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DESIGN CALCULATIONS FOR PASSIVE SOLAR BUILDINGS BY A PROGRAMMABLE HAND CALCULATOR

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ABSTRACT

The behavior of room temperature in a passive solar building without backup heat is of great interest to the building designer. This paper presents programs for card-reading programmable hand-calculators which compute room temperature over the course of a design day. Instructions for calculating the input parameters, and for running the programs are given, and a brief review of the theory is provided. The program can presently be used only for single-zone unmanaged, direct-gain buildings.

INTRODUCTION

Floating, or non-thermostat-controlled, room temperature in a passive solar building is an important measure of the building's performance. Optimally designed buildings will provide temperature which floats within the comfort range of the occupants without the use of heaters or air conditioners. Even if some supplementary heating or cooling is used, the floating performance of the building is of interest to the designer in assuring that full use is made of the solar energy collected by a building, and that the optimum window area is chosen.

Solar energy collected by a passive solar building is useful only to the extent that it either balances heating loads during sunny periods or can be stored for use at night. If solar heat gain increases room temperature beyond the comfort range, then the excess heat is either lost through ventilation or else it results in discomfort. In either case, a design whose maximum temperature is lower than the designer's upper limit, (e.g., 80°F (26.7°C)) is preferable to one which heats up beyond this limit.

The lowest floating temperature is also of interest to the designer. The magnitude of the minimum, and the time of day at which it occurs, will determine, for a given occupant's thermal preference characteristics, whether supplementary heat is needed.

Tradeoffs can be made in building design which affect different aspects of floating temperature behavior. Increasing the window area increases the maximum room temperature, but may also decrease the minimum temperature. Adding to the building's thermal mass decreases the magnitude of daily fluctuations in temperature without changing the daily average. Thermal mass can also delay the times of temperature extrema; for a Trombe wall, these delays can exceed 12 hours.

This paper presents a hand-calculator program which can be used to predict the floating temperature of a building, given a few simple building parameters and weather data. The calculations describe the building's response to a design day -- that is, a day with idealized (sinusoidal) weather. Two versions of the program are given; one for a Hewlett-Packard HP-67 calculator and the other for a Texas Instruments TI-59. Listings of the program are given in Appendix A.

The programs described here can be run in less than one-half hour; in some cases (e.g., those in which a few parameters are varied from an initial design), the run-time is considerably less. The methodology used in the programs can be generalized beyond the level of detail available in the programs. Some of these extensions can be done as hand calculations using intermediate outputs of the programs.

Use of this program will allow the building designer to easily predict the floating performance of a proposed single-zone, unmanaged passive solar building. The effect on floating temperature of varying parameters such as properties of the thermal mass and area of windows can be seen and, thus, optimal values can be chosen for such parameters.

The theoretical basis of the programs is described in Appendix B, and detailed derivations can be found in Ref. 1. Some familiarity with the theory will be helpful to the user of the programs, as it will assist in evaluating the input parameters to the program. As can be seen, the theory parallels that used in many public-domain building energy use analysis computer programs (e.g., NBSLD[2], DOE-2[3,4], TWOZONE[5], BLAST[6]), except that Fourier transformations are used instead of Laplace transformations, and some additional approximations are made.

The program has been validated by comparing its predictions with the measurements of floating temperature performed in two passive solar test cells at Los Alamos Scientific Laboratories.[7] Predictions of the temperature elevation (above average ambient temperature) were accurate to within ±10% of measurements at all hours of the day for two tests days (see figure, page 27).[8]

THEORY: RESPONSE FUNCTIONS

This section describes the use of response functions in the programs. Their derivation is discussed Appendix B. The response functions describe the response of temperatures in the building to driving forces such as sunlight and outside air temperature. They are functions of frequency (denoted by ω); that is, they give the effect of regular variations in the driving forces at a frequency ω on the variation of building temperatures at the same frequency. The most interesting frequency is generally one cycle per day.

The surface temperatures on the inside surface of building elements (e.g., walls and floors) are described by the materials response functions R_1 and R_2. These are complex-valued functions which are evaluated by the sub-programs for each building element. R_1 describes the response to sunlight inside the room, and R_2 gives the response to outside temperature.

The overall building performance is described by three building response functions A, B, and C, which are computed using the R_1 and R_2 results for each building element. A and C have the form of design heat loss rates. The A function relates the floating of room temperature to heating or cooling loads, while the C function describes the building response to ambient temperature variations. Response to sunlight is given by the B function.

Room temperature as a function of time is calculated using the building response functions A, B, and C, and weather data for a design day.

PROGRAM INPUT PARAMETERS

The passive solar programs require some simple weather inputs, and some data on building parameters. The building parameters are in a form which differs somewhat from other building models, since a simple format which accounts for most of the passive solar effects is desired.

Before any building parameters can be evaluated, the building must be divided into a small number of different types of construction sections. Each construction section is associated with a surface which faces the inside of the building. A typical house, for example, might have envelope wall sections, ceiling, floor, partition walls, and windows.

If a section is of thermally light construction (e.g., a window or a wall or floor with an insulating material on the inside) the 'U-value' (overall heat transfer coefficient) should be calculated, and no further data entries are needed. If a section is heavy (e.g., masonry construction; solid wood, gypsum board), then more detail is needed.

A heavy wall is divided conceptually into two layers: a thermally massive, inside-facing layer (e.g., concrete walls or floor) and an outside insulating layer (e.g., foam insulation plus sheathing or dry soil). For the inside thermally massive layer, data are needed on conductivity, heat capacity per unit volume, and thickness of the massive layer. For the insulating layer, only U-value of the insulation is needed. If there is no exterior insulation, the outside film coefficient is used.

For thermally massive elements connected to the room by an insulating layer, such as face brick on the outside of an insulated wall, the thermal mass of the outer layer is ignored. For walls with two layers of similar but not exactly equal thermal properties, such as drywall or plaster on solid wood, averaged parameters are used as an approximation. For example, a 4-inch wood wall covered on the outside by sheathing and on the inside by 1/2-inch plaster may be considered as a 4-1/2-inch wall with an insulating layer outside whose resistance is equal to that of the sheathing plus the outside air film. The properties of the thermal mass are the weighted average properties of the wood and plaster.*

Walls which cannot be approximated by this two-layer method are beyond the scope of the present programs because different formulas are needed to evaluate their response functions R_1 and R_2. Hand calculation methods which can compute these response functions are discussed in Ref. 1, appendix 2.4. An example of such a section might be a 1-inch-thick wood panel covering the inside of an 8-inch concrete wall or floor.

The program is fastest to run when the number of different surfaces or sections is minimized. The TI-59 program can only treat three different sections. To reduce the number of sections, different walls or floors with similar thermal properties can be combined into a single section; and sections which are much lighter than the rest of the house can be considered light.

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*To make the whole wall's U-value come out correct, one should average the inverse of conductivity rather than conductivity itself.

One can, for example, combine 4-inch frame envelope walls, 8-inch frame ceilings and floor, and 4-inch frame partition walls into a single section (call it a 6-inch frame wall) if the outside layer's U-value (U_r) is adjusted such that the heat transfer coefficient is the same as it would have been if each section were treated separately. (See Appendix E for an illustration of this process.)

In detailed building energy use analysis programs, such as DOE-2, NBSLD, BLAST and TWOZONE, frame walls are considered as two different sections; one with the properties of the solid wood (stud) fraction, and the other with the properties of the insulated section. This approach will also work in the hand-calculator programs, at the expense of adding an extra surface.

Once the number of surfaces has been established, the user must estimate how much of the solar radiation entering the house is absorbed on each surface. The fraction of solar gain absorbed on a surface is designated by α. The sum of α's for all heavy surfaces is less than one, because some fraction of the sunlight is absorbed by light surfaces or furniture. This fraction is released immediately to the room, and is called α_R. There are at present no simple methods for determining the values of the α's from theory, even for simple room geometries. For complex geometries, even the detailed computerized methods break down. Empirical evaluation is a possibility. Fortunately, building performance does not appear to depend crucially on the <u>exact</u> evaluation of these parameters.

As a rough guideline, for a dark floor and light walls and ceiling, $\alpha_{floor} \cong .45$, $\alpha_{envelope\ walls} \cong .10$, $\alpha_{partition\ walls} \cong .20$, $\alpha_{ceiling} \cong .10$, $\alpha_R \cong .15$. These estimates are based on computer runs using the "Lumen

II" program[9] to calculate radiation balance for a prototype passive solar room in winter.

If the walls are dark, this changes to: $\alpha_{floor} \cong .30$, $\alpha_{envelope\ walls} \cong .20$, $\alpha_{partition\ walls} \cong .35$, $\alpha_{ceiling} \cong .05$, $\alpha_R \cong .10$.

Following is a list of input parameters needed.

For each massive surface:

- K- The thermal conductivity of the inside massive layer (in Btu/^{o}F-ft-hr). (Note that many handbooks express K in other units, such as Btu-in/^{o}F-ft^2-hr).
- ρc - The heat capacity per unit volume (Btu/^{o}F-ft^3). This is usaly obtained by finding the density ρ and specific heat c from handbook tables, and multiplying these values. $\rho c \simeq 9$ for wood and varies from about 15 to 30 for concrete.
- d - The thickness (in feet) of the massive part of the section. For the partition walls use half the wall thickness.
- h - The inside film heat transfer coefficient coupling the surface to the room air (in Btu/^{o}F-hr-ft^2). A typical value is 1.5, although sparsely furnished buildings with few partition walls may have lower values ($\sim$1).

- U_r - The U-value of the resistance between the outside of the massive part of the section and the ambient air (in Btu/^{o}F-ft^2-hr). Typically, $U_r \cong 5$ for a bare thermal mass or about 0.1 for an insulated mass. Concrete slab floors on grade with perimeter insulation have $U_r \sim 0.01$. For partition walls, $U_r = 0$.

- α - The fraction of solar energy entering the house which is absorbed on the surface (including multiple reflections). α is dimensionless.
- A - The total area of the surface facing the room (ft^2).

<u>For the whole building:</u>

- $\hat{U}_q$ - The design heat loss of the building per degree temperature difference through all quick heat transfer mechanisms, including window heat loss, infiltration loss, and conduction through quick construction sections (in Btu/^{o}F-hr).
- α_R - The fraction of solar energy entering the house which is absorbed on light surfaces, furniture, carpet, etc. (α_R is dimensionless.)
- H - The daily average heater output plus internal loads (Btu/hr). Typically, internal loads are about 2000 Btu/hr for a residential unit in the United States.

<u>Weather parameters:</u>

- ω_o - Daily frequency: 2π radians/24 hr.
- t_d - The length of the day (in hours) from sunrise (or time of first solar gain through the window) to sunset (or time of last solar gain).
- $|S_1|$ - Amplitude of daily solar gain through the windows (Btu/hr). In practice this is obtained by requiring that daily total solar gain is correct; that is, $\int |S_1| \sin \omega_1 t$ equals the daily solar gain. Thus

$$S_1 = \frac{\pi}{2t_d} \times \text{daily solar gain} \quad .$$

Daily solar gain can be obtained for a sunny day from ASHRAE solar heat gain factors, [10] or it can be approximated by multiplying window transmissivity by measured solar heat flux on a surface oriented in the same direction as the window. Typical winter transmissivities for south-facing windows are ~ 0.85 for single-pane, 0.75 for double-pane, and 0.65 for triple-pane glass. To model cloudy days, the value of S_1 must be reduced. Solar gain through all windows is considered in computing S_1. Errors can result if east or west window area is large compared to south window area.

- $\bar{T}_A$ - Average ambient temperature in ^oF.
- $|\Delta T_A|$ - The amplitude of diurnal temperature fluctuations (^oF) or one-half the difference between maximum and minimum temperature.
- t_ϕ - The number of hours between sunrise and maximum ambient temperature. Note that minimum temperature is modeled as occurring 12 hours from maximum, so choose t_ϕ for best overall fit of sinusoidal temperature,

  $T_A = \bar{T}_A + \Delta T_A \cos(\omega_o(t-t_\phi))$

  to real temperature.
- $\bar{S}$ - For a weather cycle in which $|S_1|$ varies sinusoidally from day to day, the average value of $|S_1|$, (Btu/hr).
- ΔS_w - For a weather cycle, the amplitude of variation of $|S_1|$ over the cycle, (Btu/hr).

- ω_w - Frequency of weather variations (in radians/hr). ω_w is smaller than ω_o; typically $\omega_w \cong .1\omega_o$.
- ΔT_{A_w} - Amplitude of weather-cycle variation in ambient temperature (^oF).
- t_a - The time in the weather cycle at which the ambient temperature is maximized (hrs).

- t_s - The time in the weather cycle at which solar gain is maximized (hrs).

Results of weather cycle variations are not presently computed in the HP-67 program; however, the response functions can be obtained from the program and the weather cycle response may be computed by hand, as discussed in the HP program description.

PROGRAM OPERATION

This section describes the operation of the program from the point of view of the user. It assumes that the user has already evaluated all the building and weather parameters. A more detailed description of the HP-67 and TI-59 programs is given in Appendix C. A listing of the programs will be found in Appendix A. The HP-67 program and the TI-59 program are different in structure, so they are described separately below. The instructions must be followed exactly to assure correct output.

To check the performance of the programs and the selection of input parameters, a sample problem is set up and solved in Appendix E.

HP-67 Program

This program consists of three sub-programs. The first, sub-program 'R_{12}', calculates the R_1 and R_2 functions for a surface, given the building parameters for its construction section. The functions as evaluated at five frequencies (0, ω_w, ω_o, $2\omega_o$, and $3\omega_o$) and the results are read out on a data card. This program is run once for each surface.

The results on the data card produced by this program are valid for any material surface with the same construction as the one computed, so that if many runs are to be made on buildings of similar construction, the user may wish to build up a library of R_{12} data cards for commonly used construction sections.

For construction sections beyond the scope of the R_{12} sub-program, response functions can be computed manually and written onto data cards to be used in the rest of the calculation.

The second sub-program, 'ABC', computes the Building Response Functions from the data stored on R_{12} data cards. Each data card is read into the calculator once, some additional data are entered, and the calculator computes the effects of the new surface on A, B, and C. Any number of surfaces may be used. When the ABC sub-program is completed, the results are written on a data card; the card contains A and B (evaluated at all five frequencies) and $C(\omega_W$ and $\omega_0)$. Note that $C(0) \equiv A(0)$.

The final sub-program, 'T_R', takes the data from the 'ABC' sub-program and computes coefficients of $e^{i\omega t}$ from Eq. (B11) in Appendix B. The program next evaluates the temperature for each hour of the design day using (B11). It displays t, the time (relative to sunrise), for one second; then displays $T_R(t)$ for 5 seconds (or prints it); then displays t for the next hour and the new T_R. The first time and temperature displayed correspond to (solar) midnight, the second to 1a.m., etc. The coefficients of Eq. (B11) used to evaluate T_R are retained in memory.

At present, the effects of internal load and heater output must be added manually by reading A(0) from the ABC data card, decoding the entry using subroutine 'd' of the ABC or T_R sub-programs, and adding the temperature difference H/A(0) to the results for T_R. Response to weather cycles longer than one day must also be computed manually, as described at the end of the T_R sub-program.

R_{12} sub-program: Run this once for each material. Angle mode must be set to 'radians'.

1) Enter Input: ω_o in STO A

$\frac{\omega_o}{\omega_w}$ in STO B* $\left(\frac{\omega_o}{\omega_w}\right.$ is the period of the weather cycle in days$\left.\right)$

K in STO 1

ρc in STO 2

d in STO 3

h in STO 4

U_r in STO 5

0 (zero) in STO I.

2) Press E.

Wait ∿2 minutes.

Program will stop and read 'Crd' in display.

3) Feed in blank data card (both sides).

Note that the data card applies to this particular material.

Output:

The output R_1 and R_2 are encoded; can be interpreted with D routine of ABC program. The 'D' routine places the magnitude of R_1 or R_2 in the 'x' register and the phase angle in 'y'

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*Program will not run if this storage register contains 'zero'.

$R_1(0)$ in STO 0

$R_2(0)$ in STO 1

$R_1(\omega_w)$ in STO 2

$R_2(\omega_w)$ in STO 3

$R_1(\omega_o)$ in STO 4

$R_2(\omega_o)$ in STO 5

$R_1(2\omega_o)$ in STO 6

$R_2(2\omega_o)$ in STO 7

R_1 $(3\omega_o)$ in STO 8

R_2 $(3\omega_o)$ in STO 9.

<u>ABC sub-program</u>: Run this once for the whole building after all R_{12} data cards have been obtained. Angle mode must be set to 'radians'.

1) First: press E.

   Calculator will display '8'.

2) Press 'g' MERGE (or 'f' MERGE on HP 97).

3) Read in R_{12} data card for 1st material.

4) Input h in STO A
A in STO B
α in STO C } for the 1st material.

5) Press R/S; wait $\sim$ 2 minutes.

   Calculator will stop and display '8'.

6) Repeat steps (2) - (5) for each surface.

7) After inputting all R_{12} data cards,

enter $\hat{U}_q$ in STO A

α_R in STO B.

Set Flag 0 (press 'h' 'SF' '0').

8) Press R/S; wait ∿2 minutes.

Display will read 'Crd'.

9) Feed in blank data card for ABC data.

Note that output is encoded. Output for A and C can be decoded using by pressing 'd'. Output for B can be decoded by pressing 'D' The magnitude of the response function appears in 'x' register and the phase angle in the 'y' register.

Output:	A(0)	in STO 0
	B(0)	in STO 1
	$A(\omega_W)$	in STO 2
	$B(\omega_W)$	in STO 3
	$A(\omega_o)$	in STO 4
	$B(\omega_o)$	in STO 5
	$A(2\omega_o)$	in STO 6
	$B(2\omega_o)$	in STO 7
	$A(3\omega_o)$	in STO 8
	$B(3\omega_o)$	in STO 9
	$C(\omega_W)$	in STO 10
	$C(\omega_o)$	in STO 11.

<u>$T_R(t)$ sub-program:</u>

1) Read ABC data card. (This may be unnecessary if ABC has just been calculated.)

2) Press P ↔ S.

3) Enter weather data

0 (zero) in STO I

$|\Delta T_A|$ in STO 5

t_ϕ in STO 6

t_d* in STO 7

S_1 in STO 8

$\overline{T_A}$ in STO 9

ω_o in STO C.

4) Press 'E'.

Output: After ∿1 minute calculator will flash for 1 sec the time+ at midnight, then for 5 sec, the temperature at that time; then flash for 1 sec the time 1 hour later and flash for 5 sec the temperature at 1 a.m.; etc. Note that these temperatures do not include the effects of internal loads or supplementary heat.

On HP 97, hour will flash, T_R will be printed.

T_R also provides the following coefficients from Eq. (B11)

$|S_1|\ d_o\ \frac{B(0)}{A(0)}$	in	STO 0
$|S_1|\ d_o\ \frac{B(\omega_w)}{A(\omega_w)}$	in	STO 2 - 3‡

*Program may fail for t_d = 6 or 12; if there is a problem, try 6 or 12±.001.

+Relative to sunrise.

‡Magnitude of complex number is in the first register; argument is in the second.

$|S_1|\ d_1\ \frac{B(\omega_o)}{A(\omega_o)}$ in STO 4 - 5*

$|S_1|\ d_2\ \frac{B(2\omega_o)}{A(2\omega_o)}$ in STO 6 - 7*

$|S_1|\ d_3\ \frac{B(3\omega_o)}{A(3\omega_o)}$ in STO 8 - 9*

$\frac{C(\omega_w)}{A(\omega_w)}$ in STO 10 - 11*

$\frac{C(\omega_o)}{A(\omega_o)}$ in STO 12 - 13*

Subroutine D of T_R decodes representations of R_1, R_2, and B into polar format.

Subroutine d of T_R decodes representations of A and C.

Calculating response to weather cycles:

As discussed in Appendix D, only three terms need be added to the daily response calculation to compute response on any given day of a weather cycle. These terms are given in Eq. (D4) of Appendix D, "Long-term Weather Response."

To perform the computation; set $|S_1|$ equal to the amplitude of solar gain on the day in question:

$$|S_1| = \bar{S} + \Delta S_w \cos \omega_w (t-t_s)$$

where t is evaluated at noon of the day of interest.

*Magnitude of complex number is in the first register; argument is in the second.

$\bar{T}_A$ is set equal to the average temperature for the whole cycle. The program is then run and results recorded for T_R at each hour. To these results are added the three terms from Eq. (D4).

The first term, $(\bar{S} - |S_1|)\ B(0)/A(0)\ d_o$, is computed by pressing RCL 0 and multiplying the result by $(\bar{S} - |S_1|)/|S_1|$. This temperature change is added to T_R at all hours.

The second term, $\Delta S_w\ B(\omega_w)/A(\omega_w)\ d_o\ e^{i\omega_w(t-t_s)}$, is obtained as follows: Compute $\omega_w(t-t_s)$ and then enter '1'. Press RCL 3, then RCL 2. Press 'B' for complex multiplication. Then multiply the result by $\Delta S_w/|S_1|$ to get the complex number represented by this term. The temperature change is the real part of this number; Press →R and read the result from the x register. In principle, this result is different each hour of the day, but the variation is usually so slow that only two or three hours need be calculated; the rest can be obtained by linear interpolation.

The third term, $\Delta T_{A_w}\ C(\omega_w)/A(\omega_w)\ e^{i\omega_w(t-t_a)}$, is obtained similarly to the second. $\omega_w(t-t_a)$ is computed, then '1' is entered. The programmer presses P⟷S, then RCL 1 and RCL 0, then (subroutine) B. The result is multiplied by ΔT_{A_w} and the real part taken by pressing →R.

Addition of these three terms completes the calculation of response to long-term weather.

TI - 59 Program

This program consists of two sub-programs, PSA-1 and 2, which take input data on the building and weather and compute building response. The response is calculated at each frequency, and the results added cumulatively.

When the next term changes the results by a sufficiently small amount, usually when the frequency is greater than 3 cycles per day ($\omega > 3\omega_o$), the user terminates the calculation.

The first sub-program, PSA-1, accepts all the building parameter data and most of the weather data in its first steps. It calculates and displays the steady-state room temperature response of the building, then Fourier analyzes the solar gain function S (t) into its amplitudes d_n. These results are displayed (and printed) by the calculator, and must be retained by the user for manual entry in PSA-2.

The second sub-program, PSA-2 uses the data which were stored in the memory registers by PSA-1, and the d's generated from PSA-1. The program calculates the room temperature component at a given frequency, then adds these results to the sum of those previously computed at other frequencies. After four to five passes through this program, the results for $T_R(t)$ converge to the final answer. To get the room temperature, the user must input the steady-state term computed in PSA-1.

This program expresses the time delays somewhat differently than they are defined in the parameter section of this paper. It uses the variable ϕ to represent the phase delay (in hours) of a given weather term.

- For the ΔT_A terms, ϕ = - (the number of hours from midnight to the time of the temperature maximum).
- For the ΔT_{A_W} term, ϕ = - (the number of hours from the beginning of the temperature cycle to the time of maximum temperature).
- For the ΔS_W term ϕ = - (the number of hours from the beginning to the weather cycle to the maximum of solar amplitude S_1).

- For the $|S_1|$ solar terms, $\phi = -$ (the number of hours from midnight to sunrise).

  Note the negative signs in the definitions of all ϕ's.

<u>PSA-1 sub-program instructions</u>: (using Master Library module)

1) Read in program by entering 1, then pressing INV 2nd WRITE and feeding in the first side of the card. Next, enter 2, press the same keys, and read in side 2.

2) Choose the surface (i=0,3) for which you will enter the parameters. Surface 0 refers to general parameters such as $|S_1|$ and $\hat{U}_q$. Up to three surfaces are allowed.

2a) Enter surface number (i) and press E. Calculator will display '20'.

3) Enter parameter in order for surface i

(0)	Enter d_i	or	$\|S_1\|$	Press A
(1)	Enter $(\frac{1}{U_R})$ i	or	$\hat{U}_q$	Press A
(2)	Enter h_i	or	$\bar{T}_A$	Press A
(3)	Enter K_i	or	π/t_d	Press A
(4)	Enter $(\rho c)_i$	or	H	Press A
(5)	Enter α_i	or	α_R	Press A
(6)	Enter A_i	or	ΔT_A	Press A

Then return to Step 2a.

3b) To correct a parameter, return to step (2), and enter the number of the surface; then press E and enter the substep number shown in step 3 (e.g., '4' for ρc) and press B, then enter the correct parameter value and press 'R/S'.

4) Enter the number of surfaces used; press E. Calculator will display '20'.

5) Press C; calculator computes H/A(0) and prints result.

6) Press 'R/S'; calculator computes and adds steady-state ambient temperature and prints $\overline{T}_A$.

7) Enter t_d*; press R/S. Calculator prints $d_0|S_1|$ B(0)/A(0).

8) Press 'R/S'. Calculator prints steady-state component of T_R. Record this value for future use.

9) Compute Fourier components of solar gain d_n for each frequency of interest 'n'.

   a) Enter n (you will need n=1, 2, and 3 in most cases).

   b) Press D*.

   c) Calculator displays Im(d_n) (imaginary part of d_n). Record this value for later entry.

   d) Press x⟷t.

   e) Calculator displays Re(d_n) (real part of d_n): Record this value for later keyboard entry.

   f) Go back to step 9a) and enter another value of n.

10) Press 2nd D; calculator computes and displays d_0.

11) Keep calculator on and run sub-program PSA-2.

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*For t_d = 6, 12, or few other values, calculator will attempt to divide by zero, resulting in an error. If this happens, re-enter t_d as .001 larger or smaller and try again.

PSA-2 sub-program instructions:

1) Read in PSA-2 (after having run PSA-1 to fill the calculator memories with data). Enter '1', press INV 2nd WRITE and feed in the first side of the card. Then enter 2, press the same keys, and read in side 2.

2) Run through steps 3 - 10 once for each frequency of interest (except ω=0). Choose a frequency ω.

3) Computes $R_1(\omega)$ $R_2(\omega)$, and $A(\omega)$ $B(\omega)$ and $C(\omega)$.
Enter the period length in days (ω_0/ω); press A.
Calculator will display $Im(R_1(\omega))$ for the first surface.
To see Re $(R_1(\omega))$, press 'x⟷t'.
To see Re $(R_2(\omega))$, press RCL 10.
To see Im $(R_2(\omega))$, press RCL 11.

4) Press R/S. (If R_1 or R_2 has been examined, press R/S twice.)

5) For two or more surfaces press 'R/S' to compute R_1 and R_2 for each surface. Display is as shown in Step 3.

6) Press R/S

7) Compute ambient temperature term at frequency ω. If there is no such term, go to step 9.
Enter ΔT_A or $|\Delta T_{A_w}|$.
Press B.
Calculator displays Imaginary Part of weather term. To see real part, press 'x⟷t'. To see the term hour by hour, press 'R/S'. Calculator flashes time for 1 sec and temperature from this term for 4 sec.

8) Enter ϕ for the weather term, press E; calculator will display '2'.

9) To compute solar term at frequency ω, enter $Re(d_n)$ and press C. (If $\omega = \omega_w$, enter d_o). Enter $Im(d_n)$ and press 'R/S'. Enter $|S_1|$ or $|\Delta S_w|$ and press 'R/S'.

   Calculator will display imaginary part of solar term at frequency ω. Press 'x⟷t' to see real part.

   To see the term hour-by-hour, press R/S; calculator flashes hour for 1 sec and solar term for 4 sec.

10) Enter ϕ for solar term, press E. Calculator will display '2'.

11) Go back to Step (2) and run through the program for another frequency.

12) If all frequencies of interest have been run, enter 0 and press STO 23 and STO 4. Then enter the steady-state part of T_R obtained from Step 8 of PSA-1 and press STO 3. Press E, then press D and calculator will print $T_R(t)$ for every other hour of the day beginning with midnight. If no printer is available, these 12 temperatures are found successively in RCL 48-59.

ACKNOWLEDGMENT

This passive solar building model was developed as part of a project on analytic building calculations at Lawrence Berkeley Laboratory, initiated by Sam Berman of LBL and Robert Richardson of New York University.

We wish to thank Ray Kinoshita for her assistance in performing the calculations in support of the frame wall model used herein. Her comments on the program operating instructions and evaluation of the parameters were also helpful in improving the usability of the program.

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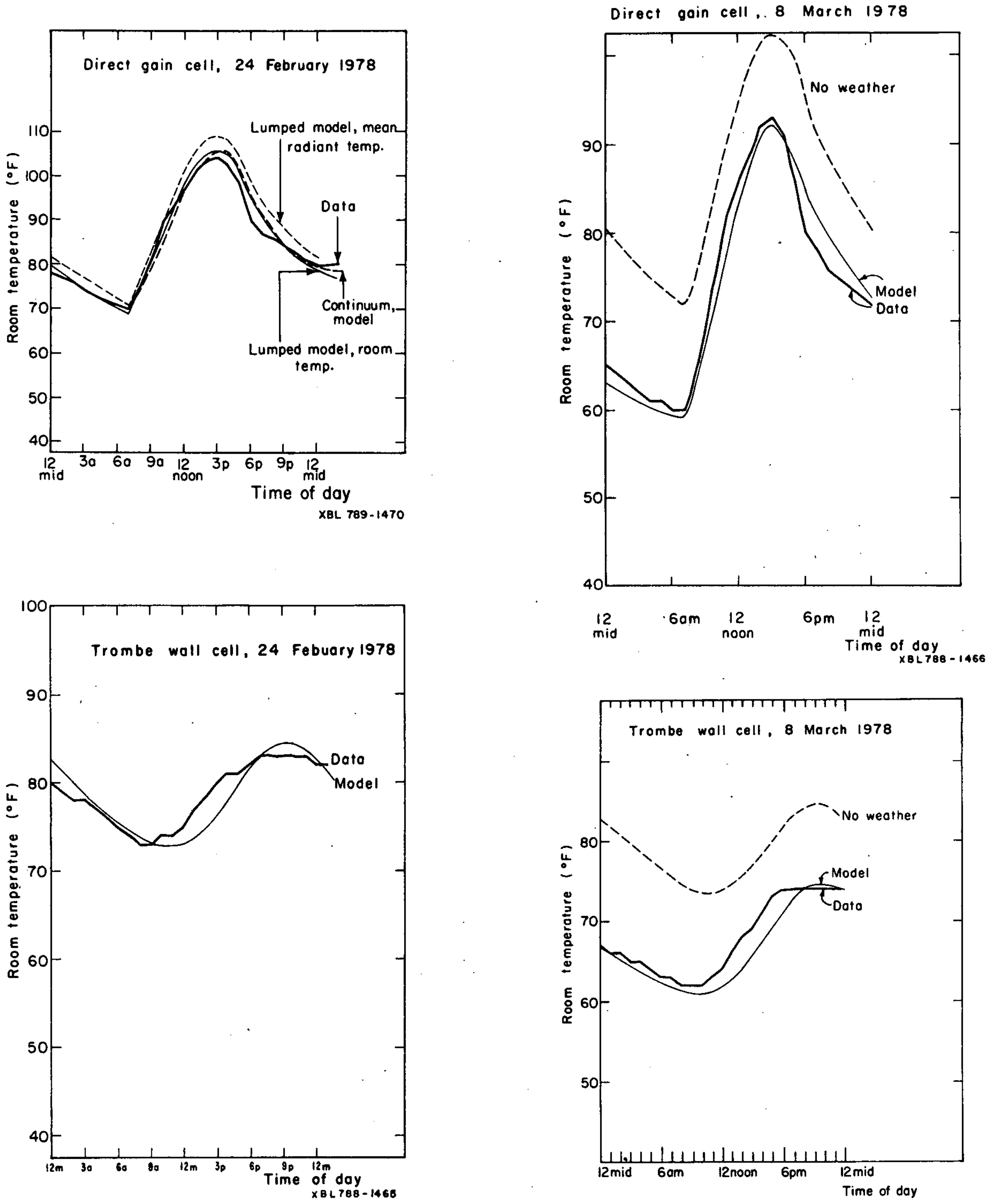


Predicted room temperature and observed data as a function of time of day for 24 February 1978 (left), and 8 March 1978 (right). Top: LASL direct gain cell; bottom: LASL Trombe wall cell. For 8 March, the curve labeled "no weather" assumes that all previous days had the same weather; the curve labeled "model" accounts for the previous two weeks' weather.

APPENDIX A: Listing of the R_{12}, ABC, $T_R(t)$, PSA-1, and PSA-2 Sub-Programs

R_{12}

001	*LBLA	21 11
002	→R	44
003	STOD	35 14
004	R↓	-31
005	STOE	35 15
006	R↓	-31
007	→R	44
008	RCLD	36 14
009	+	-55
010	X⇄Y	-41
011	RCLE	36 15
012	+	-55
013	X⇄Y	-41
014	→P	34
015	RTN	24
016	*LBLB	21 12
017	STOD	35 14
018	R↓	-31
019	STOE	35 15
020	R↓	-31
021	RCLD	36 14
022	x	-35
023	X⇄Y	-41
024	RCLE	36 15
025	+	-55
026	X⇄Y	-41
027	RTN	24
028	*LBL0	21 00
029	STOC	35 13
030	ENT↑	-21
031	e^x	33
032	RCLC	36 13
033	CHS	-22
034	ENT↑	-21
035	e^x	33
036	GSBA	23 11
037	2	02
038	÷	-24
039	STO6	35 06
040	X⇄Y	-41
041	STO7	35 07
042	X⇄Y	-41
043	RCLC	36 13
044	CHS	-22
045	ENT↑	-21
046	e^x	33
047	CHS	-22
048	GSBA	23 11
049	STO8	35 08
050	X⇄Y	-41
051	STO9	35 09
052	RTN	24
053	*LBLC	21 13
054	EEX	-23
055	4	04
056	x	-35
057	INT	16 34
058	1	01
059	0	00
060	x	-35
061	X⇄Y	-41
062	*LBL2	21 02
063	X<0?	16-45
064	GTO1	22 01
065	+	-55
066	RTN	24
067	*LBL1	21 01
068	2	02
069	Pi	16-24
070	x	-35
071	+	-55
072	GTO2	22 02
073	*LBLE	21 15
074	RCLI	36 46
075	X=0?	16-43
076	GTO9	22 09
077	2	02
078	÷	-24
079	1	01
080	-	-45
081	3	03
082	X=Y?	16-33
083	SF2	16 21 02
084	R↓	-31
085	*LBLd	21 16 14
086	RCLA	36 11
087	x	-35
088	RCL2	36 02
089	x	-35
090	RCL1	36 01
091	÷	-24
092	2	02
093	÷	-24
094	√X	54
095	STO0	35 00
096	RCL3	36 03
097	x	-35
098	GSB0	23 00
099	RCL1	36 01
100	RCL0	36 00
101	x	-35
102	ENT↑	-21
103	→P	34
104	STO0	35 00
105	X⇄Y	-41
106	STOC	35 13
107	CHS	-22
108	X⇄Y	-41
109	1/X	52
110	RCL4	36 04
111	x	-35
112	RCL5	36 05
113	x	-35
114	RCLC	36 13
115	RCL0	36 00
116	GSBA	23 11
117	RCL9	36 09
118	RCL8	36 08
119	GSBB	23 12
120	P⇄S	16-51
121	STO8	35 08
122	X⇄Y	-41
123	STO9	35 09
124	P⇄S	16-51
125	0	00
126	RCL4	36 04
127	RCL5	36 05
128	+	-55
129	RCL7	36 07
130	RCL6	36 06
131	GSBB	23 12
132	P⇄S	16-51
133	RCL9	36 09
134	RCL8	36 08
135	P⇄S	16-51
136	GSBA	23 11
137	1/X	52
138	X⇄Y	-41
139	CHS	-22
140	P⇄S	16-51
141	STO9	35 09
142	X⇄Y	-41
143	STO8	35 08
144	P⇄S	16-51
145	RCLC	36 13
146	CHS	-22
147	RCL0	36 00
148	1/X	52
149	RCL5	36 05
150	x	-35

```
151  RCL9      36 09
152  RCL8      36 08
153  GSBB      23 12
154  RCL7      36 07
155  RCL6      36 06
156  GSBA      23 11
157  P⇄S       16-51
158  RCL9      36 09
159  RCL8      36 08
160  GSBB      23 12
161  GSBC      23 13
162  RCL9      36 09
163  RCL8      36 08
164  0            00
165  R↑        16-31
166  STOi      35 45
167  ISZI   16 26 46
168  P⇄S       16-51
169  CLX         -51
170  RCL5      36 05
171  GSBB      23 12
172  GSBC      23 13
173  P⇄S       16-51
174  STOi      35 45
175  ISZI   16 26 46
176  F2?    16 23 02
177  GTOc   22 16 13
178  P⇄S       16-51
179  GTOE      22 15
180  *LBL9     21 09
181  RCL1      36 01
182  RCL3      36 03
183  ÷           -24
184  STO8      35 08
185  1/X          52
186  RCL4      36 04
187  x           -35
188  RCL5      36 05
189  x           -35
190  RCL4      36 04
191  +           -55
192  RCL5      36 05
193  +           -55
194  STOC      35 13
195  1/X          52
196  RCL5      36 05
197  RCL8      36 08
198  ÷           -24
199  1            01
200  +           -55
201  x           -35
202  RCL5      36 05
203  P⇄S       16-51
204  RCLC      36 13
205  ÷           -24
206  X⇄Y         -41
207  0            00
208  X⇄Y         -41
209  GSBC      23 13
210  STOi      35 45
211  ISZI   16 26 46
212  X⇄Y         -41
213  0            00
214  X⇄Y         -41
215  GSBC      23 13
216  STOi      35 45
217  ISZI   16 26 46
218  P⇄S       16-51
219  RCLB      36 12
220  1/X          52
221  GTOd   22 16 14
222  *LBLc  21 16 13
223  WDTA      16-61
224  P⇄S       16-51
```

ABC

```
001  *LBLA     21 11
002  →R           44
003  STOD      35 14
004  R↓          -31
005  STOE      35 15
006  R↓          -31
007  →R           44
008  RCLD      36 14
009  +           -55
010  X⇄Y         -41
011  RCLE      36 15
012  +           -55
013  X⇄Y         -41
014  →P           34
015  RTN          24
016  *LBLC     21 13
017  EEX         -23
018  4            04
019  x           -35
020  *LBL3     21 03
021  INT       16 34
022  1            01
023  0            00
024  x           -35
025  X⇄Y         -41
026  *LBL2     21 02
027  X<0?      16-45
028  GTO1      22 01
029  +           -55
030  RTN          24
031  *LBL1     21 01
032  2            02
033  Pi        16-24
034  x           -35
035  +           -55
036  GTO2      22 02
037  *LBLD     21 14
038  CF2    16 22 02
039  *LBL6     21 06
040  1            01
041  0            00
042  ÷           -24
043  STOE      35 15
044  FRC       16 44
045  1            01
046  0            00
047  x           -35
048  STOD      35 14
049  CLX         -51
050  RCLE      36 15
051  INT       16 34
052  F2?    16 23 02
053  GTO4      22 04
054  EEX         -23
055  CHS         -22
056  4            04
057  x           -35
058  *LBL5     21 05
059  RCLD      36 14
060  X⇄Y         -41
061  RTN          24
062  *LBLc  21 16 13
063  EEX         -23
064  2            02
065  x           -35
066  GTO3      22 03
067  *LBLd  21 16 14
068  SF2    16 21 02
069  GTO6      22 06
070  *LBL4     21 04
071  EEX         -23
072  CHS         -22
073  2            02
074  x           -35
075  GTO5      22 05
076  *LBLE     21 15
077  CF0    16 22 00
078  CLRG      16-53
079  P⇄S       16-51
080  CLRG      16-53
081  P⇄S       16-51
082  *LBLe  21 16 15
083  8            08
084  STOI      35 46
085  R/S          51
086  F0?    16 23 00
087  GTO0      22 00
088  *LBL7     21 07
089  0            00
090  STOI      35 46
091  RCLE      36 15
092  STO7      35 07
```

```
093  *LBLa    21 16 11
094  RCLi        36 45
095  GSBD        23 14
096  RCLA        36 11
097  CHS           -22
098  x             -35
099  0              00
100  ENT↑          -21
101  1              01
102  GSBA        23 11
103  RCLA        36 11
104  x             -35
105  RCLB        36 12
106  x             -35
107  P⇄S         16-51
108  RCLi        36 45
109  GSBd     23 16 14
110  GSBA        23 11
111  GSBc     23 16 13
112  STOi        35 45
113  P⇄S         16-51
114  RCLi        36 45
115  GSBD        23 14
116  RCLC        36 13
117  x             -35
118  RCLA        36 11
119  x             -35
120  ISZI     16 26 46
121  P⇄S         16-51
122  RCLi        36 45
123  GSBD        23 14
124  GSBA        23 11
125  GSBC        23 13
126  STOi        35 45
127  P⇄S         16-51
128  ISZI     16 26 46
129  RCLI        36 46
130  9              09
131  X≤Y?        16-35
132  GTOb     22 16 12
133  GTOa     22 16 11
134  *LBLb    21 16 12
135  RCL3        36 03
136  GSBD        23 14
137  RCLB        36 12
138  x             -35
139  RCLA        36 11
140  x             -35
141  RCL9        36 09
142  GSBd     23 16 14
143  GSBA        23 11
144  GSBc     23 16 13
145  STO9        35 09
146  RCL5        36 05
147  GSBD        23 14
148  RCLB        36 12
149  x             -35
150  RCLA        36 11
151  x             -35
152  RCL7        36 07
153  GSBd     23 16 14
154  GSBA        23 11
155  GSBc     23 16 13
156  STO7        35 07
157  F0?      16 23 00
158  GTOB        22 12
159  STOE        35 15
160  GTOe     22 16 15
161  *LBL0       21 00
162  RCLE        36 15
163  STO7        35 07
164  *LBL9       21 09
165  RCLI        36 46
166  2              02
167  +             -55
168  STOI        35 46
169  0              00
170  RCLA        36 11
171  RCLi        36 45
172  GSBd     23 16 14
173  GSBA        23 11
174  GSBc     23 16 13
175  STOi        35 45
176  ISZI     16 26 46
177  0              00
178  RCLB        36 12
179  RCLi        36 45
180  GSBD        23 14
181  GSBA        23 11
182  GSBC        23 13
183  STOi        35 45
184  RCLI        36 46
185  1              01
186  9              09
187  X=Y?        16-33
188  GTOB        22 12
189  DSZI     16 25 46
190  GTO9        22 09
191  *LBLB       21 12
192  RCL9        36 09
193  GSBd     23 16 14
194  0              00
195  RCLA        36 11
196  GSBA        23 11
197  GSBc     23 16 13
198  STO0        35 00
199  RCL7        36 07
200  GSBd     23 16 14
201  0              00
202  RCLA        36 11
203  GSBA        23 11
204  GSBc     23 16 13
205  STO1        35 01
206  P⇄S         16-51
207  WDTA        16-61
208  RTN            24
209  R/S            51
```

$T_R(t)$

```
001  *LBLB       21 12
002  STOD        35 14
003  R↓            -31
004  STOE        35 15
005  R↓            -31
006  RCLD        36 14
007  x             -35
008  X⇄Y           -41
009  RCLE        36 15
010  +             -55
011  X⇄Y           -41
012  RTN            24
013  *LBLD       21 14
014  CF2      16 22 02
015  *LBL6       21 06
016  1              01
017  0              00
018  ÷             -24
019  STOE        35 15
020  FRC         16 44
021  1              01
022  0              00
023  x             -35
024  STOD        35 14
025  CLX           -51
026  RCLE        36 15
027  INT         16 34
028  F2?      16 23 02
029  GSB4        23 04
030  EEX           -23
031  CHS           -22
032  4              04
033  x             -35
034  RCLD        36 14
035  X⇄Y           -41
036  RTN            24
037  *LBLd    21 16 14
038  SF2      16 21 02
039  GTO6        22 06
040  *LBL4       21 04
041  EEX           -23
042  2              02
043  x             -35
044  RTN            24
045  *LBLE       21 15
046  P⇄S         16-51
047  *LBLa    21 16 11
048  RCLi        36 45
049  GSBd     23 16 14
```

```
050   1/X        52
051   STOA    35 11
052   X⇌Y       -41
053   CHS       -22
054   STOB    35 12
055   ISZI 16 26 46
056   GSB0    23 00
057   RCLi    36 45
058   GSBD    23 14
059   GSBB    23 12
060   P⇌S     16-51
061   RCL8    36 08
062   P⇌S     16-51
063   x         -35
064   RCLB    36 12
065   RCLA    36 11
066   GSBB    23 12
067   DSZI 16 25 46
068   SPC     16-11
069   STOi    35 45
070   ISZI 16 26 46
071   X⇌Y       -41
072   STOi    35 45
073   9          09
074   RCLI    36 46
075   X=Y?    16-33
076   GTO9    22 09
077   3          03
078   X=Y?    16-33
079   GSB8    23 08
080   RCLI    36 46
081   5          05
082   X=Y?    16-33
083   GSB8    23 08
084   ISZI 16 26 46
085   GTOa 22 16 11
086   *LBL8   21 08
087   7          07
088   +         -55
089   STOI    35 46
090   RCLi    36 45
091   GSBd 23 16 14
092   RCLB    36 12
093   RCLA    36 11
094   GSBB    23 12
095   STOi    35 45
096   X⇌Y       -41
097   ISZI 16 26 46
098   RCLi    36 45
099   X⇌Y       -41
100   STOi    35 45
101   ISZI 16 26 46
102   X⇌Y       -41
103   STOi    35 45
104   RCLI    36 46
105   9          09
106   -         -45
107   STOI    35 46
108   RTN        24
109   *LBL0   21 00
110   P⇌S     16-51
111   RCLI    36 46
112   3          03
113   -         -45
114   2          02
115   X>Y?    16-34
116   GTO7    22 07
117   ÷         -24
118   RCLC    36 13
119   x         -35
120   STOD    35 14
121   CHS       -22
122   RCL7    36 07
123   x         -35
124   1          01
125   →R         44
126   1          01
127   +         -55
128   →P         34
129   RCLC    36 13
130   x         -35
131   RCL7    36 07
132   ÷         -24
133   Pi      16-24
134   RCL7    36 07
135   ÷         -24
136   X²         53
137   RCLD    36 14
138   X²         53
139   -         -45
140   ÷         -24
141   P⇌S     16-51
142   RTN        24
143   *LBL7   21 07
144   0          00
145   RCLC    36 13
146   RCL7    36 07
147   x         -35
148   Pi      16-24
149   X²         53
150   ÷         -24
151   P⇌S     16-51
152   RTN        24
153   *LBL9   21 09
154   P⇌S     16-51
155   RCL7    36 07
156   2          02
157   ÷         -24
158   1          01
159   2          02
160   -         -45
161   STOI    35 46
162   P⇌S     16-51
163   *LBLA   21 11
164   P⇌S     16-51
165   RCLI    36 46
166   RCL6    36 06
167   -         -45
168   RCLC    36 13
169   x         -35
170   RCL3    36 03
171   +         -55
172   RCL2    36 02
173   RCL5    36 05
174   x         -35
175   P⇌S     16-51
176   →R         44
177   STO1    35 01
178   RCLI    36 46
179   3          03
180   x         -35
181   RCLC    36 13
182   x         -35
183   RCL9    36 09
184   +         -55
185   RCL8    36 08
186   →R         44
187   ST+1 35-55 01
188   RCLI    36 46
189   2          02
190   x         -35
191   RCLC    36 13
192   x         -35
193   RCL7    36 07
194   +         -55
195   RCL6    36 06
196   →R         44
197   ST+1 35-55 01
198   RCLI    36 46
199   RCLC    36 13
200   x         -35
201   RCL5    36 05
202   +         -55
203   RCL4    36 04
204   →R         44
205   ST+1 35-55 01
206   RCL0    36 00
207   ST+1 35-55 01
208   P⇌S     16-51
209   RCL9    36 09
210   P⇌S     16-51
211   ST+1 35-55 01
212   RCLI    36 46
213   PSE     16 51
214   RCL1    36 01
215   PRTX      -14
216   ISZI 16 26 46
217   SPC     16-11
218   RCLI    36 46
219   2          02
220   4          04
221   X≤Y?    16-35
222   RTN        24
223   GTOA    22 11
224   R/S        51
```

PSA-1

```
000  76  LBL
001  43  RCL
002  22  INV
003  97  DSZ
004  09   09
005  57  ENG
006  03   3
007  03   3
008  42  STD
009  08   08
010  85   +
011  53   (
012  07   7
013  42  STD
014  07   07
015  65   ×
016  43  RCL
017  09   09
018  54   )
019  95   =
020  42  STD
021  06   06
022  76  LBL
023  32  X↕T
024  73  RC*
025  08   08
026  42  STD
027  05   05
028  73  RC*
029  06   06
030  72  ST*
031  08   08
032  43  RCL
033  05   05
034  72  ST*
035  06   06
036  69  OP
037  38   38
038  69  OP
039  36   36
040  97  DSZ
041  07   07
042  32  X↕T
043  76  LBL
044  57  ENG
045  92  RTN
046  76  LBL
047  15   E
048  42  STD
049  00   00
050  65   ×
051  07   7
052  95   =
053  42  STD
054  01   01
055  02   2
056  00   0
057  44  SUM
058  01   01
059  91  R/S
060  76  LBL
061  11   A
062  72  ST*
063  01   01
064  69  OP
065  21   21
066  91  R/S
067  76  LBL
068  12   B
069  44  SUM
070  01   01
071  91  R/S
072  72  ST*
073  01   01
074  69  OP
075  21   21
076  91  R/S
077  76  LBL
078  13   C
079  43  RCL
080  00   00
081  42  STD
082  09   09
083  43  RCL
084  21   21
085  42  STD
086  10   10
087  43  RCL
088  25   25
089  42  STD
090  11   11
091  76  LBL
092  59  INT
093  71  SBR
094  43  RCL
095  43  RCL
096  28   28
097  85   +
098  43  RCL
099  29   29
100  35  1/X
101  85   +
102  53   (
103  43  RCL
104  27   27
105  55   ÷
106  43  RCL
107  30   30
108  54   )
109  95   =
110  35  1/X
111  42  STD
112  08   08
113  65   ×
114  43  RCL
115  33   33
116  95   =
117  44  SUM
118  10   10
119  01   1
120  75   -
121  43  RCL
122  08   08
123  55   ÷
124  43  RCL
125  29   29
126  95   =
127  65   ×
128  43  RCL
129  32   32
130  95   =
131  44  SUM
132  11   11
133  69  OP
134  29   29
135  71  SBR
136  43  RCL
137  69  OP
138  29   29
139  97  DSZ
140  09   09
141  59  INT
142  03   3
143  07   7
144  02   2
145  03   3
146  69  OP
147  04   04
148  43  RCL
149  24   24
150  55   ÷
151  43  RCL
152  10   10
153  95   =
154  42  STD
155  12   12
156  42  STD
157  14   14
158  69  OP
159  06   06
160  91  R/S
161  03   3
162  07   7
163  01   1
164  03   3
```

```
165  69  OP
166  04  04
167  43  RCL
168  22  22
169  44  SUM
170  14  14
171  69  OP
172  06  06
173  91  R/S
174  42  STO
175  08  08
176  02  2
177  03  3
178  69  OP
179  04  04
180  43  RCL
181  08  08
182  69  OP
183  06  06
184  35  1/X
185  65  ×
186  89  π
187  95  =
188  42  STO
189  23  23
190  65  ×
191  01  1
192  02  2
193  95  =
194  35  1/X
195  65  ×
196  43  RCL
197  20  20
198  65  ×
199  43  RCL
200  11  11
201  55  ÷
202  43  RCL
203  10  10
204  95  =
205  42  STO
206  13  13
207  44  SUM
208  14  14
209  03  3
210  07  7
211  03  3
212  06  6
213  69  OP
214  04  04
215  43  RCL
216  13  13
217  69  OP
218  06  06
219  91  R/S
220  03  3
221  07  7
222  03  3
223  05  5
224  00  0
225  01  1
226  69  OP
227  04  04
228  43  RCL
229  14  14
230  69  OP
231  06  06
232  98  ADV
233  91  R/S
234  76  LBL
235  14  D
236  42  STO
237  05  05
238  65  ×
239  89  π
240  55  ÷
241  01  1
242  02  2
243  95  =
244  42  STO
245  02  02
246  33  X²
247  94  +/-
248  85  +
249  43  RCL
250  23  23
251  33  X²
252  95  =
253  65  ×
254  01  1
255  02  2
256  55  ÷
257  43  RCL
258  23  23
259  95  =
260  35  1/X
261  42  STO
262  03  03
263  01  1
264  06  6
265  03  3
266  01  1
267  69  OP
268  04  04
269  43  RCL
270  05  05
271  69  OP
272  06  06
273  43  RCL
274  08  08
275  94  +/-
276  49  PRD
277  02  02
278  00  0
279  42  STO
280  01  01
281  42  STO
282  04  04
283  36  PGM
284  05  05
285  17  B'
286  69  OP
287  21  21
288  36  PGM
289  04  04
290  13  C
291  03  3
292  05  5
293  69  OP
294  04  04
295  43  RCL
296  01  01
297  69  OP
298  06  06
299  32  X⇌T
300  02  2
301  04  4
302  69  OP
303  04  04
304  43  RCL
305  02  02
306  69  OP
307  06  06
308  22  INV
309  37  P/R
310  99  PRT
311  32  X⇌T
312  99  PRT
313  98  ADV
314  32  X⇌T
315  37  P/R
316  91  R/S
317  76  LBL
318  19  D'
319  01  1
320  06  6
321  00  0
322  01  1
323  69  OP
324  04  04
325  43  RCL
326  23  23
327  65  ×
328  01  1
329  02  2
```

```
330   95   =
331   35  1/X
332   69  OP
333   06   06
334   98  ADV
335   91  R/S
336   35  1/X
337   99  PRT
338   98  ADV
339   91  R/S
```

PSA-2

```
000   76  LBL
001   43  RCL
002   22  INV
003   97  DSZ
004   09   09
005   57  ENG
006   03   3
007   03   3
008   42  STO
009   08   08
010   85   +
011   53   (
012   07   7
013   42  STO
014   07   07
015   65   X
016   43  RCL
017   09   09
018   54   )
019   95   =
020   42  STO
021   06   06
022   76  LBL
023   32  X:T
024   73  RC*
025   08   08
026   42  STO
027   05   05
028   73  RC*
029   06   06
030   72  ST*
031   08   08
032   43  RCL
033   05   05
034   72  ST*
035   06   06
036   69  OP
037   38   38
038   69  OP
039   36   36
040   97  DSZ
041   07   07
042   32  X:T
043   76  LBL
044   57  ENG
045   92  RTN
046   76  LBL
047   65   X
048   00   0
049   42  STO
050   01   01
051   36  PGM
052   05   05
053   17  B'
054   36  PGM
055   04   04
056   13   C
057   92  RTN
058   76  LBL
059   39  COS
060   42  STO
061   02   02
062   94  +/-
063   42  STO
064   01   01
065   36  PGM
066   06   06
067   13   C
068   42  STO
069   10   10
070   42  STO
071   12   12
072   32  X:T
073   42  STO
074   11   11
075   42  STO
076   13   13
077   43  RCL
078   06   06
079   42  STO
080   02   02
081   94  +/-
082   42  STO
083   01   01
084   36  PGM
085   06   06
086   12   B
087   94  +/-
088   42  STO
089   02   02
090   32  X:T
091   42  STO
092   01   01
093   92  RTN
094   76  LBL
095   11   A
096   35  1/X
097   65   X
098   89   π
099   55   ÷
100   01   1
101   02   2
102   95   =
103   42  STO
104   23   23
105   43  RCL
106   21   21
107   42  STO
108   14   14
109   42  STO
110   18   18
111   43  RCL
112   25   25
113   42  STO
114   16   16
115   00   0
116   42  STO
117   15   15
118   42  STO
119   17   17
120   42  STO
121   19   19
122   43  RCL
123   00   00
124   42  STO
125   09   09
126   76  LBL
127   23  LNX
128   71  SBR
129   43  RCL
130   43  RCL
131   31   31
132   65   X
133   43  RCL
134   23   23
135   55   ÷
136   43  RCL
137   30   30
138   55   ÷
139   02   2
140   95   =
141   34  √X
142   42  STO
143   08   08
144   65   X
145   43  RCL
146   28   28
147   65   X
148   43  RCL
149   30   30
150   65   X
```

```
151  02  2
152  95  =
153  35  1/X
154  42  STO
155  07  07
156  43  RCL
157  08  08
158  65  ×
159  43  RCL
160  27  27
161  95  =
162  42  STO
163  06  06
164  71  SBR
165  39  COS
166  43  RCL
167  07  07
168  42  STO
169  03  03
170  94  +/-
171  42  STO
172  04  04
173  36  PGM
174  04  04
175  13  C
176  44  SUM
177  12  12
178  32  X:T
179  44  SUM
180  13  13
181  43  RCL
182  28  28
183  65  ×
184  43  RCL
185  30  30
186  33  X²
187  65  ×
188  43  RCL
189  08  08
190  33  X²
191  65  ×
192  02  2
193  95  =
194  42  STO
195  04  04
196  43  RCL
197  29  29
198  42  STO
199  03  03
200  36  PGM
201  04  04
202  13  C
203  43  RCL
204  28  28
205  35  1/X
206  44  SUM
207  03  03
208  43  RCL
209  03  03
210  42  STO
211  04  04
212  43  RCL
213  10  10
214  49  PRD
215  03  03
216  43  RCL
217  11  11
218  49  PRD
219  04  04
220  36  PGM
221  04  04
222  12  B
223  36  PGM
224  05  05
225  15  E
226  42  STO
227  10  10
228  32  X:T
229  42  STO
230  11  11
231  43  RCL
232  28  28
233  35  1/X
234  49  PRD
235  10  10
236  49  PRD
237  11  11
238  43  RCL
239  12  12
240  42  STO
241  03  03
242  43  RCL
243  13  13
244  42  STO
245  04  04
246  36  PGM
247  04  04
248  13  C
249  42  STO
250  12  12
251  32  X:T
252  42  STO
253  13  13
254  91  R/S
255  43  RCL
256  29  29
257  65  ×
258  43  RCL
259  32  32
260  95  =
261  49  PRD
262  12  12
263  49  PRD
264  13  13
265  43  RCL
266  12  12
267  44  SUM
268  16  16
269  43  RCL
270  13  13
271  44  SUM
272  17  17
273  43  RCL
274  29  29
275  94  +/-
276  49  PRD
277  01  01
278  49  PRD
279  02  02
280  69  OP
281  21  21
282  65  ×
283  43  RCL
284  33  33
285  94  +/-
286  95  =
287  49  PRD
288  01  01
289  49  PRD
290  02  02
291  49  PRD
292  10  10
293  49  PRD
294  11  11
295  43  RCL
296  01  01
297  44  SUM
298  14  14
299  43  RCL
300  02  02
301  44  SUM
302  15  15
303  43  RCL
304  10  10
305  44  SUM
306  18  18
307  43  RCL
308  11  11
309  44  SUM
310  19  19
311  69  OP
312  29  29
313  71  SBR
314  43  RCL
315  69  OP
```

```
316   29   29
317   97  DSZ
318   09   09
319   23  LNX
320   91  R/S
321   76  LBL
322   55   ÷
323   43  RCL
324   14   14
325   42  STO
326   03   03
327   43  RCL
328   15   15
329   42  STO
330   04   04
331   36  PGM
332   04   04
333   18  C'
334   42  STO
335   03   03
336   32  X:T
337   42  STO
338   04   04
339   92  RTN
340   76  LBL
341   15   E
342   42  STO
343   05   05
344   01   1
345   02   2
346   42  STO
347   09   09
348   04   4
349   08   8
350   42  STO
351   08   08
352   76  LBL
353   38  SIN
354   43  RCL
355   05   05
356   42  STO
357   02   02
358   43  RCL
359   23   23
360   49  PRD
361   02   02
362   71  SBR
363   65   ×
364   74  SM*
365   08   08
366   02   2
367   44  SUM
368   05   05
369   69  OP
370   28   28
371   97  DSZ
372   09   09
373   38  SIN
374   91  R/S
375   76  LBL
376   99  PRT
377   43  RCL
378   23   23
379   55   ÷
380   02   2
381   55   ÷
382   89   π
383   95   =
384   35  1/X
385   42  STO
386   07   07
387   76  LBL
388   90  LST
389   43  RCL
390   23   23
391   65   ×
392   43  RCL
393   07   07
394   66  PAU
395   99  PRT
396   95   =
397   42  STO
398   02   02
399   71  SBR
400   65   ×
401   66  PAU
402   66  PAU
403   66  PAU
404   66  PAU
405   99  PRT
406   97  DSZ
407   07   07
408   90  LST
409   92  RTN
410   76  LBL
411   12   B
412   42  STO
413   01   01
414   42  STO
415   02   02
416   43  RCL
417   18   18
418   49  PRD
419   01   01
420   43  RCL
421   19   19
422   49  PRD
423   02   02
424   71  SBR
425   55   ÷
426   91  R/S
427   71  SBR
428   99  PRT
429   91  R/S
430   76  LBL
431   13   C
432   42  STO
433   01   01
434   91  R/S
435   42  STO
436   02   02
437   91  R/S
438   49  PRD
439   01   01
440   49  PRD
441   02   02
442   43  RCL
443   16   16
444   42  STO
445   03   03
446   43  RCL
447   17   17
448   42  STO
449   04   04
450   36  PGM
451   04   04
452   13   C
453   71  SBR
454   55   ÷
455   91  R/S
456   71  SBR
457   99  PRT
458   91  R/S
459   76  LBL
460   14   D
461   98  ADV
462   04   4
463   08   8
464   22  INV
465   90  LST
466   91  R/S
```

APPENDIX B: Theory

This section describes the mathematical model which is the basis for the programs. The model looks at the response of interior (that is, room-side) surfaces of building elements, such as envelope and partition walls, on which solar heat may be absorbed. An equation is developed to describe the response of material surface temperatures to weather variables in terms of Fourier materials response functions R_1 and R_2, which are evaluated in the programs.

Next, the room temperature response is computed from the surface temperature results. Room temperature can be given as a function of weather variables in terms of three Building Response Functions, called A, B, and C. These functions are also evaluated in the programs.

The final results for room temperature are derived from the weather description and the Building Response Functions, and hourly temperature is displayed or printed by the programs.

We now proceed to the theory.

Solar energy entering a direct-gain building passes through the room air until it reaches a surface (e.g., a floor). When it strikes the surface, some fraction is absorbed and the rest is reflected. The reflected component eventually is absorbed on some surface. The fraction of sunlight which is absorbed a given surface depends on the sun angles and room geometry in a very complex way. We assume that the internal solar radiation balance is already known, either from direct measurement or through simulation; and that the amount of sunlight absorbed on a surface "j" is given by

α_j S, where S is the total amount of sunlight entering the building (in watts or Btu hr^{-1}).

For each surface "j", the surface temperature, T_{sj}, can be determined by a surface heat balance, which is expressed as:

$$h_j A_j (T_{sj} - T_R) - A_j K_j \left. \frac{\partial T_j(x,t)}{\partial x} \right|_{x=o} = \alpha_j S \qquad \text{(B1)}$$

where

h_j is the combined radiation/convection film heat transfer coefficient for the jth surface (watts m^{-2} $^oC^{-1}$ or Btu hr^{-1} ft^{-2} $^oF^{-1}$),

A_j is the area of the surface (m^2 or ft^2),

T_R is the room temperature, (°C or °F),

$T_j(x,t)$ is the temperature distribution within the j^{th} material, (^oC or ^oF),

K_j is the conductivity of the j^{th} material (watts m^{-1} $^oC^{-1}$ or Btu hr^{-1} $^oF^{-1}$ ft^{-1} ,

α_j is the fraction of sunlight absorbed on the j^{th} surface,

x is the distance into the material (m or ft), and
S is the solar gain transmitted through all the windows (watts or Btu/hr).

This equation sets heat losses from the surface (left-hand side of (B1)) equal to heat gains (right-hand side). It assumes that the surface transfers heat directly to the room air, rather than being in radiative contact with other surfaces, which results in a substantial simplification of the computation effort.[B2]

Heat-flows within a material satisfy the diffusion equation:

$$K_j \frac{\partial^2 T_j(x,t)}{\partial x^2} = (\rho c)_j \frac{\partial T_j(x,t)}{\partial t} \qquad \text{(B2)}$$

where $(\rho c)_j$ is the heat capacity per unit volume of the j^{th} material (watt-hr $^oC^{-1}m^{-3}$ or Btu $^oF^{-1}$ ft^{-3}).[B3]

Equations (B1) and (B2) describe heat-flows at the inside surface of a material and in its interior; at the outside surface we assume that the material is coupled to the ambient air (at temperature T_A) by a pure resistance which can be described by a heat transfer coefficient U_r. (For an uninsulated material this coefficient is just equal to an exterior surface film coefficient).

This description allows the solution for surface temperatures in terms of the driving forces of solar gain and ambient temperature. This solution can be written in simple form if we look at the amplitudes of temperature (and solar) fluctuations at a steady harmonic frequency. The result can be expressed as:

$$T_{sj} = (h_j T_R + \alpha_j \ S/A_j) \ R_{1j} + T_A R_{2j} \qquad \text{(B3)}$$

where R_{1j} and R_{2j} are frequency-dependent response functions whose forms are given in Table B1. These response functions give all the information needed to describe the thermal behavior of the material and its surface. They are evaluated and can be displayed in the programs.

Response functions are computed for all surfaces of materials with significant thermal mass. They would be needed, for example, for surfaces of masonry materials, and also for surfaces of wood or drywall if the house is relatively lightweight. For surfaces of materials with little thermal mass, such as upholstery furniture, thin wood, carpet, insulation, etc., no computation of response functions is necessary. Solar absorption on these surfaces is accounted for by the parameter α_R, which appears below in Eq. (B4), and represents the portion of solar energy absorbed on light surfaces. If the light weight surface is an envelope wall, the U-value of the wall is computed by conventional methods. The sum of U-value times areas for all thermally light elements, including windows, is added to the heat loss rate due to infiltration, given by the heat capacity of air (0.018 Btu/^{o}F-ft^3 at sea level) times the volume of the building (in ft^3) times the air change rate (in air changes per hour), to produce a single term, called $\hat{U}_q$, which describes all quick heat transfer. Heat losses through massive elements are already taken into account in the R_2 functions.

In most cases, only a few sets of response functions need be evaluated. For example, if both walls and floor are made of masonry with similar thermal properties, they can be combined into a single surface. No distinction need be made between directly solar-illuminated materials and those in the shade -- all that is required is that the total solar absorption on the surface be correctly specified (as a fraction of total solar heat gain S).

We combine the surface temperature results into an expression for room temperature using a heat balance for the room air. This is given by:

$$\sum_{j=1}^{N} \hat{h}_j \left(T_R - T_{sj}\right) + \hat{U}_q \left(T_R - T_A\right) = H + \alpha_R S \qquad \text{(B4)}$$

where

$\hat{h}_j = h_j A_j$,

H is the heater output,

α_R is the fraction of sunlight absorbed directly into the room air or on the surfaces of light-weight objects (e.g., upholstery),

$\hat{U}_q$ is the quick heat transfer coefficient, the sum of U values time areas for all pure conductances (e.g., windows) plus the loss rate due to infiltration.

This heat balance says that heat losses from the room air to material surfaces plus losses directly to the outside air are equal to heat gains from the heater or from solar absorption on light material surfaces (which conduct immediately into the room air).

We can use Equations (B4) and (B3) to derive the room temperature; at any frequency, the amplitude of room temperature is given by:

$$T_R \cdot A(\omega) = S \cdot B(\omega) + T_A \cdot C(\omega) + H \qquad \text{(B5)}$$

where

$$A(\omega) = \sum_{j=1}^{N} \hat{h}_j \, (1-h_j \, R_{1j}) + \hat{U}_q \quad , \tag{B6a}$$

$$B(\omega) = \sum_{j=1}^{N} \alpha_j \, h_j \, R_{1j} + \alpha_R \ , \quad \text{and} \tag{B6b}$$

$$C(\omega) = \sum_{j=1}^{N} \hat{h}_j \, R_{2j} + \hat{U}_q \ . \tag{B6c}$$

For the case of Trombe wall buildings, an additional term is included in each of the three building response functions; this term is not presently computed in the programs.

The building response functions are combined with weather data to give an expression for room temperature at each hour. Weather data are expressed in idealized (approximate) form, as simple sine waves. Ambient temperature is given by

$$T_A(t) = \bar{T}_A + |\Delta T_A| \cos\,(\omega_o(t - t_\phi)) = \bar{T}_A + \underset{\sim}{\Delta T_A} \, e^{i\omega_o t} \tag{B7}$$

where

$\bar{T}_A$ is the average temperature for the day,

$|\Delta T_A|$ is the amplitude of diurnal temperature variation,

t_ϕ is the number of hours from sunrise until the hour at which temperature reaches its maximum,

$\underset{\sim}{\Delta T_A}$ is the complex number given by $|\Delta T_A| \, e^{-i\omega_o t_\phi}$, and

$\omega_o = 2\pi/24$ hrs.

Solar heat gain is given by a half-sine wave corresponding to a period of t_d hours of sunlight per day.

$$S(t) = \begin{cases} |S_1| \sin \omega_1 t & \text{daytime} \\ 0 & \text{night} \end{cases} \tag{B8}$$

where

$$\omega_1 = \pi/t_d \quad .$$

To perform the calculations, S(t) is Fourier analyzed into components at frequencies of zero, one, two, and three cycles per day. Further terms are unnecessary for two reasons: their size is smaller than the first few terms, and the building response functions reduce their effect on room temperature. This Fourier analysis allows us to express S(t) as:

$$S(t) = |S_1| \sum_{n=0}^{3} d_n \, e^{in\omega_0 t} \tag{B9}$$

where $\omega_0 = 2\pi/24$ hours

and t is measured beginning at sunrise.

This expression requires that

$$d_n = \frac{\omega_0 t_d}{\pi^2} \qquad n=0$$

$$\frac{\omega_0}{t_d} \; \frac{1 + e^{-in\omega_0 t_d}}{\left(\frac{\pi}{t_d}\right)^2 - (n\omega_0)^2} \qquad n \neq 0 \quad . \tag{B10}$$

The programs calculate d_n and evaluate T_R using the equation

$$T_R(t) = |S_1| \sum_{n=0}^{3} \frac{B(n\omega_o)}{A(n\omega_o)} d_n e^{in\omega_o t} + \bar{T}_A + \Delta \underset{\sim}{T}_A \frac{C(\omega_o)}{A(\omega_o)} e^{i\omega_o t} . \tag{B11}$$

If a heater is present, then T_R is increased by the constant H/A(0) where H is the heater output and A(0) turns out to equal the conventional design heat loss of the building per degree.

Notes and References

B1. Simulating radiant energy interchange in buildings in very complex. One computer program which performs this calculation is G.P. Mitalas and D. G. Stephenson, "Fortran IV Programs to Calculate Radiant Energy Interchange Factors." Computer Program No. 25 of the Division of Building Research, National Research Council of Canada, Ottawa, Canada, 1966.

B2. D. B. Goldstein. Some Analytic Models of Passive Solar Building Performance. Lawrence Berkeley Laboratory, LBL-7811, November, 1978, and Garland Press, New York City, 1979. See Section 2.2.5 and Appendix 2.4.

B3. If the material consists of several layers, a separate diffusion equation is needed for each layer; but in practice, all layers beyond the first (inside) layer can usually be modeled as pure resistances, as is done in the programs.

Table B1. Equations for response functions.

<u>Materials Response Functions R_1 and R_2:</u>

$$R_1(\omega) = \frac{\cosh kd + \frac{1}{RKk} \sinh kd}{(h+\frac{1}{R}) \cosh kd + (Kk + \frac{h}{RKk}) \sinh kd}$$

$$R_2(\omega) = \frac{\frac{1}{R}}{(h+\frac{1}{R}) \cosh kd + (Kk + \frac{h}{RKk}) \sinh kd}$$

where

K is the thermal conductivity of the material,

d is the thickness of the material,

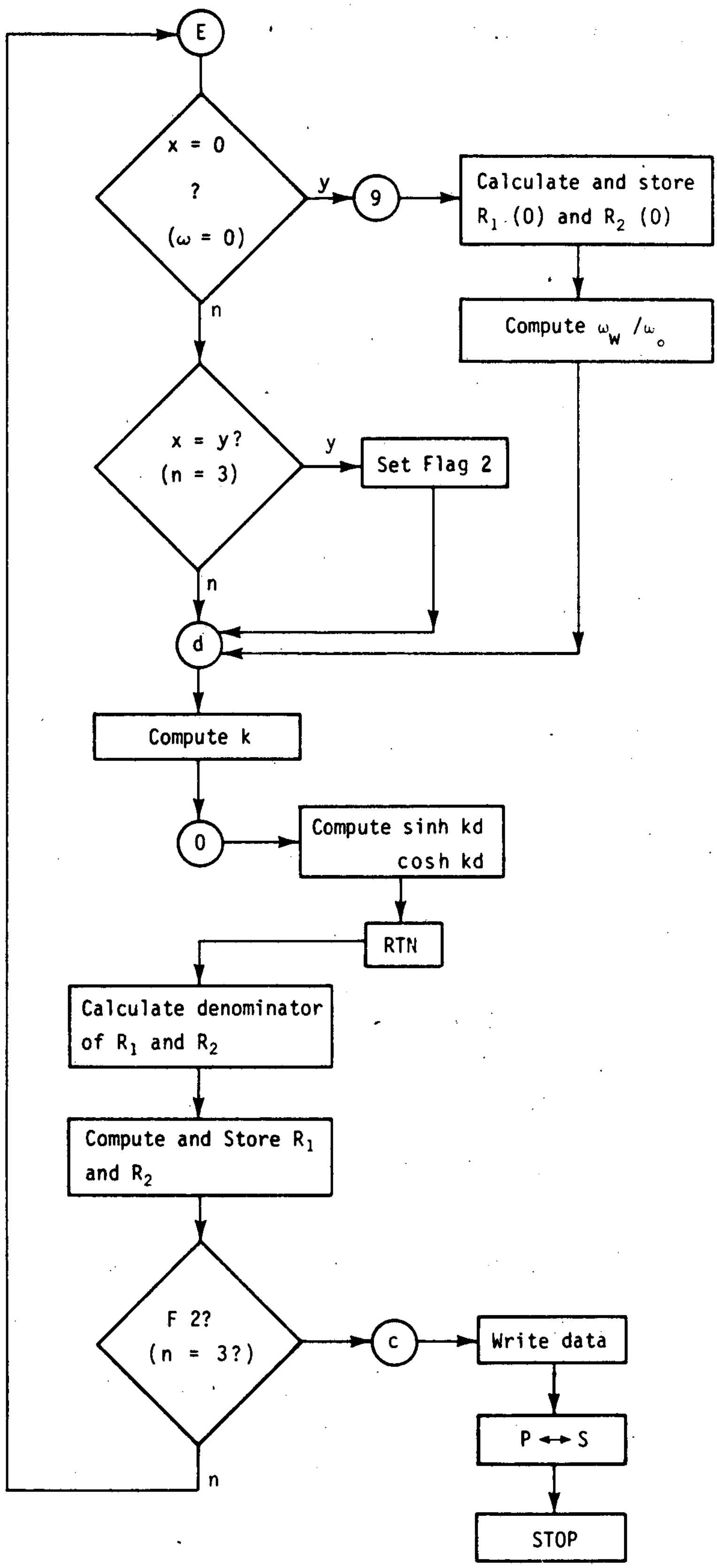
R is the thermal resistance of the exterior insulating layer, $(R = U_r^{-1})$,

$k = \sqrt{i\omega\rho c/K}$ with ρc = the volumetric heat capacity of the material, and

h is the film heat transfer coefficient for the surface.

APPENDIX C: Detailed Description of the Programs

R_{12} Program



XBL 7910-4517

R_{12} Sub-Program

Calculates $R_1(\omega)$ and $R_2(\omega)$ for a surface, at $\omega = 0$, ω_w, ω_o, $2\omega_o$, $3\omega_o$

Inputs:	STO	1	K
	STO	2	ρc
	STO	3	d
	STO	4	h
	STO	5	U_r
	STO	A	ω_o
	STO	B	ω_o/ω_w (or 1; can't be zero)
	STO	I	0

Outputs: (encoded on data card)

STO	0	$R_1(0)$
STO	1	$R_2(0)$
STO	2	$R_1(\omega_w)$
STO	3	$R_2(\omega_w)$
STO	4	$R_1(\omega_o)$
STO	5	$R_2(\omega_o)$
STO	6	$R_1(2\omega_o)$
STO	7	$R_2(2\omega_o)$
STO	8	$R_1(3\omega_o)$
STO	9	$R_2(3\omega_o)$

Outputs are decoded with subroutine D of ABC or T_R programs. Note that outputs are calculated in registers 10-19 and switched to the primary registers for readout. Note also that $R_2(2\omega_o)$ and $R_2(3\omega_o)$ will not be used in subsequent calculations.

R_{12} Sub-Program: Subroutines

A- Complex addition: $R_1 e^{i\theta_1} + R_2 e^{i\theta_2}$

Requires STO D and STO E

Input: x R_1

y θ_1

z R_2

t θ_2

Output: complex sum (polar) R in x, θ in y.

B- Complex multiplication: $R_1 e^{i\theta_1} \times R_2 e^{i\theta_2}$

input, output, and storage requirements like A.

0- Cosh and sinh: Calculates cosh r (1+i) and sinh r (1+i) for r real. Requires STO C, D, E; calls subroutine A

Input: x r

Output: STO 6-7 cosh r (1+i) (polar)

STO 8-9 sinh r (1+i) (polar).

C- Complex encode: Concatenates two components (R and θ) of a complex number into one number for storage. Decoding is accurate to 5-place accuracy; can be done with subroutine D of ABC and T_R programs. Valid for R $\tilde{<}$ 10 and $\theta <$ 10 radians.

Loses one significant figure for R $\gtrless 10^{\pm 1}$; two for R $\gtrless 10^{\pm 2}$, etc.

Accuracy is lost from θ for large R and from R for small R.

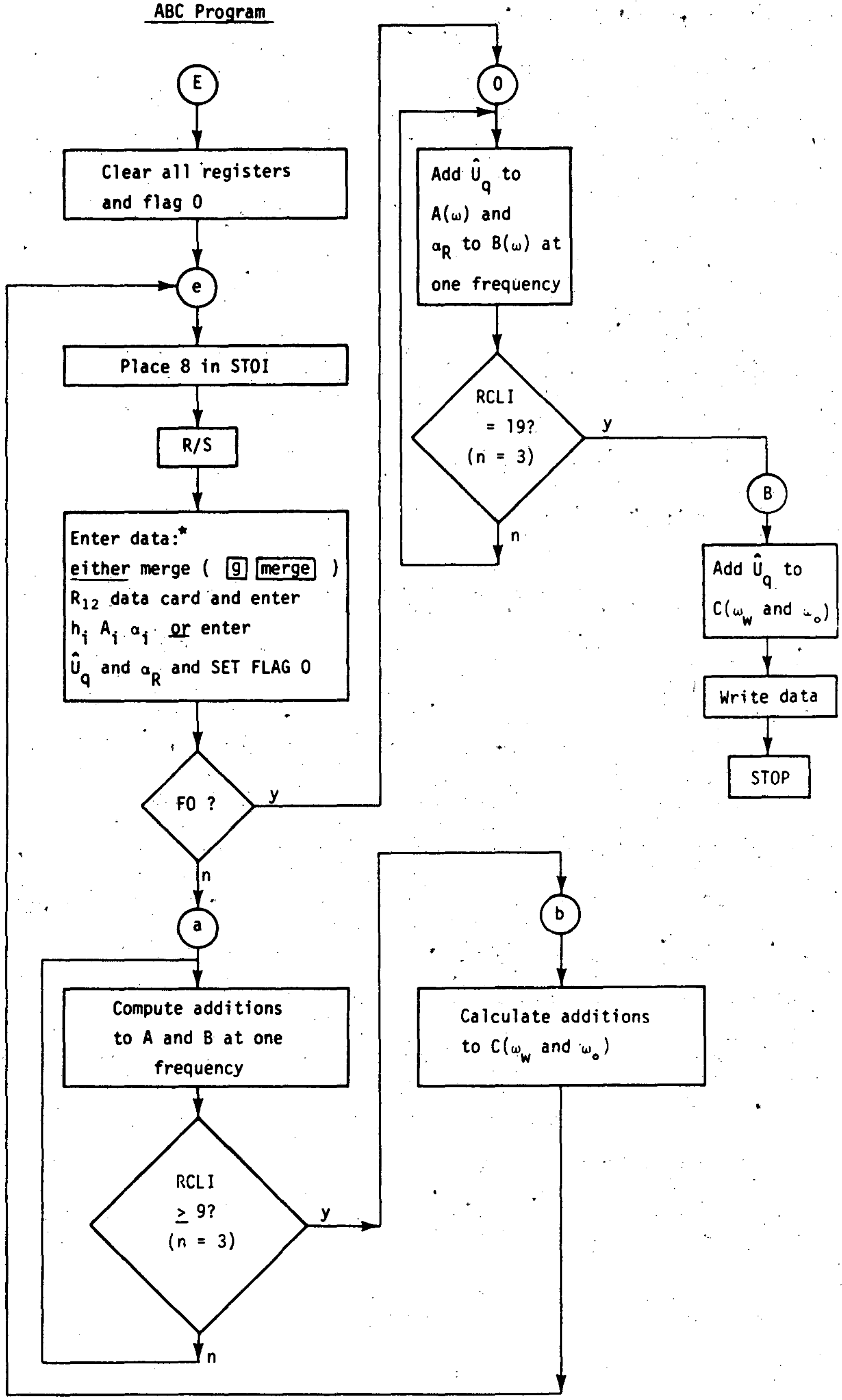
requires LBL 1 and LBL 2

Input: x - R

y - θ

Output: x - encoded number.

ABC Program



* - performed by user

XBL 7910-4515

ABC Sub-Program

Calculates $A(0,\omega_w,\omega_o,2\omega_o,3\omega_o)$, $B(0,\omega_w,\omega_o,2\omega_o, 3\omega_o)$, and $C(\omega_w,\omega_o)$ given R_1 and R_2.

Inputs: R_{12} data card and h_i A_i α_i for each surface

Data card fills STO 0 - STO 8

STO A - h_i

STO B - A_i

STO C - α_i

Outputs: (encoded on data card)

STO 0	$A(0)$
STO 1	$B(0)$
STO 2	$A(\omega_w)$
STO 3	$B(\omega_w)$
STO 4	$A(\omega_o)$
STO 5	$B(\omega_o)$
STO 6	$A(2\omega_o)$
STO 7	$B(2\omega_o)$
STO 8	$A(3\omega_o)$
STO 9	$B(3\omega_o)$
STO 10	$C(\omega_w)$
STO 11	$C(\omega_o)$

A's and C's are decoded with 'd' subroutine; B's are decoded with 'D'.

Note that program reverses $p \longleftrightarrow s$ before output is finished, so

$A(0) \rightarrow B(3\omega_o)$ are accumulated in STO 10-19.

$C(\omega_w)$ is accumulated in STO 9.

$C(\omega_0)$ is accumulated in STO 7, but transferred to STO E during data or card entry.

All storage registers are used.

<u>ABC Sub- Program: Subroutines</u>

A- Complex addition: $R_1 e^{i\theta_1} + R_2 e^{i\theta_2}$

requires STO D and STO E

Input: x R_1

y θ_1

z R_2

t θ_2

Output: complex sum (polar form in x-y)

C- Complex encode:

Concatenates two components (R and θ) of a complex number into one number to 5-digit accuracy.

Good for $R \lesssim 10$ and $\theta < 10$ radians.

Loses one significant figure to $R \gtrless 10^{\pm 1}$, two for $R \gtrless 10^{\pm 2}$, etc.

Accuracy is lost from θ for large R and from R for small R.

Requires LBL 1, LBL 2, and LBL 3.

Input: x - R

y - θ

Output: encoded number - x

C- complex encode

like C only for $R \lesssim 1000$ and $\theta < 10$.

D- Complex decode - inverse of C.

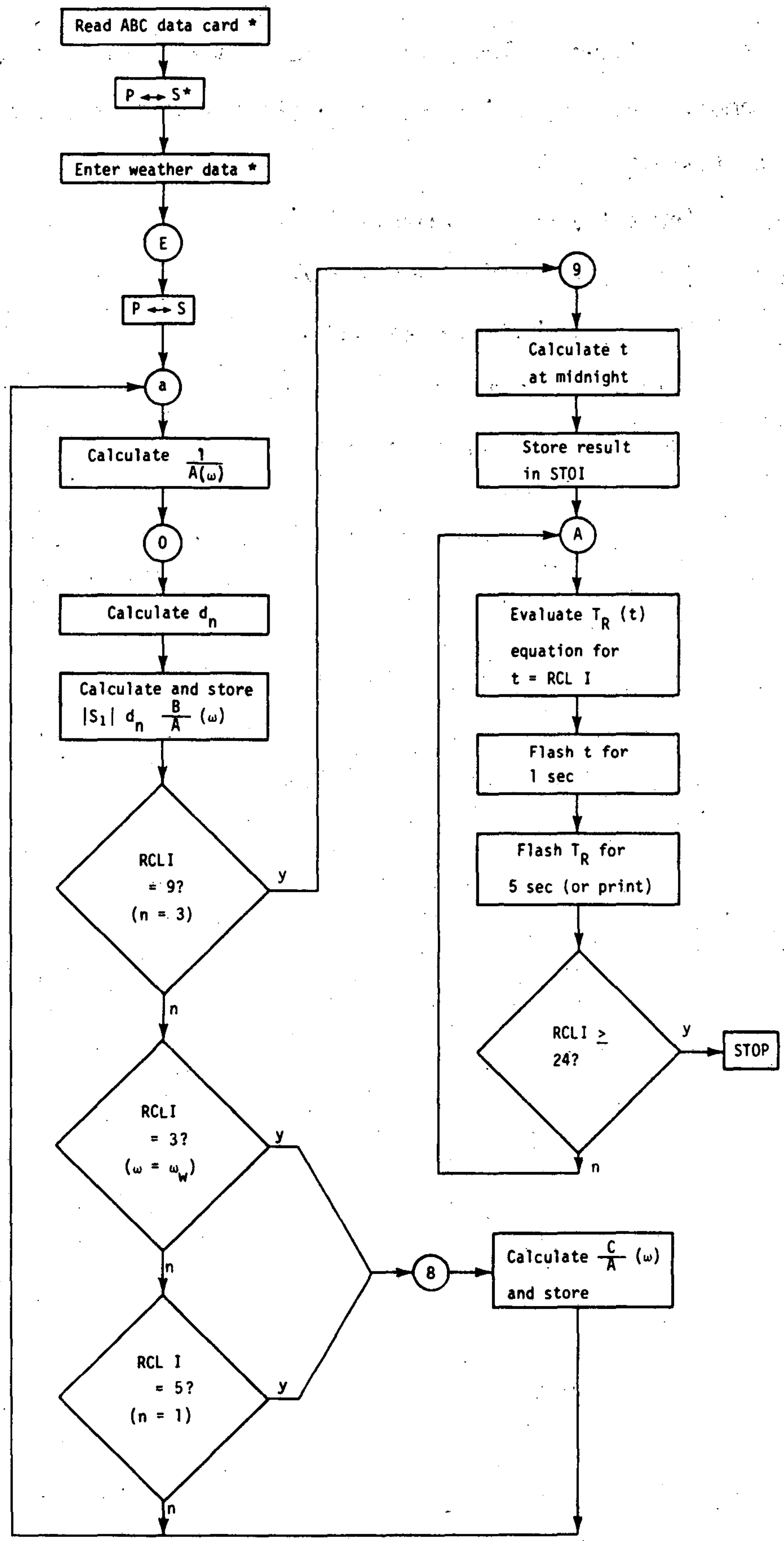
d- inverse of c.

D and d both require flag 2, LBL 4, LBL 5, LBL 6, STO D, and STO E.

D and d are structured so as to preserve the contents of the y and z stack registers. Thus, two complex numbers can be recalled from memory, decoded, and added or multiplied using these routines, as follows:

RCL 1	(recall encoded number)
D	(decode and place in x and y)
RCL 2	(recall second encoded number)
D	(decode and place in x-y; number from RCL 1 now in z-t)
A	(add the two complex numbers)

T_R (t) Program



* Performed by user

XBL 7910-4516

$T_R(t)$ Sub-Program

Evaluates coefficients of $e^{in\omega_o t}$ in the following equation:

$$T_R(t) = |S_1| \sum_{n=0}^{3} \frac{B(n\omega_o)}{A(n\omega_o)} d_n\, e^{in\omega_o t} + \Delta T_A \frac{C(\omega_o)}{A(\omega_o)} e^{i\omega_o t} + \overline{T}_A$$

$$+\left(|S_1| \frac{B(\omega_w)}{A(\omega_w)} d_o\, e^{i\omega_w t} + \frac{C(\omega_w)}{A(\omega_w)} e^{i\omega_w t} \right) \tag{C1}$$

(Terms in parenthesis are not correct, but are evaluated in the form given.)

Evaluates $T_R(t)$ using Eq. (C1) (but not the part in parentheses) for each hour of the day, beginning at solar midnight.

Inputs: ABC data card fills STO 0-11

STO	I	0
STO	15	$\|\Delta T_A\|$
STO	16	t_ϕ
STO	17	t_d
STO	18	$\|S_1\|$
STO	19	$\overline{T}_A$
STO	C	ω_o

Outputs: Time (t) at midnight flashed for 1 sec, then room temperature

$T_R(t)$ flashed for 5 sec (or printed)

Time (t) at 1 a.m....

Iterates for $\tilde{>}$ 24 hours.

Outputs of $T_R(t)$:

STO	0	$\|S_1\|\ d_o\frac{B}{A}(0)$
STO	2-3	$\|S_1\|\ d_o\frac{B}{A}(\omega_w)$
STO	4-5	$\|S_1\|\ d_1\frac{B}{A}(\omega_o)$
STO	6-7	$\|S_1\|\ d_2\frac{B}{A}(2\omega_o)$
STO	8-9	$\|S_1\|\ d_3\frac{B}{A}(3\omega_o)$
STO	10-11	$\frac{C}{A}(\omega_w)$
STO	12-13	$\frac{C}{A}(\omega_o)$

Subroutines:

D- decodes complex numbers $\sim 10^0$ (e.g., $B(\omega)$)

requires flag 2, LBL 4, LBL 5, LBL 6, STO D and STO E

Input: encoded number in x

Output: complex number (polar) in x-y

d- like D; decodes complex numbers $\sim 10^2$ (e.g., $A(\omega)$, $C(\omega)$)

B- complex multiply $R_1e^{i\theta_1} \times R_2e^{i\theta_2}$

requires STO D, E

Input; x R_1

y θ_1

z R_2

t θ_2

Output: complex product (polar) in x-y

D- calculates d_n using contents of STO I for n,

requires STO D

Input: STO I; STO I = 1,3; calculates d_o

5	d_1
7	d_2
9	d_3

STO C - ω_o

STO 17 - t_d

Output: d_n in x-y

A- calculates $T_R(t)$ given coefficients of Eq. (B11) in STO 10 - STO 19 and STO 0 - STO 3, as described in "$T_R(t)$ Program Output" above, with primary and secondary storage registers reversed, and

$|\Delta T_A|$ in STO 5

t_ϕ in STO 6

$\bar{T}_A$ in STO 9

ω_o in STO C

t in STO I

Iterates until t > 24 hr.

Sub-Program PSA-1

PSA-1 stores weather and building data in memories 20-46. It then computes and prints the steady-state temperature terms that arise from the heater, ambient temperature, and solar gain. The sum is printed last. Finally the Fourier components for the solar gain component can be calculated, but are not stored.

Procedure:

LBL E (46-59) sets the counter at 20 + (i x 7) where i is the index of the surface (equal to 0 for the weather terms).

LBL A (60-66) stores an input parameter at the location specified by the counter and adds one to the counter.

If the user wishes to change a parameter the surface number is re-entered (LBL E) and then the parameter number and its new value and entered via LBL B (67-76). LBL B also adds one to the counter so further changes can be made through LBL A. After all parameters are entered the number of surfaces used is stored via LBL E.

LBL C (77-233) computes, stores, and prints the steady-state components from the input parameters entered in LBL E and LBL A. Steps 77-90 are used for initialization. A loop from steps 91-140 computes $A_j(0)$, $B_j(0)$ and U_j for each surface j and keeps a running sum. A subroutine call (93-94) to SBR RCL (00-45) is used to swap the locations of the jth surface parameter with those of the first surface so that direct address arithmetic can be used in the loop. A second call at 135-136 swaps the locations back again. Steps 142-233 print an identifier for each steady-state term and compute and print their values. The hours of sunshine must be entered at step 174-175 and is echoed by print-out at steps 180-183.

The formulas used in this section are:

$$A_i(0) \equiv (\text{Area})_i\left(\left(\frac{1}{U_r}\right)_i + \frac{1}{h_i} + \frac{d_i}{K_i}\right)^{-1} \qquad (95\text{-}118)$$

$$A(0) = \sum_{i=1}^{i_{max}} A_i(0) + \hat{U}_q$$

(A(0) was initialized $\hat{U}_q$ is steps 83-86).

$$B_i(0)\ (1 - U_i/h_i)\ \alpha_i$$

(119-132, with U_i computed in steps 110-112. U_i is the U-value of the i^{th} construction section.)

$$B(0) = \sum_{i=1}^{i_{max}} B_i(0) + \alpha_R$$

(B(0) was initialized to α_R in steps 87-90)

$$d_o = t_d/12\pi$$

(174-194 with input and echo to print of t_d)

$$T_H = H(0)/A(0)$$

(142-160 with print commands)

$T_s = |S_1|\ d_o B(0)/A(0)$ (195-219 with print command)

The definition of the variables can be found in the input list. In addition to the above calculations LBL C also recalls and display T_A (161-173) and prints $T_R = T_H + T_A + T_S$ (220-233)

LBL D computes the Fourier components for sunshine from the following formula

$$d_n = \frac{\omega_1}{12(\omega_1^2 - (n\pi/12)^2)} \qquad (1 + e^{-in\omega_o t_d})$$

where $\omega_o = \pi/12$ and $\omega_1 = \pi/t_d$

n is entered in steps 236-237 and the first factor accumulated in steps 236-262. The master-library routines for complex exponentiation and multiplication are then used in steps 273-290 to compute d_n. Steps 291-316 generate the label and calculate d_n in both polar and rectangular form. Only the latter is automatically printed.

LBL D' (317-339) comutes and displays the zeroth Fourier component $d_o = \omega_o/\pi\omega_1 = t_d/12\pi$

Storage registers:

10 - ΣA_j (0)	27 d_1
11 - ΣB_j (0)	28 $(1/U_r)_1$
12 - T_H (0)	⋮
13 - T_S (0)	34 d_2
14 - T_R (0)	⋮
20 - S	41 d_3
21 - $\hat{U}_q$	⋮

Sub-Program PSA-2

PSA-2 calculates the frequency dependence of the response functions. From these functions the time dependent temperature terms can be calculated and summed with specified phase lags between them. The results for every other hour of a sample day are summed into memories 48-59.

Procedure:

LBL A (94-320): Computes the material response functions, and the building response functions for a specified ω. ω is calculated in steps 94-104 from the period entered by the user. After counters and initializations are performed (95-125) the imaginary and real components of the R_1, R_2 A, B and C are calculated in the loop from 126-320. The subroutine calls 128-129 and 313-374 to SBR RCL(00-45) perform the identical purpose as in LBL C of PSA-1. The formulas for R_1 and R_2 are taken from Table B1. R_1 is calculated in steps 130-219 and 238-254.

The complex hyperbolic functions cosh and sinh are evaluated by a subroutine call to SBR cos (58-93) which uses the master library's complex trigonometric functions and the relationships cosh z = cos iz and sinh z = - i sin iz. The numerator is calculated in steps 130-180 and the denominator is calculated in re-arranged form in steps 181-219. R_1 (ω) is then calculated with a complex divide in steps 238-254.

$R_2(\omega)$ is calculated in steps (130-237). Since $R_2(\omega)$ is so similar to $R_1(\omega)$ the initial sums are carried out in the same location. The final complex divide is calculated in steps 220-237.

A 'R/S' is present at step 254 so that the $R'(\omega)$ can be examined if it is so desired. $A(\omega)$, $B(\omega)$, and $C(\omega)$ are computed as:

$$A(\omega) = \sum_{j=1}^{j_{max}} A_j h_j (1-h_j R_{1j}) + \hat{U}_q \qquad (273\text{-}302)$$

$$B(\omega) = \sum_{j=1}^{j_{max}} (\alpha_j h_j R_{1j}) + \alpha_R \qquad (255\text{-}272)$$

$$C(\omega) = \sum_{j=1}^{j_{max}} (A_j h_j R_{2j}) + \hat{U}_q \qquad (273\text{-}294 \text{ and } 303\text{-}310)$$

where the A_j are the $_j$th sunface areas. Steps 311-320 merely control the loop.

LBL B (410-429) computes a frequency dependent weather term;

$T_R(\omega) = \Delta T_A \; C(\omega)/A(\omega)$.

The complex divide $(C(\omega)/A(\omega))$ is computed by a call (424-425) to SBR '÷' (321-339) which divides a complex number in registers 01 and 02 by $A(\omega)$ via the complex divide routine of the master library.

LBL C (430-458) computes a frequency dependent solar term:

$$\Delta T(\omega) = |S_1| \frac{d_n \; B(n\omega_o)}{A(n\omega_o)} \qquad (438\text{-}455)$$

The real and imaginary components of d_n and $|S_1|$ are entered in steps 430-438. If a weather frequency solar term is examined, substitute the magnitude of this term for $|S_1|$ and use d_o in place of d_n. The complex divide by $A(n\omega_0)$ is performed by a call to SBR '÷'.

Both LBL B and LBL C can call SBR PRT (375-404). SBR PRT prints hourly temperature increments over the period of the term.

LBL E (340-374): This subroutine calculates bihourly temperature increments for a given frequency dependent solar or weather term. The results are summed into registers 48-59. The phase of the term (ϕ) is entered at the beginning. The subroutine calculates the temperatures in a loop from the formula:

$$T(t) = \text{Real part}\left(\Delta T e^{i\omega(t + \phi)}\right) \quad (352\text{-}373)$$

The ΔT from LBL C or LBL D are complex numbers. LBL E calls SBR X (46-57) to perform the complex multiply.

LBL D (459-466): LBL D advances the paper and prints the contents of registers 48-59.

- A note on speeding up the program. If the program is not intended to be modified it can be noticeably sped up by replacing labels by absolute addresses. In particular, the loops to LBL SIN and LBL LIST in LBL B and LBL PRT respectively, are very slow. Delete LBL SIN (352-353) and insert one pause. Insert one step at 370 and rewrite the decrement statements as DSZ, 9, 352 (steps 370-373). Do the same with LBL LIST. Since these labels are called many times, this replacement is very noticeable in terms of program execution time.

APPENDIX D: Long Term Weather Responses

To evaluate the effect of multi-day cycles of temperature and sunlight, we assume that the cycle can be described by sinusoidal terms. Ambient temperature is taken to be of the form.

$$T_A(t) = \bar{T}_A + |\Delta T_{A_W}|\, e^{i\omega_W(t-t_a)} + \underset{\sim}{\Delta T_A}\, e^{i\omega_o t} \qquad \text{(D1)}$$

where ω_W is the frequency at which weather variations take place (typically $2\pi/1$ week),

$|\Delta T_{A_W}|$ is the amplitude of weather-variation of temperature,

t_a is the time at which the ambient temperature is at its maximum,

and $\bar{T}_A$ is now the average temperature over the whole cycle.

Solar gain is still taken as a half sine-wave for each day, but the amplitude is assumed to be sinusoidally modulated, as shown:

$$S(t) = \begin{cases} \left(\bar{S} + \Delta S_W \cos\omega_W\,(t-t_s)\right) \sin \omega_1(t-t_{sr}) & \text{day} \\ 0 & \text{night} \end{cases} \qquad \text{(D2)}$$

where

$\bar{S}$ is the average amplitude of solar gain (average $|S_1|$),

ΔS_W is the amplitude of modulation of $|S_1|$ over the cycle,

t_s is the time at which the solar energy in the cycle is at its maximum, and

T_{sr} is the time of the most recent sunrise.

It can be shown [D1] that room temperature is then given by:

$$T_R(t) = (S + \Delta S_w \cos\omega_w\,(t-t_s)) \sum_{n=1}^{3} \frac{B(n\omega_o)}{A(n\omega_o)}\, d_n\, e^{in\omega_o t} + \bar{S}\,\frac{B(0)}{A(0)}\, d_o$$

$$+ \Delta S_w \frac{B(\omega_w)}{A(\omega_w)}\, d_o\, e^{i\omega_w(t-t_s)} + \bar{T}_A + \Delta T_{A_w} \frac{C(\omega_w)}{A(\omega_w)}\, e^{i\omega_w(t-t_a)}$$

$$+ \Delta \underset{\sim}{T}_A \frac{C(\omega_o)}{A(\omega_o)}\, e^{i\omega_o t} + \frac{H}{A(0)} \quad . \qquad \text{(D3)}$$

Comparing Eq. (B11) with Eq. (D1), it can be seen that if we set $S_1 = \bar{S} + \Delta S_w \cos \omega_w(t-t_s)$, Eq. (D1) says

$$T_R(t) = (\text{old terms}) + (\bar{S} - |S_1|)\ \frac{B(0)}{A(0)}\, d_o$$

$$+ \Delta S_w \frac{B(\omega_w)}{A(\omega_w)}\, d_o\, e^{i\omega_w(t-t_s)} + \Delta T_{A_w} \frac{C(\omega_w)}{A(\omega_w)}\, e^{i\omega_w(t-t_a)} \quad .$$

(D4)

Thus only three new terms and only two new combinations of Building Response functions are needed to account for multi-day weather cycles.

Notes and References

D1. D. B. Goldstein, Some Analytic Models of Passive Solar Building Performance. Lawrence Berkeley Laboratory, LBL-7811, November, 1978, and Garland Press, New York City, 1979. See Section 2.4 and Appendix 2.4.

APPENDIX E: Example Problem Using the Program

This section sets up a simple problem of modelling a passive solar house on a sunny winter design day, and computes the input parameters to the programs. Intermediate and final results are given. The user can employ this example to check the answers he computes using the programs.

The example house is a single-story wood-frame residential structure of conventional American construction. Its only passive solar features are its bare (or tile-covered) slab-on-grade floor and its large south-facing collector window area. As the results in Table E.4 show, it should not be considered "optimized."

The use of a wood-frame house for the example illustrates some of the simplifications that can be made in using the model to describe a house. In principle, seven material surfaces would be needed for a model of this building: the stud portion of envelope walls, of partition walls, and of the ceiling; the cavity portion of envelope walls, partition walls, and ceiling; and the floor. But to increase computational speed and convenience, in this problem we use only three surfaces. This procedure can be shown to lead to less than 1^{o}F errors.

Consider a 30 X 50 foot single-story house with 8-foot ceiling. The walls have R-11 insulation between 2 X 4 studs; the ceiling uses R-30 insulation and 2 X 6 joists; while the floor is a bare concrete slab on grade. There are 250 ft^2 of south-facing double-glazing and 30 ft^2 of glazing on each of the other elevations. We assume 1.5 ft^2 of partition wall per ft^2 of envelope wall.

We consider three surfaces:

1) Wall and ceiling studs (25% of envelope wall area,
   + 15% of partition wall area
   + 10% of ceiling area)

2) Wall and ceiling cavities

3) Floor

For the first two, we consider a wall section composed of the following layers (from inside to outside)

1) Inside film resistance
   $R = .68$ hr - ft^2- ^oF/Btu (walls)
   $.61$ (ceiling)

2) Gypsum wall board
   $K = .075$ Btu/^oF-ft-hr $\rho c = 13$ Btu/ft^3 - ^oF
   $d = 1/24$ ft

3) Insulation   R - 11 wall
   R - 30 ceiling

or

studs: $K = .068$ Btu/^oF - ft - hr $\rho c = 9$ Btu/ft^3 - F
   $d = .292$ envelope wall
   $= .146$ partition wall
   $= .458$ ceiling

4) (Ceiling studs only) Insulation over studs   R - 11

5) Stucco plus exterior film coefficient $R = .41$

or attic (for the ceiling) $R \cong 3$

Areas are:

1) Studs: $.25 \times (\underbrace{8 \times (30 + 50) \times 2}_{\text{gross wall area}} - \underbrace{(250 + 3 \times 30)}_{\text{window area}})$

$$+ .15 \times 1.5 \times 8 \times (30 + 50) \times 2$$

$$+ .10 \times 1500$$

$$= 235 + 288 + 150$$

$$= 673 \text{ ft}^2$$

2) Cavities .75 X (8 X (30 + 50) X 2 - (250 + 3 X 30) + .85 X 1.5

X 8 X (30 + 50) X 2 + .90 X 1500

= 705 + 1632 + 1350

= 3687 ft^2

3) Floor: 1500 ft^2

For two-layer wall approximation; we average the parameters as follows:

1) Studs

The massive layer has the average properties of the gypsum board and the studs.

For the envelope walls, we take the weighted average of 1/K and ρc for 1/2" of gypsum and 3 1/2" of wood

$$K_e = ((1/2'' \times \frac{1}{.075} + 3\ 1/2'' + \frac{1}{.068}) \div 4'')^{-1} = .0688$$

$$(\rho c)_e = (1/2'' \times 13 + 3\ 1/2'' \times 9) \div 4'' = 9.5$$

$$d_e = 1/3 \text{ ft}$$

For the ceiling, we take the weighted average of 1/K and ρc for 1/2" gypsum and 5 1/2" wood:

$$K_c = (1/2'' \times \frac{1}{.075} + 5\ 1/2'' \times \frac{1}{.068}) \div 6'')^{-1} = .06853$$

$$(\rho c)_c = (1/2'' \times 13 + 5\ 1/2'' \times 9) \div 6'' = 9.333$$

The thickness d is 1/2 foot

For the partition walls, we do the same average for 1 3/4" of wood (half the stud thickness)

$$K_p = (1/2" + \frac{1}{.075} \times 1\ 3/4" \times \frac{1}{.068}) \div 2\ 1/4")^{-1} = .06944$$

$$(\rho c)_p = (1/2" \times 13 + 1\ 3/4" \times 9) \div 2\ 1/4" = 9.889$$

The thickness d is 2 1/4 inches or .1875 ft

We take the area-weighted average of K, ρc, d, and h to derive the properties of the first material (the studs).

$$K_1 = (235\ ft^2 \times .0688 + 288\ ft^2 \times .06944 + 150\ ft^2 \times .06853) \div 673\ ft^2$$

$$= .06901\ Btu/{}^oF\text{-}hr\text{-}ft$$

$$(\rho c)_1 = (235\ ft^2 \times 9.5 + 288\ ft^2 \times 9.889 + 150\ ft^2 \times 9.333) \div 673\ ft^2$$

$$= 9.629\ Btu/{}^oF\text{-}ft^3$$

$$d_1 = (235\ ft^2 \times 1/3 + 288\ ft^2 \times .1875 + 150\ ft^2 \times .5) \div 673\ ft^2$$

$$= .3081\ ft$$

$$h_1 = (235\ ft^2 \times \frac{1}{.68} + 288\ ft^2 \times \frac{1}{.68} + 150\ ft^2 \times \frac{1}{.61}) \div 673\ ft^2$$

$$= 1.508\ Btu/ft^2\text{-}hr\text{-}{}^oF$$

$$A_1 = 673\ ft^2$$

To compute U_r, we require that the steady-state heat loss through the approximate version of material 1 is the same as for the exact case.

For the exact case, the heat loss rate is

$$U_eA_e + U_cA_c$$

or

$$\frac{1}{\left(\frac{.075}{1/24}\right)^{-1} + \left(\frac{.068}{.292}\right)^{-1} + .41} \times 235\ ft^2 + \frac{1}{\left(\frac{.075}{1/24}\right)^{-1} + \left(\frac{.068}{.458}\right)^{-1} + 11 + 3} \times 150\ ft^2$$

$= \quad 51.22\ Btu/^{o}F\text{-}hr$

For the model, the heat loss rate is

$$\left(\left(\frac{K_1}{d_1}\right)^{-1} + U_{r1}^{-1}\right)^{-1} A_1$$

so $\left(\frac{A_1}{51.22} - \frac{d_1}{K_1}\right)^{-1} = U_{r1}$

or $U_{r1} = .117\ Btu/^{o}F\text{-}hr\text{-}ft^2$

Note that we have ignored the inside film resistance in computing U_r for both cases.

2) Cavities

The same process is followed. The massive layer is the gypsum board and the outside layer contains everything else. Since the massive layer has only one component, the averaging process is unnecessary and K_2, $(\rho c)_2$, and d_2 are just given by the parameters of gypsum board. A_2 is 3687 ft^2. We calculate the average h for this surface as the area-weighted average:

$$h_2 = (705\ ft^2 \times \frac{1}{.68} + 1632\ ft^2 \times \frac{1}{.68} + 1350\ ft^2 \times \frac{1}{.61}) \div 3687\ ft^2$$

$= \quad 1.532\ Btu/ft^2\text{-}hr\text{-}^{o}F$

We still have to calculate U_{r2}

For the exact case, the heat loss rate is

$$U_eA_e + U_cA_c$$

or

$$\frac{1}{\left(\frac{.075}{1/24}\right)^{-1} + 11 + .41} \text{ X } 705 \text{ ft}^2 + \frac{1}{\left(\frac{.075}{1/24}\right)^{-1} + 30 + 3} \text{ X } 1350 \text{ ft}^2$$

$$= \quad 99.15 \text{ Btu/}^{o}\text{F-hr}$$

For the model, the heat loss rate is

$$\left(\left(\frac{K_2}{d_2}\right)^{-1} + U_r^{-1}\right)^{-1} A_2$$

so

$$\left(\frac{A_2}{99.15} - \frac{d_2}{K_2}\right)^{-1} = U_{r2}$$

or

$$U_{r2} = .0273 \text{ Btu/}^{o}\text{F-ft}^2\text{-hr}$$

3) Floor

For the floor parameters, we assume that the slab floor is coupled to the ground, and that the ground-water migration is not important in the area of the house. The mean length of a path of heat flow from the floor through the ground to the outside air is on the order of 20 feet, so, we assume a 20-foot floor thickness. (The results are not sensitive to exact floor thickness.)

We use typical materials properties for concrete (and soil) of

$K = .8\ Btu/^{o}F\text{-}ft\text{-}hr$, $\rho c = 20\ Btu/^{o}F\text{-}ft^3$; $h = \frac{1}{.61}$

$= 1.639\ Btu/^{o}F\text{-}ft^2\text{-}hr$

For U_r, we take the outside film coefficient at the soil, $h = 6\ Btu/ft^2\text{-}^{o}F\text{-}hr$, though the results would not change if h were changed from this value.

To compute the α's, we use the "typical" values given in the text for a dark floor and light walls. Thus $\alpha_3 = .45$, $\alpha_R = .15$.

To compute α_1, and α_2, we average the solar gain on the envelope walls, partition walls, and ceiling over surfaces 1 and 2 on an area-weighted basis. The total solar gain absorbed on these surfaces is

$$.10 + .20 + .10 = .40, \text{ so } \alpha_1 = .40 \times \frac{673\ ft^2}{673\ ft^2 + 3687\ ft^2}$$

and

$$\alpha_2 = .40 \times \frac{3687\ ft^2}{673\ ft^2 + 3687\ ft^2} \text{ or } \alpha_1 = .062 \text{ and } \alpha_2 = .338$$

We next compute the final remaining parameter for the house, $\hat{U}_q$. This term is composed of infiltration losses (assumed to be at 0.6 air changes per hour) and window losses.

Infiltration loss rate is $(8 \times 30 \times 50)\ ft^3 \times 0.018\ Btu/^{o}F\text{-}ft^3 \times 0.6$ air changes/hr = $129.6\ Btu/^{o}F\text{-}hr$. Window losses through double glazing with $U = 0.49\ Btu/^{o}F\text{-}ft^2\text{-}hr$ (1/2" air space) are given by UA or $0.49 \times (250\ ft^2 + 3 \times 30\ ft^2) = 166.6\ Btu/^{o}F\text{-}hr$. So $\hat{U}_q = 296.2\ Btu/^{o}F\text{-}hr$.

Weather Parameters

The example we take describes a northern California climate on a relatively cool but clear winter design day in January. We set $\overline{T}_A = 45^0$ and $\Delta T_A = 10^0F$, so the daily high temperature is 55 and the low is 35. We set the length of day at nine hours ($t_d = 9$ hr). Maximum ambient temperature is at 3 pm or 7 1/2 hours after sunrise ($t_\phi = 7.5$ hr).

Solar gain is taken from Ashrae solar heat gain tables for 40°N. latitude for January and summed over all four elevations:

S:	$250\ ft^2$ X 1630 Btu/day	= 407,500 Btu/day
E & W:	2 X 30 ft^2 X 508 Btu/day	= 30,480 Btu/day
N:	30 ft^2 X 118 Btu/day	= 3,540 Btu/day
	Total	441,520 Btu/day

This total is modified by a shading coefficient of about 0.85 for double-pane glass and multiplied by 0.9 to account for opaque window frame area, so total daily solar gain entering the house is 337,760 Btu/day. S_1 is given by: (daily solar heat gain) X $\frac{\pi}{2t_d}$ so S_1 = 58,950 Btu/hr.

We set $\omega_w = \frac{1}{10}\omega_o$ to calculate response functions; however, this will not be used in the calculation. To run this problem on the TI-59, the phase terms are: $\phi = -7.5$ for the solar terms and $\phi = -15$ for the temperature term.

This completes the derivation of the building and weather parameters. They are summarized in Table E.1.

Results of the Calculation

We next display intermediate results to check the operation of each of the programs. The parameters for each surface are used to compute R_1 and R_2. Table E.2 gives the values of the response functions for each of the three surfaces, while Table E.3 gives the building response functions. The room temperature results are given in Table E.4 under the assumption of no heater output. To include the effect of 2000 Btu/hr of internal loads, add 4°F $\left(= 2000 \text{ Btu/hr}/A(0)\right)$ to each entry in Table E.4. (This effect is calculated directly in the TI-59 program).

As seen in the results from Table E.4, the response of the example building is not optimal. Afternoon temperatures are too warm and morning temperatures too cool. As an alternate to this design, one could consider the effects of more insulation and smaller collector area, or more thermal mass.

Table E.1

House Parameters for Example Problem

	Surface 1	Surface 2	Surface 3
K	.06901	.075	0.8
ρc	9.629	13	20
d	.3081	.04167	20
h	1.508	1.532	1.639
U_r^*	.117	.0273	6
A	673	3687	1500
α	.062	.338	.45

$\hat{U}_q$ = 296.2

α_R = .15

ΔT_A = 10

t_ϕ = 7.5

t_d = 9

S_1 = 58950

$\overline{T}_A$ = 45

$^*U_r^{-1}$ is used for input in the TI-59 program.

Table E.2

Response Functions for the Example House*

R_1 (hr-^{o}F-ft^2/Btu)

ω	Surface 1	Surface 2	Surface 3
0	.6309	.6414	.5956
ω_o	$.5473e^{-i0.1851}$	$.6374e^{-i0.0891}$	$.2933e^{-i0.4385}$
$2\omega_o$	$.5022e^{-i0.2163}$	$.6258e^{-i0.1752}$	$.2372e^{-i0.5069}$
$3\omega_o$	$.4780e^{-i0.2443}$	$.608e^{-i0.2558}$	$.2064e^{-i0.54383}$

R_2

ω	Surface 1	Surface 2	Surface 3
0	.0484	.0172	.0236
ω_o	$.0291e^{-i1.2979}$	$.0171e^{-i0.1281}$	0
$2\omega_o$	$.0161e^{-i1.9614}$	$.0167e^{-i0.2531}$	0
$3\omega_o$	$.0102e^{-i2.4303}$	$.0162e^{-i0.3725}$	0

*Note that $e^{x} = e^{x-2\pi}$ for any x.

Table E.3

Building Response Functions for the Example House

ω	A(Btu/^{o}F-hr)	B	C(Btu/^{o}F-hr)
0	502.2	.9802	502.2*
ω_o	$2332.3e^{i0.514}$	$.7364e^{-i0.1783}$	$402.0e^{-i0.1016}$
$2\ \omega_o$	$2953.0e^{i0.562}$	$.6847e^{-i0.2231}$	‡
$3\ \omega_o$	$3493.4e^{i0.588}$	$.6504e^{-i0.2631}$	‡

*Set equal to A(0)

‡Not computed

Table E.4

Room Temperature as a Function of Time

t	Solar Time	T_R (no heater or internal loads)
-7.5	12 m	68.0
-6.5	1 am	67.1
-5.5	2	66.4
-4.5	3	65.9
-3.5	4	65.3
-2.5	5	64.6
-1.5	6	64.1
-.5	7	64.2
.5	8	65.5
1.5	9	68.4
2.5	10	72.7
3.5	11	77.7
4.5	12 n	82.1
5.5	1 pm	85.0
6.5	2	85.7
7.5	3	84.3
8.5	4	81.5
9.5	5	78.4
10.5	6	75.6
11.5	7	73.6
12.5	8	72.3
13.5	9	71.3
14.5	10	70.3
15.5	11	69.2
16.5	12 m	68.0

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