t = 0 is the time at which the target originally settled into the given position, τ is the average time that the target remains in place, σ^2 is the variance in the distribution, and the normalization constant

$$C = \frac{1}{2} + \frac{1}{2} \quad \mathrm{erf}\left(\frac{\tau}{\sigma\sqrt{2}}\right)$$

is chosen such that

$$\int_0^\infty P_1(t)dt = 1$$

Sixty-eight percent of the targets will leave between $\tau-\sigma$ and $\tau+\sigma$. Ninety-five percent of the targets will leave between $\tau-2\sigma$ and $\tau+2\sigma$.

We now assume that the target is detected at some arbitrary time $t = t_1 > 0$ which is completely uncorrelated to the movements of the target, and we wish to know the probability density $P_2(t_2)$ of the time t_2 between detection and the departure of the target.

This turns out to be one of the main problems of a branch of probability theory called renewal theory. The random variable t_2 is called the residual waiting time or the excess lifetime. Using the results of renewal theory (Ref. A1) it can be shown that the probability that the target will leave at a time t_2 after it is detected is

$$P_2(t_2)dt_2 = \frac{1 - F_1(t_2)}{\mu} dt_2$$

where

$$F_1(t_2) = \int_0^{t_2} P_1(t)dt$$

and

$$\mu = \int_0^{\infty} t\ P_1(t)dt$$

or, integrating by parts and using $F_1(\infty) = 1$,

$$\mu = \int_0^{\infty} [1 - F_1(t)]dt$$

so that

$$P_2(t_2)dt_2 = \frac{[1 - F_1(t_2)]dt_2}{\int_0^{\infty} [1 - F_1(t)]dt}$$

Using the original expression for $P_1(t)$ we now have

$$1 - F_1(t_2) = \int_{t_2}^{\infty} P_1(t)dt$$

$$= \frac{1}{C\,\sigma\sqrt{2\pi}} \int_{t_2}^{\infty} e^{-(t-\tau)^2/2\sigma^2}\, dt$$

Setting $t-\tau = \sigma\sqrt{2}x$, this becomes

$$1 - F_1(t_2) = \frac{1}{C\sqrt{\pi}} \int_{\frac{t_2-\tau}{\sigma\sqrt{2}}}^{\infty} e^{-x^2}\, dx$$

which gives (Ref. A2)

$$1 - F_1(t_2) = \frac{1}{2C}\left\{1 - \text{erf}\left[\frac{(t_2-\tau)}{\sigma\sqrt{2}}\right]\right\}$$

where erf(x) is the error function, so that the expression for $P_2(t_2)dt_2$ becomes

$$P_2(t_2)dt_2 = \frac{\left\{1 - \operatorname{erf}\left[\frac{(t_2 - \tau)}{\sigma\sqrt{2}}\right]\right\}dt_2}{\int_0^{\infty}\left\{1 - \operatorname{erf}\left[\frac{(t-\tau)}{\sigma\sqrt{2}}\right]\right\}dt}$$

This can be further simplified. Using the formulas (Ref. A3)

$$\int \operatorname{erf}(ax)\,dx = x \operatorname{erf}(ax) + \frac{e^{-a^2x^2}}{a\sqrt{\pi}}$$

and

$$\int_0^{\infty} [1 - \operatorname{erf}(ax)]dx = \frac{1}{a\sqrt{\pi}}$$

we obtain

$$\int_y^{\infty} [1 - \operatorname{erf}\ (ax)]dx = \frac{e^{-a^2y^2}}{a\sqrt{\pi}} - y[1 - \operatorname{erf}(ay)]$$

Using this [illegible] have

$$\int_0^{\infty}\left\{1 - \operatorname{erf}\left[\frac{(t-\tau)}{\sigma\sqrt{2}}\right]\right\} dt = \sigma\sqrt{\frac{2}{\pi}}\, e^{-\tau^2/2\sigma^2}$$

$$+ \tau\left[1 + \operatorname{erf}\left(\frac{\tau}{\sigma\sqrt{2}}\right)\right]$$

so that the final result for $P_2(t_2)dt_2$ is

$$P_2(t_2)dt_2 = \frac{\left\{1 - \text{erf}\left[\frac{(t_2-\tau)}{\sigma\sqrt{2}}\right]\right\} dt_2}{\sigma\sqrt{\frac{2}{\pi}}\, e^{-\tau^2/2\sigma^2} + \tau\left[1 + \text{erf}\left(\frac{\tau}{\sigma\sqrt{2}}\right)\right]}$$

The probability that the target will still be present at a time t_3 after it was detected is now given by

$$P_3(t_3) = 1 - \int_0^{t_3} P_2(t_2)dt_2$$

or, since $\int_0^{\infty} P_2(t_2)dt_2 = 1$,

$$P_3(t_3) = \int_{t_3}^{\infty} P_2(t_2)dt_2$$

Using the expression for $P_2(t_2)$, we obtain after integrating

$$P_3(t_3) = \frac{\sigma\sqrt{\frac{2}{\pi}}\, e^{-(t_3-\tau)^2/2\sigma^2} - (t_3-\tau)\left\{1 - \text{erf}\left[\frac{(t_3-\tau)}{\sigma\sqrt{2}}\right]\right\}}{\sigma\sqrt{\frac{2}{\pi}}\, e^{-\tau^2/2\sigma^2} + \tau\left[1 + \text{erf}\left(\frac{\tau}{\sigma\sqrt{2}}\right)\right]}$$

REFERENCES TO APPENDIX A

A1. Feller, W., An Introduction to Probability Theory and Its Applications, Vol. 2, Second Edition, John Wiley & Sons, New York, 1971, p. 370.

A2. Abramowitz, M., and Stegun, I. A., Handbook of Mathematical Functions, Dover Publications, Inc., New York, 1965, p. 297.

A3. Gradshteyn, I. W., and Ryzhik, I. M., Table of Integrals, Series, and Products, Academic Press, New York, 1965, pp. 633 and 648.

APPENDIX B. TI-59 CODE FOR MOBILE TARGET CALCULATIONS

The equation for the probability of a target being present at an observed location as a function of time where it is not known when the observation took place relative to the time the target stopped is

$$P(t) = \frac{\sigma\sqrt{\frac{2}{\pi}}\, e^{-\frac{(t-\tau)^2}{2\sigma^2}} - (t-\tau)\left[1-\mathrm{erf}\left(\frac{t-\tau}{\sigma\sqrt{2}}\right)\right]}{\sigma\sqrt{\frac{2}{\pi}}\, e^{-\frac{\tau^2}{2\sigma^2}} + \tau\left[1+\mathrm{erf}\left(\frac{\tau}{\sigma\sqrt{2}}\right)\right]}$$

where erf = error function

τ = mean dwell time

σ = standard deviation in dwell time.

The derivation of this equation is presented in Appendix A. An approximation to the error function suitable for use in the TI-59 is

$$\text{If } |X| < 1.18\text{: } \mathrm{erf}(X) = \frac{2}{\sqrt{\pi}}\left(X - \frac{X^3}{3} + \frac{X^5}{10} - \frac{X^7}{42}\right)$$

$$\text{If } |X| \geq 1.18\text{: } \mathrm{erf}(X) = 1 - \frac{1}{\sqrt{\pi}}\,\frac{e}{X}$$

With this approximation the maximum error in the error function (about 5 percent) occurs at X = 1.18. The resultant error in the calculated probability of target presence is less than 1 percent for cases resulting in probabilities greater than 5 percent.

The following pages present TI-59 coding of the above equations and directions for running the program.

INSTRUCTIONS

The program uses standard partioning (479/59) and will fit on two magnetic cards (four "sides"). After entering the program, the procedures are as follows:

Step	Instruction	Data*	Keys
1	Set initial time	0.	STO 5
2	Set stop criteria**	.001	STO 16
3	Initialize		INV 2nd FIX
4	Enter average dwell time	6	STO 3
5	Enter standard deviation	2	STO 2
6	Enter time increment	1	STO 1
7	Begin run		A

---

*Data for sample case illustrated.

**Program stops when the probability of a target being present is less than this value.

The output from the printer is illustrated on the next page. The quantities are:

ADP = average dwell time (input)
DEV = standard deviation of the dwell time (input)
TIME = time
PROB = probability of target being present at given time.

RESULTS FOR SAMPLE CASE

```
 ADT
            6.
 DEV
            2.


TIME                                 TIME
        0.000                                6.000
PROB                                 PROB
        1.000                                0.133

TIME                                 TIME
        1.000                                7.000
PROB                                 PROB
        0.833                                0.066

TIME                                 TIME
        2.000                                8.000
PROB                                 PROB
        0.667                                0.028

TIME                                 TIME
        3.000                                9.000
PROB                                 PROB
        0.508                                0.008

TIME                                 TIME
        4.000                               10.000
PROB                                 PROB
        0.361                                0.000

TIME
        5.000
PROB
        0.233
```

PROGRAM CODING

```
000   76  LBL
001   11   A
```

```
002   02   2
003   34  √X
004   65   ×
005   43  RCL
006   02   02
007   95   =
008   42  STO
009   06   06
```

$\sigma\sqrt{2}$

```
010   89   π
011   34  √X
012   42  STO
013   07   07
```

$\sqrt{\pi}$

```
014   43  RCL
015   06   06
016   55   ÷
017   43  RCL
018   07   07
019   95   =
020   42  STO
021   08   08
```

$A/\sqrt{\pi}$

```
022   43  RCL
023   02   02
024   33  X²
025   65   ×
026   02   2
027   95   =
028   42  STO
029   09   09
```

$2\sigma^2$

```
030   43  RCL
031   03   03
032   55   ÷
033   43  RCL
034   06   06
035   95   =
036   42  STO
037   12   12
038   42  STO
039   10   10
```

$x = \tau/A$

```
040   01   1
041   93   .
042   01   1
043   08   8
044   42  STO
045   04   04
```

1.18

```
046   61  GTO
047   03   03
048   49   49
049   61  GTO
050   03   03
051   68   68
052   58  FIX
053   03   03
```

Print τ and σ

PROGRAM CODING (CONTINUED)

```
054   43  RCL
055   12   12
056   71  SBR
057   12   B
058   42  STO
059   20   20
```

$E_1 = \text{erf}(x)$

```
060   98  ADV
061   43  RCL
062   05   05
063   42  STO
064   15   15
```

Initialize time

```
065   98  ADV
066   43  RCL
067   15   15
068   75   -
069   43  RCL
070   03   03
071   95   =
072   55   ÷
073   43  RCL
074   06   06
075   95   =
076   42  STO
077   10   10
```

$x^1 = \frac{t-\tau}{\sigma\sqrt{2}}$

```
078   50  I×I
079   42  STO
080   12   12
081   71  SBR
082   12   B
083   42  STO
084   21   21
```

$E_2 = \text{erf}(x^1)$

```
085   71  SBR
086   13   C
```

E_2 sign check

```
087   43  RCL
088   15   15
089   55   ÷
090   43  RCL
091   03   03
092   95   =
093   68  NOP
```

t/τ

```
094   61  GTO
095   03   03
096   87   87
```

Print time

```
097   01   1
098   85   +
099   43  RCL
100   20   20
101   95   =
102   65   ×
103   43  RCL
104   03   03
105   95   =
106   42  STO
107   23   23
```

$\tau[1+E_1] = d_1$

PROGRAM CODING (CONTINUED)

```
108  43 RCL
109  03  03
110  33 X²
111  55  ÷
112  43 RCL
113  09  09
114  95  =
115  71 SBR
116  14  D
117  65  ×
118  43 RCL
119  08  08
120  95  =
```

$$\sigma\sqrt{2/\pi}\ e^{-\tau^2/2\sigma^2} = d_2$$

```
121  85  +
122  43 RCL
123  23  23
124  95  =
125  42 STO
126  23  23
```

$$d_1 + d_2$$

```
127  43 RCL
128  15  15
129  75  -
130  43 RCL
131  03  03
132  95  =
133  33 X²
134  55  ÷
135  43 RCL
136  09  09
137  95  =
138  71 SBR
139  14  D
140  65  ×
141  43 RCL
142  08  08
143  95  =
144  42 STO
145  24  24
```

$$\sigma\sqrt{2/\pi}\ e^{\frac{-(t-\tau)^2}{2\sigma^2}} = n_1$$

```
146  01  1
147  75  -
148  43 RCL
149  21  21
150  95  =
151  65  ×
152  53  (
153  43 RCL
154  15  15
155  75  -
156  43 RCL
157  03  03
158  54  )
159  95  =
160  94 +/-
161  42 STO
162  25  25
```

$$(t-\tau)(1-E_2) = n_2$$

PROGRAM CODING (CONTINUED)

```
163  43 RCL
164  24  24
165  85  +
166  43 RCL
167  25  25
168  95  =
169  55  ÷
170  43 RCL
171  23  23
172  95  =
173  42 STO
174  29  29
```

$$P(t) = \frac{(n_1 + n_2)}{(d_1 + d_2)}$$

```
175  61 GTO
176  04  04
177  08  08
178  32 X:T
179  43 RCL
180  16  16
181  77  GE
182  01  01
183  97  97
```

Stop check

```
184  98 ADV
185  68 NOP
186  43 RCL
187  15  15
188  85  +
189  43 RCL
190  01  01
191  95  =
192  42 STO
193  15  15
194  61 GTO
195  00  00
196  66  66
197  91 R/S
```

Increment time

PROGRAM CODING (CONTINUED)

```
198  76 LBL
199  12  B
200  43 RCL
201  04  04
202  32 X:T
```

```
203  43 RCL
204  12  12
205  77  GE
206  02  02
207  59  59
```

Check for x > or < 1.18

$x < 1.18$

```
208  43 RCL
209  12  12
210  45 YX
211  05  5
212  95  =
213  55  -
214  01  1
215  00  0
216  95  =
217  42 STO
218  13  13
```

$x^5/10$

```
219  43 RCL
220  12  12
221  45 YX
222  07  7
223  95  =
224  55  -
225  04  4
226  02  2
227  95  =
228  94 + -
229  85  +
230  43 RCL
231  13  13
232  95  =
233  42 STO
234  13  13
```

$x^7/42$

```
235  43 RCL
236  12  12
237  45 YX
238  03  3
239  95  =
240  55  -
241  03  3
242  95  =
243  94 + -
244  85  +
245  43 RCL
246  13  13
247  85  +
248  43 RCL
249  12  12
250  95  =
251  65  ×
252  02  2
253  95  =
254  55  -
255  43 RCL
256  07  07
257  95  =
258  92 RTN
```

$x^3/3$

$\mathrm{erf}(x) = 2/\sqrt{\pi}\,(x - x^3/3 + x^5/10 + x^7/42)$

PROGRAM CODING (CONTINUED)

```
259  03   3
260  32  X:T
261  43  RCL
262  12   12
263  77   GE
264  02   02
265  88   88
```

x ≥ 3?

```
266  43  RCL
267  12   12
268  33  X²
269  94  +/-
270  27  INV
271  23  LNX
272  55   -
273  43  RCL
274  12   12
275  95   =
276  55   ÷
277  43  RCL
278  07   07
279  95   =
280  42  STO
281  13   13
282  01   1
283  75   -
284  43  RCL
285  13   13
286  95   =
287  92  RTN
288  01   1
289  92  RTN
```

$$\text{erf}(x) = 1 - \frac{e^{-x^2}}{\sqrt{\pi}\,x}$$

x ≥ 1.18

```
290  00   0
291  76  LBL
292  13   C
293  00   0
294  32  X:T
295  43  RCL
296  12   12
297  67   EQ
298  03   03
299  12   12
```

x = 0?

```
300  43  RCL
301  12   12
302  55   ÷
303  43  RCL
304  10   10
305  95   =
306  65   ×
307  43  RCL
308  21   21
309  95   =
310  42  STO
311  21   21
312  43  RCL
313  21   21
314  92  RTN
```

Sign check for E

PROGRAM CODING (CONTINUED)

```
315   76 LBL
316   14  D
317   42 STO
318   26  26
319   50 I×I
320   42 STO
321   27  27        Limit exponent to 99
322   09  9
323   09  9
324   32 X:T
325   43 RCL
326   27  27
327   77  GE
328   03  03
329   36  36
330   43 RCL
331   26  26
332   94 +/-
333   22 INV
334   23 LNX
335   92 RTN
336   43 RCL
337   27  27
338   55  ÷
339   43 RCL
340   26  26        e^Y, Y<99
341   95  =
342   65  ×
343   09  9
344   09  9
345   95  =
346   61 GTO
347   03  03
348   32  32
349   25 CLR
350   69 OP
351   00  00
352   01  1
353   03  3
354   01  1
355   06  6
356   03  3
357   07  7
358   69 OP         Label "ADT"
359   01  01
360   69 OP
361   05  05
362   43 RCL
363   03  03
364   99 PRT
365   61 GTO
366   00  00
367   49  49
```

PROGRAM CODING (CONCLUDED)

```
368  25 CLR
369  69 OP
370  00   00
371  01   1
372  06   6
373  01   1
374  07   7
375  04   4
376  02   2                Label "DEV"
377  69 OP
378  01   01
379  69 OP
380  05   05
381  43 RCL
382  27   27
383  99 PRT
384  61 GTO
385  00   00
386  52   52
---------------------------------------
387  25 CLR
388  69 OP
389  00   00
390  03   3
391  07   7
392  02   2
393  04   4
394  03   3
395  00   0
396  01   1
397  07   7                Label "Time"
398  69 OP
399  01   01
400  69 OP
401  05   05
402  43 RCL
403  28   28
404  99 PRT
405  61 GTO
406  02   02
407  47   47
---------------------------------------
408  25 CLR
409  69 OP
410  00   00
411  03   3
412  03   3
413  03   3
414  05   5
415  03   3
416  02   2
417  01   1
418  04   4                Label "Prob"
419  69 OP
420  01   01
421  69 OP
422  05   05
423  43 RCL
424  29   29
425  99 PRT
426  61 GTO
427  01   01
428  78   78
```

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DEPARTMENT OF THE NAVY

Cruiser Destroyer Group One
ATTN: N321

Fleet Intelligence Ctr, Pacific
Department of the Navy
ATTN: FICPAC, Code 21

Fleet Intelligence Ctr, Europe & Atlantic
Department of the Navy
ATTN: Code 222

Marine Corps Dev & Ed Cmd
Department of the Navy
ATTN: Commander

Naval Intelligence Cmd
ATTN: NIC-01

Naval Intelligence Support Ctr
ATTN: NISC-30

Naval Postgraduate Sch
ATTN: Code 1424, Lib

Naval War College
ATTN: Code E-11, Tech Svc

Naval Weapons Center
ATTN: Code 32607

Naval Field Op Intell Ofc
ATTN: Commanding Officer

Nuclear Weapons Tng Group, Pacific
Department of the Navy
ATTN: Code 32

Nuclear Weapons Tng Group, Atlantic
Department of the Navy
ATTN: Code 222

DEPARTMENT OF THE NAVY (Continued)

Office of the Chief of Naval Ops
ATTN: OP-OOK

Commander-in-Chief
U.S. Atlantic Fleet
Department of the Navy
ATTN: Code N-2
ATTN: Code N-3

Commander-in-Chief
U.S. Naval Forces, Europe
ATTN: N54, Nuc Warfare Off

DEPARTMENT OF THE AIR FORCE

Air University Library
Department of the Air Force
ATTN: AUL/LSE

Assistant Chief of Staff
Intelligence
Department of the Air Force
ATTN: INA

Assistant Chief of Staff
Studies & Analyses
Department of the Air Force
ATTN: AF/SAG, H. Zwemer
ATTN: AF/SASB, R. Mathis
ATTN: AF/SASF
ATTN: AF/SASM
ATTN: AF/SAMI
ATTN: AF/SASM, W. Adams

Headquarters Space Div
Air Force Systems Command
ATTN: YKD

Commander-in-Chief
U.S. Air Forces in Europe
ATTN: USAFE/INA

Commander-in-Chief
U.S. Air Forces in Europe
ATTN: USAFE/XPX

OTHER GOVERNMENT AGENCY

Central Intelligence Agency
ATTN: OSWR/NED

DEPARTMENT OF ENERGY CONTRACTORS

Sandia Labs
Livermore Lab
ATTN: L. Hostetler
ATTN: A. Kernstein
ATTN: T. Gold

DEPARTMENT OF DEFENSE CONTRACTORS

BDM Corp
ATTN: J. Braddock
ATTN: F. Conant

Boeing Co
ATTN: J. Russel

DEPARTMENT OF DEFENSE CONTRACTORS (Continued)

66th MI Group
4 cy ATTN: D. Welch

General Research Corp
ATTN: Tac Warfare Ops

Hudson Institute, Inc
ATTN: C. Gray
ATTN: H. Kahn

IRT Corp
ATTN: J. Hengle
ATTN: W. Macklin

JAYCOR
ATTN: R. Sullivan

Kaman Sciences Corp
ATTN: F. Shelton

Kaman Sciences Corp
ATTN: E. Daugs

Kaman Tempo
ATTN: DASIAC

Martin Marietta Corp
ATTN: F. Marion

Martin Marietta Corp
ATTN: J. Donathan

Pacific-Sierra Research Corp
ATTN: H. Brode
ATTN: G. Lang

R & D Associates
ATTN: P. Haas
2 cy ATTN: J. Hurley

DEPARTMENT OF DEFENSE CONTRACTORS (Continued)

Rand Corp
ATTN: V. Jackson

Santa Fe Corp
ATTN: D. Paolucci

Science Applications, Inc
ATTN: C. Whittenbury/W. Yengst
ATTN: M. Drake
ATTN: J. Martin

Science Applications, Inc
ATTN: R. Craver
ATTN: W. Layson
ATTN: J. Shannon

SRI International
ATTN: J. Scholz
ATTN: J. Sloss

System Planning Corp
ATTN: J. Douglas

Systems, Science & Software, Inc
ATTN: K. Pyatt

Tetra Tech, Inc
ATTN: F. Bothwell

TRW Defense & Space Sys Group
ATTN: N. Lipner
ATTN: D. Scally
ATTN: R. Burnett

TRW Defense & Space Sys Group
ATTN: P. Dai