EARTH DAM SEEPAGE ANALYSIS WITH A PROGRAMMABLE CALCULATOR

by

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ABSTRACT

A model was developed for the Texas Instruments Company TI-59 programmable calculator for estimation of seepage from a reservoir. The model, DAM SEEPAGE, separates flow through the dam and flow under the dam by assuming that the boundary between the dam and foundation is a streamline. The algorithm used is based on Darcy's law, Dupuit's assumptions, and Bear's hydraulic approach.

DAM SEEPAGE was applied to three situations, two proposed dams in eastern Montana and one existing dam in central Oklahoma. The two proposed dams were compared on the basis of seepage loss. The existing dam, a well-documented site, was chosen to check the accuracy of DAM SEEPAGE. The results of the study are within 2 percent of previously modeled results and within 13 percent of previously measured results.

CHAPTER 1

INTRODUCTION

Dams, structures that serve to store water for use during periods of drought or to protect land areas, represent one of civilizations oldest engineered structures. In current times, dams are an essential component of water supply systems, hydroelectric power facilities, and flood-control projects. Dams can also serve to create reservoirs for recreational and navigational use and for sediment retention. A dam frequently serves as a multipurpose facility.

Dams for permanent facilities (as opposed to temporary dams associated with construction projects) are typically constructed of masonry, concrete, earth fill, or rock fill, although timber and steel materials can also be used. Certain advantages and disadvantages are associated with the construction, function, and operation of dams built with each of these materials. Concrete, masonry and rock-fill dams require strong foundations of rock or firm incompressible soil. Earth-fill dams derive their stability from a massive cross section and as a result can be built at almost any site with either a rock or soil foundation.

Earth-fill dams for water storage have been used since the early days of civilization as attested by history and ruins of past cultures. Some of these structures built in antiquity were of considerable size. One earth-fill dam, which measured 11 miles long and 70 feet high and contained approximately 17 million cubic yards of embankment, was completed in Ceylon in 504 B.C. (U.S. Department of the Interior, Bureau of Reclamation, 1977).

Early dams were constructed simply by heaping earthen materials across the area to be blocked with human traffic often producing all of the compacting. Many were washed out by overtopping, underseepage, or other destructive forces. Standards of practice eventually emerged that had no theoretical bases except that a method had worked at a number of locations. In the last 50 years, analytical and experimental methods have played an important part in the design and construction of earth-fill dams.

The foundations of earth-fill dams often consist of alluvial deposits, composed of relatively pervious sands and gravels overlying impervious geological formations. The pervious materials may range in size from fine sand to gravels and usually consist of stratified mixtures.

Two basic problems are found in pervious foundations. One pertains to the amount of seepage through the foundation and the other is concerned with the forces exerted by the seepage on the structure. The type and extent of treatment justified to decrease the amount of seepage are determined by the purpose of the dam and the amount of inflow relative to the reservoir capacity. Water loss from seepage may be of economic concern for a storage dam but of little consequence for a detention dam. An economic study of the value of the water and the cost of limiting the amount of seepage may be required in some cases to determine the appropriate extent of treatment.

The control of water loss is not the only reason for analyzing seepage. Justin (1936) tabulated earth-dam failures and among those that did not fail from overtopping, about 80 percent failed because of piping, sloughing at the toe, or other results of uncontrolled seepage. Horsky (1969) reported on a reservoir that was constructed on a limestone foundation that had such a high rate of seepage that the reservoir would not hold water. A large percentage of earth dam failures reported by Sherard and others (1963) were also seepage failures.

Therefore, the greatest design need, after considering overtopping, is prevention of internal damage from water seeping through both the structure and foundation. The methods of analyzing the influence of seepage are among the most important aspects of designing and construction of earth dams.

Estimation of seepage loss from a reservoir under steady-state conditions where a free surface is present has traditionally been performed by the graphical method of flow nets (Cedergren, 1977), because exact analytical methods are often difficult to apply where complex boundary conditions exist. Graphical techniques, however, can be quite cumbersome.

These difficulties have led to the development of numerical methods that permit seepage analysis using high-speed digital computers. Finnemore and Perry (1968) adapted the relaxation techniques of Shaw and Southwell (1941) to the computer to analyze seepage through an earth dam. Another finite-difference approach to steady-state seepage loss from a reservoir has been described by Jeppson

(1968, 1969). Neuman and Witherspoon (1970) described a finite-element approach that assures rapid convergence in all cases.

Analytical equations provide a powerful tool for preliminary seepage analysis. Harr (1962) and Polubarinova-Kochina (1962) presented several analytical solution to the partial differential equations of flow through porous media. In many cases, however, it is not practical to obtain an analytical solution because of the complexity of the expressions describing the system. It is possible, though, to exactly solve a simplified representation of a complex system. These techniques include the use of Darcy's law, Dupuit's assumptions, and Bear's hydraulic approach with simple boundary conditions and simplifying assumptions (Harr, 1962; Bear, 1979). Often, these assumptions do not restrict the use of the equation in a preliminary investigation or in a field situation. For example, hydraulic conductivities in the vicinity of a proposed dam are rarely known with any certainty. A preliminary estimate of seepage from a proposed reservoir can be obtained by assuming a homogeneous and isotropic porous medium. As more data are collected during the design phase of a site investigation, a more accurate estimate of seepage can be obtained with a two-dimensional finite-difference or finite-element model.

Analytical equations often use relatively complex mathematical functions and can be quite lengthy. If a number of hydraulic conductivities are used to obtain a range of seepage values, calculations can be quite cumbersome. Use of a programmable calculator can simplify evaluation of these equations. The equations can be programmed into the calculator and stored on magnetic cards for rapid application to various situations.

The use of programmable calculator models in surface-water and ground-water hydrology is common. Warner and Yow (1979, 1980a, 1980b) presented calculator models that predicted head changes in response to pumping from or injection into wells completely penetrating fully confined and semiconfined aquifers and partially penetrating fully confined aquifers. Sandberg and others (1981) developed several surface-water and ground-water models for the evaluation of the hydrologic impacts of mining. Kelly (1982) presented a calculator model that analytically calculates two-dimensional dispersion utilizing the complementary error function. Rayner (1980), Grimestead (1981), Sayed (1982), and Pashetto and McElwee (1982) presented calculator models that evaluated transmissivity and storativity from aquifer test data.

The objective of this study was to develop a model suitable for use on a programmable calculator to estimate seepage from a reservoir. The model outlined here was developed for the TI-59 programmable calculator (Texas Instruments Incorporated, Dallas, Texas) but can also be adapted to similar calculators of other brands with minor modifications. The model is based on the Dupuit assumption for flow through a dam and a modification of the Laplace equation for flow under a dam. To solve these equations, simplified boundary conditions were applied to the equations.

The model, DAM SEEPAGE, was then applied to two proposed earth dams in eastern Montana to compare the sites on the basis of seepage loss from the reservoirs. The model was also applied to a well-documented site in central Oklahoma to check the accuracy of DAM SEEPAGE.

CHAPTER 2

MATHEMATICAL CONSIDERATIONS

Darcy's Law

A mathematical treatment of water flow through porous media can best be accomplished by beginning with an examination of Darcy's law. To obtain the fundamental relation for the quantity of flow through a porous medium, consider figure 1. The cross-sectional area is equal to A and the specific discharge is q.

According to Bernoulli's theorem, the total head at any section can be given as the sum of the elevation head, the pressure head, and the velocity head (Vennard and Street, 1975). The velocity head for flow through a porous medium is very small and can be neglected (Freeze and Cherry, 1979). Therefore the total head at P_1 and P_2 can be given by:

$$\text{Total head at } P_1 = z_1 + h_1$$

$$\text{Total head at } P_2 = z_2 + h_2$$

where z_1 and z_2 are the elevation heads and h_1 and h_2 are the pressure head at P_1 and P_2, respectively.

The lost of head, Δh, between P_1 and P_2 is

$$\Delta h = (z_1 + h_1) - (z_2 + h_2) \qquad (2.1)$$

and the hydraulic gradient, i, can be written as

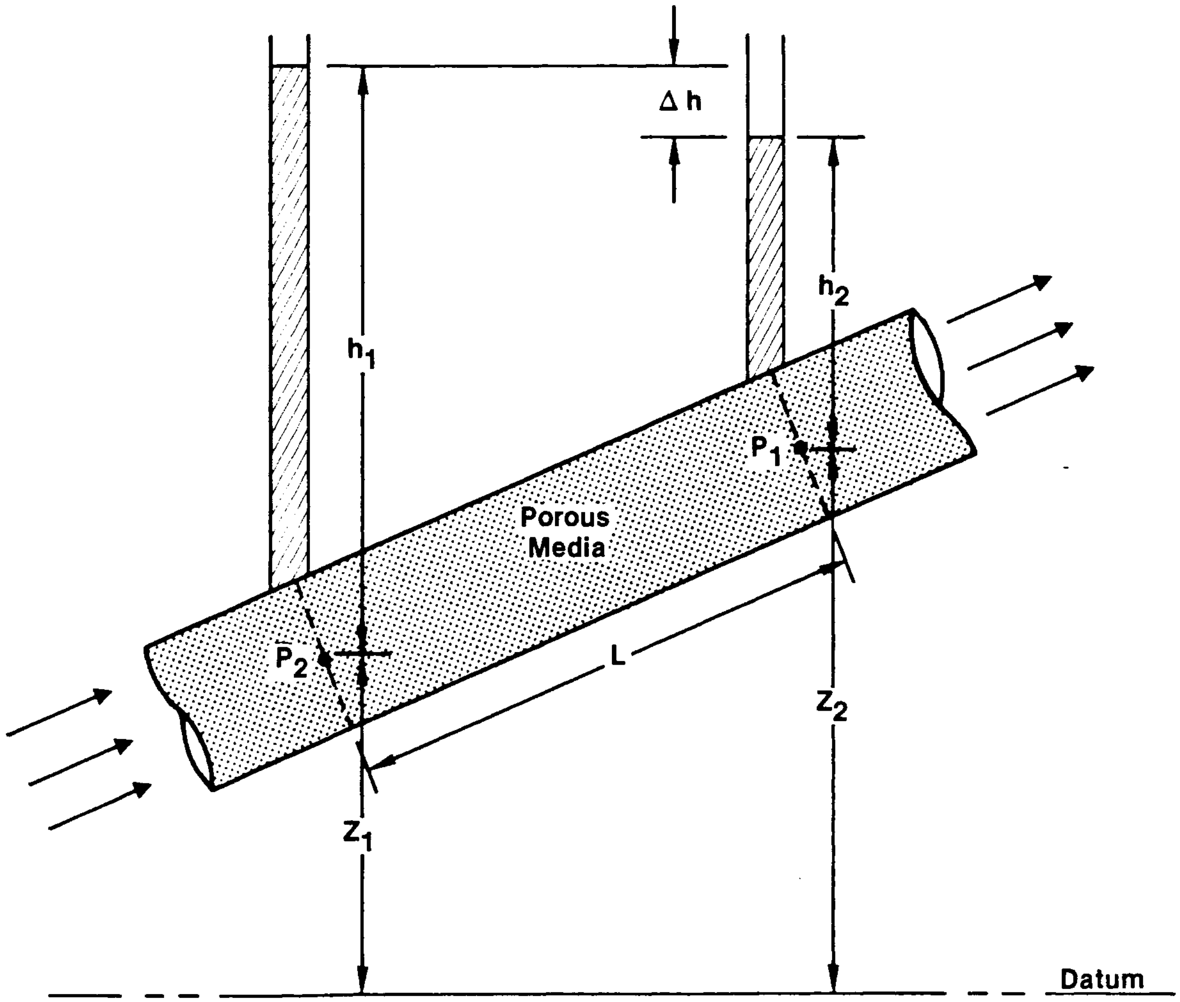


Figure 1. Flow system to illustrate Darcy's law

$$i = \frac{\Delta h}{L} \tag{2.2}$$

where L is the distance between P_1 and P_2.

Darcy (1856) published a simple relationship between specific discharge and hydraulic gradient:

$$q = - K \frac{\Delta h}{L} \tag{2.3}$$

or in one-dimensional differential form:

$$q = - K\frac{dh}{dx} \tag{2.4}$$

Darcy's law states that the specific discharge is directly proportional to the hydraulic gradient. The constant of proportionality, K, is known as the hydraulic conductivity.

Although Darcy's law is presented in differential form in equation (2.4), it does not describe flow through an individual pore. Specific discharge is a macroscopic concept and is different from microscopic velocities associated with actual flow paths through individual pores. In other words, the porous medium with its individual grains, pores, and flow paths is replaced by a representative continuum for which macroscopic parameters such as specific discharge and hydraulic conductivity can be defined (Freeze and Cherry, 1979).

With this definition in mind, specific discharge can be defined as:

$$q = \frac{Q}{A} \tag{2.5}$$

where Q is discharge and A is the cross-sectional area. Substituting equation (2.5) into equation (2.4) yields an alternative form of Darcy's law:

$$Q = - KA\frac{dh}{dx} \tag{2.6}$$

Darcy's law is an empirical relationship, derived solely on experimental results. Many attempts have been made to derive Darcy's law from more fundamental physical laws. The most successful approaches attempt to apply the Navier-Stokes equations, which are widely known in the study of fluid mechanics, to the flow of water through the pore channels of idealized conceptual models of porous media.

Laplace's Equation

In many practical situations, flow through porous media is such that the velocity and hydraulic gradient vary throughout the medium. For these situations, a general differential equation that describes the steady-flow condition for a given point in the medium is desirable.

To derive this equation, known as Laplace's equation, consider a unit volume of porous media shown in figure 2. Following continuity, the flow entering the unit volume in the x,y,and z directions is the flow leaving the unit volume. Therefore,

$$\begin{aligned} \rho q_x + \rho q_y + \rho q_z = (\rho q_x + \frac{\partial}{\partial x}(\rho q_x) + (\rho q_y + \frac{\partial}{\partial y}(\rho q_y) \\ + (\rho q_z + \frac{\partial}{\partial z}(\rho q_z + \frac{\partial}{\partial z}(\rho q_z) \end{aligned} \tag{2.7}$$

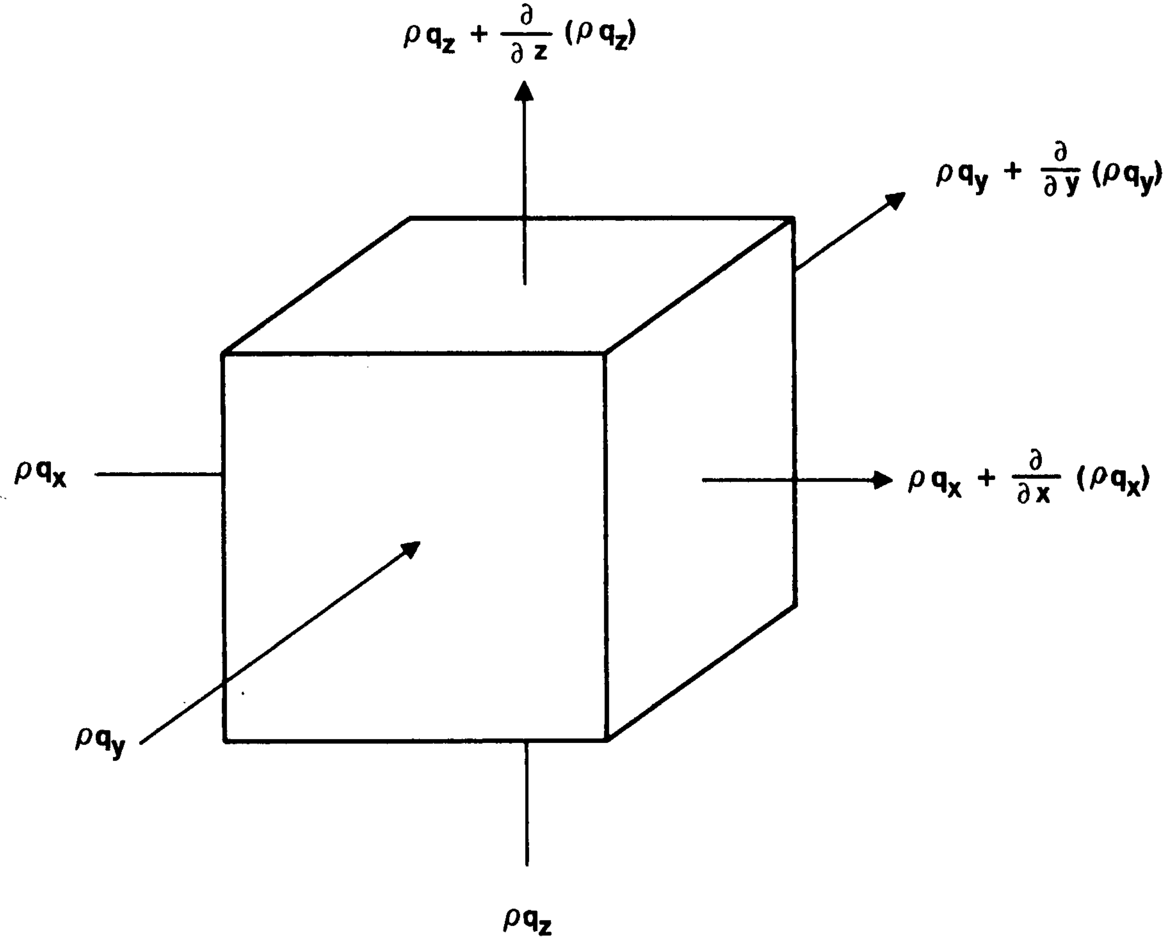


Figure 2. Three-dimensional flow through a unit volume of porous media

where: ρ = density of water.

q_x, q_y, q_z = specific discharge in x,y,z direction

Rearranging yields:

$$\frac{\partial}{\partial x}(\rho q_x) + \frac{\partial}{\partial y}(\rho q_y) + \frac{\partial}{\partial z}(\rho q_z) = 0 \tag{2.8}$$

If the fluid is incompressible, ρ is constant and

$$\frac{\partial q_x}{\partial x} + \frac{\partial q_y}{\partial y} + \frac{\partial q_z}{\partial z} = 0 \tag{2.9}$$

Even if the fluid is compressible and $\rho \neq$ constant, Freeze and Cherry (1979) have shown that equation (2.9) is a good approximation, because:

$$\rho \frac{\partial q_x}{\partial x} >> q_x \frac{\partial \rho}{\partial x}$$

Substitution of Darcy's law for q_x, q_y, and q_z into equation (2.9) yields:

$$\frac{\partial}{\partial x}(K_x \frac{\partial h}{\partial x}) + \frac{\partial}{\partial y}(K_y \frac{\partial}{\partial y}) + \frac{\partial}{\partial z}(K_z \frac{\partial h}{\partial z}) = 0 \tag{2.10}$$

For a homogeneous porous medium, K is not a function of space, therefore,

$$K_x(\frac{\partial^2 h}{\partial x^2}) + K_y(\frac{\partial^2 h}{\partial y^2}) + K_z(\frac{\partial^2 h}{\partial z^2}) = 0 \tag{2.11}$$

If the porous medium is also isotopic, $K_x = K_y = K_z$; therefore

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} + \frac{\partial^2 h}{\partial z^2} = 0 \tag{2.12}$$

Equation (2.12) is known as Laplace's equation.

The two-dimensional form of Laplace's equation may be represented by two families of curves that intersect at right angles to form a pattern known as a flow net. One set of lines is called the streamlines or flow lines, the other the equipotentials. A streamline or flow line defines the locus of the flow path of an individual particle of water, and an equipotential represents a line of equal head. A flow net thereflow represents a graphical solution to Laplace's equation. However, as previously mentioned, flow nets can be cumbersome and time consuming to construct.

Moderately Confined System

A moderately confined flow system is essentially a fully confined system except that the confining layers above and/or below the main flow system are moderately pervious. The most common occurrence of this system is the so-called leaky aquifer. However, the material under a dam can be considered a moderately confined system. The reservoir sediment or foundation grout can be represented by a confining layer with a hydraulic conductivity that is relatively low in comparison to an alluvial foundation.

For this type of system, Bear (1979) assumed that the flow in the main system is horizontal and therefore, by the law of refraction, the flow in the moderately pervious layer is vertical. Storage in the moderately pervious layer is assumed negligible. A simplification known as the "hydraulic approach" can be employed if horizontal flow can be assumed. With this approach, the problem is treated with average head ($\tilde{h}$) as the dependent variable rather than a problem in h(x,y,z):

$$\tilde{h} = \tilde{h}(x,y) = \frac{1}{H} \int_0^H h(x,y,z)\, dz \qquad (2.13)$$

where H is the thickness of the main flow system. The reduction by only one dimension simplifies the problem by requiring less information on the flow system.

Figure 3 represents a moderately confined system in which flow can be rigorously described by Laplace's equation. The simplified equation utilizing the hydraulic approach can be derived by multiplying Laplace's equation (equation 2.12) by dz and integrating over the vertical and applying the concept of average head.

$$\int_0^H \left(\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} + \frac{\partial^2 h}{\partial z^2}\right) dz = 0$$

$$H\left(\frac{\partial^2 \tilde{h}}{\partial x^2} + \frac{\partial^2 \tilde{h}}{\partial y^2}\right) + \left[\frac{\partial h}{\partial z}\right]_{z=H} - \left[\frac{\partial h}{\partial z}\right]_{z=0} = 0$$

Applying Darcy's law to the change of head in the vertical direction yields:

$$\left[\frac{\partial h}{\partial z}\right]_{z=H} = \frac{1}{K}\left[q_z\right]_{z=H} \qquad (2.14)$$

and

$$\left[\frac{\partial h}{\partial z}\right]_{z=0} = \frac{1}{K}\left[q_z\right]_{z=0} \qquad (2.15)$$

Because the vertical component of specific discharge at the upper side of the main flow system equals the vertical component of the flow

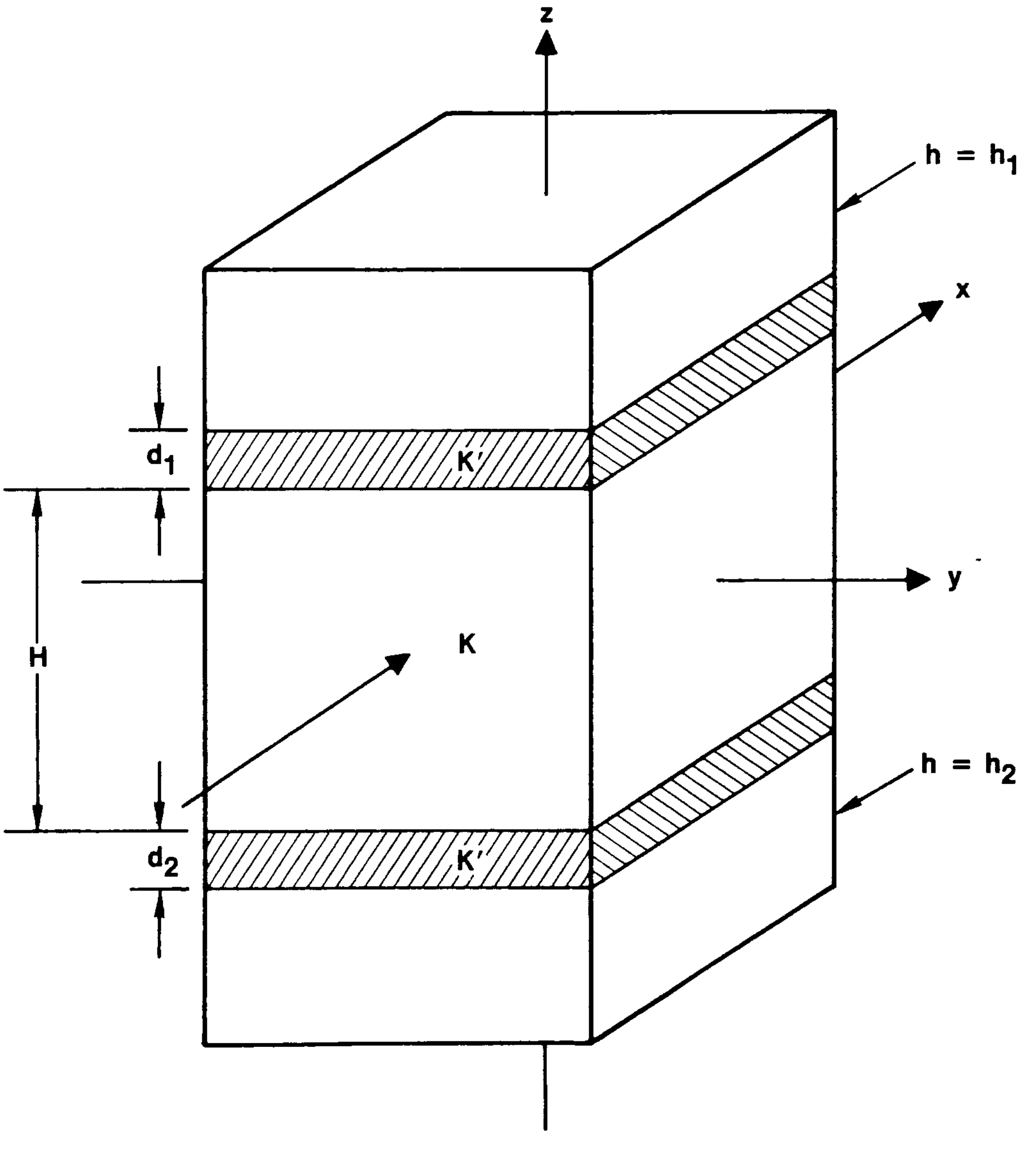


Figure 3. Flow through a unit volume of porous media with moderately confined layers

through the confining layer, equations (2.14) and (2.15) can be expanded to.

$$\left[\frac{\partial h}{\partial z}\right]_{z=H} = \frac{1}{K}[q_z]_{z=H} = \frac{K'}{Kd_1}\left([h]_{z=H} - h_1\right) \qquad (2.16)$$

$$\left[\frac{\partial h}{\partial z}\right]_{z=0} = \frac{1}{K}[q_z]_{z=0} = \frac{K'}{Kd_2}\left([h]_{z=0} - h_2\right) \qquad (2.17)$$

where: K' = hydraulic conductivity of the moderately confined layer

d = thickness of the moderately confined layer.

If instead of $[h]_{z=H}$, the value of head at the upper side of the main flow system, the average head value is used, equation (2.16) becomes

$$\left[\frac{\partial h}{\partial z}\right]_{z=H} = \frac{\tilde{h} - h_1}{Kc_1}$$

where: $c_1 = d_1/K'$ = hydraulic resistance of the confining layer.

Similarly,

$$\left[\frac{\partial h}{\partial z}\right]_{z=0} = \frac{\tilde{h} - h_1}{Kc_2}$$

and the basic equation becomes

$$KH\left(\frac{\partial^2 \tilde{h}}{\partial x^2} + \frac{\partial^2 \tilde{h}}{\partial y^2}\right) + \frac{\tilde{h} - h_1}{c_1} - \frac{\tilde{h} - h_2}{c_2} = 0 \qquad (2.18)$$

The only step of an approximate nature in the derivation is the replacement of $[h]_{z=H}$ and $[h]_{z=0}$ by $\tilde{h}$. Therefore, equation (2.18) is applicable provided that the values of head on both the upper and lower faces of the main flow system differ only slightly from the average value over z. In other words, the last terms, representing the leakage, are relatively small because of large values of c_1 and c_2.

CHAPTER 3

CALCULATOR MODEL

If an earth dam is constructed on an alluvial foundation, the hydraulic conductivity of the foundation may differ considerably from that of the dam. In this structure, two problems must be considered: flow through the dam and flow under the dam. If it is assumed that the boundary between the dam and the foundation is a flow line, no flow occurs between the dam and the foundation. Therefore, two flow fields may be considered separately. In other words, the flow through the dam can be treated as if there was an impervious foundation and the flow under the dam can be treated as if there was an impervious dam. In assuming the boundary between the dam and foundation to be a flow line, error can be introduced. The magnitude of the error depends on the difference in hydraulic conductivity between the dam and the foundation. It should be emphasized that the purpose here is to provide an estimate of seepage during the preliminary phase of the investigation. The magnitude of this error cannot be evaluated during the early phases of an investigation but should be considered when additional information becomes available during the design phase.

Flow through Dam

To analyze flow through the dam, information on the location of the free surface or upper boundary of saturated flow is required. The flow system with the unknown boundary condition can be analyzed by using an approximate method developed by Dupuit (1863) and advanced by Forchheimer (1930). The Dupuit theory of unconfined flow stems from two assumptions: (1) for small inclinations of a free surface in a gravity flow system, the streamlines can be taken as horizontal and the equipotential lines as vertical and (2) the velocity associated with these streamlines is proportional to the slope of the free surface (the proportionality factor is the hydraulic conductivity of the porous media), but is independent of the depth of the water-saturated porous media.

To calculate seepage through a homogeneous and isotropic earth dam, Shaffernak (1917) utilized the Dupuit assumptions and accounted for the development of the free surface (Harr, 1962). For the solution, consider figure 4 where line AB represents a parabolic free surface that intersects the downstream slope at a distance l_s from the foundation of the dam.

The quantity of seepage through a unit length of the dam at right angles to the cross section (Dupuit's first assumption) can be given by Darcy's law as:

$$Q = KiA \tag{3.1}$$

From Dupuit's second assumption, the hydraulic gradient, i, is equal to the slope of the seepage face and is constant with depth:

$$i = \frac{dz}{dx} = \tan \beta \tag{3.2}$$

The area per unit length of the dam can be expressed as:

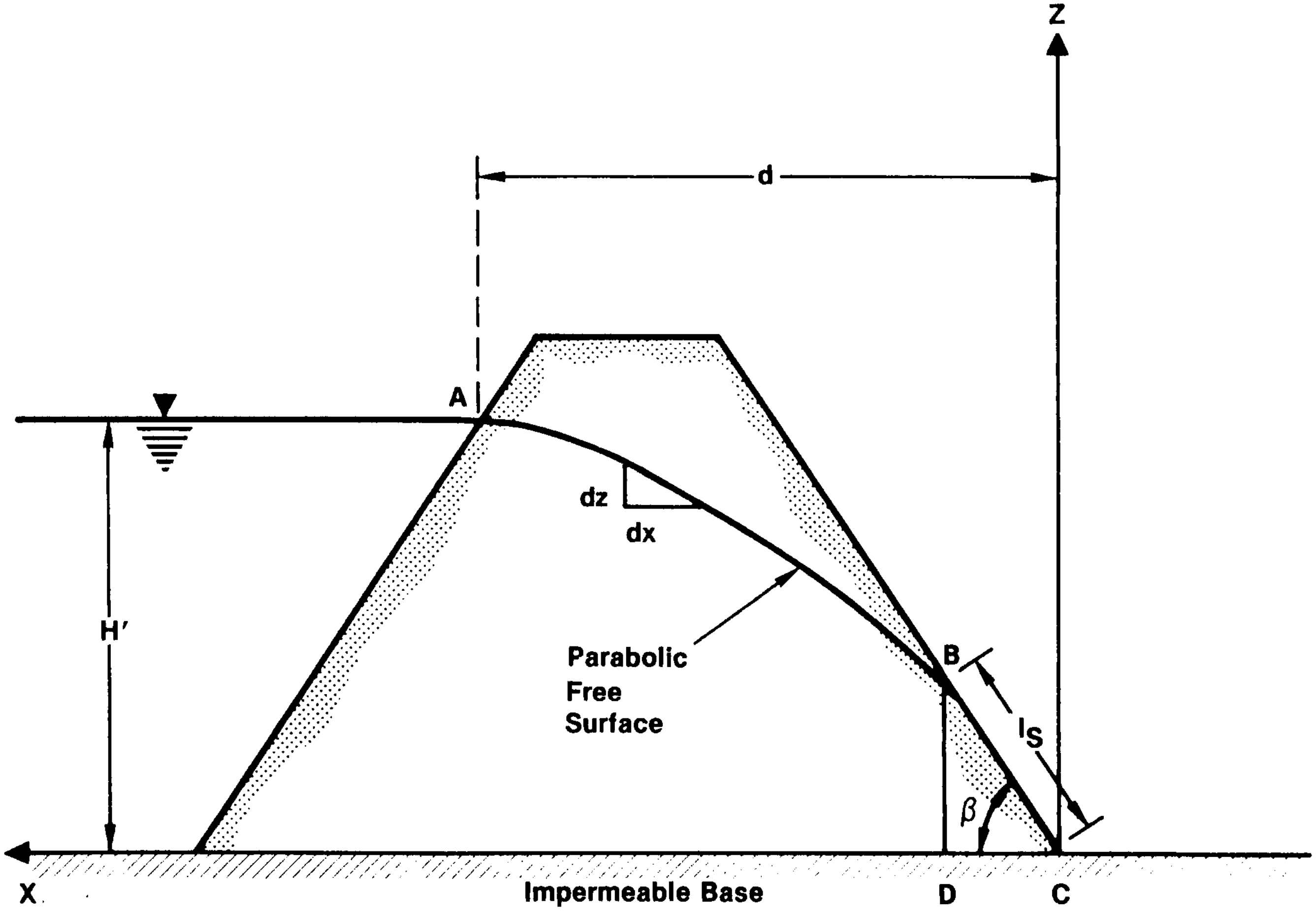


Figure 4. Schaffernak's approach for flow through an earth dam

$$A = z = l_s \sin \beta \tag{3.3}$$

where l_s is the length of the surface of seepage. Substituting equations (3.2) and (3.3) into equation (3.1) yields

$$Q = Kz \frac{\partial z}{\partial x} = Kl_s \sin \beta \tan \beta \tag{3.4}$$

Because l_s is a function of both d and H', Schaffernak (1917) suggested a graphical procedure to determine the value of l_s. This procedure is outlined in figure 5. However, l_s can also be determined by solving equation (3.4) in terms of l_s. Setting the limits of integration yields:

$$\int_{l_s \sin \beta}^{H'} Kz\,dz = \int_{l_s \cos b}^{d} K\,(l_s \sin \beta)(\tan \beta)\,dx$$

Integration and application of the fundamental theorem of calculus yields:

$$\frac{1}{2}\,(H'^2 - l_s^2 \sin^2 \beta) = (l_s \sin \beta)(\tan \beta)(d - l_s \cos \beta)$$

Expressing tan β as sin β/cos β yields:

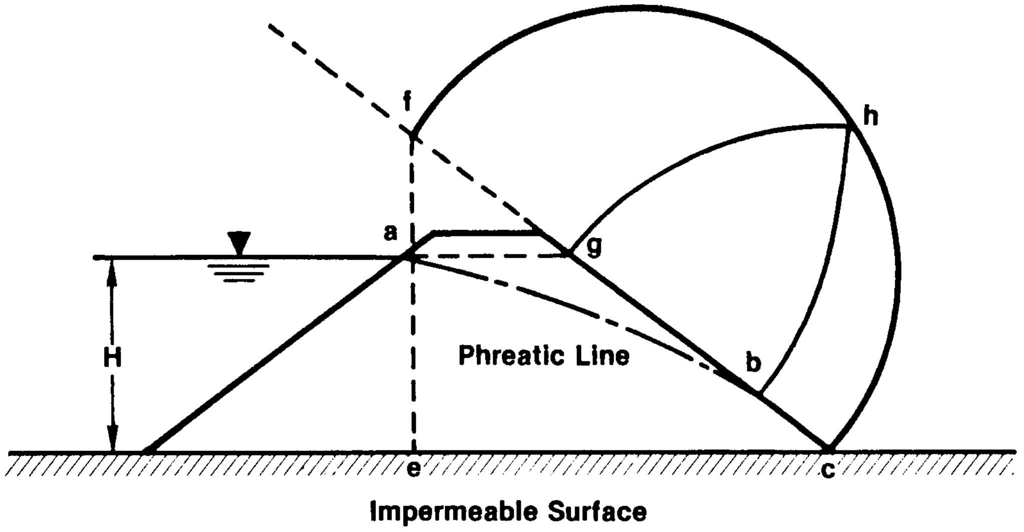
$$\frac{1}{2}\,(H'^2 - l_s^2 \sin^2 \beta) = l_s\left(\frac{\sin^2 \beta}{\cos \beta}\right)(d - l_s \cos \beta)$$

Multiplying by cos β/sin² β and simplifying yields:

$$\frac{H'^2 \cos \beta}{2\sin^2 \beta}\;\frac{l_s^2 \cos \beta}{2} = l_s d - l_s^2 \cos \beta$$

Expressing as a quadratic equation in l_s yields:

$$(\cos \beta)\, l_s^2 - (2d)l_s + \frac{H'^2 \cos \beta}{\sin^2 \beta} = 0$$



1. Extend the downstream slope line *bc* upwards.
2. Draw a vertical line *ae* through the point *a*. This will intersect the projection of line *bc* (step 1) at point *f*.
3. With *fc* as diameter, draw a semicircle *fhc*.
4. Draw a horizontal line *ag*.
5. With c as the center and *cg* as the radius, draw an arc of a circle, *gh*.
6. With *f* at the center and *fh* as the radius, draw an arc of a circle, *hb*.
7. Measure *bc* = *l*.

Figure 5. Scheffenck's graphical method to determine l_s

Applying the quadratic solution formula yields:

$$l_s = 2d \pm [4d^2 - \frac{4((H'^2 \cos^2 \beta)/(\sin^2 \beta)]^{\frac{1}{2}}}{2 \cos \beta}$$

Because the positive root would give a physically meaningless answer, simplifying the negative root yields:

$$l_s = \frac{d}{\cos \beta} - [\frac{d^2}{\cos^2 \beta} - \frac{H'^2}{\sin^2 \beta}]^{\frac{1}{2}} \qquad (3.5)$$

The distance d can now be determined by a method using trigonometry. This procedure is outlined in figure 6 and yields:

$$d = W - H' \cot \beta \qquad (3.6)$$

Therefore, the calculation of seepage per length of the dam can be summarized as follows:

$$Q_d = Kl_s \sin \beta \tan \beta \qquad (3.7)$$

where

$$l_s = \frac{d}{\cos \beta} - [\frac{d^2}{\cos^2 \beta} - \frac{H'^3}{\sin^2 \beta}]^{\frac{1}{2}} \qquad (3.8)$$

where: $d = W - H' \cot \beta$ (3.9)

Flow under Dam

An idealized case of flow under a dam is presented in figure 7. On one side of the dam, water infiltrates vertically from the reservoir into the foundation through the confining layer. On the

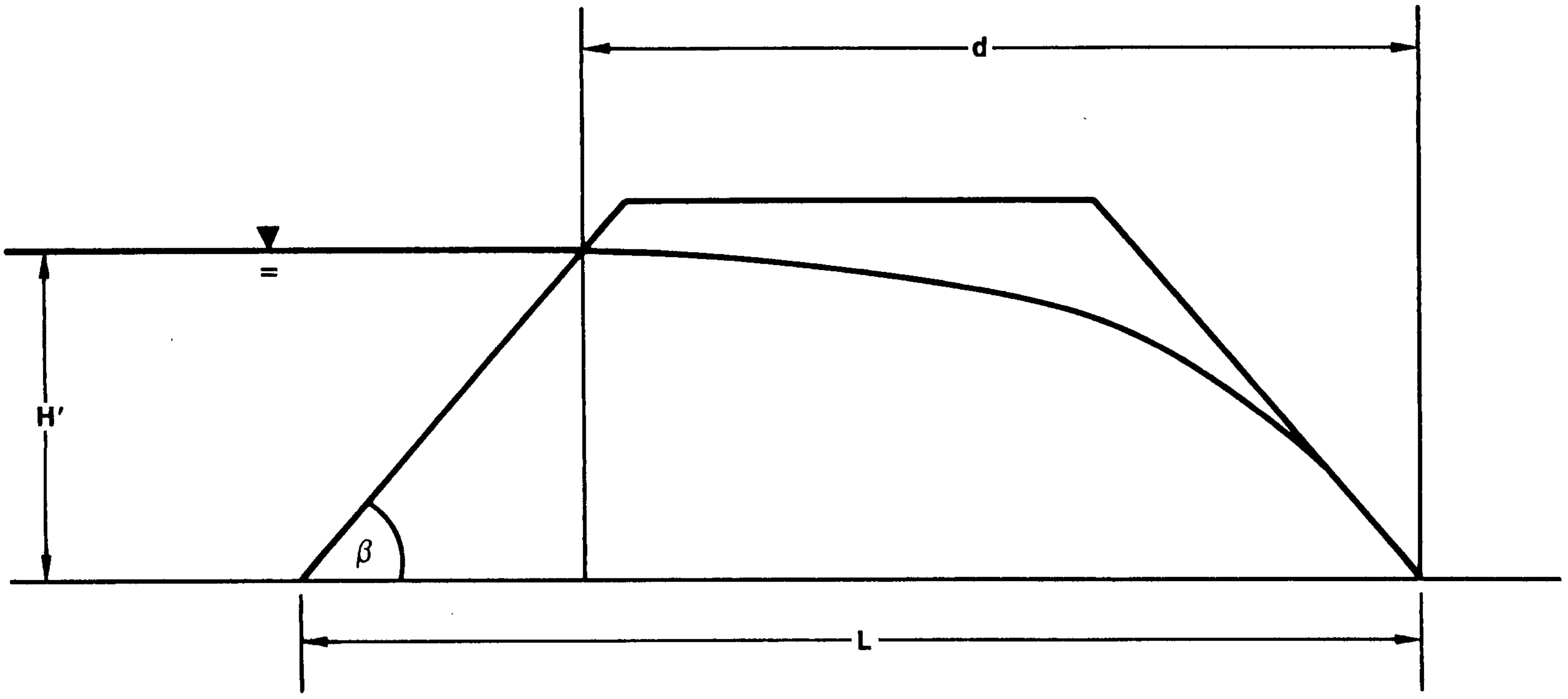


Figure 6. Method to determine d

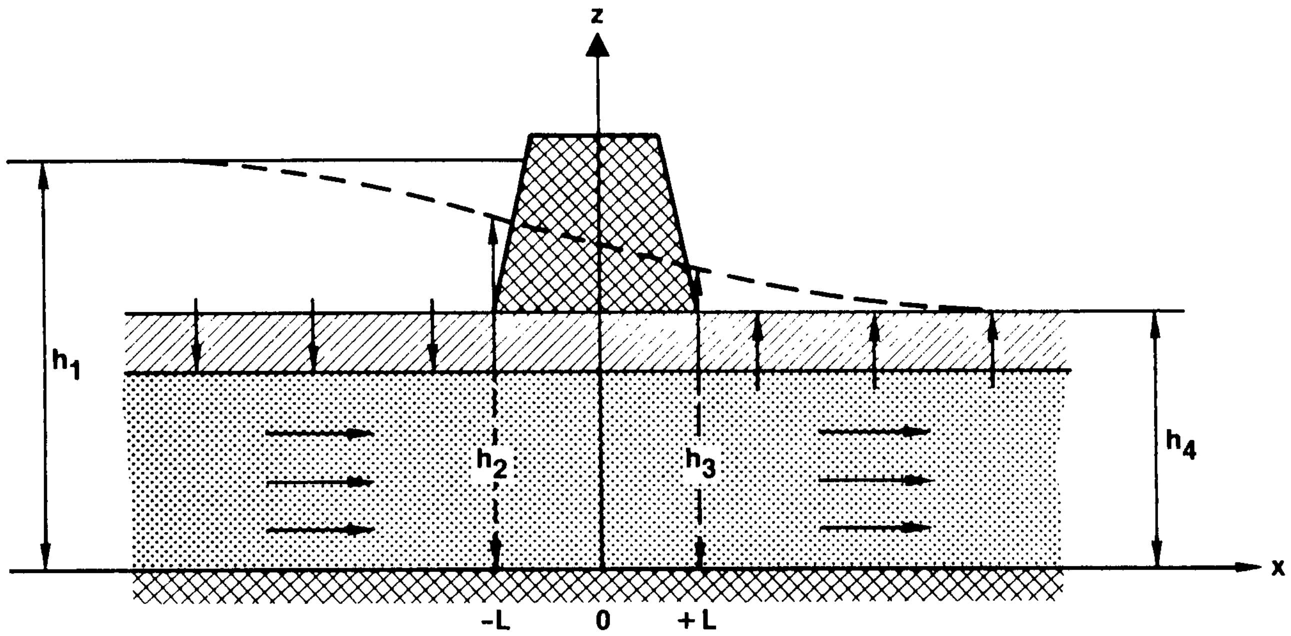


Figure 7. Flow under a dam, idealized case

other side of the dam, water will seep upward through the confining layer.

Because flow under the dam is in the x-direction, there is no variation of head with y, that is dh/dy = 0. Therefore, equation (2.18), derived earlier, reduces to:

$$KH \frac{\partial^2 h}{\partial x^2} + \frac{h - h_1}{c} = 0 \tag{3.10}$$

where c is the hydraulic resistance of the confining layer. Equation (3.10) can also be written as:

$$\frac{\partial^2 h}{\partial x^2} + \frac{h - h_1}{\lambda^2} = 0 \tag{3.11}$$

where

$$\lambda = (KHc)^{\frac{1}{2}} \tag{3.12}$$

is defined as the leakage factor.

In this form, the general solution of the differential equation is:

$$h - h_1 = Ae^{x/\lambda} - Be^{-x/\lambda} \tag{3.13}$$

where A and B are arbitrary constants. That equation (3.13) is a solution of equation (3.10) is easily verified by direct substitution. The two constants A and B, characteristic of a second-order differential equation, must be determined from the boundary conditions. To obtain the boundary conditions and the particular solution, the foundation is separated into three parts (refer to fig. 7):

Region 1: $-\infty < x < -L$

Region 2: $-L < x < +L$

Region 3: $+L < x < +\infty$

Solutions will be established for these three regions, thereby introducing the values h_2 and h_3, which represent the head at $x = -L$ and $x = +L$, respectively, as unknown parameters.

Region 1

In the region $-\infty < x < -L$, infiltration from the reservoir occurs. The boundary conditions are:

$x = -\infty \quad : \quad h = h_1$

$x = -L \quad : \quad h = h_2$

Therefore, the particular solution for region 1 is:

$$h = h_1 - (h_1 - h_2)e^{(x+L)/\lambda} \tag{3.14}$$

Specific discharge, q, can be calculated by Darcy's law:

$$q_1 = -K\frac{dh}{dx} = K\frac{h_1 - h_2}{\lambda} e^{(x+L)/\lambda} \tag{3.15}$$

Therefore, total discharge through a unit length of the dam at $x = -L$ is:

$$Q_1 = KH\frac{h_1 - h_2}{\lambda} \tag{3.16}$$

Region 2

In the region $-L < x < +L$, there is no infiltration. the basic differential equation in this part is therefore:

$$\frac{\partial^2 h}{\partial x^2} = 0 \tag{3.17}$$

with the general solution:

$$h = Ax + B \tag{3.18}$$

The boundary conditions are:

$x = -L \quad : \quad h = h_2$
$x = +L \quad : \quad h = h_3$

Hence, the particular solution for region 2 is:

$$h = -\left(\frac{h_2 - h_3}{2L}\right)x + \tfrac{1}{2}(h_2 + h_3) \tag{3.19}$$

Specific discharge, q, can be calculated by Darcy's law:

$$q_2 = -K\frac{\partial h}{\partial x} = \frac{h_2 - h_3}{2L} \tag{3.20}$$

Therefore, total discharge through a unit length of the dam is:

$$Q_2 = KH\,\frac{h_2 - h_3}{2L} \tag{3.21}$$

Region 3

In the region $+ L < x < + \infty$, leakage takes place in an upward direction. The general solution is:

$$h - h_4 = Ae^{x/\lambda} + Be^{-x/\lambda} \tag{3.22}$$

The boundary conditions are:

$x = +\infty \quad : \quad h = h_4$
$x = +L \quad : \quad h = h_3$

Therefore the particular solution for region 3 is:

$$h = h_4 + (h_3 - h_4)e^{-(x-L)/\lambda} \tag{3.23}$$

Specific discharge, q, can be calculated by Darcy's law:

$$q_3 = - K\frac{dh}{dx} = K\frac{h_3 - h_4}{\lambda} \quad e^{-(x-L)/\lambda} \tag{3.24}$$

Therefore, total discharge through a unit length of the dam at $x = +L$ is:

$$Q_3 = KH\frac{h_3 - h_4}{\lambda} \tag{3.25}$$

To determine the values of h_2 and h_3, it should be noted that, as continuity demands at the common boundaries, $x = \pm L$, the values of total discharge, Q_f, in equations (3.16), (3.21), and (3.25) must be equal, that is, $Q_f = Q_1 = Q_2 = Q_3$. In fact, these three equations contain the three unknown quantities: Q_f, h_2, and h_3. Solution of these equations gives:

$$h_2 = h_1 - \frac{(h_1 - h_4)\lambda}{W + 2\lambda} \tag{3.26}$$

$$h_3 = h_4 + \frac{(h_1 - h_4)\lambda}{W + 2\lambda} \tag{3.27}$$

$$Q_f = \frac{KH(h_1 - h_4)\lambda}{W + 2\lambda} \tag{3.28}$$

where: $W = 2L$

Again, that these equations are the solution can be easily verified by substitution. Equation (3.28) gives the total seepage under the dam per unit length of the dam. Also, because h_2 and h_3 are known, the head in any point of the foundation can be calculated by equations (3.14), (3.19), or (3.23), depending on the region of interest.

User Instructions and Calculator Code

The calculator model, DAM SEEPAGE, calculates:

1. Line of seepage l_s by equation (3.8).
2. Distance d by equation (3.9).
3. Flow through the dam Q_d by equation (3.7).
4. Leakage factor λ by equation (3.12).
5. Head values, h_2 and h_3, by equations (3.26) and (3.27).
6. Flow under the dam Q_f by equation (3.28).
7. Total flow through the system.

User instructions and calculator codes are presented in Appendix A.

DAM SEEPAGE estimates seepage loss from a reservoir using any consistent units. For example, if conductivities are expressed in ft/min and lengths in feet, flow rates will be expressed as ft^3/min.

Idealized Case

An idealized case is presented in figure 8, and parameters, program labels, and input/output are given in table 1.

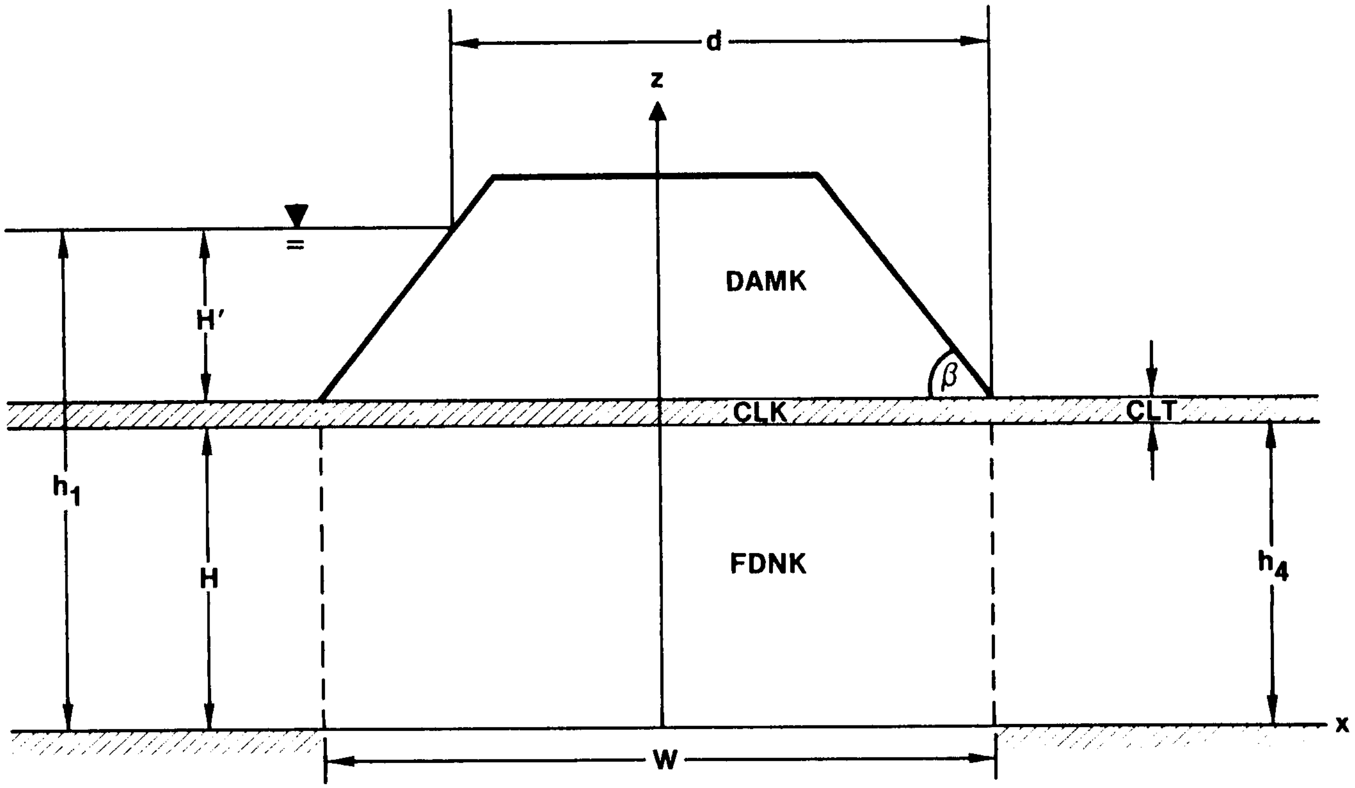


Figure 8. Application of model to idealized case

Table 1. Idealized case with program labels and input/output

Parameter	Program Label	Input (Output)
Hydraulic conductivity		
Foundation	FDNK	10^{-5} ft/min
Dam	DAMK	10^{-7} ft/min
Confining layer	CLK	10^{-8} ft/min
Foundation thickness	H4	100 ft
Reservoir depth	H'	50 ft
Confining layer thickness	CLT	5 ft
Dam width	W	300 ft
Dam length	LNTH	500 ft
Embankment face angle (β)	BETA	20°
Main flow system under dam	H	(95 ft)
Confining layer resistance (c)	RES	(5×10^{-8} min)
Leakage factor (λ)	LAM	(689 ft)
Line of seepage (l_s)	LOS	(80 ft)
Head values		
h_1	H1	(150 ft)
h_2	H2	(129 ft)
h_3	H3	(120 ft)
h_4	H4	(100 ft)
Flow through dam (Q_p)	QDAM	(5×10^{-4} ft³/min)
Flow under dam (Q_f)	QFDN	(0.014 ft³/min)
Total flow	QTOT	(0.014 ft³/min)

CHAPTER 4

DESCRIPTION OF STUDY AREAS

The model developed in this study was applied to three cases, two proposed off-channel storage structures on Thirteenmile Creek and Burns Creek in eastern Montana and the Site 13 flood-water-retarding structure on Sugar Creek in central Oklahoma.

The proposed structures in eastern Montana would be used to provide a water supply for a coal-to-gasoline conversion facility proposed by Mobil Oil Corporation. These sites, considered for off-channel storage of Yellowstone River water, were selected on the basis of storage potential and proximity to the proposed coal-conversion facility (Law Engineering, 1982; Harza Engineering, 1981).

The Sugar Creek structure was chosen by Naney (1974) and Naney and Thompson (1979) to study the impact of an earth dam on the ground-water flow regime. These studies presented comparison of two flow nets. One flow net was prepared from data obtained through extensive instrumentation of the site. The other flow net was prepared from data generated by the finite-difference model STEADY, developed by Nelson (1962). Results of the model compared favorably with results of the piezometric surface map.

Eastern Montana Sites

The Thirteenmile Creek and Burns Creek sites are located in the Northern Great Plains physiographic province near Glendive, Montana (fig. 9). The area is a dissected tableland with average relief near the mouths of the creeks on the order of 200 to 400 feet (Law Engineering, 1982).

The area lies on the southwestern flank of the Williston basin, a major sedimentary basin centered in western North Dakota. Bedrock strata dip gently northeast (1-2 degrees) toward the center of the basin (Torrey and Kohout, 1956).

Tertiary sedimentary rocks of the Fort Union Formation occur at the surface or beneath a relatively thin mantle of Quaternary deposits throughout the study area. The Fort Union Formation is a continental clastic deposit widely exposed throughout the region. The Formation is generally composed of soft carbonaceous shales, claystones, clayey sandstones, and lightly cemented sandstones. The outcropping member in the study areas, the Tongue River, was deposited in a cyclic deltaic environment, which existed during the Paleocene Epoch along a continental marine margin (Howard, 1960; Matson and Pinchook, 1977).

The lower reaches of Thirteenmile Creek, located 2 miles west of Intake, Montana, is near the limit of the advance of Quaternary continental glaciers, which moved southward up the Yellowstone Valley. North of the glacial margin, the topography is typified by gently rolling hills. In these areas, the Fort Union Formation is covered, except in erosional valleys, by a mantle of sediments of glacial origin. South of the glacial margin, the land surface rises to higher elevations and the Fort Union Formation occurs at the surface. Badlands-type topography, characteristic of the erodible Fort Union sediments, is common south of the glacial margin but occurs only along erosional valleys north

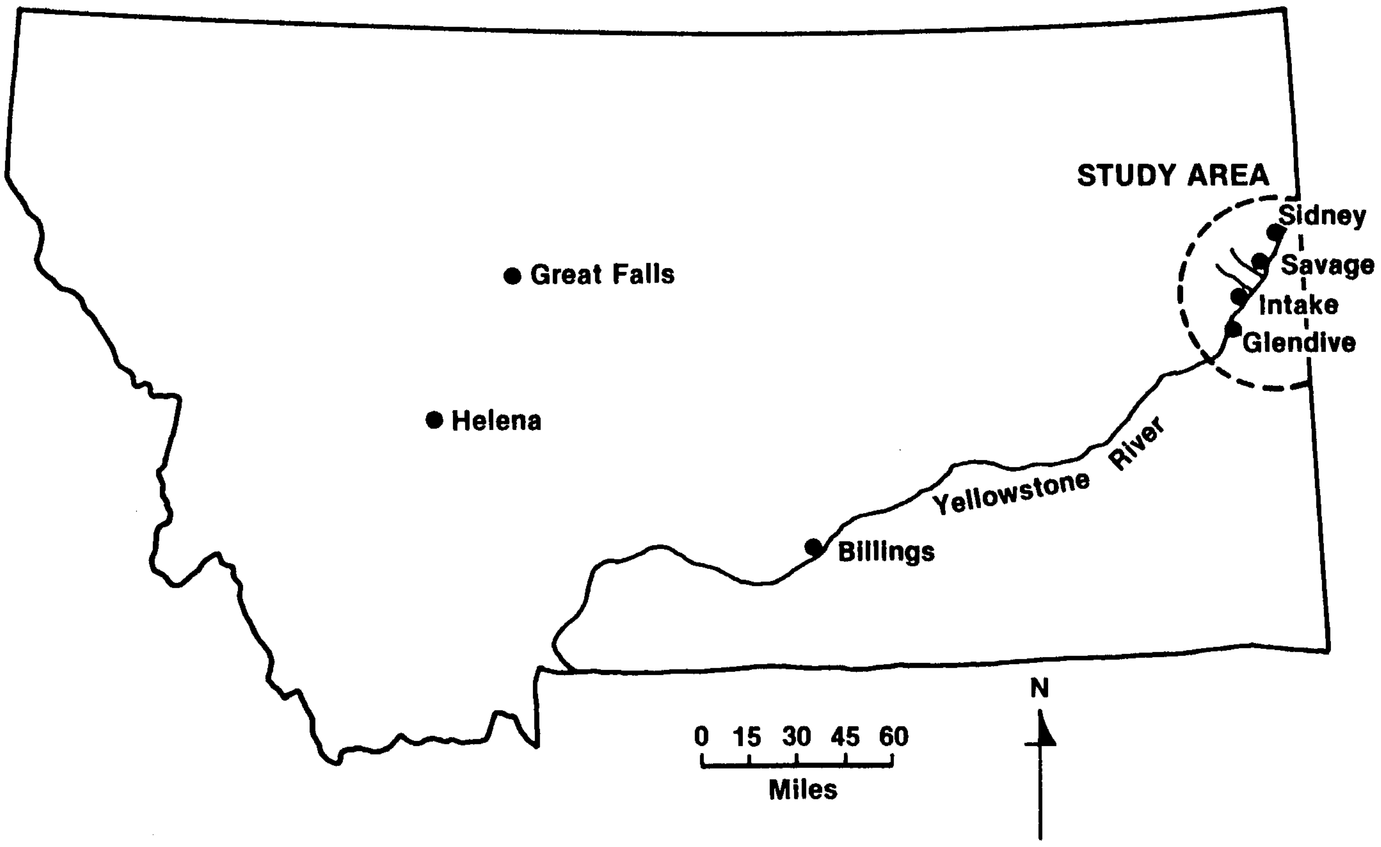


Figure 9. Location map for eastern Montana study areas

of the margin. The postglacial erosion and deposition in the area have resulted in the presence of alluvial terraces, colluvial debris, recent stream-bed alluvium, and a thin veneer of windblown silts in the valleys of Thirteenmile and Burns Creeks (Howard, 1960).

Both the bedrock formations and the unconsolidated materials mantling the bedrock contain ground water and are the source of water for industrial and municipal use as well as for domestic and stock water in the rural area (Torrey and Kohout, 1956).

Central Oklahoma Site

The Site 13 structure is located in the Wahita River watershed near Hinton, Oklahoma (fig. 10). The area is moderately to sharply rolling. The erosion-resistant sandstone in the Permian Rush Springs Formation acts as the controlling feature and generally exhibits steep-sided gullies and canyons with relief on the order of 100 feet (Naney, 1974).

The Permian Marlow Formation and the Permian Rush Springs Formation, which together make up the White Horse Group, crop out in the study area (Davis, 1955). The Marlow Formation, characterized as a red-brown siltstone with lenses of very fine sandstone and gypsiferous stringers, crops out in the area in badly eroded gullies and along roadsides. Naney (1974) reported that the Marlow Formation is in contact with the alluvium in the study area and serves as a lower boundary for determining the extent of ground-water seepage out of the reservoir. The Marlow Formation is disconformably overlain by the Rush Springs Formtion. However, the Rush Springs thins to only a

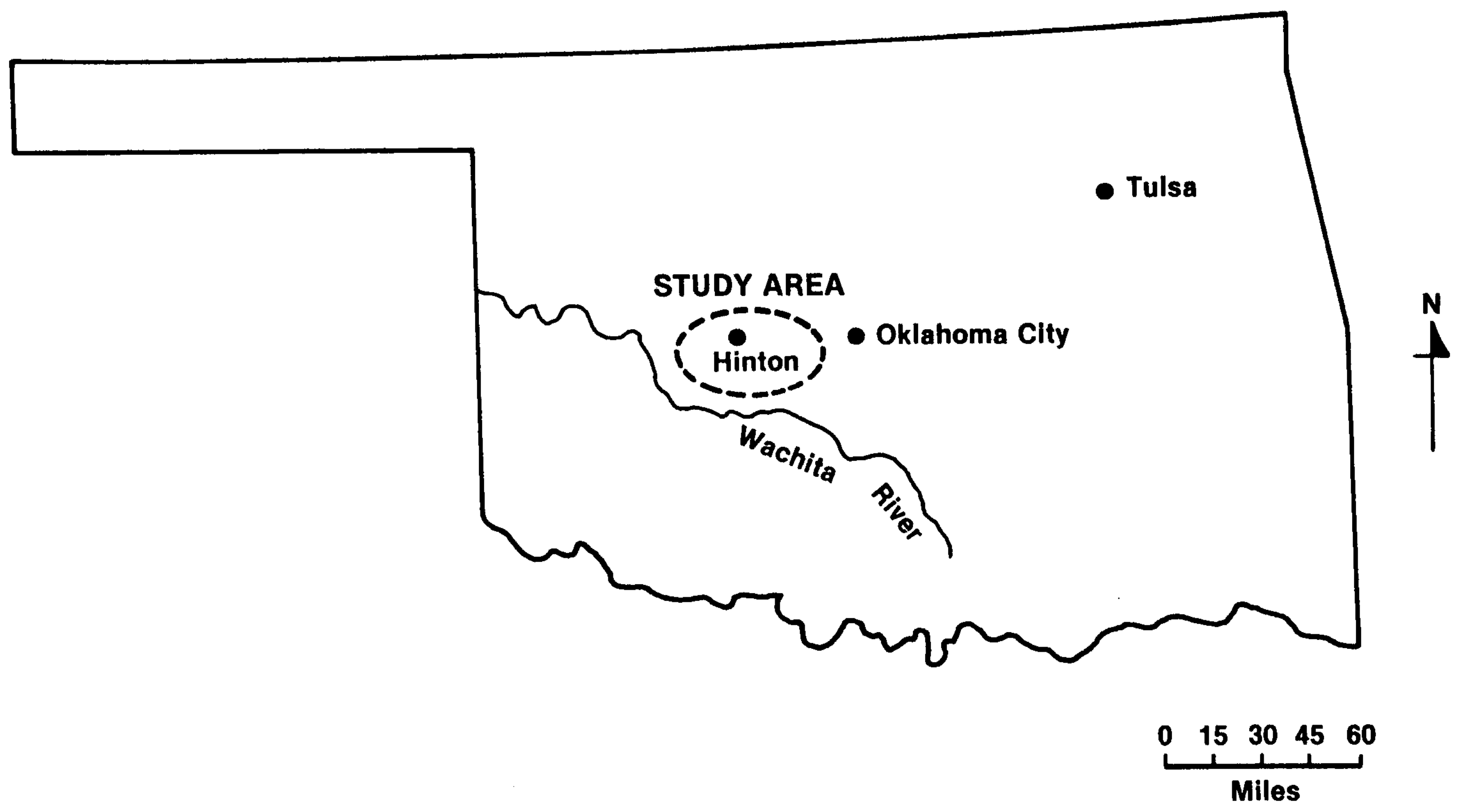


Figure 10. Location map for Site 13 structure, central Oklahoma

few feet in the area of the dam site and is not considered significant to the study (Naney, 1974).

The Quaternary sediments exposed in the area are made up entirely of nonindurated flood-plain alluvium. These materials range in size from clays through coarse sands but are primarily in the clay, silt, and very fine sand range. Gradational variations of grain size of the sediments within the alluvium commonly occur both vertically and horizontally (Naney, 1974).

CHAPTER 5

APPLICATION OF MODEL DAM SEEPAGE

Model DAM SEEPAGE developed and outlined in Chapter 2 and 3 was applied to the Thirteenmile Creek and Burns Creek sites in eastern Montana and the Site 13 in central Oklahoma described in Chapter 2. The input parameters for the model are derived from data presented by Law Engineering (1982), Naney (1974), and Naney and Thompson (1979). Simplifying assumptions such as a homogeneous and isotopic dam and foundation were used in model applications.

Because the sites in eastern Montana involve proposed structures, various values of hydraulic conductivity (dam, foundation, and confining layer), confining layer thickness, and reservoir depth were used to generate a range of seepage values. Again, the purpose was to produce a preliminary estimate for seepage based on limited subsurface information. Because the site in central Oklahoma has been extensively monitored and modeled, DAM SEEPAGE was applied with input parameters from Naney (1974) and Naney and Thompson (1979) to check the accuracy of DAM SEEPAGE.

Eastern Montana Sites

In evaluating the potential for off-channel storage of Yellowstone River water to provide a water supply for a proposed coal-conversion facility, a geotechnical feasibility analysis was performed by Law Engineering (1982) for Mobil Oil Corporation. Law Engineering

(1982) reported on two potential dam sites near Glendive, Montana: one on Thirteenmile Creek near Intake, Montana, in Dawson County and the second on Burns Creek in Richland County. Each dam and reservoir site provides a storage capacity greater than 35,000 acre-feet.

Thirteenmile Creek Site

The proposed dam on the Thirteenmile Creek site would be about 2,700 feet long at the crest elevation (2,164 feet MSL) and would extend across the valley in a generally east-west direction. At maximum section, the dam would be about 125 feet above the natural valley floor (Law Engineering, 1982).

As part of the geotechnical investigation, Law Engineering determined or estimated the physical properties of the subsurface materials at the Thirteenmile Creek site through field and laboratory tests. Table 2 summarizes the stratigraphic column, and table 3 summarizes the data pertinent to this study for each stratigraphic horizon.

On the basis of their investigation, Law Engineering (1982) recommended:

1. The colluvium, alluvium, and terrace silts be removed from the embamkment area.
2. The lacustrine clays be utilized as the embankment materials.
3. The alluvium underlying the terrace silts be removed from a portion of the embankment sufficient to preclude seepage.
4. The interbedded sands, clays, and lignites of the Fort Union Formation constitute the foundation for the proposed dam.

In addition, an embankment with 3H:1V (horizontal to vertical) slopes

Table 2. Stratigraphic column for Thirteenmile Creek site. -- After Law Engineering (1982)

Geologic Age	Stratigraphic Unit	Thickness Range (ft)	Lithologic Description
Holocene	Loess	0-1	Fine sandy silt, buff to tan; soft.
	Alluvium	0-13	Fine to coarse sandy gravel with cobbles; brown to dark-brown; very firm to dense.
	Colluvium	0-5	Clayey silt with subangular rock fragments: tan; stiff.
	Terrace silt	11-24	Fine to medium sandy silts with thin, sandy, gravel lenses: light brown to brown; loose to firm.
Pleistocene	Upper glacio-fluvial deposits	0-35	Lacustrine silts and clays; stiff, alluvial silts, sands, and gravels; firm to dense; interbedded; occasional organic-rich zones.
	Glacial lacustrine deposits	57-84	Upper portion: slightly fine sandy, silty clay and clayeye silt; dark gray to brown; stiff. Lower portion: thinly varved clay; gray.
	Lower glacio-fluvial deposits and colluvium(?)	70-76	Slightly silty, fine to coarse sandy gravel and silty, fine to medium sand; dark brown, very dense
Paleocene	Fort Union Formation, Tongue River Member	1,000 (estimated)	Alternating cyclic strata consisting laminated to thin-bedded shale, weakly cemented; carbonaceous shale, fractured fractured lignite; siltstone, and fine-grained sandstone; massive.

Table 3. Subsurface material properties, Thirteenmile Creek site. -- Data from Law Engineering (1982)

Stratigraphic Unit	Void Ratio	Porosity	Hydraulic Conductivity ft/min
Colluvium	0.78	0.44	10^{-5} to 10^{-7}
Alluvium	N/A	N/A	10^{0} to 10^{-2}
Terrace silts	1.01	0.50	10^{-5} to 10^{-6}
Lacustrine silty clay	0.50	0.33	$<10^{-6}$
Lacustrine varved clay	0.50	0.33	$<10^{-6}$
Lower alluvium	N/A	N/A	10^{-2}
Fort Union clay (shale)	0.45	0.31	[a] 10^{-5} to 10^{-6}
Fort Union "erodible sand"	1.05	0.51	[b] 10^{-4} to 10^{-5}

a. In lignites, 10^{-3} to 10^{-4}.

b. Much higher in joints.

was recommended based on the strength of the lignite, the most problematic foundation element. A summary of model input and output is presented in table 4.

Burns Creek Site

The proposed dam on the Burns Creek site would be about 3,200 feet long at the crest elevation (2,177 feet MSL) and would extend across the valley in a northeast-southwest direction. At maximum section, the dam would be about 110 feet above the natural valley floor (Law Engineering, 1982).

Law Engineering (1982) determined or estimated the physical properties of the subsurface materials on the basis of field and laboratory tests. The stratigraphic column is summarized in table 5 and the data pertinent to this study for each stratigraphic horizon is summarized in table 6.

On the basis of their investigation, Law Engineering (1982) recommended:

1. Removal of colluvium and surficial fine-grained soils.
2. Embankment be either a traditional zoned-earth fill embankment or an earth-fill embankment with an impermeable upstream face.
3. Proposed dam be founded in the gravelly alluvium.

Although the gravelly alluvium would present a stable foundation, the permeability and depth of the gravel would result in excessive seepage beneath the dam. However, complete removal of the gravelly alluvium to preclude seepage would be impractical as this would require extensive dewatering and excavation approaching 50 feet deep (Law

Table 4. Input and output values for Thirteenmile Creek site. -- Data from Law Engineering (1982)

Parameter	Values
Hydraulic conductivity	
Foundation	10^{-4} ft/min
Dam	10^{-7} ft/min
Confining layer	10^{-5} ft/min
Foundation thickness	1,000 ft
Reservoir depth	110 ft
Confining layer thickness	20 ft
Dam width	1,000 ft
Dam length	2,700 ft
Embankment face angle	18.43°
Flow through dam	0.002 ft^3/min
Flow under dam	1.00 ft^3/min
Total flows	1.00 ft^3/min

Table 5. Stratigraphic column for Burns Creek site. -- After Law Engineering (1982)

Geologic Age	Stratigraphic Unit	Thickness Range (ft)	Lithologic Description
Holocene	Loess	0-1	Fine sandy silt; buff to tan; loose.
	Alluvium	0-50	Slightly fine to coarse sandy gravel with cobbles; brown to dark brown; very firm to dense.
	Colluvium	1-15	Fine sandy, clayey silt with subangular rock fragments; tan to brown; firm to stiff.
	Terrace silt	0-15	clayey, silty, fine to medium sand with gravel lenses; light brown to brown, loose to firm.
Pleistocene	Glacio-lacustrine deposits	5-20	Clayey, fine sandy silt with thinly varbed, slightly silty, clay zones; gray and dark brown; stiff to very stiff.
	Glaciofluvial deposits	9-17	Very fine to fine sand with fine to coarse sandy, gravel zones; dark gray to black; stiff to very stiff.
Paleocene	Fort Union Formation, Tongue River	1,000 (estimated)	Alternating cyclic strata consisting of laminated to thin-bedded shale. weakly cemented; carbonaceous shale; fractured lignite; siltstone and fine-grained sandstone; massive.

Table 6. Subsurface material properties, Burns Creek site. -- Data from Law Engineering (1982)

Stratigraphic Unit	Void Ratio	Porosity	Hydraulic Conductivity ft/min
Colluvium	0.77	0.43	10^{-4} to 10^{-6}
Alluvium	N/A	N/A	10^{0} to 10^{-5}
Terrace silts	0.78	0.44	10^{-3} to 10^{-5}
Lacustrine	0.77	0.44	$<10^{-6}$
Fort Union clay (shale)	0.39	0.28	[a]10^{-5} to 10^{-6}
Fort Union "very fine grained sand"	0.92	0.48	10^{-4} to 10^{-5}

a. In lignites, 10^{-3} to 10^{-4}.

Engineering, 1982). Therefore, Law Engineering recommended installation of a relatively narrow cutoff wall through the alluvium. This is simulated in the model by a relatively thick and tight confining layer.

Law Engineering (1982) concluded that the upstream embankment alternative offers no cost advantage over the traditional zoned design. Therefore, the model was applied to the traditional zoned design using an average value for conductivity. A summary of model input and output for the Burns Creek site is presented in table 7.

Central Oklahoma Site

Naney (1974) and Naney and Thompson (1979), in cooperation with the Southern Great Plains Research Watershed at Chickasha, Oklahoma, investigated the impact of an earth dam on the ground-water flow regime in a typical upstream tributary of the Washita River. The Site 13 dam was chosen for extensive instrumentation, and results were compared to a mathematical model applied to predict the impact of the dam on the ground-water flow regime.

The Site 13 dam is about 1,000 feet long and extends across a broad, flat alluvial flood plain in an east-west direction and is founded on 50 feet of alluvium (Naney, 1974). A summary or model input and output for the Site 13 dam is presented in table 8.

The thickness of the confining layer is estimated at 0.5 feet. Naney (1974) gave no indication of grouting or other foundation preparation to reduce seepage as would be expected in a flood-water retention structure. Conductivity of the confining layer is therefore assumed to be 0.2 ft/d, the same as that of the dam. Dam width,

Table 7. Input and output values for Burns Creek site. -- Data from Law Engineering (1982)

Parameter	Values
Hydraulic conductivity	
Foundation	10^{-3} ft/min
Dam	10^{-7} ft/min
Confining layer	10^{-8} ft/min
Foundation thickness	80 ft
Reservoir depth	110 ft
Confining layer thickness	10 ft
Dam width	600 ft
Dam length	3,000 ft
Embankment face angle	18.43°
Flow through dam	0.002 ft^3/min
Flow under dam	1.42 ft^3/min
Total flow	1.42 ft^3/min

Table 8. Input and output values for Site 13 dam

Parameter	Values
Hydraulic conductivity	
Foundation	[a]1.43 ft/d
Dam	[c]0.2 ft/d
Confining layer	[c]0.2 ft/d
Foundation thickness	[a]50 ft
Reservoir depth	[b]15 ft
Confining layer thickness	[c]0.5 ft
Dam width	[b]40 ft
Dam length	[a]1,000 ft
Embankment face angle	[b]45°
Flow through dam	1,000 ft^3/day
Flow under dam	15,941 ft^3/day
Total flow	16,941 ft^3/day

a. Reported by Naney (1974).

b. Estimated from Naney (1974)

c. Assumed value.

reservoir depth, and embankment angle were estimated from diagrams, maps, and cross sections reported by Naney (1974).

Results

Eastern Montana Sites

Several model runs were made on the Thirteenmile Creek and Burns Creek sites in eastern Montana to obtain a range of seepage values. The following input parameters were varied based on data from Law Engineering (1982):

1. Foundation hydraulic conductivity.
2. Dam hydraulic conductivity.
3. Reservoir depth.
4. Confining layer thickness.
5. Confining layer conductivity.

A sensitivity analysis was conducted by varying the values for items (4) and (5) to evaluate the effect of foundation grouting and cutoffs to reduce seepage. Although the quantitative relationship between the extent of foundation grouting and depth of cutoffs and magnitude of confining layer thickness and conductivity is not known, it can be easily seen that they are related. Summaries of the model runs are presented in Appendices B through F.

Results of the sensitivity analysis indicate that changes in the foundation hydraulic conductivity have the most effect on seepage. Also, changes in the dam hydraulic conductivity have the least effect. This can be expected because, as dam hydraulic conductivity decreases, seepage through the dam becomes insignificant compared to seepage under the dam. In other words, the medium with the higher hydraulic conductivity has more effect on seepage than the medium with the lower hydraulic conductivity.

At the Thirteenmile Creek site, the lowest seepage rate was 0.079 ft^3/min and the highest was 3.25 ft^3/min. These represent seepage rates from 1 to 39 acre-feet/yr. At the Burns Creek site, the seepage rates of 0.11 ft^3/min and 46.52 ft^3/min represent a seepage range of 1 to 561 acre-feet/yr.

To achieve estimates for worst-case scenarios, input values that yielded the highest seepage rates were combined. Summaries of input and output are presented in table 9. At the Thirteenmile Creek site, the worst-case seepage rate was 20.48 ft^3/min (247 acre-feet/yr) and for the Burns Creek site, 224.26 ft^3/min (2,706 acre-feet/yr).

On the basis of these results, it appears that the Thirteenmile Creek site has an advantage over the Burns Creek site (highest rate: 39 vs. 424 acre-feet/yr; worst case: 247 vs. 2,093 acre-feet/yr). Law Engineering (1982) also recommended the Thirteenmile Creek site on the basis of estimated embankment construction costs.

Central Oklahoma Site

The Site 13 structure was selected for this study to check the accuracy of DAM SEEPAGE. Naney (1974) and Naney and Thompson (1979) reported a measured seepage rate of 19,460 ft^3/day and a modeled seepage rate of 16,674 ft^3/day. Results (table 12) of DAM SEEPAGE yield a seepage rate of 16,941 ft^3/day (142 acre-feet/yr).

Table 9. Input and output values for the Thirteenmile Creek and Burns Creek sites, worst case

Parameter	Thirteenmile Creek Site Values	Burns Creek Site Values
Hydraulic conductivity		
Foundation	10^{-3} ft/min	10^{0} ft/min
Dam	10^{-6} ft/min	10^{-6} ft/min
Confining layer	10^{-7} ft/min	10^{-7} ft/min
Foundation thickness	1,000 ft	50 ft
Reservoir depth	115 ft	115 ft
Confining layer thickness	5 ft	5 ft
Dam width	1,000 ft	600 ft
Dam length	2,700 ft	3,200 ft
Embayment face angle	18.43°	18.43°
Flow through dam	0.03 ft^3/min	0.008 ft^3/min
Flow under dam	20.45 ft^3/min	224.26 ft^3/min
Total flow	20.48 ft^3min	224.26 ft^3/min

This represents a value within 2 percent of previously modeled results and within 13 percent of previously measured values.

The assumed values for dam hydraulic conductivity, confining layer thickness, and confining layer conductivity may introduce some error into the estimate of seepage rate. However, examination of results in Appendices B through F indicates that dam hydraulic conductivity has a small effect on total seepage rate. Confining layer conductivity, especially if it is equal to or less than than of the dam, also has a small effect on the seepage rate. Small values for confining layer thickness, however, can have a significant effect on seepage rate. Because the Site 13 structure is a flood-water retaining structure (Naney, 1974; Naney and Thompson, 1979), seepage reduction in the foundation would probably not be a major design consideration. Thickness of the confining layer would probably consists of reservoir sediment and would likely be small; a value of 0.5 ft was assumed.

CHAPTER 6

SUMMARY AND CONCLUSIONS

The purpose of this study was to develop a model suitable for use on a TI-59 programmable calculator to estimate seepage from a reservoir. The model, DAM SEEPAGE, was designed to obtain a preliminary estimate of seepage. DAM SEEPAGE's basic approach is the separation of flow through the dam and flow under the dam by assuming that the boundary between the dam and the foundation is a streamline and that no seepage between the dam and foundation occurs. This, and other simplifying assumptions used in the development of the model algorithm, does not severely restrict the use of DAM SEEPAGE in a preliminary investigation or in a field situation where detailed subsurface information is limited.

Although a modeling effort should be conducted with a method that maintains the integrity of the hydrologic and geologic characteristics of the flow system, the output of a model is no better than the input. Therefore, the use of a sophisticated model such as STEADY developed by Nelson (1962) and used by Nancy (1974) and Naney and Thompson (1979) in their study would be inappropriate for a proposed dam where subsurface information is limited. In other words, the effort involved in modeling should be in proportion to the expected accuracy of the results.

DAM SEEPAGE was applied to three situations, two in eastern Montana and one in central Oklahoma. Several model runs were made for the eastern Montana cases to obtain a range of seepage values in order to compare the two proposed dam sites on the basis of seepage loss from the reservoirs.

Because data for the sites in eastern Montana were incomplete, several model runs were made for each site to obtain a range of seepage values. One model parameter was varied while holding the others constant. Results of these runs indicate that changes in foundation hydraulic conducitivity has the greatest effect on seepage and changes in dam hydraulic conductivity has the least effect. DAM SEEPAGE was then used for data from an earth dam in central Oklahoma and the results compared to the results of two studies (Naney, 1974; Naney and Thompson, 1979) to check the accuracy of the calculator model. The comparison yielded seepage values within 2 percent of those from the finite-difference model STEADY and within 13 percent of the results of piezometric surface results.

This study has shown that simplified representations of a flow system and the differential equations that describe it can be modeled with a programmable calculator to obtain a preliminary estimate of seepage from a reservoir. In the early stages of a project, DAM SEEPAGE can prove to be a cost effective alternative to cumbersome flow-net analysis and expensive finite-difference or fine-element modeling. A series of runs to obtain a range of expected seepage values takes only a few minutes and costs essentially nothing.

APPENDIX A

MODEL DAM SEEPAGE

Step	Procedure	Enter	Press	Display
1	Partition storage	3	2nd OP 17	719.29
2	Read side 1 of card 1	1		1.
3	Read side 2 of card 1	2		2.
4	Read side 1 of card 2	3		3.
5	Read side 2 of card 2	4		4.
6	Enter foundation hydraulic conductivity	FDNK	A	FDNK
7	Enter dam hydraulic conductivity	DAMK	2nd A	DAMK
8	Enter foundation thickness	H4	B	H4
9	Enter reservoir depth	H'	2nd B	H'
10	Enter confining layer thickness	CLT	C	CLT
11	Enter confining layer conductivity	CLK	2nd C	CLK
12	Enter dam width	W	D	W
13	Enter embankment face angle	BETA	2nd D	BETA
14	Enter dam length	LNTH	E	LNTH
15	Watch printer			
16	Repeat steps 6-13 to change any parameter			
17	Reenter dam length whether changed or not	LNTH	E	LNTH

```
000    76    LBL
001    11     A
002    42    STO
003    01     01        Enter FDNK
004    91    R/S   ____
005    76    LBL
006    16    A"
007    42    STO
008    02     02        Enter DAMK
009    91    R/S   ____
010    76    LBL
011    12     B
012    42    STO
013    06     06        Enter foundation thickness
014    91    R/S   ____
015    76    LBL
016    17    B"
017    42    STO
018    04    R/S        Enter reservoir depth
019    91    R/S   ____
020    76    LBL
021    13     C
022    42    STO
023    22     22        Enter confining layer thickness
024    91    R/S   ____
025    76    LBL
026    18    C"
027    42    STO        Enter confining layer conductivity
028    21     21
029    91    R/S   ____
030    76    LBL
031    14     D
032    42    STO        Enter dam width
033    07     07
034    91    R/S   ____
035    76    LBL
036    19    D"
037    42    STO        Enter embankment face angle
038    08     08
039    91    R/S   ____
040    76    LBL
```

```
041   15    E
042   42   STO        Enter dam length
043   23    23
                ____
044   25   CLR
045   43   RCL
046   06    06
047   75    -
048   42   RCL
049   22    22        Calculate H
050   95    =
051   42   STO
052   03    03
                ____
053   25   CLR
054   43   RCL
055   22    22
056   55    ÷
057   43   RCL        Calculate c
058   21    21
059   95    =
060   42   STO
061   05    05
                ____
062   25   CLR
063   43   RCL
064   07    07
065   75    -
066   53    (
067   43   RCL
068   04    04
069   55    ÷         Calculate d
070   43   RCL
071   08    08
072   30   TAN
073   54    )
074   95    =
075   42   STO
076   09    09
                ____
077   25   CLR
078   53    (
079   43   RCL        Calculate ls
080   09    09
```

```
081   33   X²
082   55   ÷
083   43   RCL
084   08    08
085   39   COS
086   33   X²
087   54    )
088   75    -
089   53    (
090   43   RCL
091   04    04
092   33   X²
093   55    ÷
094   43   RCL
095   08    08
096   38   SIN
097   33   X²
098   54    )
099   95    =          Calculate l_s (con.)
100   34   √X
101   42   STO
102   10    10
103   24   CLR
104   53    (
105   43   RCL
106   09    09
107   55    ÷
108   43   RCL
109   08    08
110   39   COS
111   54    )
112   75    -
113   43   RCL
114   10    10
115   95    =
116   42   STO
117   11    11    ———
118   25   CLR
119   43   RCL         Calculate QDAM
120   02    02
```

```
121   65    X
122   43   RCL
123   11    11
124   65    X
125   43   RCL
126   08    08
127   38   SIN
128   65    X
129   43   RCL
130   08    08
131   30   TAN          Calculate QDAM (con.)
132   95    =
133   42   STO
134   12    12
135   43   RCL
136   12    12
137   65    X
138   43   RCL
139   23    23
140   95    =
141   42   STO
142   24    24    ——
143   25   CLR
144   43   RCL
145   04    04
146   85    +
147   43   RCL          Calculate h1
148   06    06
149   95    =
150   42   STO
151   13    13    ——
152   25   CLR
153   43   RCL
154   01    01
155   65    X
156   43   RCL
157   03    03
158   65    X           Calculate λ
159   43   RCL
160   05    05
```

161	95	=	
162	34	√X	Calculate λ
163	42	STO	
164	14	14	
165	25	CLR	
166	43	RCL	
167	07	07	
168	85	+	
169	53	(	
170	02	2	
171	65	X	
172	43	RCL	
173	14	14	
174	54	)	
175	95	=	
176	42	STO	
177	15	15	
178	25	CLR	
179	53	(	
180	43	RCL	
181	13	13	
182	75	-	
183	43	RCL	
184	06	06	
185	54	)	
186	65	X	Calculate QFDN
187	48	RCL	
188	01	01	
189	65	X	
190	43	RCL	
191	03	03	
192	95	=	
193	42	STO	
194	16	16	
195	25	CLR	
196	43	RCL	
197	16	16	
198	55	÷	
199	43	RCL	
200	15	15	

```
201   95    =
202   42   STO
203   28    28
204   25   CLR
205   43   RCL
206   28    28         Calculate QFDN (con.)
207   65    X
208   43   RCL
209   23    23
210   95    =
211   42   STO
212   25    25   ____
213   25   CLR
214   43   RCL
215   24    24
216   85    +
217   43   RCL
218   25    25         Calculate QTOT
219   95    =
220   42   STO
221   17    17   ____
222   25   CLR
223   52    (
224   43   RCL
225   13    13
226   75    -
227   43   RCL
228   06    06
229   54    )
230   65    X
231   53    (
232   43   RCL
233   14    14         Calculate h2 + h3
234   54    )
235   95    =
236   55    ÷
237   53    (
238   43   RCL
239   07    07
240   85    +
```

241	53	(	
242	02	2	
243	65	X	
244	43	RCL	
245	14	14	
246	54	)	
247	54	)	
248	95	=	
249	42	STO	
250	20	20	
251	25	CLR	
252	43	RCL	
253	20	20	
254	94	+/-	
255	85	+	Calculate $h_2 + h_3$ (con.)
256	43	RCL	
257	13	13	
258	95	=	
259	42	STO	
260	18	18	
261	25	CLR	
262	43	RCL	
263	20	20	
264	85	+	
265	43	RCL	
266	06	06	
267	95	=	
268	42	STO	
269	19	19	
270	25	CLR	
271	69	OP	
272	00	00	
273	00	0	
274	00	0	
275	00	0	
276	00	0	Print header
277	00	0	
278	00	0	
279	00	0	
280	00	0	

```
281   69   OP
282   01    01
283   01    1
284   03    3
285   03    3
286   00    0
287   00    0
288   00    0
289   03    3
290   06    6
291   01    1
292   07    7
293   69   OP
294   02    02
295   01    1
296   07    7
297   03    3
298   03    3
299   01    1
300   03    3           Print header (con.)
301   02    2
302   02    2
303   01    1
304   07    7
305   69   OP
306   03    03
307   69   OP
308   05    05
309   68   OP
310   00    00
311   00    0
312   00    0
313   00    0
314   00    0
315   00    0
316   00    0
317   04    4
318   01    1
319   69   OP
320   01    01
```

```
321   03     3
322   01     1
323   02     2
324   04     4
325   04     4
326   02     2
327   00     0
328   00     0
329   03     3
330   02     2
331   69    OP
332   02     02
333   02     2
334   01     1
335   00     0
336   00     0
337   01     1
338   03     3
339   03     3
340   05     5
341   02     2         Print header (con.)
342   04     4
343   69    OP
344   03     03
345   04     4
346   06     6
347   00     0
348   00     0
349   00     0
350   00     0
351   00     0
352   00     0
353   00     0
354   00     0
355   69    OP
356   04     04
357   69    OP
358   05     05
359   69    OP
360   00     00
```

```
361   00    0
362   00    0
363   00    0
364   00    0
365   00    0
366   00    0
367   04    4
368   03    3
369   69   OP
370   01    01
371   03    3
372   00    0
373   00    0
374   00    0
375   02    2
376   03    3
377   04    4
378   01    1
379   03    3
380   07    7          Print header (con.)
381   69   OP
382   02    02
383   01    1
384   05    5
385   02    2
386   03    3
387   02    2
388   04    4
389   03    3
390   06    6
391   03    3
392   02    2
393   69   OP
394   03    03
395   03    3
396   01    1
397   00    0
398   00    0
399   00    0
400   00    0
```

```
401   00    0
402   00    0
403   00    0
404   00    0
405   69   OP
406   04    04
407   69   OP
408   05    05           Print header (con.)
409   69   OP
410   00    00
411   98   ADV
412   98   ADV
413   98   ADV
414   02    2
415   01    1
416   01    1
417   06    6
418   03    3
419   01    1
420   02    2
421   06    6            Print FDNK
422   69   OP
423   04    04
424   43   RCL
425   01    01
426   69   OP
427   06    06
428   69   OP
429   00    00
430   01    1
431   06    6
432   01    1
433   03    3
434   03    3            Print DAMK
435   00    0
436   02    2
437   06    6
438   69   OP
439   04    04
440   43   RCL
```

```
441    02    02
442    69    OP            Print DAMK (con.)
443    06     06    ____
444    69    OP
445    00     00
446    01     1
447    -5     5
448    02     2
449    07     7
450    02     2
451    06     6            Print CLK
452    68    OP
453    04     04
454    43    RCL
455    21     21
456    69    P¼
457    06     06    ____
458    69    OP
459    00     00
460    01     1
461    05     5
462    02     2
463    06     7
464    03     3
465    07     7            Print CLT
466    69    OP
467    04     04
468    43    RCL
469    22     22
470    69    OP
471    06     06    ____
472    69    OP
473    00     00
474    02     2
475    03     3
476    06     6
477    05     5            Print H'
478    69    OP
479    04     04
480    43    RCL
```

```
481   04 04
482   69   OP         Print H' (con.)
483   06    06
484   68   OP
485   00    00
486   02    2
487   03    3
488   69   OP
489   04    04        Print H
490   43   RCL
491   03    03
492   69   OP
493   06    06
494   69   OP
495   00    00
496   03    3
497   05    5
498   01    1
499   07    7
500   03    3
501   06    6         Print resistance (c)
502   69   OP
503   04    04
504   43   RCL
505   05    05
506   69   OP
507   06    06
508   69   OP
509   00    00
510   02    2
511   07    7
512   01    1
513   03    3
514   03    3
515   00    0         Print λ
516   69   OP
517   04    04
518   43   RCL
519   14    14
520   69   OP
```

```
521   06    06   ____
522   69   OP
523   00    00
524   04    4
525   03    3
526   02    2
527   04    4
528   01    1          Print  width
529   06    6
530   69   OP
531   04    04
532   43   RCL
533   07    07
534   69   OP
535   06    06   ____
536   69   OP
537   00    00
538   02    2
539   07    7
540   03    3
541   01    1
542   03    3
543   07    7
544   02    2          Print  length
545   03    3
546   69   OP
547   04    04
548   43   RCL
549   23    23
550   69   OP
551   06    06   ____
552   69   OP
553   00    00
554   02    2
555   07    7
556   03    3
557   02    2          Print  l_s
558   03    3
559   05    6
560   69   OP
```

```
561   04   04
562   43   RCL
563   11   11          Print l_s (con.)
564   69   OP
565   06   06
                ----
566   69   OP
567   00   00
568   01   1
569   04   4
570   01   1
571   07   7
572   02   3
573   07   7
574   01   1           Print β
575   03   3
576   69   OP
577   04   04
578   43   RCL
579   08   08
580   63   OP
581   06   06
                ----
582   69   OP
583   00   00
584   01   1
585   06   6
586   69   OP
587   04   04          Print d
588   43   RCL
589   90   09
590   69   OP
591   06   06
                ----
592   69   OP
593   00   00
594   02   2
595   03   3
596   00   0           Print h_1
597   02   2
598   69   OP
599   04   04
600   43   RCL
```

```
601   13   13
602   69   OP            Print h1 (con.)
603   06    06     ____
604   69   OP
605   00    00
606   02    2
607   03    3
608   00    0
609   03    3
610   69   OP            Print h2
611   04    04
612   43   RCL
613   18    18
614   69   OP
615   06    06     ____
616   69   OP
617   00    00
618   02    2
619   03    3
620   00    0
621   04    4
622   68   OP            Print h3
623   04    04
624   43   RCL
625   19    19
626   69   OP
627   06    06     ____
628   69   OP
629   00    00
630   02    2
631   03    3
632   00    0
633   05    5            Print h4
634   69   OP
635   04    04
636   43   RCL
637   06    06
638   69   OP
639   06    06     ____
640   69   OP
```

```
641   00    00
642   03    3
643   04    4
644   01    1
645   06    6
646   01    1
647   03    3            Print  QDAM
648   03    3
649   00    0
650   68    OP
651   04     04
652   43    RCL
653   24     24
654   69    OP
655   06     06   _____
656   69    OP
657   00     00
658   03    3
659   04    4
660   02    2
661   01    1
662   01    1
663   06    6            Print  QFDN
664   03    3
665   01    1
666   69    OP
667   04     04
668   43    RCL
669   25     25
670   69    OP
671   06     06   _____
672   69    OP
673   00     00
674   03    3
675   04    4
676   03    3
677   07    7            Print  QTOT
678   03    3
679   02    2
680   03    3
```

```
681   07    7
682   69   OP
683   04    04
684   43   RCL
685   17    17
686   69   OP           Print QTOT (con.)
687   06    06
688   69   OP
689   00    00
690   91   R/S    ____
691   00    0
692   00    0
693   00    0
694   00    0
695   00    0
696   00    0
697   00    0
698   00    0
699   00    0
700   00    0
701   00    0
702   00    0
703   00    0
704   00    0
705   00    0
706   00    0
707   00    0
708   00    0
709   00    0
710   00    0
711   00    0
712   00    0
713   00    0
714   00    0
715   00    0
716   00    0
717   00    0
718   00    0
719   00    0
720   00    0
```

APPENDIX B

FOUNDATION HYDRAULIC CONDUCTIVITY VS. SEEPAGE RATE

	Seepage Rate (ft^3/min)	
FCNK(ft/min)	Thirteenmile Creek Site	Burns Creek Site
1	--	46.52
10^{-1}	--	14.68
10^{-2}	--	4.61
10^{-3}	3.25	1.42
5×10^{-4}	2.29	--
10^{-4}	1.01	0.42
5×10^{-5}	0.70	0.11
10^{-5}	0.30	--
10^{-6}	0.08	--

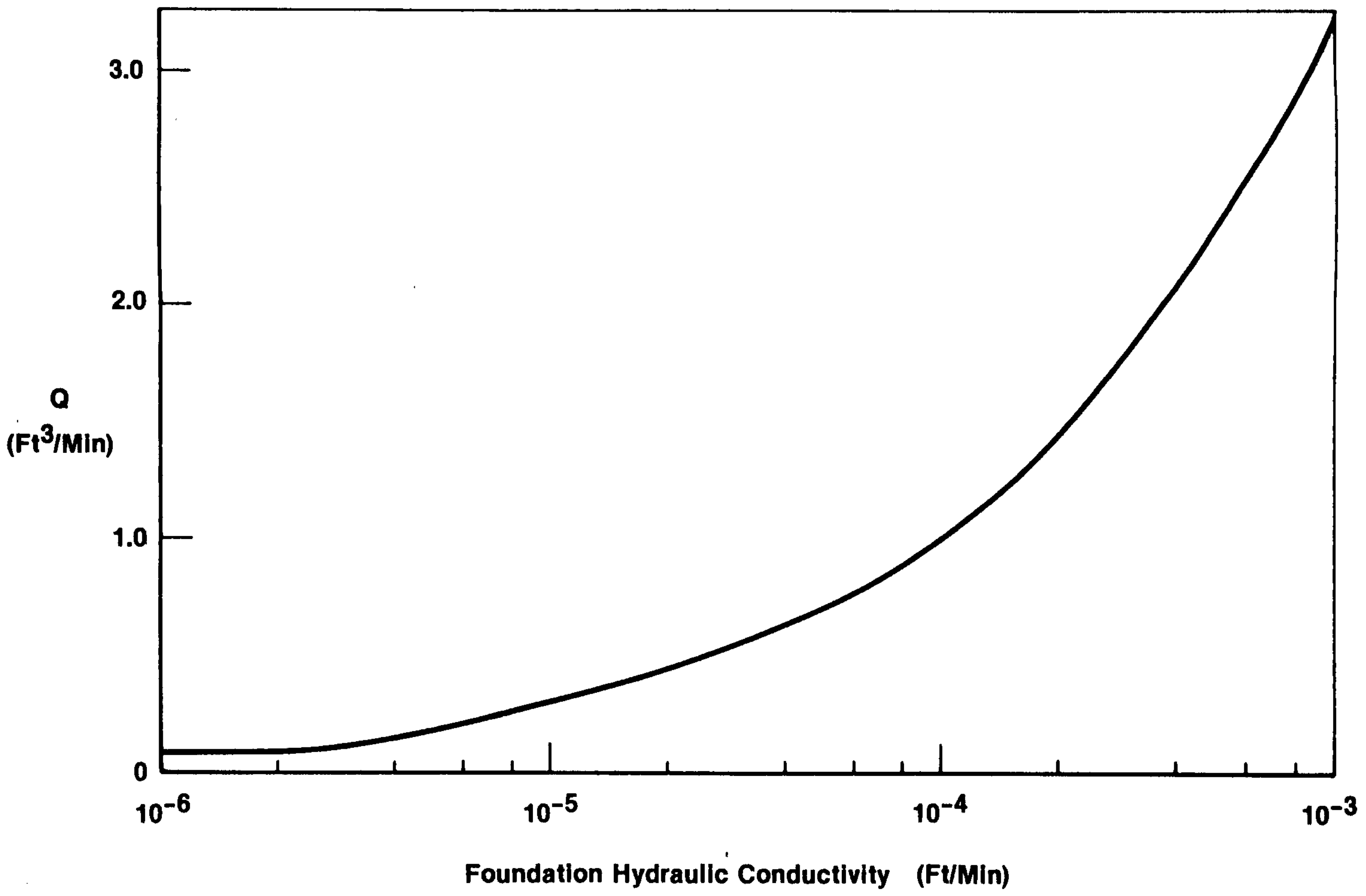


Figure B-1. Foundation hydraulic conductivity vs. seepage rate, Thirteenmile Creek site

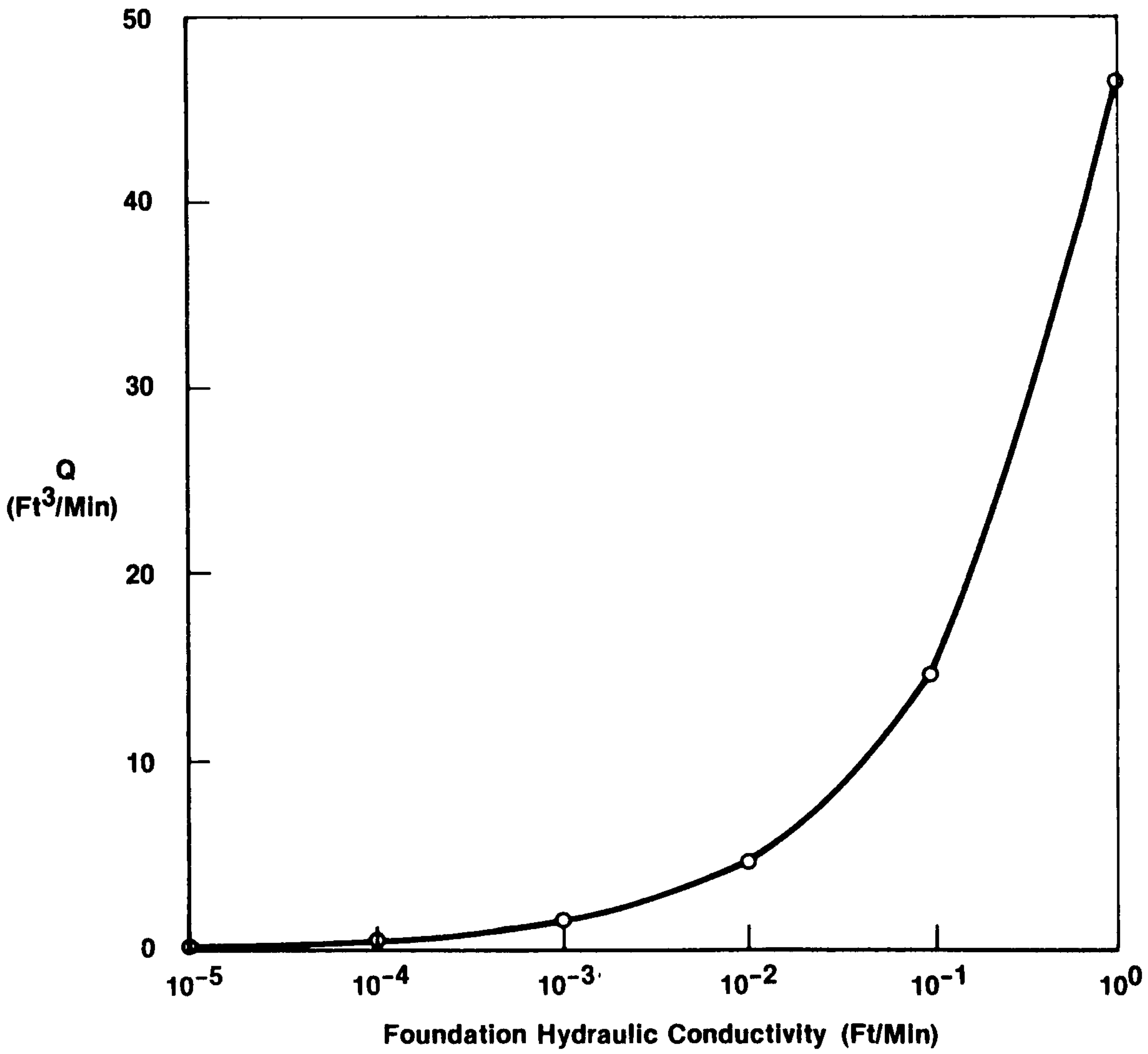


Figure B-2. Foundation hydraulic conductivity vs. seepage rate, Burns Creek site

APPENDIX C

DAM HYDRAULIC CONDUCTIVITY VS. SEEPAGE RATE

	Seepage Rate (ft^3/min)	
DAMK(ft/min)	Thirteenmile Creek Site	Burns Creek Site
10^{-6}	1.03	1.45
5×10^{-7}	1.02	1.44
10^{-7}	1.01	1.42
5×10^{-5}	1.00	1.42
10^{-8}	1.00	1.42
5×10^{9}	1.00	1.42

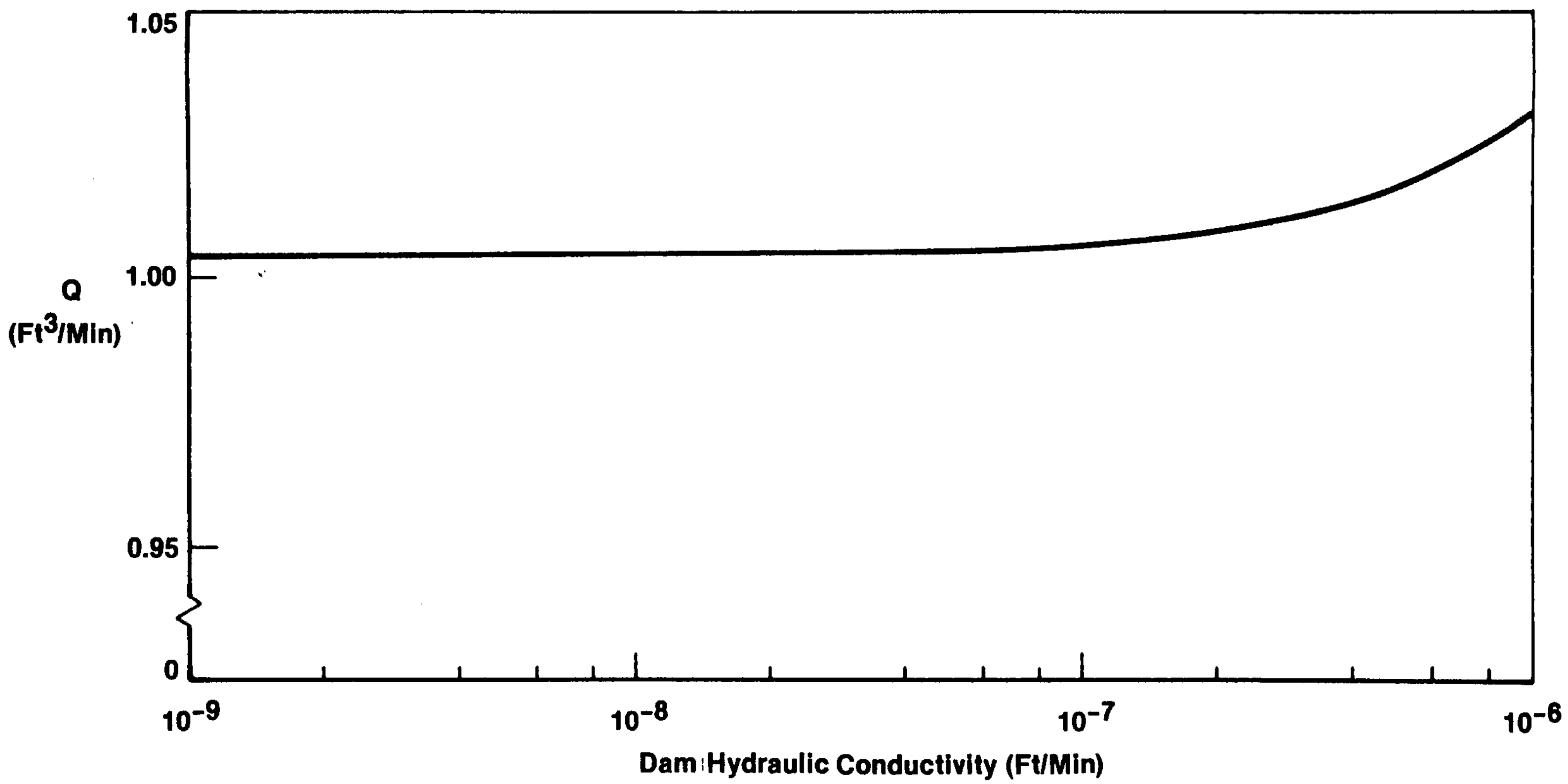


Figure C-1. Dam hydraulic conductivity vs. seepage rate, Thirteenmile Creek site

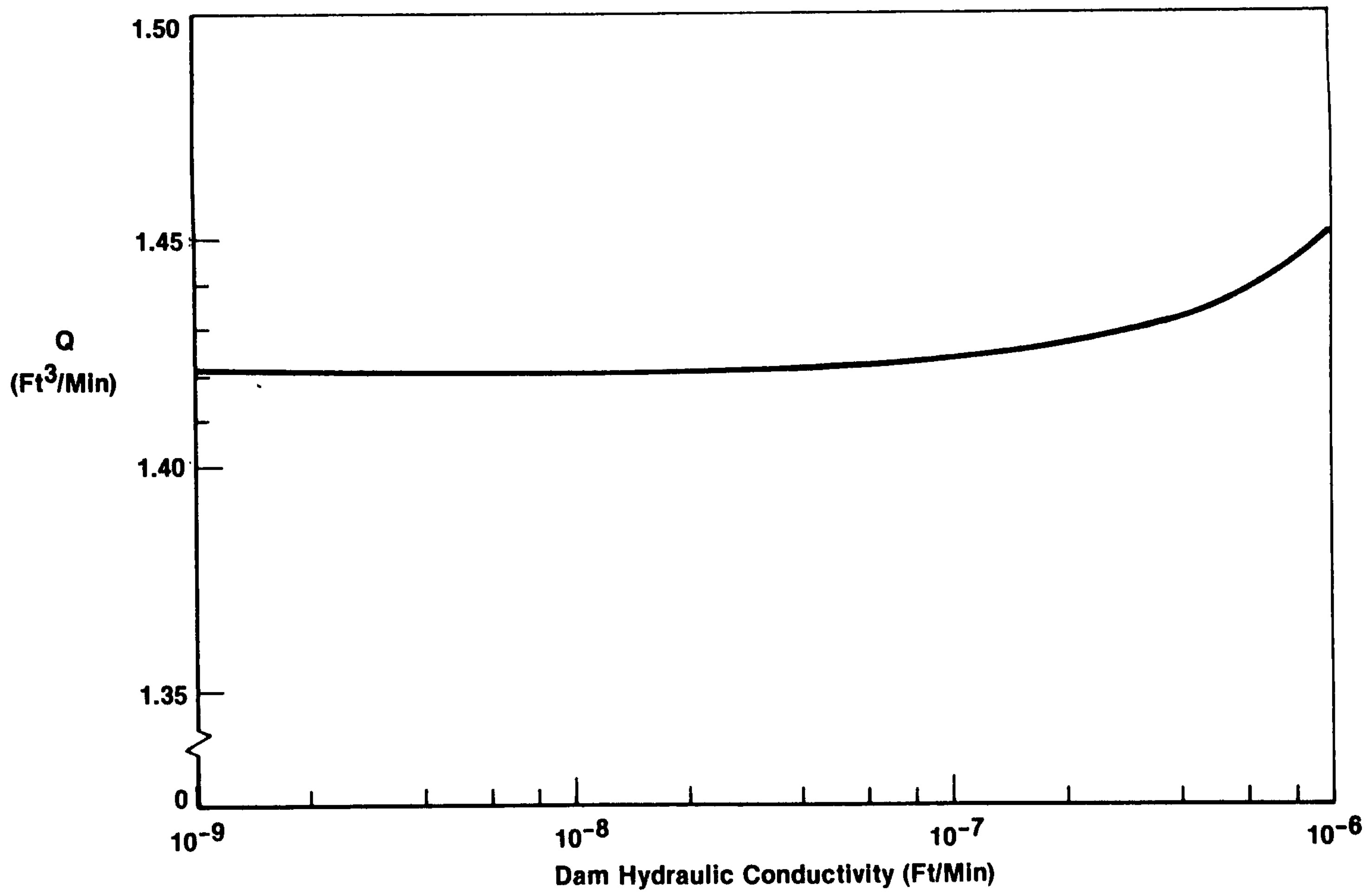


Figure C-2. Dam hydraulic conductivity vs. seepage rate, Burns Creek site

APPENDIX D

RESERVOIR DEPTH VS. SEEPAGE RATE

Reservoir (feet)	Seepage Rate (ft^3/min)	
	Thirteenmile Creek Site	Burns Creek Site
115	1.05	1.48
110	1.00	1.42
105	0.96	1.36
100	0.91	1.30

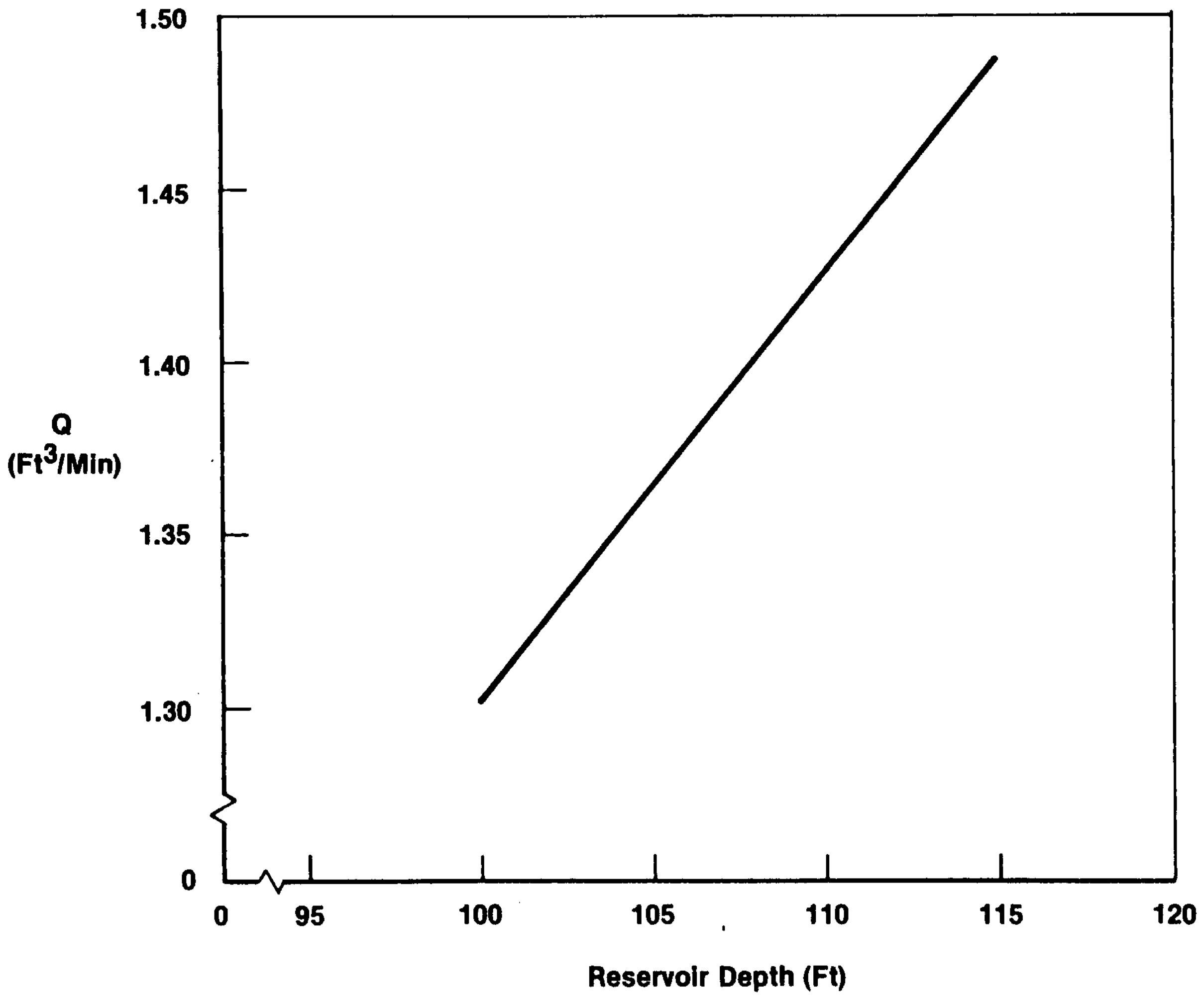


Figure D-1. Reservoir depth vs. seepage rate, Thirteenmile Creek site

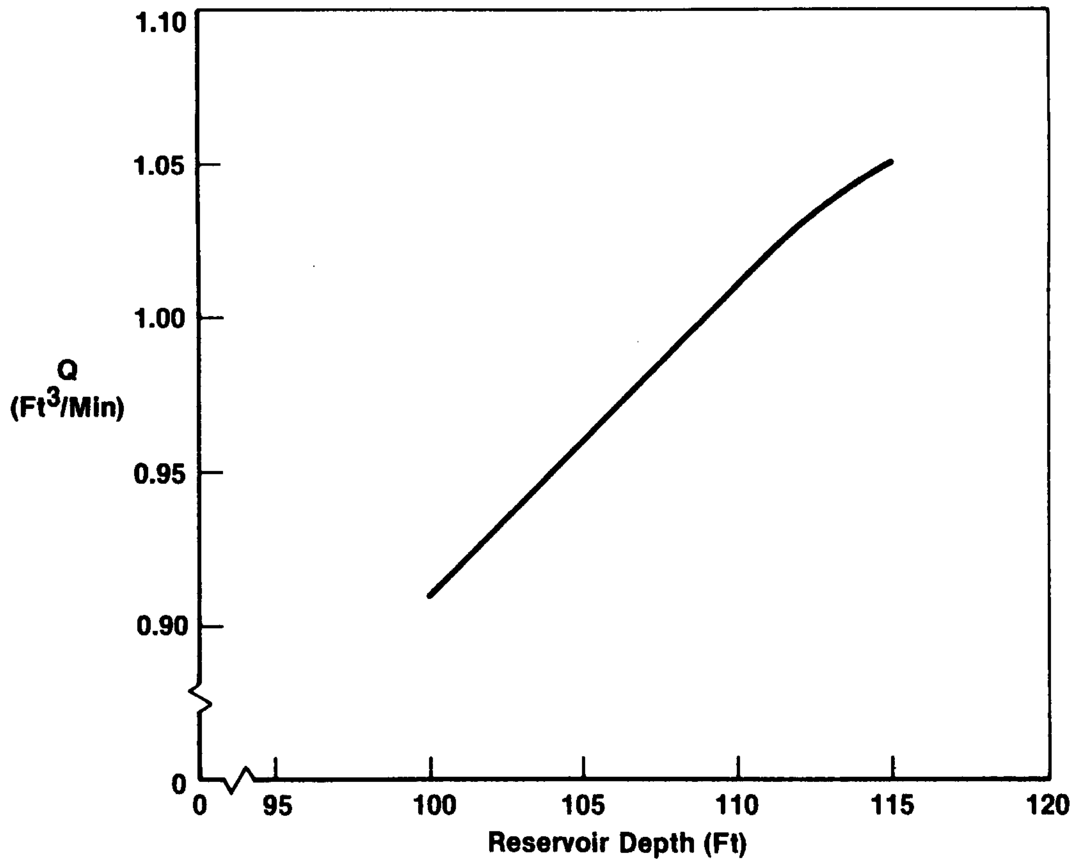


Figure D-2. Reservoir depth vs. seepage rate, Burns Creek site

APPENDIX E

CONFINING LAYER THICKNESS VS. SEEPAGE RATE

	Seepage Rate (ft^3/min)	
CLT (ft)	Thirteenmile Creek Site	Burns Creek Site
5	1.96	2.06
10	1.41	1.42
15	1.16	1.13
20	1.01	0.94
25	0.90	--
30	0.82	--

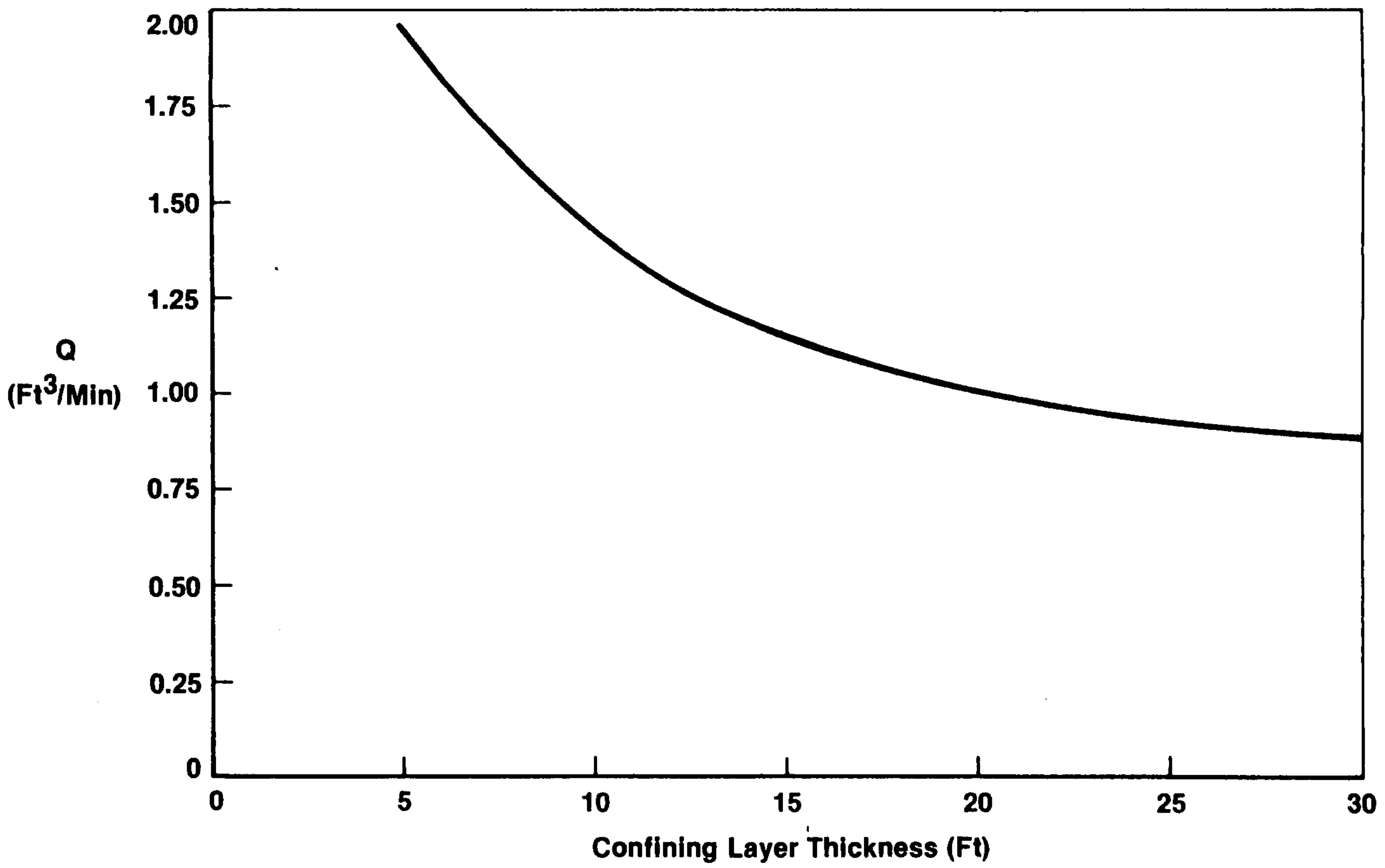


Figure E-1. Confining layer thickness vs. seepage rate, Thirteenmile Creek site

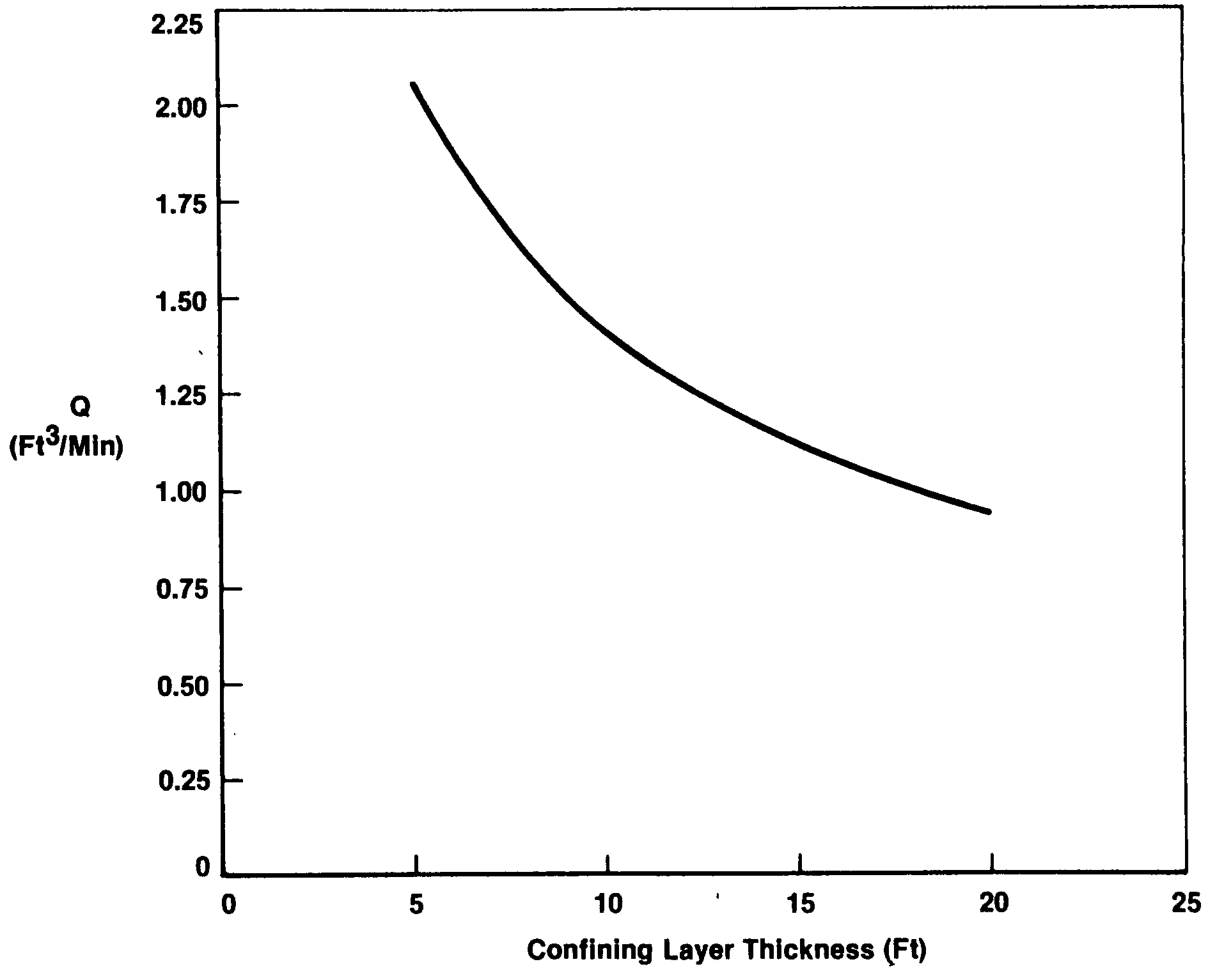


Figure E-2. Confining layer thickness vs. seepage rate, Burns Creek site

APPENDIX F

CONFINING LAYER CONDUCTIVITY VS. SEEPAGE RATE

	Seepage Rate (ft^3/min)	
CLK(ft)	Thirteenmile Creek Site	Burns Creek Site
10^{-7}	2.96	4.19
5×10^{-8}	2.16	3.05
10^{-8}	1.00	1.42
5×10^{-9}	0.72	1.01
10^{-9}	0.33	0.46
5×10^{-10}	0.23	0.33

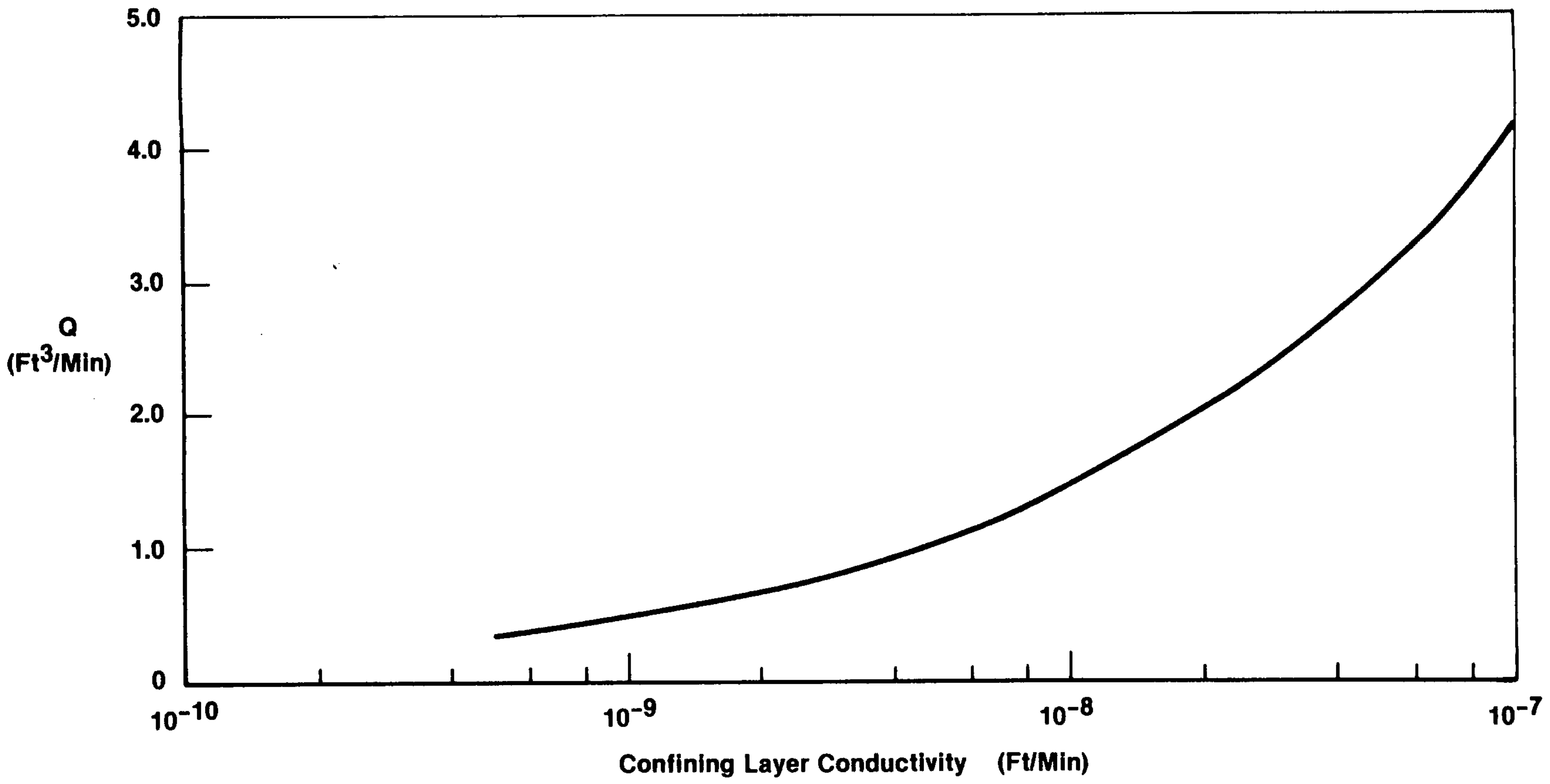


Figure F-1. Confining layer conductivity vs. seepage rate, Burns Creek site

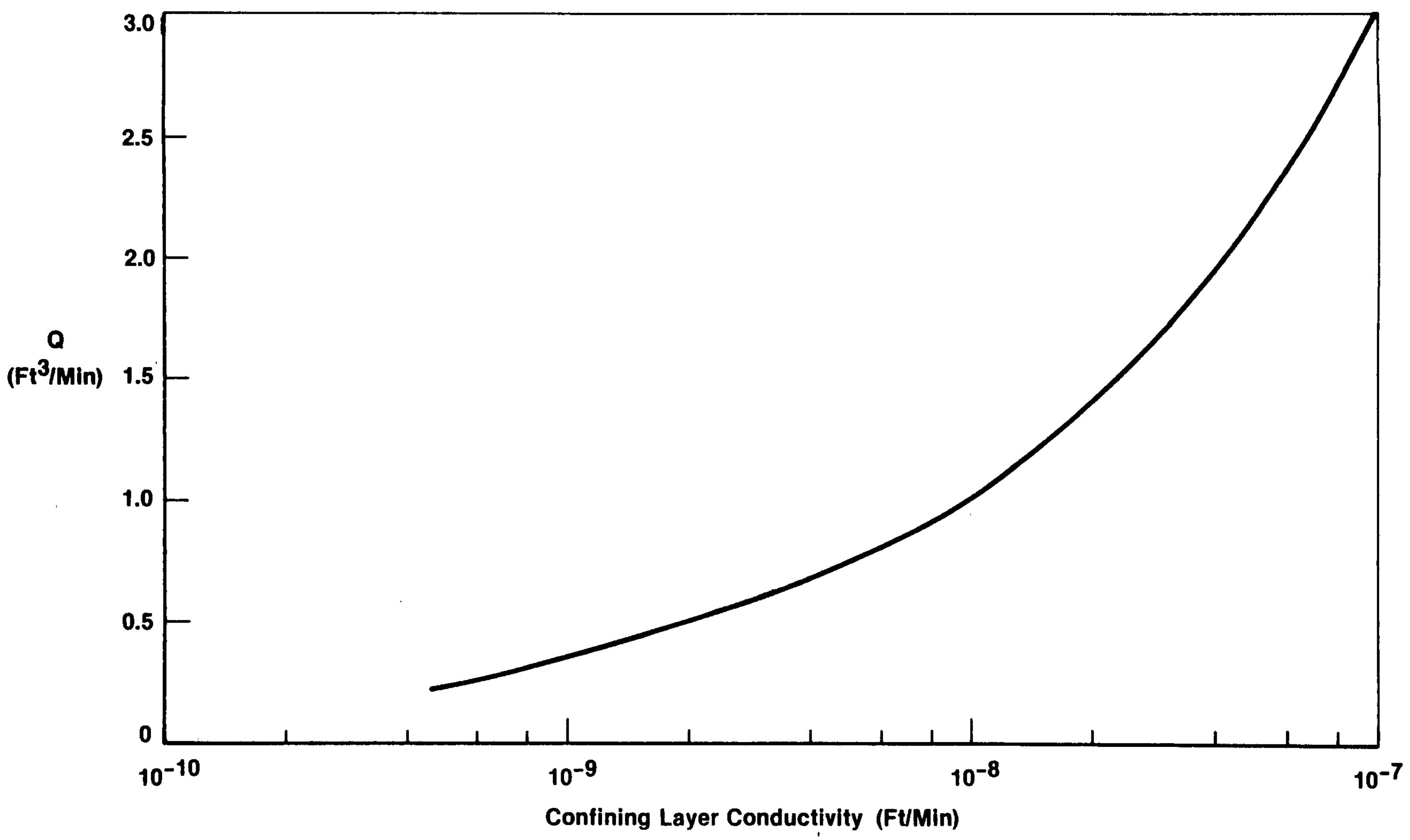


Figure F-1. Confining layer conductivity vs. seepage rate, Thirteenmile Creek site

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