DEVELOPMENT OF A LIBRARY MODULE FOR THE ANALYSIS OF ADVANCED COMPOSITE MATERIALS

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Abstract: Because of their orthotropic nature, advanced composite materials present some special problems to the engineer. The relation between the elastic constants of an orthotropic material is more complex than that of an isotropic material. In addition, composites exhibit coupling between normal and shear deformations as well as between bending and stretching load responses. The more complicated behavior of advanced composites is usually handled using techniques of matrix mathematics, but the required bookkeeping often overwhelms the isotropic-trained engineer who is used to working with fewer terms and less complicated equations. This paper discusses the development of a module, designed to be used with the Texas Instruments TI-58 and TI-59 desktop calculators which can be used to handle much of the required "bookkeeping". Given such input as the elastic constants of a single ply of composite material, the stacking sequence of a laminate, and the applied loads (or strains), the module will output desired information such as the physical properties of a laminate, the strains (or loads) in both the whole laminate and the individual plies, and the values of the laminate properties as the laminate is rotated. The module will also provide "strength ratios" for a particular laminate and load vector to indicate how close the material is to failure. It is hoped that this module will make the analysis and use of advanced composites seem less formidable to both research and developmental engineers.

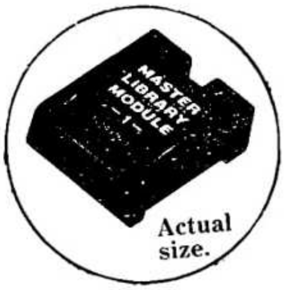
Key words: Advanced composite materials; composite materials analysis; orthotropic materials; programmable calculators; solid state software.

The orthotropic behavior of advanced composite materials can be described using techniques of matrix mathematics. However, the bookkeeping required by such techniques can soon become overwhelming. The Air Force Materials Laboratory at Wright-Patterson Air Force Base has recently completed a set of calculator programs capable of handling much of the required bookkeeping. These programs are presently being fabricated into a library module which can interface with a standard TI-58 or TI-59 calculator. (A library module is a solid state, plug-in set of prewritten, read-only programs.) See Figure 1. This paper discusses the structuring and potential applications of the composite materials module.

The module consists of nine user-accessible programs which calculate material properties, initial stresses and strains, mechanical properties, strength ratios, and translated moduli for both single plies and laminates of a composite material. Because the calculator's data storage registers are unaffected by the call for a program, the results of one program can easily be used as input for another. Several of the programs in the composite materials module are designed to be used in conjunction with one another.

Program 1 of the composite materials module provides data which is essential to all remaining programs. This program partitions the calculator to insure the proper number of data storage registers, clears the calculator memory, and performs a test to determine whether or not the calculator is connected to the PC-100A printer (since all programs have been designed to work on or off the printer). It then allows the user to input the engineering constants - the longitudinal, transverse, and shear moduli and the major Poisson's ratio - and the thickness of a single ply for the desired material. Program 1 also allows the user to convert his input data into English or SI units. This conversion can be performed on an individual entry or on the entire list as desired. Upon completion of input data entry, Program 1 calculates the reduced stiffness moduli (Q's), the compliances (S's), and the invariants required to calculate rotated values of moduli for a single ply. These results are then used by the remaining eight programs in the module.

Programs 2 and 3 deal with initial stresses and strains induced by changes in temperature and moisture concentration. Program 2 is concerned with stresses and strains in a single ply of composite material. The user inputs four environmental coefficients - one each in the longitudinal and transverse directions for thermal and for moisture effects. Again, these values can be converted into English or SI units either singly or as a list. Next, the user must input temperature change, moisture concentration, and curing strains. The program will then calculate initial stresses and strains for the ply. Program 2 can provide the user with single-ply initial stresses and strains in both the ply-axis system and in a coordinate system rotated thru any angle from the ply



Actual size.

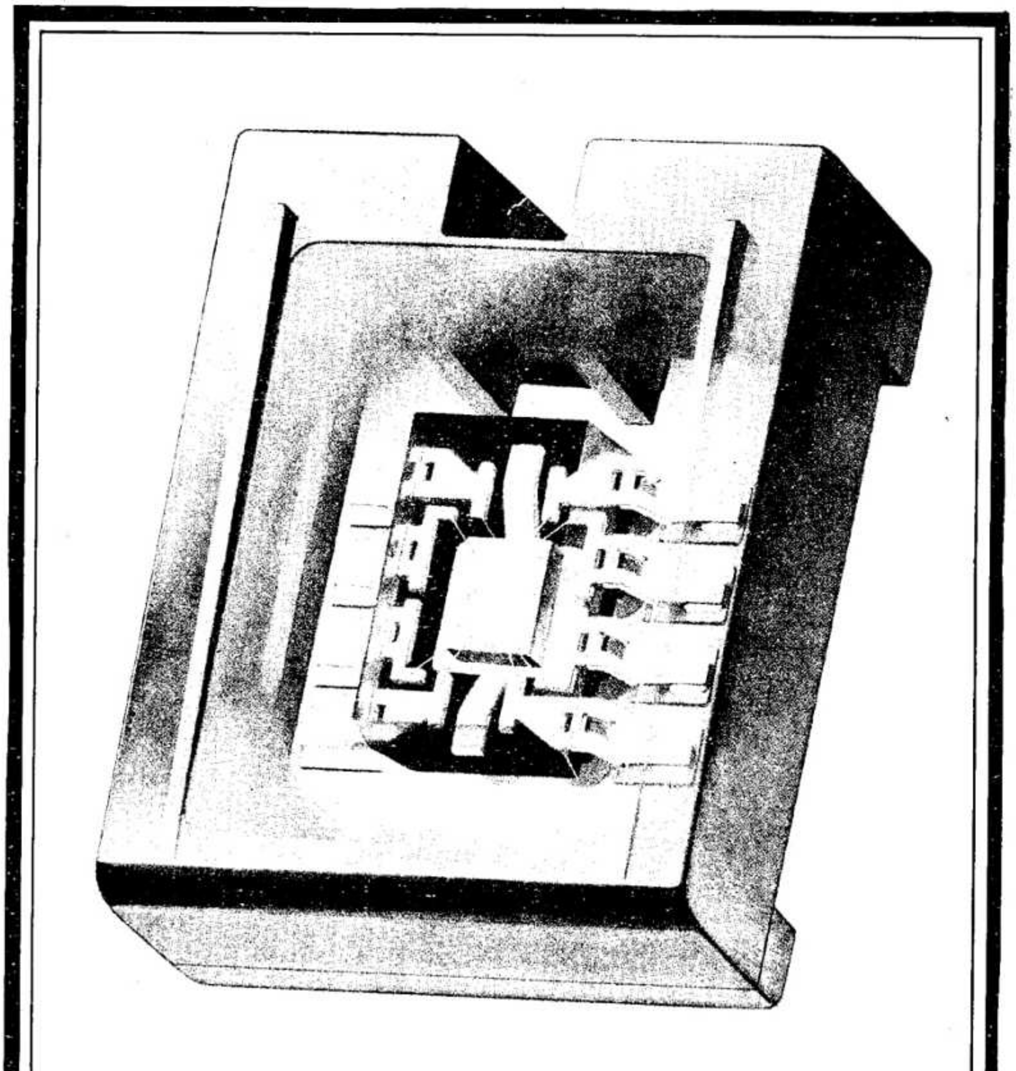


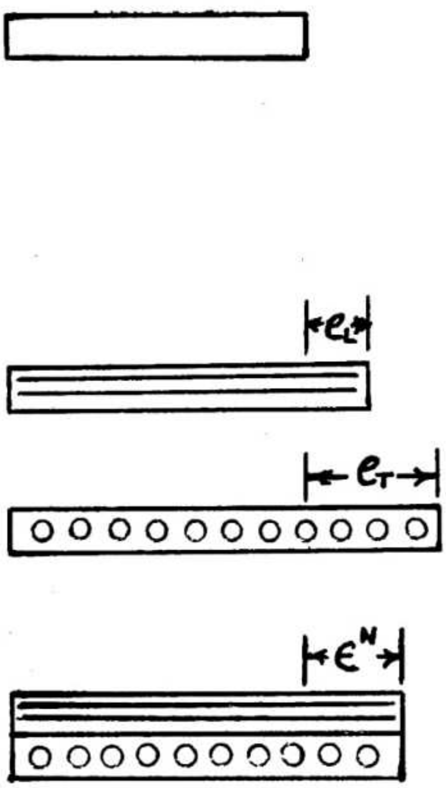
FIGURE 1: ENLARGED VIEW OF A LIBRARY MODULE

axes. Program 3 is used to calculate non-mechanical stresses and strains in a laminate. Non-mechanical stresses and strains are caused by the fact that plies at different orientations want to expand different amounts but are prevented from doing so by their presence in the laminate. See Figure 2. After the user inputs the stacking sequence of the laminate of interest, the program calculates the non-mechanical loads and strains for the given laminate. Program 3 is also capable of providing the stresses and strains in each ply of the laminate in both the ply- and laminate-axis systems. The initial strains from Program 2 as well as the non-mechanical strains from Program 3 are retained thruout the execution of Programs 5 and 6 so that they are available for use in the calculation of the strength ratios.

Program 4 calculates the mechanical properties of a single ply of composite material. This program calculates both the on- and off-axis values of the stiffness moduli and compliances. Rotation of stresses and strains is also accomplished by Program 4. In addition, multiplication of stiffness moduli and strains or compliances and stresses can be performed for both the on- and off-axis cases.

Program 5 is used to handle in-plane properties of laminates. Elements of the in-plane moduli (A-matrix) and their inverses can be calculated for any given laminate (symmetric or unsymmetric). Program 5 also provides values of the A- and inverse A-matrices rotated from the original laminate axes thru any desired angle. For symmetric laminates, this program can be used to multiply the A-matrix and the in-plane strains or the inverse A-matrix and the applied loads. Program 5 can also provide the stresses and strains in each ply of the laminate in both the ply- and laminate-axis systems.

Program 6 is similar to Program 5 except that it deals with the flexural rather than the in-plane case. This program can be used to calculate elements of the flexural moduli (D-matrix) and their inverses for any given laminate (symmetric or unsymmetric). Values of the D- and inverse D-matrices rotated thru any desired angle from the original laminate axes can also be calculated. For symmetric laminates, Program 6 can be used to multiply the D-matrix and laminate curvatures or the inverse D-matrix and the applied moments. Stresses and strains in each ply of the laminate in both the ply- and laminate-axes systems at any given distance from the laminate midplane can be calculated by Program 6.



Uncured ply - no environmental effects

When heat or moisture is applied, the ply expands. The amount of expansion is related to the ply orientation.
For example:

Zero-degree ply wants to expand an amount equal to e_L

90-degree ply wants to expand an amount equal to e_T

However, when the two are placed together in a laminate, each acts on the other and they "compromise" so that both expand an amount equal to the non-mechanical strain ϵ^N

FIGURE 2: Non-mechanical strains in a laminate

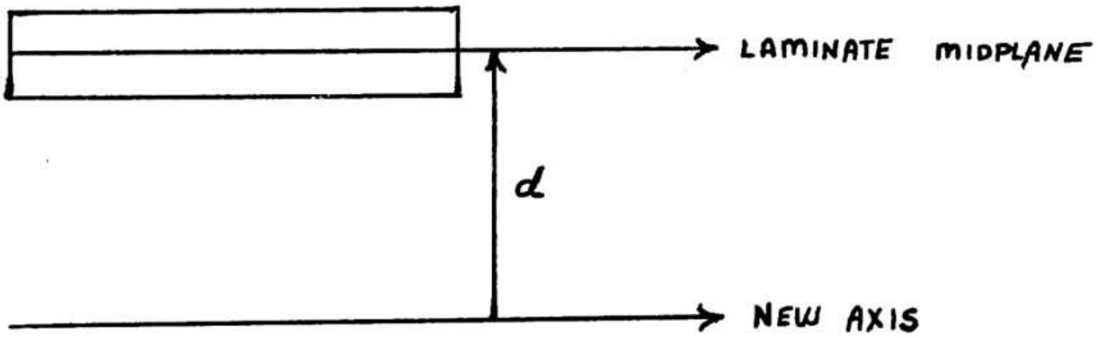


FIGURE 3: Translation of moduli by a distance "d"

Program 7 calculates values for the coupling moduli (B-matrix) and their inverses for unsymmetric laminates. This program can also rotate the values of the B- and inverse B-matrices thru any desired angle from the original laminate axes. Although Program 7 does not perform any multiplication, its output can be used as input for programs (written by the module user) dealing with unsymmetric laminates.

Program 8 uses the strains generated in previous programs (both mechanical and non-mechanical) to calculate values for the strength ratios of each ply in a given laminate. The user should use Program 1 to provide the necessary material properties, Programs 2 and 3 to calculate non-mechanical strains, and Program 5 for in-plane mechanical loads or Program 6 for bending. It is then necessary to input ultimate strengths for the material of interest. Program 8 allows the user to convert the input ultimate strengths into English or SI units either singly or as a complete list. After all of the ultimate strengths for the material have been input, the values of the F's and G's - coefficients for the strength ratio equation - are calculated and output. Program 8 then calculates the strength ratio for any ply of the given laminate when supplied with the ply orientation (for the in-plane case) or the ply orientation and distance of the ply from the laminate midplane (for the bending case). When the non-mechanical strains are negligible, the user may skip their calculation and proceed directly from Program 1 to Program 5 or 6.

Program 9 calculates the values of the A-, B-, and D-matrices and their inverses when translated a distance, d, from the midplane of the laminate. See Figure 3. After using Program 1 to calculate the needed material properties, the user inputs the stacking sequence and the distance from the midplane of the laminate. Program 9 then calculates the translated values of the A-, B-, and D-matrices using the following relationships:

$$A_{ij}' = A_{ij}$$

$$B_{ij}' = dA_{ij} + B_{ij}$$

$$D_{ij}' = d^2A_{ij} + 2dB_{ij} + D_{ij}$$

where the primed variables a anslated values and unprimed variables are those which have been calculated from the laminate midplane. Program 9 will also invert the translated matrices. This program can be used to calculate moduli for stiffeners or sandwich beams.

To demonstrate the use of the module, I'll describe a sample problem done with the programs used in the development of the module. Figure 4 illustrates the keyboard of a standard TI-59 calculator. (A TI-58 is the same except that there is no "write" command). Figure 5 shows the output produced by the PC-100A printer. (The symbols typed next to the printed output have been added for this explanation. They are not produced by the PC-100A printer). If this problem were run off the printer the numerical answers would appear in the display of the calculator and the user would be able to step thru them using the Run/Stop (R/S) key.

The problem solved in Figure 5 is to find the mechanical stresses in the center of each ply (in both the ply- and laminate-axes systems) in a $[0/\mp 45]_s$ laminate of AS/3501 when a bending moment of 1000 Nm is applied in the 1-direction. After inserting the composite materials module and turning the calculator on, the user should press 2nd ENG to put the calculator in engineering notation. If another type of output (e.g. FIX 3) is desired, the user may set the calculator in the desired notation and the numerical output will be printed (or displayed) in the desired notation. However, some of the labels may be incorrectly printed if the calculator is in other than engineering notation.

To solve the sample problem, the user must first press 2nd PGM 01 to call the first program. This program allows him to input the engineering constants and ply thickness for AS/3501 and to calculate the material properties which will be required by the moment/curvature program. After calling Program 1, the user presses E which partitions and clears the calculator memory, tests for the printer connection, prints "ENT" (to tell the user to enter his data), and places a "1" in the display to inform the user that the calculator is ready for the first engineering constant (E_L). The user must then key in the value of E_L for AS/3501 and press R/S to enter this value. The calculator then prints the value of E_L and places a "2" in the display indicating that it now requires the value of the second engineering constant (E_T). The user continues entering the engineering constants and the ply thickness for AS/3501. Following entry of the fifth value, the calculator advances the paper

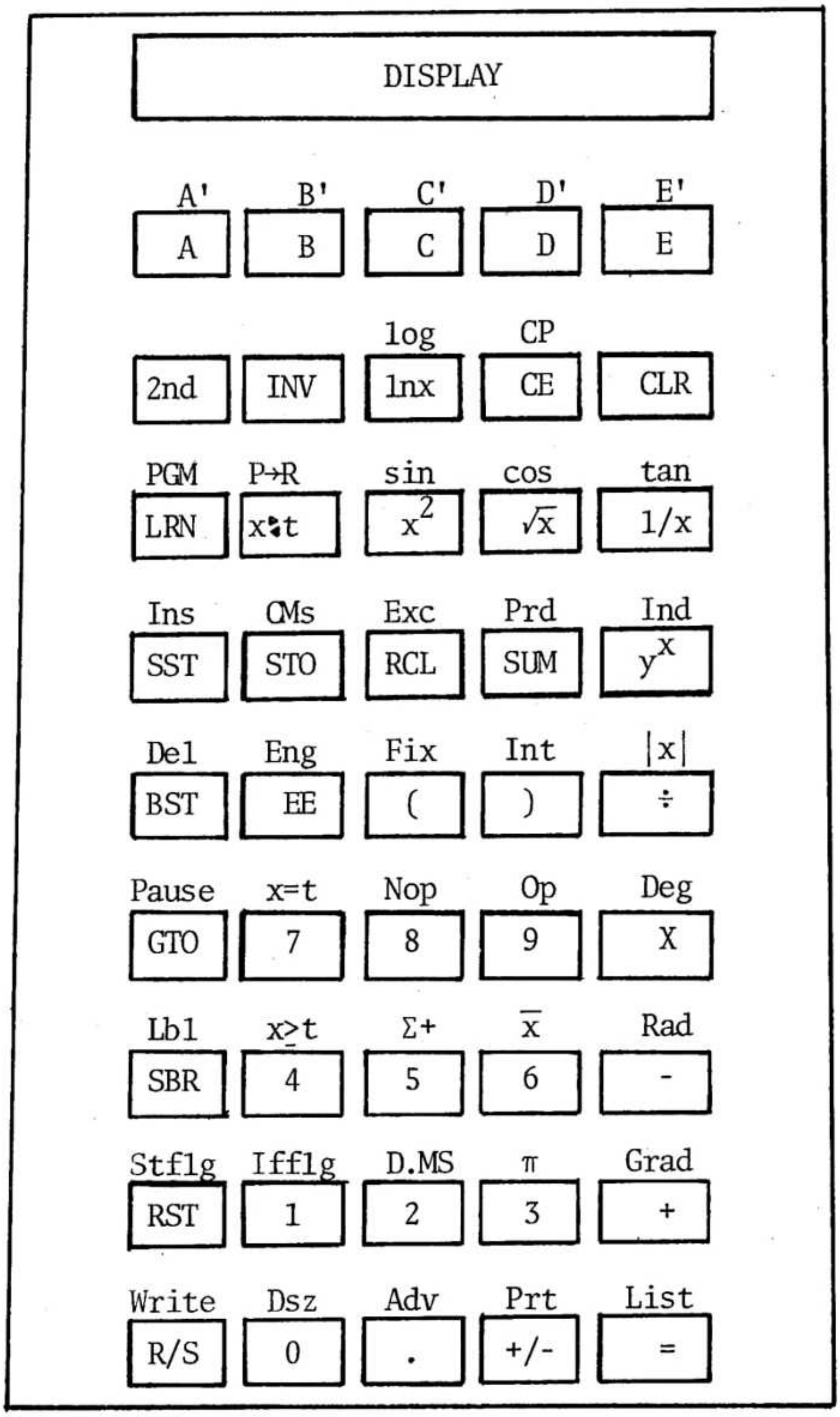


FIGURE 4: Keyboard of TI-59 calculator

PROGRAM 1

```
  ENT
 127.55  09 E(L)
 11.032  09 E(T)
 4.4816  09 G(LT)
   250. -03 ν(LT)
 133.35-06  h_o


      Q
128.24325  09 Q(11)
 11.09196  09 Q(22)
  2.77299  09 Q(12)
   4.4816  09 Q(66)
       0.  00 Q(16)
       0.  00 Q(26)


       S
 7.8400627-12 S(11)
 90.645395-12 S(22)
-1.9600157-12 S(12)
 223.13459-12 S(66)
        0.  00 S(16)
        0.  00 S(26)


      U
 55.18475  09 U(1)
58.575644  09 U(2)
14.482853  09 U(3)
17.255843  09 U(4)
18.964453  09 U(5)


        I
36.220297  09 I(1)
18.964453  09 I(2)
58.575644  09 R(1)
14.482853  09 R(2)
```

FIGURE 5A: SAMPLE PROBLEM

```
PROGRAM 6

        6.  00 N

        0.  00
      -45.  00
       45.  00
       45.  00
      -45.  00
        0.  00

      L
        1.  03 M(1)
        0.  00 M(2)
        0.  00 M(6)

      DI
  246.35097-03 d(11)
  1.4681115 00 d(22)
 -120.23466-03 d(12)
  2.1592145 00 d(66)
  62.830219-03 d(16)
  671.50235-03 d(26)

      L
        1.  03 M(1)
        0.  00 M(2)
        0.  00 M(6)

      E
  246.35097 00 κ(1)
 -120.23466 00 κ(2)
  62.830219 00 κ(6)
```

```
PROGRAM 6
         0.  00 θ
       2.5   00 z

        E'
  82.127254-03 ε(1)'
 -40.083231-03 ε(2)'
  20.946024-03 ε(6)'

        E
  82.127254-03 ε(1)
 -40.083231-03 ε(2)
  20.946024-03 ε(6)

        Q
  128.24325 09 Q(11)
   11.09196 09 Q(22)
    2.77299 09 Q(12)
     4.4816 09 Q(66)
         0.  00 Q(16)
         0.  00 Q(26)

        E'
  82.127254-03 ε(1)'
 -40.083231-03 ε(2)'
  20.946024-03 ε(6)'

        L'
  10.421115 09 σ(1)'
 -216.86354 06 σ(2)'
  93.871702 06 σ(6)'

        L
  10.421115 09 σ(1)
 -216.86354 06 σ(2)
  93.871702 06 σ(6)
```

FIGURE 5B:
SAMPLE PROBLEM

```
PROGRAM 6

     -45.  00 θ
      1.5  00 z

      E'
6.3293996-03 ε(1)'
18.897014-03 ε(2)'
73.326291-03 ε(6)'

      E
49.276352-03 ε(1)
-24.049939-03 ε(2)
12.567615-03 ε(6)

      Q
128.24325  09 Q(11)
 11.09196  09 Q(22)
  2.77299  09 Q(12)
   4.4816  09 Q(66)
       0.  00 Q(16)
       0.  00 Q(26)

      E'
6.3293996-03 ε(1)'
18.897014-03 ε(2)'
73.326291-03 ε(6)'

      L'
  864.104  06 σ(1)'
227.15629  06 σ(2)'
328.61911  06 σ(6)'

      L
874.24925  06 σ(1)
217.01104  06 σ(2)
-318..47385 06 σ(6)
```

```
PROGRAM 6

      45.  00 θ
     500. -03 z

      E'
6.2990047-03 ε(1)'
2.1097999-03 ε(2)'
-24.442097-03 ε(6)'

      E
16.425451-03 ε(1)
-8.0166462-03 ε(2)
4.1892048-03 ε(6)

      Q
128.24325  09 Q(11)
 11.09196  09 Q(22)
  2.77299  09 Q(12)
   4.4816  09 Q(66)
       0.  00 Q(16)
       0.  00 Q(26)

      E'
6.2990047-03 ε(1)'
2.1097999-03 ε(2)'
-24.442097-03 ε(6)'

      L'
813.65527  06 σ(1)'
40.868893  06 σ(2)'
-109.5397  06 σ(6)'

      L
536.80179  06 σ(1)
317.72238  06 σ(2)
386.39319  06 σ(6)
```

FIGURE 5C: SAMPLE PROBLEM

and awaits further instruction. If the user wishes to convert the units of any entry, he must key in the entry and press C (to convert from English to SI) or 2nd C (to convert from SI to English) before he presses the R/S key; to convert all five entries, the user should press C or 2nd C after entering the fifth value. The calculator will then print out the converted values for all five input values. Upon completion of data entry for AS/3501, the user presses key A and the calculator calculates and prints out the reduced stiffness moduli (Q), compliances (S), the U's (constants required for the calculation of off-axis values of Q), and the invariants (I).

After using Program 1 to calculate the required values of the material properties for AS/3501, the user presses 2nd PGM 06 to call Program 6 which deals with mechanical bending of a laminate. To enter the laminate's stacking sequence, the user presses A and a zero is displayed. The user keys in the total number of plies in the laminate (in this case, 6) and presses R/S. The total number of plies is printed, the paper is advanced, and a "6" is displayed to indicate that the ply orientation of the sixth (top) ply is required. The user keys in a "0" and presses R/S to enter this value. The zero is printed and a "5" is displayed indicating that the fifth ply orientation is now needed. The user continues entering all ply orientations in the laminate (as the display counts down). Upon entry of the last angle, the calculator advances the paper and awaits instruction.

To enter the applied bending moments, the user presses SBR STO D and a "1" is displayed. The user keys in 1000 and presses R/S to enter the value of M_1. A "2" is then displayed to indicate that the value of M_2 should now be entered. Following entry of all three bending moments (M_1, M_2, and M_6), the user may press SBR RCL D to output the stored values thereby allowing him to check his input data.

Once the stacking sequence and bending moments have been entered, the laminate curvatures can be calculated by pressing SBR RCL SBR INV B SBR X SBR RCL D. SBR RCL SBR INV B calculates and outputs the values of d_{ij} (the inverse of the D-matrix). SBR X tells the calculator it will be performing a matrix multiplication. SBR RCL D outputs the stored bending moments. Upon completion of this sequence, the matrix multiplication $d_{ij}M_j$, which results in the values of κ_i, is automatically performed. The user then presses SBR RCL E to output κ_i. If the user wishes to simply perform the matrix multiplication $d_{ij}M_j$ without outputting the values of d_{ij} and M_j, he can omit the SBR RCL before SBR INV B and before D.

Once the laminate curvatures have been calculated, the stresses and strains in each ply of the laminate (in both the ply- and laminate-axis systems) can be found. Since all ply stresses and strains are calculated in the same manner, only the -45 degree calculations (Figure 5C) will be explained here. To describe a desired ply the user presses 2nd C and a zero is displayed indicating that the ply's orientation is required. The ply angle should be entered using the R/S key and a "1" will be displayed to tell the user that a value of Z (distance from the laminate midplane to the point of interest) is needed. Since we are interested in the stresses and strains in the center of the upper -45 degree ply, a 1.5 (the value of Z in units of ply thickness) is keyed in and R/S is pressed. Now that the ply of interest has been identified, the ply strains may be recalled by pressing SBR RCL E for ply strains in the laminate axes and SBR RCL 2nd E for ply strains in the ply axes. Ply stresses can be calculated by multiplying the on-axis values of Q and the ply strains in the ply-axes system. This multiplication is performed by pressing SBR RCL C SBR X SBR RCL 2nd E. The resulting ply stresses can be output by pressing SBR RCL D for ply stresses in the laminate axes and SBR RCL 2nd D for ply stresses in the ply axes. Once the stresses and strains in the -45 degree ply have been output, the user may calculate the ply stresses and strains in any other ply by pressing 2nd C, identifying the new ply, and repeating the sequence. The results of all three ply orientations of our sample laminate are shown in Figures 5B and 5C. Once again, if the user did not want to output the values of Q_{ij} and ε_j when he performed the matrix multiplication to calculate the stresses, he could have omitted the SBR RCL before C and before 2nd E.

As can be seen in Figures 5B and 5C, the symbol "L" is used to identify force-related parameters (stress, loads, and moments) and "E" is used for displacement-related parameters (strains and curvatures). This symbology is used in all programs in the module. Also, in programs dealing with laminates, primed values indicate "ply axes" and unprimed values mean "laminate axes" where stresses and strains are concerned.

Though calculation of the ply stresses in the sample problem may seem complicated the first time thru, Figure 6

KEYS	NOTES
2nd PGM 01	Calls Program 1
E	Enter E_L, E_T, G_{LT}, ν_{LT}, h_o (Use keys C and 2nd C for unit conversion)
A	Calculates Q, S, U, I
2nd PGM 06	Calls Program 6
A	Enter laminate stacking sequence
SBR STO D	Enter M_1, M_2, M_6
SBR INV B SBR X D	Multiplies $d_{ij}M_j$ (Result: k_i)
SBR RCL E	Recalls curvatures (k_i)
2nd C	Enter ply orientation and z
SBR RCL 2nd E	Recalls ply strains (ply axes)
SBR RCL E	Recalls ply strains (laminate axes)
C SBR X 2nd E	Multiplies $Q_{ij}\epsilon'_j$ (Result: σ'_i)
SBR RCL 2nd D	Recalls ply stresses (ply axes)
SBR RCL D	Recalls ply stresses (laminate axes)

FIGURE 6: Summary of solution to sample problem to find stresses in the center of each ply of a $[0/\mp 45]_s$ laminate to which a moment (M_1=1000 Nm) is applied

illustrates that the problem solution is actually quite straightforward. To simplify program use, labels identifying the user-defined keys will be provided for each program. A user who has become familiar with the composite materials module can calculate ply stresses in each ply in less than fifteen minutes. When I solved the same program using a non-programmable calculator, the solution required several hours and much more paper. In addition, the possibilities for error were limitless.

As mentioned earlier, the programs for the composite materials module have been turned over to Texas Instruments Incorporated for fabrication of the module. The library modules are scheduled for delivery sometime in September of 1979.

The Air Force Materials Laboratory foresees several potential applications for the composite materials module. Though computers will still be needed for detailed analyses, the calculator/module combination can provide the engineer with a fast, economical way to perform preliminary calculations which can give him a "feel" for various trends. Armed with this information, he can then make more efficient use of his computer resources.

By relieving the user of the necessity of having to set up all of the necessary calculations on his own, the module enlarges the group of personnel able to perform composite material calculations. Calculations which were once performable only by a small number of highly paid engineers or consultants can be done by junior engineers or technicians. Figure 7 lists some of the calculations carried out by the composite materials module.

Once the number of people able to do composite material analyses has increased, the need for many of the expensive tests being done today in order to obtain empirical results for use in design specifications may decrease. The module may also provide a quick and easy means for technicians to check test results as testing is being done. In addition, engineers will have a readily available method for obtaining information concerning proposed designs during the course of their planning sessions.

Finally, the module can be an aid to a short-course in composite materials. By relieving students of the need to laboriously perform all of the long and somewhat tedious

$$m = 1/(1-(\nu_{LT})^2 E_L/E_T)$$
$$Q_{11} = mE_L \qquad Q_{22} = mE_T$$
$$Q_{12} = \nu_{LT}Q_{22}$$
$$Q_{16} = 0 \qquad Q_{26} = 0$$

$$U_1 = (1/8)(3Q_{11}+3Q_{22}+2Q_{12}+4Q_{66})$$
$$U_2 = (1/2)(Q_{11}-Q_{22})$$
$$U_3 = (1/8)(Q_{11}+Q_{22}-2Q_{12}-4Q_{66})$$
$$U_4 = (1/8)(Q_{11}+Q_{22}+6Q_{12}-4Q_{66})$$
$$U_5 = (1/2)(U_1-U_4)$$

$$I_1 = U_1-U_5$$
$$I_2 = U_5$$
$$R_1 = |U_2|$$
$$R_2 = |U_3|$$

$$e_L = \alpha_L^T\Delta T+\alpha_L^H c+e_L^c$$
$$e_T = \alpha_T^T\Delta T+\alpha_T^H c+e_T^c$$
$$\sigma_L = Q_{11}e_L+Q_{12}e_T$$
$$\sigma_T = Q_{12}e_L+Q_{22}e_T$$

$$N_1^N = p_\sigma^N V_{0A}+q_\sigma^N V_{1A}$$
$$N_2^N = N_1^N-2q_\sigma^N V_{1A}$$
$$N_6^N = -q_\sigma^N V_{2A}$$

$$F_{11} = 1/(X_L X_L')$$
$$F_{22} = 1/(X_T X_T')$$
$$F_{12} = F_{12}^*\sqrt{F_{11}F_{22}}$$
$$F_{66} = 1/(X_{LT})^2$$
$$F_1 = 1/X_L-1/X_L'$$
$$F_2 = 1/X_T-1/X_T'$$

$$G_{11} = F_{11}Q_{11}^2+2F_{12}Q_{11}Q_{12}+F_{22}Q_{12}^2$$
$$G_{22} = F_{11}Q_{12}^2+2F_{12}Q_{12}Q_{22}+F_{22}Q_{22}^2$$
$$G_{12} = F_{11}Q_{11}Q_{12}+F_{12}Q_{11}Q_{22}+F_{12}Q_{12}^2+F_{22}Q_{12}Q_{22}$$
$$G_{66} = F_{66}Q_{66}^2$$
$$G_1 = F_1Q_{11}+F_2Q_{12}$$
$$G_2 = F_1Q_{12}+F_2Q_{22}$$

$$a = G_{66}(\varepsilon_6^M)^2+G_{11}(\varepsilon_1^M)^2+2G_{12}(\varepsilon_1^M)(\varepsilon_2^M)+G_{22}(\varepsilon_2^M)^2$$
$$b = G_1(\varepsilon_1^M)+G_2(\varepsilon_2^M)+2G_{11}(\varepsilon_1^M)(\varepsilon_1^N-e_L)+2G_{12}((\varepsilon_1^M)(\varepsilon_2^N-e_T)+(\varepsilon_2^M)(\varepsilon_1^N-e_L))$$
$$+2G_{22}(\varepsilon_2^M)(\varepsilon_2^N-e_T)+2G_{66}(\varepsilon_6^M)(\varepsilon_6^N)$$
$$c = 1-G_{11}(\varepsilon_1^N-e_L)^2-2G_{12}(\varepsilon_1^N-e_L)(\varepsilon_2^N-e_T)-G_{22}(\varepsilon_2^N-e_T)^2-G_{66}(\varepsilon_6^N)^2-G_1(\varepsilon_1^N-e_L)$$
$$-G_2(\varepsilon_2^N-e_T)$$
$$S_-^M = (b/2a)-\sqrt{(b/2a)^2-(c/a)}$$
$$S^M = (b/2a)+\sqrt{(b/2a)^2-(c/a)}$$

$$X_{11}' = U_1V_0+U_2V_1\cos 2\phi+U_2V_2\sin 2\phi+U_3V_3\cos 4\phi+U_3V_4\sin 4\phi$$
$$X_{22}' = U_1V_0-U_2V_1\cos 2\phi-U_2V_2\sin 2\phi+U_3V_3\cos 4\phi+U_3V_4\sin 4\phi$$
$$X_{12}' = U_4V_0-U_3V_3\cos 4\phi-U_3V_4\sin 4\phi$$
$$X_{66}' = U_5V_0-U_3V_3\cos 4\phi-U_3V_4\sin 4\phi$$
$$X_{16}' = -\tfrac{1}{2}U_2V_2\cos 2\phi+\tfrac{1}{2}U_2V_1\sin 2\phi-U_3V_4\cos 4\phi+U_3V_3\sin 4\phi$$
$$X_{26}' = -\tfrac{1}{2}U_2V_2\cos 2\phi+\tfrac{1}{2}U_2V_1\sin 2\phi+U_3V_4\cos 4\phi-U_3V_3\sin 4\phi$$

$$Q_{11} = U_1+U_2+U_3$$
$$Q_{22} = U_1-U_2+U_3$$
$$Q_{12} = U_4-U_3$$
$$Q_{66} = U_5-U_3$$
$$Q_{16} = 0 \qquad Q_{26} = 0$$

$$A_{ij}' = A_{ij} \qquad B_{ij}' = dA_{ij}+B_{ij} \qquad D_{ij}' = d^2A_{ij}+2dB_{ij}+D_{ij}$$

FIGURE 7

matrix mathematics required to solve composites problems, more time can be devoted to grasping concepts without sacrificing the valuable experience of obtaining numerical solutions. The possibility of arithmetic error in the calculation of results is also greatly reduced.

It is hoped that with the aid of the composite materials module, the orthotropic material calculations which right now seem so formidable to many will eventually become as familiar and straightforward as those for isotropic materials.

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REFERENCES

1. "Personal Programming", Texas Instruments Incorporated (1977).

2. S. W. Tsai and H. T. Hahn, "Composite Materials Workbook", Technical Report AFML-TR-78-33 (1978).

3. S. W. Tsai and H. T. Hahn, "Introduction to Composite Materials, Volume 1: Deformation of Unidirectional and Laminated Composites", Technical Report AFML-TR-78-201, Volume 1 (1978).

4. S. W. Tsai and H. T. Hahn, "TI-59 Magnetic Card Calculator Solution to Composite Materials", Unpublished (1979).