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LUBRICANT COMPACT WEAR RATE TECHNIQUES

K. R. Mecklenburg

Midwest Research Institute
425 Volker Boulevard
Kansas City, Missouri 64110

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Materials Laboratory
Air Force Wright Aeronautical Laboratories
Air Force Systems Command
Wright-Patterson Air Force Base, Ohio 45433

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B.D. McConnell

B. D. McConnell
Project Engineer

R.J. Benzing

R. J. Benzing
Chief, Fluids, Lubricants and
Elastomers Branch

FOR THE COMMANDER

F D Cherry

F. D. Cherry
Chief, Nonmetellic Materials Division

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The need for solid lubrication of cruise missile engine bearings prompted a study on the wear rate of a solid lubricant compact under various conditions of load and velocity. The experimental procedure and test specimen configuration were detailed in the report. Mathematical handling of the weight loss data, discussed in detail, allowed the representation of the wear rate as a function of various combinations of load and velocity to be presented in a single equation, the equation of the wear rate "surface". Two equations were developed on the lubricant compact, one for oscillatory motion and one for unidirectional motion.

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20. ABSTRACT (Concluded)

Alternate forms for the wear equation were discussed with the reasons for non-use. Detailed computer programs developed for data handling were also presented.

FOREWORD

This final report describes the work performed by the Midwest Research Institute, 425 Volker Boulevard, Kansas City, Missouri 64110, under U. S. Air Force Contract F33615-78-C-5227. The work was monitored by the Materials Laboratory, Air Force Wright Aeronautical Laboratories (AFWAL/MLBT), Air Force Systems Command, Wright-Patterson Air Force Base, Ohio and was sponsored by the Defense Advanced Research Projects Agency, Department of Defense, Arlington, Virginia, under DARPA Order Number 3576. Mr. B. D. McConnell, AFWAL/MLBT, was the Project Engineer. The report covers the period from 15 September 1978 to 15 October 1980.

This technical report was submitted by the author in November 1980.

The author wishes to thank Mr. Carl Windisch for his assistance in the performance of the experimental work and Mrs. Wanda Wildinger for her patience and understanding in the typing associated with this report.

TABLE OF CONTENTS

LIST OF ILLUSTRATIONS

LIST OF TABLES

I. INTRODUCTION

A cruise missile engine was designed using conventional oil lubrication of the bearing. The concept of the design was adequately demonstrated and the missiles were accepted as part of the overall warhead delivery system.

After the concept was demonstrated and the logistics of delivery were resolved, some problems emerged. One of the envisioned mechanical problems was that the oil lubrication system for the engine would not have adequate storage capability. The oil itself would chemically change during the storage time. The oil would also evaporate and migrate to other parts of the missile, where oil contamination is not desired.

Part of the solution to the oil lubrication problem was to develop a solid lubrication system as a replacement for the oil system. Other parts of the solution were to improve the rolling element quality by surface modification, thereby reducing the need for lubrication, and to change the basic materials of the rolling element bearing to eliminate the need for lubrication.

In developing the solid lubrication system, several material characteristics had to be determined for the best candidate lubricant materials. Some of these characteristics included the material integrity at the elevated temperatures expected in the engine, basic friction characteristics, and the rate of wear of the material under various conditions of rubbing speed and applied load.

The need for wear rate information formed the basic requirement for this program. The wear rates for individual conditions of speed and load were to be determined for the solid lubricant compact material. These wear rates were then to be combined into one mathematical expression that could be used by a computer program in the prediction of the life and stability of the solid lubricated bearing system.

The program was divided into three parts: mathematical technique development, experimental wear rate determinations, and material equation representation. During the mathematical technique development, existing data from three lubricant compact materials were used to formulate the necessary handling technique and data reduction programs. The single equation representing the wear rate as a function of both speed and load was developed for each of these three compact materials for which adequate and relatively accurate wear rate data were available. During the experimental wear rate determination phase, specialized handling techniques had to be developed for the new lubricant compact material. When the preliminary weight loss measurements were determined and found to be inconsistent, the lubricant material was examined and found to be hygroscopic, requiring special handling and careful experimental timing techniques. During the material equation representation, the data were simply placed in the various computer programs that had been developed and the single wear rate surface equation was extracted.

The results of the program have been presented in this report. The report has been organized to present the results as the lubricant material would have progressed; from being received, through testing, and then into data reduction. However, the actual working timetable was more efficient. While the experimental wear rate determinations were being made on the new material, previous data from three lubricant compact materials were used to develop the mathematical techniques and definitions. After these two phases were simultaneously completed the final material equation was determined for the oscillatory mode of operation. More specimens were made available, and the experiments in the unidirectional mode of operation were performed while the data from the oscillatory mode were examined for agreement with various other equation formats. Finally, the material equation for the unidirectional mode of operation was determined.

This report presents the experimental techniques, including the equipment and test specimen descriptions, operating procedure, and test results; the mathematical techniques, including the wear surface concept, scattering coefficient, weighting factor and weighting value, curve-fitting, surface equation extraction, and results; alternate equation forms, including curve-fitting and various wear equations; and the conclusions.

II. EXPERIMENTAL TECHNIQUES

This part of the report describes the equipment used during the experimental weight loss determinations, the lubricant compact test specimens that were subjected to various loads and rubbing velocities while at 315°C (600°F), the operating procedure that had to be developed to accommodate the hygroscopic nature of the lubricant specimens, and the results of the weight loss experiments, including some of the frictional data.

A. Equipment Description

The equipment used for the experimental wear rate determination phase of the program was basically a dual rub shoe tester. The test element configuration is shown in Figure 1. The machines used (there were two, one for oscillatory motion and one for unidirectional motion) were Hohman A-6 testers, using the downward-facing test shaft for the test element location. In this configuration, a disc was either rotated or oscillated between two rub blocks or rub shoes. The speed of the shaft, which was controlled, was one of the variables of the test conditions.

Testing was done in two modes: oscillatory and unidirectional. The oscillatory work was done first to provide the information needed to augment another program dealing with the same rub shoe material. In the oscillatory mode, motion of ±30 degrees was provided through an eccentric connection between the rotating motor shaft and oscillating test shaft. Another machine from the same manufacturer was used for the unidirectional motion tests.

Another variable was the load on the rub shoes. For this work, the rub shoes were conforming, as opposed to plain or flat. Conforming shoes means that the entire rubbing shoe face is mated with the outer surface of the moving disc, as opposed to the initially line contact that would exist with flat rub shoes. The loads on the conforming shoes were reported in units of pressure, and, for the purists, the pressure is in force per unit area of apparent contact and not area of asperity contact. Actual loads were applied to the shoes through a deadweight, lever, and cone system with an overall multiplication factor of 50.

The last of the variables was temperature. All work was done at 315°C (600°F). The test zone was maintained at 315°C by an oven half supported below the test elements. The oven extended upward around the shoes and disc, enclosing as much of the configuration as possible. Temperature was monitored and controlled through an iron-constantan, Type J, thermocouple mounted in the metal backing of the left rub shoe. Although the absolute accuracy of any thermocouple is questionable unless the thermocouple is accurately calibrated, the repeatability from one experiment to another with this technique produced very good results. That is, the same thermocouple and the same oven, used in the same manner for every experiment, produced test zones of better than ± 4°C from the control value of 315°C.

The room in which the experiments were conducted had controlled temperature, and the relative humidity varied about ± 10% at the normal room temperature of 20°C. The humidity variation at the test temperature of 315°C was not significant to the experiments. As the specimens cooled from 315°C to 30°C, the humidity was found to influence the recorded weight loss. This will be discussed further under the Operating Procedure section of this report.

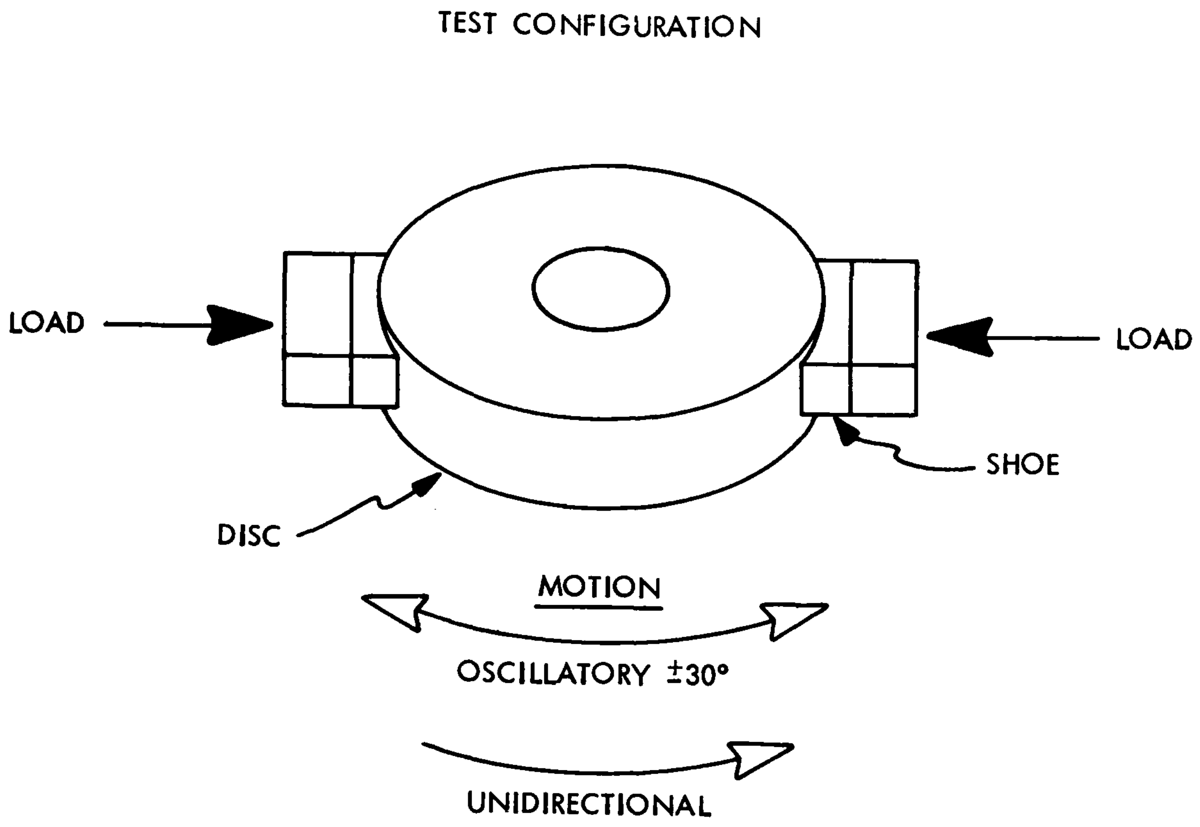


Figure 1 - Dual Rub Shoe Configuration

B. Test Specimens

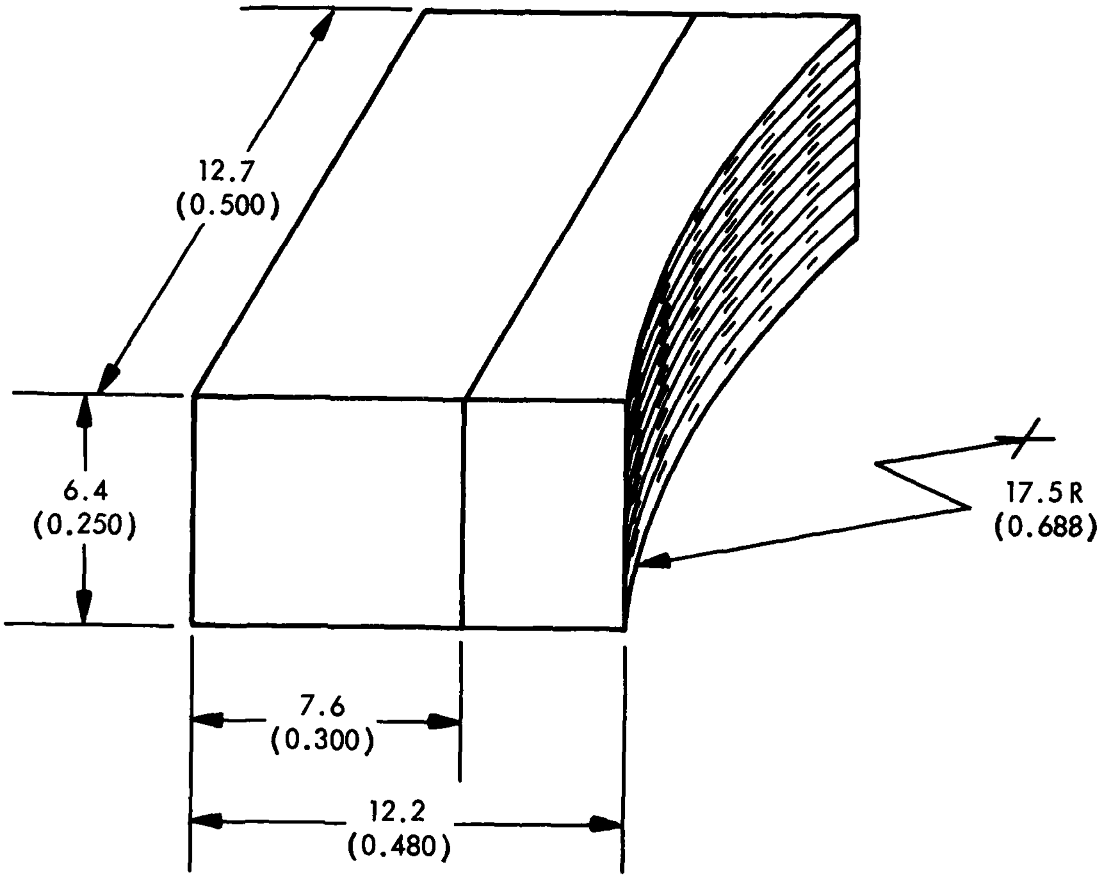
The rub shoe specimens used for this work had the configuration and dimensions shown in Figure 2. The shoes were basically rectangular parallelepipeds, 12.7 mm by 12.2 mm by 6.4 mm, with a 17.5-mm radius machined into one of the 12.7-mm by 6.4-mm faces, as shown.

The composite material that comprised the working part of the shoe was made from a three-dimensional weave of carbon fibers reinforced with Thermid 600 polyimide resin containing powdered Westinghouse composite and dibasic ammonium phosphate additives. The composite material was glued to a 440°C stainless steel backing block. The glued, two-piece construction facilitated handling and tightening in the test apparatus and permitted more testing with less of the specialized composite material. The rub shoes were supplied by M. N. Gardos of the Hughes Aircraft Company.

Two rub shoes were used for each test. The shoes were designated simply the left shoe and the right shoe as the test apparatus was normally viewed. The stainless steel backing block of the left shoe had a 1.2-mm hole drilled into it approximately 6 mm deep for insertion of the thermocouple used for temperature control.

The mating surface was a disc of 440°C, hardened to Rockwell C 57 ± 3 and ground to an initial surface finish of less than 200 nm, rms (8 μin., rms). The actual surface finish was measured and found to be 100 nm, rms (4 μin., rms) in the direction of the centerline axis and 125 nm, rms (5 μin., rms) around the outer, rubbing surface. The disc dimensions were 35 mm outside diameter (1.375 in.), 16 mm inside diameter (0.625 in.), and 9.5 mm thick (0.375 in.). The 9.5-mm thickness of the disc more than adequately covered the entire 6.4-mm thickness of the composite portion of the rub shoe.

Before use, the block was washed with acetone. After the initial operation, the disc was not recleaned, thus maintaining the lubricant transfer film.



Lengths shown in mm (inch).

Figure 2 - Rub Block Dimensions

C. Operating Procedures

As previously stated, the objective of this work was to obtain data on the weight loss of the rub shoes as functions of time, load, and speed. Weight loss as a function of time produced the wear rate, and, by using several time periods, any wear rate variation as a function of time was found. Using selected levels of operating speeds, wear rate as a function of load for each speed was determined by varying the load. Using selected levels of loads, the wear rate as a function of speed for each load was determined from the same data with the varied levels of speed. Thus, the operating procedure had to allow the accurate determination of weight loss data under various conditions of speed, load, and time.

Frictional coefficients were also desired from this test work, and some frictional values were obtained for the oscillatory mode of operation. However, the old frictional force measuring system failed before the new recording system was received. By the time the new system was received, all of the test specimens for the unidirectional testing had been used. No frictional coefficients were available from the unidirectional testing.

The operating procedure for the test sequence started with the establishment of the correct temperature environment around the previously installed test specimens. Temperature was monitored by an iron-constantan, Type J, thermocouple installed in a drilled hole in the left rub shoe. When the temperature controller had cycled twice, the test zone was considered to be adequately stabilized.

The testers (one for oscillatory motion and one for unidirectional motion) had been previously calibrated for operating speed versus control setting, so that when a particular control setting was established, the speed of operation was known. The speed control system had a feedback system that was quite responsive. Speed variation with load did not exceed ± 0.5%.

When both speed and temperature were established satisfactorily, the load was applied and the timer started. Since loading is by deadweight and through a lever system, it is necessary that the loading arm remain level throughout the test period. This was the operator's function. The Hohman A-6 tester has a screw-controlled pivot for the loading lever in which wear compensation adjustments can be made.

After the required test time, the motion was stopped, the lever system was unloaded, the heating oven was turned off, and the oven was lowered away from the test zone. It had been previously determined that 45 min under these conditions allowed the test specimens to cool to approximately 30°C. The rub shoes were then removed and weighed. After weighing, the shoes were either reinstalled for the next test or placed in a desiccator for storage.

It was found in the early portion of the test work with the composite shoes that erratic results in weight loss information were noted when the shoes were stored in normal laboratory air. With this information, a special experiment was conducted that confirmed that the composite material was hygroscopic. Desiccator storage demonstrated no weight change with time, while storage in laboratory air (even on the balance pan) produced a weight gain as moisture entered the composite material.

It was also found in the early portion of the test work that new rub shoes produced inconsistent weight loss data the first time they were used. There seemed to be a "breaking in" time for the new shoes. Thus, it was established that each new rub shoe had to have a certain amount of operation before it could be used in an experiment. For the oscillatory condition, break-in operation was conducted at 200 cpm for 60 min with 6.9-MPa loading. For the unidirectional mode, break-in operation was at 100 rpm for 30 min with 4.1-MPa loading. As with the actual test conditions, the break-in was done at 315°C.

During the testing program, rub shoe failures were occasionally encountered. The composite material seemed literally to come apart, as the wear rate became excessive, and the test had to be terminated. Under some of these conditions (usually high load and high speed), the actual time to failure was recorded as the operating time, but only the data for the other shoe was used in the calculations. The "bad" shoe data were not included. It will be noted at this point that the nonexistent data ("zero" weight loss) had to be handled in a special manner in the data reduction portion; the computer programming required the recognition of "zero" input without bombing.

Information was recorded from each experiment on a data sheet, as shown in Figure 3. The shoe dimensions were taken in an attempt to correlate weight loss with dimensional changes. The attempt was fruitless, as mushrooming, disintegration, and generally inconsistent dimensional changes made correlation extremely difficult. The prime variable was the weight change.

Friction values were read from the recording chart and converted to friction coefficients. The readings were made shortly after the test stabilized, then 15 min into the test, 60 min into the test, and finally at the end of the test. Not all values were determined for each test, as some tests failed rapidly and others were scheduled for only one hour.

Date: ____________

LUBRICANT COMPACT WEAR RATE STUDIES

	Start		Finish	
Shoe Dimensions	Left	Right	Left	Right
Width				
Depth				
Thickness				
Weight				
Overall Dimension				

Load ________ Temperature Set at 600°F

Speed, Dial Setting ________ Desired Speed ________

Cycles ________ ________

Time of Operation ________

Average Speed ________

Frictional Torque

	Lines	Torque
Start-up		
15 Minutes		
60 Minutes		
Finish		

Comments:

Figure 3

D. Test Results

The weight loss data from the oscillatory tests are presented in Table 1. The table includes the conditions of test and the resulting weight loss information for 68 of the tests performed.

The loads used for these experiments were 1.4 MPa (200 psi), 4.1 MPa (600 psi), 6.9 MPa (1,000 psi), 13.8 MPa (2,000 psi), 20.7 MPa (3,000 psi), and 27.6 MPa (4,000 psi). The ball-to-ball pocket loads in an actual operating ball bearing have been found to be approximately 22 N (5 lb) (Reference 1). With the material being tested, in the bearing configuration, the calculated Hertz contact stress was between 195.8 MPa (28,400 psi) and 16.0 MPa (2,300 psi), depending upon the assumptions regarding bearing geometry. If some wear is experienced so that the wear scar is 1.5 mm (0.060 in.) diameter, the average unit stress drops to 12.2 MPa (1,770 psi). If the 22-N (5-lb) force is off by 4 to 9 N (1 to 2 lb), the Hertz stress levels would vary according to the following values:

Force		Stress	
N	lb	MPa	psi
31.1	7	17.93	2,600
26.7	6	17.02	2,469
22.2	5	16.02	2,324
17.8	4	14.88	2,158
13.3	3	13.51	1,960

The stress levels used for the wear rate determinations fairly well bracketed the expected stress level in the operating bearing.

The velocity factor (the oscillation rate) was 100, 200, 400, or 800 cycles per minute (cpm). The velocity for an oscillating motion cannot be constant, and the variation in surface velocity was basically sinusoidal. For the 35-mm (1.375-in.) diameter disc, moving through an arc of ± 30 degrees, the average surface velocity and the maximum surface velocity varied regularly, as shown in Table 2.

TABLE 1

WEIGHT LOSS DATA, OSCILLATORY MODE

	Conditions			Weight Loss	
Test Number	Load (MPa)	Velocity (cpm)	Length of Test (min)	Left Shoe (g)	Right Shoe (g)
109	1.4	100	98.5	0.0030	0.0040
119			120.0	0.0035	0.0045
124			180.0	0.0035	0.0030
101			420.0	0.0040	0.0060
120		200	120.0	0.0030	0.0035
111			180.0	0.0040	0.0040
102			420.0	0.0050	0.0060
121		400	120.0	0.0050	0.0045
115			180.0	0.0035	0.0030
106			195.2	0.0030	0.0035
125			420.0	0.0090	0.0075
118	4.1	100	60.0	0.0028	0.0015
112			120.0	0.0050	0.0037
103			180.0	0.0035	0.0035
122		200	60.0	0.0030	0.0025
113			120.0	0.0037	0.0035
104			180.0	0.0045	0.0060
123		400	60.0	0.0037	0.0023
116			120.0	0.0060	0.0051
108			180.0	0.0025	0.0025
114	6.9	100	60.0	0.0035	0.0005
105			120.0	0.0055	0.0045
117			180.0	0.0045	0.0035
46		200	60.0	0.0030	0.0075
107			60.0	0.0105	0.0070
48			180.0	0.0115	0.0611
49			235.0	0.0200	0.1674
51		400	60.0	0.0031	0.0210
126			120.0	0.0100	0.0075
52			180.0	0.0150	0.1011
55		800	60.0	0.0501	0.0320
137			60.0	0.0050	0.0065
57			151.1	0.0018	--
56			180.0	0.0464	0.0355
59	13.8	200	60.0	0.0185	0.0165
135			60.0	0.0065	0.0075
62			88.4	0.0060	—
86			180.0	0.0005	0.0125
66			420.0	0.0245	--
65		400	60.0	0.0080	--
136			60.0	0.0045	0.0055
72			68.0	0.0125	0.0162
64			180.0	0.0167	0.1036
67		800	40.0	0.0145	0.0069
68			60.0	0.0215	0.0475
71			180.0	0.0160	0.0844
91	20.7	200	60.0	0.0005	0.0060
92			120.0	0.0008	0.0285
127			120.0	0.0060	0.0060
93			180.0	--	0.0230
94		400	60.0	0.0080	0.0175
95			120.0	0.0160	0.0661
96			180.0	0.0205	0.0059
97		800	35.7	0.0170	0.0380
128			60.0	0.0210	0.0185
129			120.0	0.0490	0.0440
75	27.6	200	32.6	0.0005	0.0044
80			60.0	0.0488	0.0135
74			180.0	0.0486	0.0889
76		400	38.6	0.0010	--
81			60.0	0.0443	0.0390
130			120.0	0.0225	0.0195
87			180.0	0.0176	0.0839
82		800	6.0	0.0001	—
77			26.0	0.0119	—
131			30.0	0.0185	0.0180
84			50.6	0.0490	--
89			55.9	0.0611	0.0854

TABLE 2

SURFACE VELOCITY VALUES FOR VARIOUS OSCILLATION RATES

Velocity Factor (cpm)	Maximum Surface Velocity (mm/s)	Maximum Surface Velocity (ft/min)	Average Surface Velocity (mm/s)	Average Surface Velocity (ft/min)
100	182.9	36	61.0	12
200	365.8	72	121.9	24
400	731.5	144	243.8	48
800	1,463.0	288	487.7	96

Conditions of Motion:

Arc	± 30°
Diameter of Disc	35 mm (1.375 in.)
Rubbed Distance	36.6 mm/cycle (1.44 in./cycle)

Forty-four other oscillatory tests were conducted in the preliminary work, in which the hygroscopic nature of the composite material was discovered and in which the effects of the cooling rate and heating procedure were established. Twenty-five break-in runs and "false starts" were also conducted. A false start was a test in which something unusual caused the test to be terminated. One type of false start was the unexplained uneven wear of the shoe (or shoes) in the test in which the shoe wore along one edge and allowed the counter-balanced holders to turn. The load then forced the shoes down and away from the test disc. Another type of false start resulted when the glue between the composite material and the backing block failed. If the shoe had been positioned properly and was operating satisfactorily, such a glue failure might not have been noticed until the shoes were unloaded at the conclusion of the test. Under these circumstances, the composite part of the shoe usually fell to the bottom of the test apparatus and broke into several segments with a resulting loss of powder and weight loss data accuracy. When the shoes were unloaded, they were still at 315°C. No attempts were made to handle or catch the rub shoe components at that temperature. The test chamber was also loaded with the oven half, on its elevating mechanism, and safety from fires was considered more important than salvaging an already destroyed rub shoe.

It should be mentioned that the Hohman A-6 tester is a very rugged machine, with adequate power to drive the test specimens. At the conditions of test (315°C, ± 30 degree motion), the maximum load used was 2,224 N (500 lb), producing the 27.6-MPa (4,000-psi) pressure. The operation under these conditions was quite impressive, with the heat, motion, and strain of the mechanism. Also, this load and motion were being applied to a plastic-type compound that demonstrated a reasonable rate of wear.

As mentioned in the Operating Procedure section, the dimensional changes of the rub shoe were not consistent enough to allow the changes to be correlated with the weight loss information. The dimensional changes were influenced by mushrooming and disintegration. Even the "overall dimension" was not consistent. The overall dimension was an attempt to measure the wear of the shoes by recording the distance between the backs of the rub shoe holders. A difference in overall dimension was supposed to provide the wear into the face of the shoe. Attempts to record the overall dimension were influenced by the thermal state of the test zone as well as the condition of the shoes. The attempts were quickly abandoned as nonproductive.

The weight loss data from the unidirectional tests are presented in Table 3. The table includes the conditions of test and the resulting weight loss data for 36 of the tests. As with the oscillatory testing, other operations were performed on these rub shoes. However, these other operations were primarily break-in, as the number of false starts was reduced to two and only four preliminary tests were required. The rub shoe supply was meager, and the testing was restricted to the low load and low speed regimes. Only a few tests were run at the moderate load condition of 6.9 MPa (1,000 psi) and none at the 13.8-MPa (2,000-psi), 20.7-MPa (3,000-psi), or 27.6-MPa (4,000-psi) levels.

TABLE 3

WEIGHT LOSS DATA, UNIDIRECTIONAL MODE

	Conditions			Weight Loss	
Test Number	Load (MPa)	Velocity (rpm)	Length of Test (min)	Left Shoe (g)	Right Shoe (g)
161	1.4	50	60.0	0.0015	0.0035
167			60.0	0.0239	0.0096
155			68.0	0.0035	0.0025
174			120.0	0.0200	0.0125
141			180.0	0.0080	0.0075
172		100	60.0	0.0045	0.0040
165			120.0	0.0315	0.0140
146			180.0	0.0060	0.0040
162		200	60.0	0.0290	0.0210
168			60.0	0.0065	0.0003
157			120.0	0.0070	0.0065
176			120.0	0.0268	0.0179
139			180.0	0.0075	0.0070
179		400	60.0	0.0340	0.0300
177			120.0	0.0543	0.0450
150			180.0	0.0090	0.0075
159		600	15.0	0.0085	0.0075
143			180.0	0.0165	0.0150
145	4.1	50	120.0	0.0085	0.0170
170			180.0	0.0355	0.0205
156		100	60.0	0.0060	0.0050
164			60.0	0.0480	0.0215
175			60.0	0.0125	0.0100
140			120.0	0.0055	0.0040
163			180.0	0.0315	0.0210
169			180.0	0.0510	0.0360
166		200	60.0	0.0475	0.0297
148			120.0	0.0115	0.0080
173			180.0	0.0324	0.0202
178		400	110.0	0.1083	0.0920
142			120.0	0.0150	0.0075
153		600	5.1	0.0135	0.0125
144	6.9	50	60.0	0.0055	0.0055
147		100	60.0	0.0035	0.0040
149		200	60.0	0.0065	0.0060
151		400	60.0	0.0195	0.0170

The velocity factor for the unidirectional testing was simply the rate of rotation. Values of 50, 100, 200, 400, and 600 rpm were used for the unidirectional testing, corresponding to surface velocities of 11 m/min (36 fpm), 22 m/min (72 fpm), 44 m/min (144 fpm), 88 m/min (288 fpm), and 132 m/min (432 fpm).

The friction coefficients for most of the oscillatory mode testing are presented in Table 4. As mentioned earlier, the old frictional recording system failed and the new system was received and installed after the unidirectional test program was completed. Other than presenting the frictional coefficient values for the various conditions, no other analysis was attempted on the frictional values. The data were not considered to be sufficiently accurate to warrant further analysis, due to the inaccuracy in the data recording system and the variation in material from shoe to shoe.

TABLE 4

FRICTION COEFFICIENTS, OSCILLATORY MODE

Test Number	Conditions Load (MPa)	 Velocity (cpm)	Length of Test (min)	Frictional Coefficient Start	 15 min	 60 min	 Final
46	6.9	200	60.0	0.143	0.140		0.140
48			180.0	0.175	0.113	0.099	0.094
49			235.0	0.148	0.070	0.079	0.112
51	6.9	400	60.0	0.113	0.131		0.079
52			180.0	0.122	0.087	0.070	0.058
55	6.9	800	60.0	0.113	0.087		0.087
57			151.1	0.244	0.157	0.087	0.079
56			180.0	0.113	0.094	0.140	0.157
59	13.8	200	60.0	0.157	0.134		0.119
62			88.4	0.161	0.161	0.166	0.148
86			180.0	0.209	0.122	0.100	0.096
66			420.0	0.188	0.136	0.105	0.105
65	13.8	400	60.0	0.144	0.087		0.096
72			68.0	0.148	0.113	0.105	0.105
64			180.0	0.113	0.135	0.096	0.070
67	13.8	800	40.0	0.214	0.144		0.148*
68			60.0	0.151	0.086		0.091
71			180.0	0.155	0.087	0.101	0.079
91	20.7	200	60.0	0.084	0.058		0.052
92			120.0	0.076	0.073	0.060	0.064
93			180.0	0.083	0.056	0.048	0.044
94	20.7	400	60.0	0.105	0.073		0.055
95			120.0	0.095	0.055	0.045	0.038
96			180.0	0.087	0.058	0.047	0.044
97	20.7	800	35.7	0.122	0.035		0.040
75	27.6	200	32.6	0.144	0.098		0.094
80			60.0	0.157	0.122		0.103
74			180.0	0.144	0.131	0.109	0.106
76	27.6	400	38.6	0.177	0.120		0.131
81			60.0	0.153	0.116		0.104
87			180.0	0.185	0.144	0.109	0.100
82	27.6	800	6.0	0.157			0.092
77			26.0	0.131	0.088		0.095
84			50.6	0.125	0.109		0.092
89			55.9	0.113	0.096		0.100

Notes: All testing done at 316°C (600°F) and ± 30 degree motion.
All shoes were pre-run.
*Frictional transducer trouble. Test stopped.

In both the weight loss measurements and the frictional force measurements, some variation in recorded performance was expected. However, some of the variation that was found could only be attributed to the variation of material from shoe to shoe. The weight loss tables, Tables 1 and 3, show that some test conditions were used more than others. When more than three experiments were run at any one set of conditions (and both shoes provided weight loss data), the variation in material performance was the reason for the additional test operation.

Analysis of the wear rate (the weight loss per unit of time) provided no set pattern of performance. The wear rates for the shorter time periods were compared to the wear rates for the longer time periods. No significant trend was found. Indeed, in the analysis for different formats for the final wear rate surface equation, time was found to be a linear factor in the wear rate. That is, the coefficient on the modified Archard equation (or Rhee equation) was found to be very close to unity, which means that material performance varies directly with time. Hence, this report deals with wear rate and not wear.

Any reported wear rate or weight loss experimental work has shown variation for identical conditions or has shown inconsistencies or variations from linear relationships. The variations, which seemingly cannot be eliminated, can only be reduced in magnitude by careful experimental procedures. This work also produced variations in wear rate, some of which were attributable to the material variations. Whatever the source of the variation, some degree of repeatability existed with these data. The degree of repeatability was calculated and incorporated into the data reduction techniques. The degree of repeatability has been termed "data scatter," and the concept of a "scattering coefficient" was originated and has been defined in the following section of this report.

III. MATHEMATICAL TECHNIQUES

This part of the report introduces the concepts of: a wear rate surface and how the surface concept was developed; a scattering coefficient and how the variation in wear rate was handled; and the weighting factor and weighting value and how the variation in accuracy of the data were handled. Also included in this part of the report are the curve-fitting processes, the surface equation extraction, and the resulting wear rate surface equations.

A. Wear Surface Concept

After the weight loss data had been generated, the data processing procedure began. In this procedure, the data were to be reduced to a single equation that would provide the rate of wear of the lubricant compact for any load and velocity combination (within the tested regime). The desired result from this data processing procedure was a single, empirical formula that could be used by a specialized computer program in predicting the operating life and stability of a solid lubricated bearing system based upon the Thermid 600 polyimide, 3D weave composite material.

The concept of what was done has been broken down into a series of perceptions or steps. These perceptions are explained, one by one, until the entire concept has been explained. Any mathematical relationships that were found to be required are explained individually.

In the handling of the data derived from three controlled inputs (in this case: time, load, and velocity), a pictorial representation of the inputs and output has often been found necessary. For this work specifically, time was removed as an input since its effect was found to be directly related to wear. That is, if the time had been doubled or halved, the expected weight loss would have doubled or halved, within the degree of accuracy justified from the original weight loss data. No significant deviation from this concept was found in any of the subsequent analysis work.

The representation of wear rate (in which the time element for the weight losses has been removed) as a function of load and velocity has been represented by a three-dimensional graph in which load and velocity were the x and y coordinates and wear rate was the z coordinate, the vertical height. If only one condition of load and velocity had been considered, the average wear rate and the data scatter would have been represented as shown in Figure 4, Step 1 of the Three-Dimensional Wear Rate Surface Development. Both terms--average wear rate and data scatter--are mathematically defined in detail in the following section of this report. Meanwhile, an intuitive definition of an average wear rate and data scatter will suffice.

The average wear rate has been represented as an x on the vertical line that originated from one condition of load and velocity. The data scatter has been represented by the upper and lower lines parallel to the load axis. This representation technique was used so that the relative magnitudes of average values and amount of scatter could be more easily comprehended than low numbers with several zeros after the decimal point.

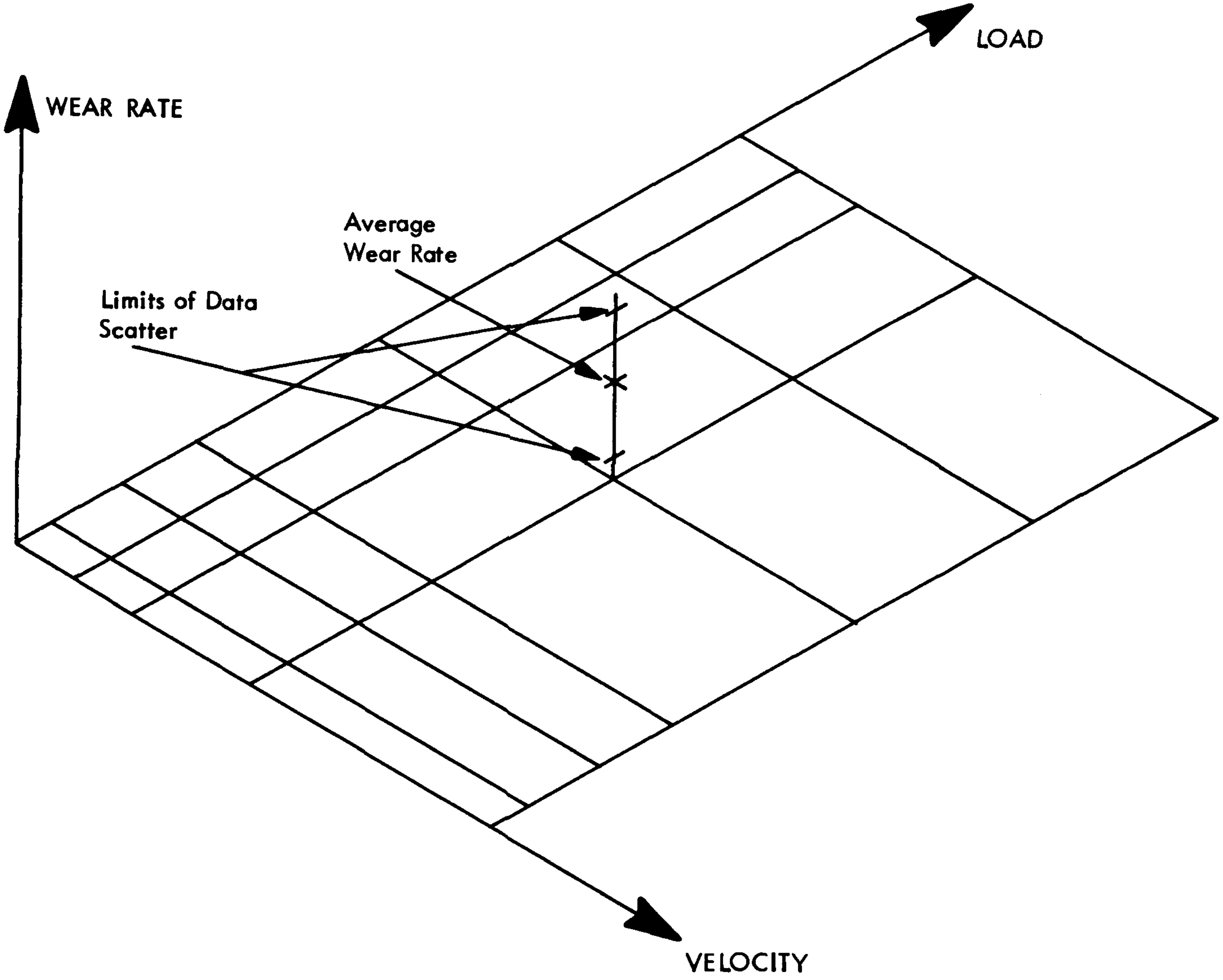


Figure 4 - Step 1 of the Three-Dimensional Wear Rate Surface Development

The next step was to take more test results and continue the plotting of average wear rate and data scatter. Using two more conditions at different velocities with the same load produced the representation shown in Figure 5. In addition to the fact that two more vertical lines with their averages and limits of data scatter were added, two additional factors were shown. The limit of data scatter for one of the points has been shown as below zero. Mathematically, such a value was possible but the physical interpretation for this phenomenon was that any wear rate value up to the value of the upper representation was possible. Prolonged wear rates below zero do not exist for this solid lubrication system, although temporary weight gains are possible as material migrates from one element to the other. The mathematical definition of data scatter or scatter coefficient included the concept of probability and expected maximum limits as applied to the wear rate. It is to be noted that no weight gains were recorded for any of the actual experiments.

The second new factor introduced, shown in Figure 5, was the representative curve of the best fit to the data. This curve represented the line that was found to best satisfy the requirements that were imposed upon it. These requirements have been presented in the development of the entire concept. For this stage in the concept presentation, the fact that the curve intercepted the vertical lines, within the band of the data scatter, was sufficient.

Taking more of the wear rate data for various loads at the same velocity and adding them to the graph produced the results shown in Figure 6. It has been emphasized in this figure that the two curves, one for wear rate as a function of velocity and the other for wear rate as a function of load, have a common point. This common point has been shown to be the same for both curves. In fact, it was the purpose of the mathematical techniques involved to make this point the same for both curves.

It was also demonstrated in Figure 6 that the data for some conditions of load and velocity were determined with greater accuracy and less variation than the data for other conditions. The data from some of the conditions that produced large scatter were determined early in the program. As the experimental technique was repeated, less variation due to operating procedure was encountered. The test data from the conditions that provided the smaller data scatter were also considered to be more significant in the curve-fitting operation. The process was termed "weighting" and has been fully explained in the section on Weighting Factor and Weighting Value.

As more and more of the wear rate data were added to the graph, the pictorial representation looked very "busy," as shown in Figure 7. Each of the curves for wear rate as a function of load intersected each of the curves for wear rate as a function of velocity. Not all intersections were within the band of data scatter, but for this set of data (which were the oscillatory wear rate data) only 2 of the 19 intersections were outside the data scatter bands and both were shown in the lower end of the wear rate as a function of load portion shown in Figure 6.

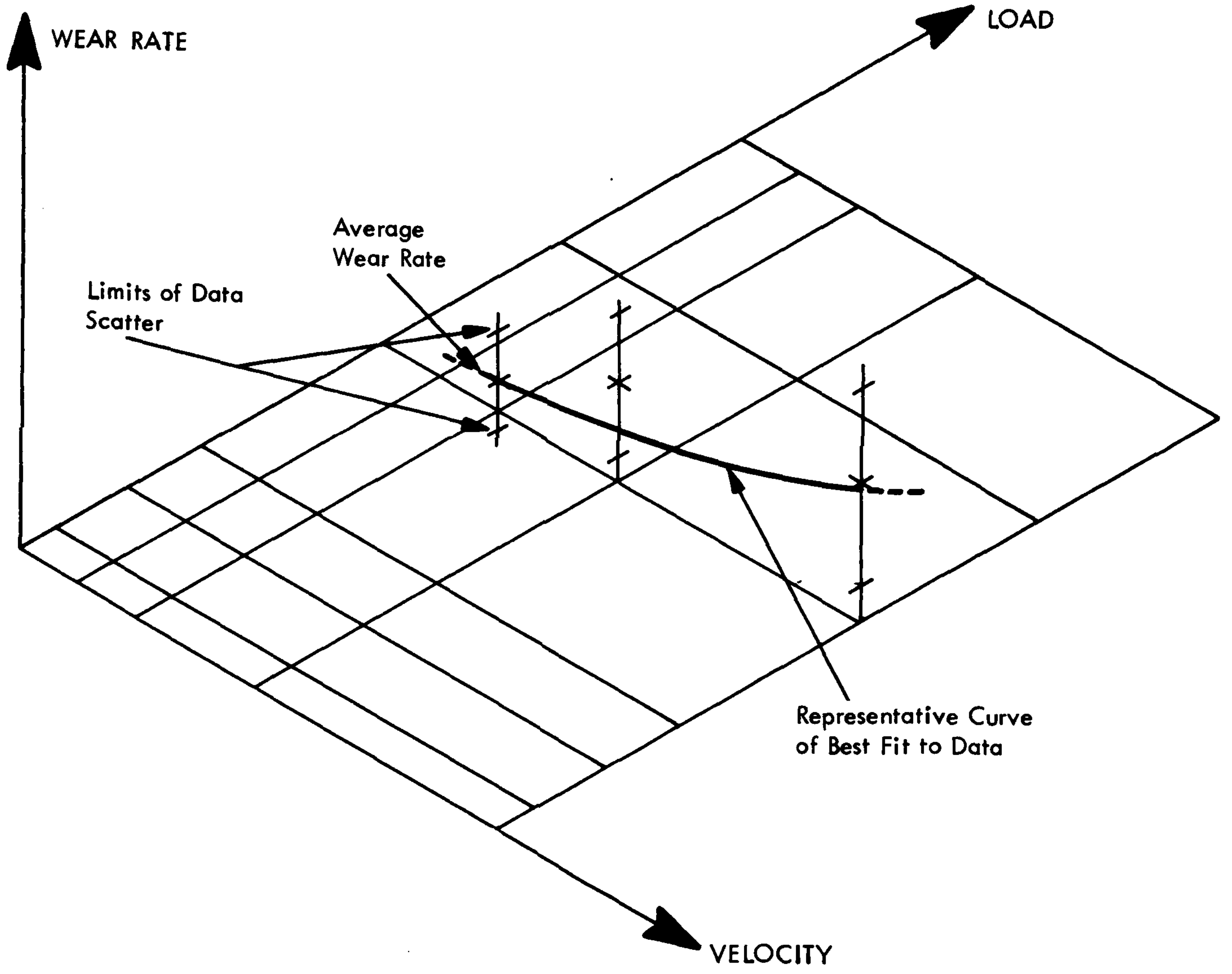


Figure 5 - Step 2 of the Three-Dimensional Wear Rate Surface Development

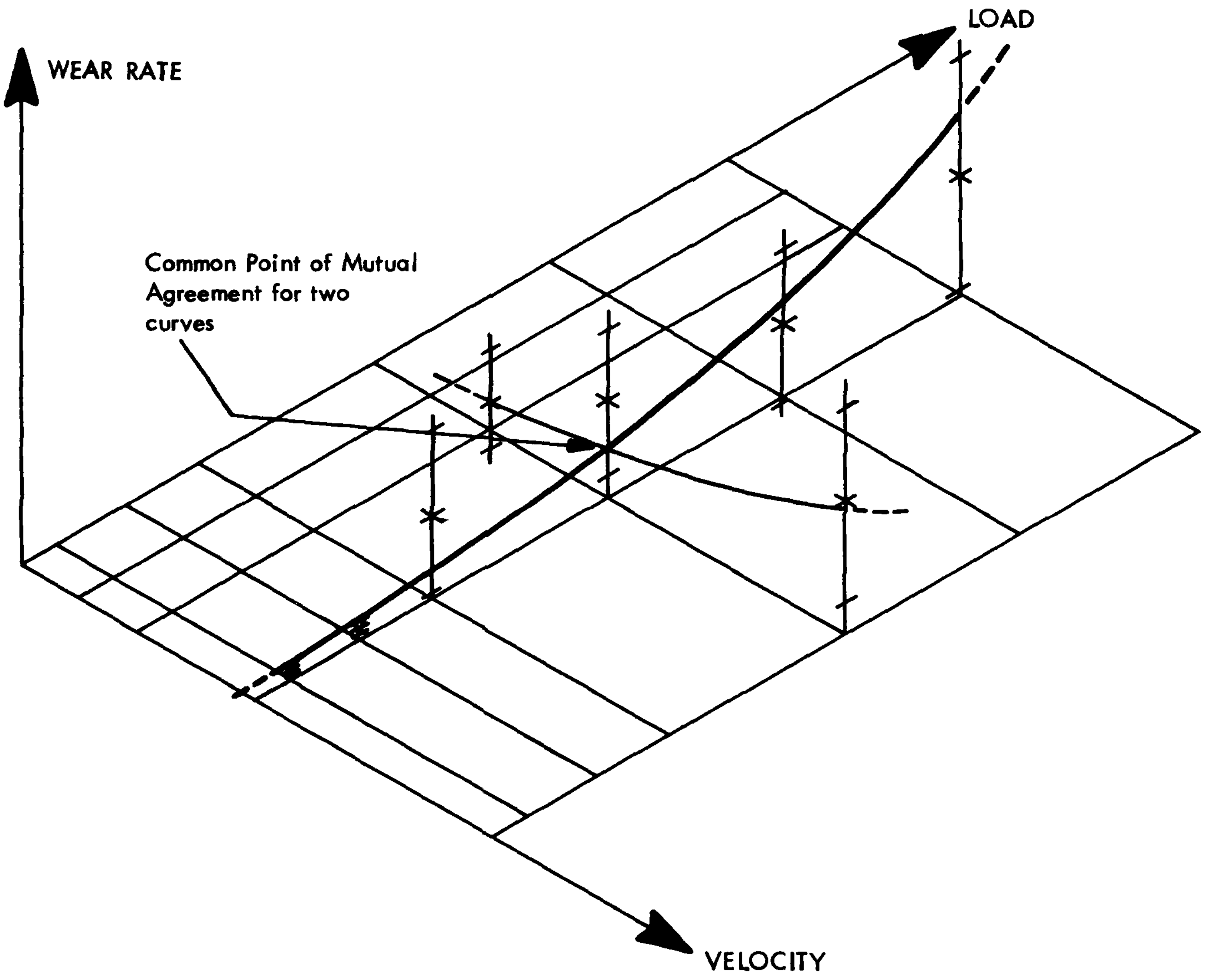


Figure 6 - Step 3 of the Three-Dimensional Wear Rate Surface Development

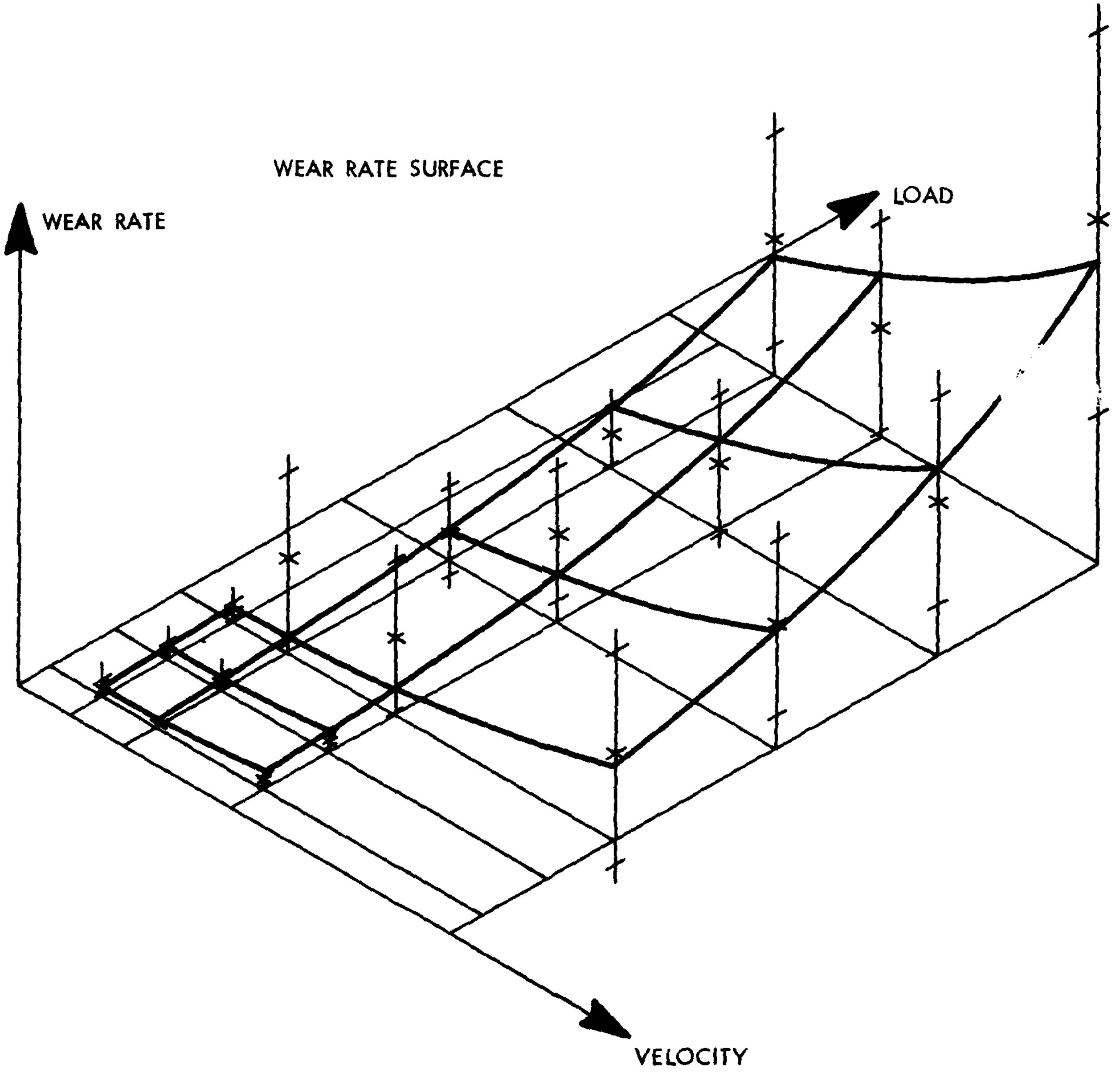


Figure 7 - Step 4 of the Three-Dimensional Wear Rate Surface Development showing a complete data set

Part of the difficulty in the data manipulation was in obtaining the equations for each of the curves so that the curves provided the same wear rate value from both "directions." That is, the value for the wear rate from a curve of wear rate as a function of load had to be the same as the value for the wear rate from a curve of wear rate as a function of velocity, for the same conditions of load and velocity. To fit 19 intersecting lines was an iteration procedure that converged, albeit not very rapid.

As Figure 7 was viewed, the concept of a wear rate surface became apparent. The height of this "surface" represented the wear rate, so that whatever conditions of load and velocity were chosen, the vertical height represented the wear rate at that set of conditions.

Proceeding from the state of conditions represented in Figure 7, the final step was to develop a mathematical equation for the wear rate surface and the program would be complete. There were, however, several steps that required further effort.

The equation for the wear rate surface was developed from the experimental data. There were inaccuracies in the experimental results that produced the data scatter. Interpretation of the surface equation developed for this family of curves should include some degree of reliability of the equation. The equation was found to be the best representation from the optional forms that were available, without producing a resulting expression that contained more coefficients than could be readily handled in a computer subroutine. Experience has shown that a tenth-order polynomial equation can be used to fit a line to a given set of data, but to attempt to cross-fit several tenth-order polynomial equations into one surface equation was considered beyond the scope of the program.

B. Scattering Coefficient

In this section of the report, the concept of scattering coefficient has been defined. Scattering coefficient was the mathematical term used to represent the amount of data scatter that was found in the wear rate data. This scatter has been shown as the band around each of the wear rate average values in Figures 4 through 7. Certain other definitions were required prior to defining the scattering coefficient and these definitions have also been presented.

The work described herein dealt with wear rate, a weight loss per unit of time. Wear rate is thus an average, and averages cannot be mathematically averaged themselves unless the base is common. That is, if each and every test had been run for 1 hr, the wear rates would have had a common base, 1 hr. Under these conditions, wear rates could have been averaged. This work did not use a common time base. The program goals included the desire to determine if the wear rate did vary as a function of time, and it was found that no significant variation in wear rate as a function of time did occur.

The average wear rate was first calculated for each condition of load and velocity. The average wear rate was the sum of the weight losses divided by the sum of the applicable times of operation. It will be noted that, for some experiments, one shoe was destroyed and did not produce data, resulting in a "zero" weight loss input. For each of these "zero" weight loss inputs, the corresponding operating time was also omitted. In the computer program written to handle the data reduction, the "zero" input condition required several operations that were not otherwise necessary.

In the statistical operation, the standard deviation of data around a base has been defined (Reference 2) from:

$$S^2 = \frac{1}{n} \sum_{i=1}^{n} (x_i - \bar{x})^2 \qquad (1)$$

where

S^2 = the sample variance
n = the number of terms
x_i = the ith term
$\bar{x}$ = the arithmetic average of the terms.

The sample standard deviation was defined as the positive square root of the sample variance, the S from above. Since averaging averages is an illegal operation, $\bar{x}$ (above) could not be simply the average of the wear rates. However, if T was defined as the average wear rate from:

$$T = \frac{\Sigma \text{ Weight Losses}}{\Sigma \text{ Operational Time}} \qquad (2)$$

then a term, scattering coefficient, could be defined as:

$$SC = \sqrt{\frac{\Sigma (WR_i - T)^2}{n-1}} \qquad (3)$$

where

SC = scattering coefficient
WR_i = wear rate for the ith condition
T = average wear rate
n-1 = one less than the number of terms involved, accommodating the correction required for small sample size.

The term scattering coefficient was used in this work in a manner analogous to the use of the standard deviation in statistical work. The analogy was that the actual wear rate had a 68.26% probability of being within the band formed by the experimentally determined average wear rate plus or minus the scattering coefficient. Roughly speaking, if numerous other wear rate experiments had been conducted and the average wear rate had been determined from all of the experimental data, the chances are two out of three that the average wear rate would fall between the value of the present wear rate plus the scattering coefficient and the present wear rate minus the scattering coefficient.

The use of the scattering coefficient in this work was to demonstrate and to give a numerical value to the degree of repeatability of the wear rate for different experiments at the same condition of load and velocity. If the scattering coefficient had been relatively small, the data were considered to be more precise. If the scattering coefficient had been relatively large, the data were considered to be less precise than desirable. Obviously, the curve-fitting operation performed on the average wear rates must consider the more accurate data to have more emphasis on the resulting curve equation than the less accurate data. Hence, the terms weighting factor and weighting value were defined, as discussed in the next section of this report.

C. Weighting Factor and Weighting Value

The scattering coefficient produced a numerical value related to the degree of accuracy of the wear rate data. A small value would seemingly represent a very precise value for the average wear rate and a large value would seemingly represent a sloppy value. However, if the basic wear rate average value was very small, a small scattering coefficient would not necessarily mean a very precise value for the average. A "normalizing" of the scattering coefficient was required, and this need led to the term, "weighting factor."

The weighting factor was defined as

$$Wt = \frac{T}{SC} \tag{4}$$

where

Wt = weighting factor

T = average wear rate, previously defined

SC = scattering coefficient, previously defined.

This weighting factor thus provided a ratio that would give generally equal treatment to the average wear rate for each condition of load and velocity.

The weighting factor usually produced a value ranging from 0.360 to 3.324. The variation was not great enough to use in influencing the curve-fitting operation. Hence, another term was arbitrarily established; this term was the "weighting value."

The weighting value was defined as 10 times the weighting factor, rounded to the nearest whole number. The use of the value 10 was entirely arbitrary, but was used from the consideration of practicality. Extremely good agreement in the individual wear rate values would produce a weighting value of 33 or less, while even the values of poor agreement would produce a weighting value of 4.

A computer program was written that handled all of the weight loss data and produced the average wear rate, the scattering coefficient, the weighting factor, and other information about the data for each individual condition of load and velocity. Rather than present the program within the report text, the program has been presented in its entirety in Appendix A. The program would accept the weight loss information and the times of operation for a single condition of load and velocity and would calculate and printout the following:

Individual Wear Rates, Left Side Shoes
Average Wear Rate, Left Side
Scattering Coefficient, Left Side
Individual Wear Rates, Right Side Shoes
Average Wear Rate, Right Side
Scattering Coefficient, Right Side
Average Wear Rate, Both Sides (WR)
Scattering Coefficient (SC)
Upper Band Value (WR + SC)
Lower Band Value (WR - SC)
Weighting Factor (Wt)

The printout of the answers also included the original inputted data for verification. The computer program was written to handle the "zero" weight loss input condition for those experiments in which the data on one shoe were lost.

After the terms were defined and the reasonableness assured, the use of the weighting factor influenced the curve-fitting operation. The curve-fitting operation has been described in the following section of the report.

D. Curve-Fitting

The average wear rates and weighting values were ready for the next step in the mathematical processing on the way to a single equation for wear rate as a function of both load and velocity. In order to understand the next operation, a review of Figure 5, entitled, Step 2 of the Three-Dimensional Wear Rate Surface Development, seemed appropriate. In this figure (page 23), three values of wear rate for three conditions of input velocity and one condition of load were shown. A curved line in the figure was also displayed and entitled "Representative Curve of Best Fit to Data." These curves were the necessary precursor for the wear rate surface equation, and their derivation has been presented in this section of the report.

Several computer programs are available for determining the equation of a line through a series of data points. The process has been termed least squares fit, linear regression, or curve-fitting. Whatever the name, the results are that the computer minimizes the point-by-point errors between the individual data points and the resulting curve. Generally speaking, the operator must select the form of the resulting equation prior to determining the coefficients for the curve. It is to be noted that computer programs exist for determining the best fit for the data for several different forms of the line equation. These programs not only select the best form, but they also determine the coefficients. One such program was used for this work to select the best form for the resulting line equations.

After the form of the line equation was selected using the data that would describe each of the lines, such as in the graph of Figure 7, the data were handled on a curve-by-curve basis. The computer program developed for this work has been presented in Appendix B, Weighted Curve-Fitting Program. The program of Appendix B was written for the Texas Instruments Programmable 59 computer/calculator (TI-59), which has various interchangeable software libraries. This computer or calculator has a built-in linear regression program. The purpose of the weighted curve-fitting work was to incorporate the requirement for weighting into the normal linear regression program. Details have been presented in Appendix B, but the program basically directed the computer to accept each wear rate data point the number of times specified by the weighting value. Since the curve-fitting operation attempts to minimize the point-to-curve errors, inputting one point several times placed more emphasis on that point and forced the resulting curve equation to emphasize that point more than another point with a lower weighting value.

After the coefficients for the curve were determined, the program also provided for the output of calculated points on the curve. These points were the calculated wear rates for the same conditions of either load or velocity that were inputted. For example, the input condition of constant load (200 psi) and velocity factors of 100, 200, and 400 were used with corresponding wear rates of 0.00001924, 0.00002682, and 0.00004145. The equation coefficients were determined as 0.00001548 and 0.002503. Recalculating the wear rates for 100, 200, and 400 velocity factors provided "new" wear rates of 0.00001988, 0.00002554, and 0.00004213.

The curve-fitting operation continued with the first iteration of the data. When calculated wear rates for the "load" curves (one load and various velocity factors, as above) (0.00001988, 0.00002554, and 0.00004213) were compared to the calculated wear rates from the "velocity" curves (one velocity and various loads) (0.00001928, 0.00002838, and 0.00004339), a difference would exist. The two values for each condition would be averaged (0.00001958, 0.00002696, and 0.00004276) and the curve-fitting operation would be repeated.

The iteration process was concerned with the situation simplistically depicted in Figure 6, entitled, Step 3 of the Three Dimensional Wear Rate Surface Development. In the figure (page 24), two curves were represented, one for the load and one for velocity, with one mutual point at the intersection. The iteration process modified both curves shown in this graph until both of the curves crossed the vertical line from the mutual condition at the same value of wear rate.

The use of the weighting value was restricted to the first two cycles of iteration. After that, the effect of weighting was insignificant. The same computer program was used, with a weighting value of one entered for each datum point.

The iteration process was continued until the difference between the two calculated wear rate values decreased to an acceptable level, usually defined by a correlation coefficient on the curve-fitting of 0.99999 or better for each curve.

To physically understand what was done mathematically, one might imagine that there were two wear rate surfaces, one coming from the wear rate as a function of load and the other coming from the wear rate as a function of velocity. The iteration process produced a "warping" or "twisting" of each of the two surfaces until they became one surface. Each surface was required to "give" by the same amount until merging was accomplished.

When the two surfaces matched, point by point, the surface equation coefficients were determined, as explained in the following section of the report.

E. Surface Equation Extraction and Results

After the curve-fitting operation was completed and all of the up to 19 intersections agreed, the extraction of the coefficients for the surface equation was relatively simple.

The curve-fitting operation previously described was for a curve that had the basic format (see Appendix B):

$$WR = a\,\epsilon^{bx} \tag{5}$$

where

WR = wear rate

ϵ = natural base

x = either load or velocity, whichever was being analyzed

a,b = constants determined from the regression analysis.

In the curve-fitting operation, various outputs could be requested. One of the significant requested outputs at the conclusion of the iteration process was the intercept point, the "zero" condition. The intercept point would theoretically be the wear rate when the variable (either load or velocity) was equal to zero. The load values used during testing ranged from 1.4 MPa (200 psi) to 27.6 MPa (4,000 psi), but the number for the wear rate at zero load was needed. Likewise, the velocity factors ranged from 50 to 800, but the number for the wear rate at zero velocity was needed. Thus, in the last iteration, when the recalculated wear rate values for the specific input conditions were being requested, the zero value was also requested.

Looking at the basic equation for the curve-fitting operation, if the load or velocity factor had been zero, the wear rate value, WR, would have been equal to "a." The value of b was indeterminant at the zero condition and, for the present discussion, immaterial.

These intercepts, the "a" values, were then processed in the curve-fitting program, with both the load set and the velocity set. The "zero" point was requested from both sets of data, with the resulting "zero-zero" point the same for both sets of data. That is, the same value of "a" was found for the zero condition from the load set and from the velocity set. This common "a" value for the zero-zero condition was termed "A" and its importance will be discussed in the latter part of this section.

The curve-fitting operation also produced various values of "b" from the resulting fitted curve equations. These "b" values for the load set of curves were placed into a linear regression analysis program for the straight-line equation:

$$y = m_1 x_1 + f_1 \qquad (6)$$

where
y was not defined
x_1 = load values
m_1 = slope of line = D
f_1 = intercept of line = C.

The "b" values from the velocity set of curves were also placed into the same linear regression analysis for the straight line equation:

$$y = m_2 x_2 + f_2 \qquad (7)$$

where
y was not defined
x_2 = velocity factors
m_2 = slope of line = D
f_2 = intercept of line = B.

The same value of "D" was found from both sets of curves, but the values of B and C were not the same.

These values of A, B, C, and D were placed in the following equation, representing the wear rate surface *equation:*

$$WR = A\ \epsilon^{(BL + CV + DLV)} \qquad (8)$$

and the significance of each of the determined values was established.

One of the distinguishing features of the equation was the interrelationship of load and velocity, the load-velocity product. This feature highlighted the fact that the best-fit equation for the data could not separate the combined effect. Any statements regarding a single-factor relationship seemed to be inappropriate. Any attempt to fit an alternate equation was going to be marginally accurate because the alternate equation forms do not incorporate any type of interrelationship. The physical interpretation was that the effect of the combined load and velocity would produce greater wear than might be expected from the effect of either input applied separately.

The values that were determined for the oscillatory mode of operation were:

$A = 0.00001423$
$B = 0.1021$
$C = 0.004504$
$D = -0.00006866$

The resulting wear rate surface equation for the oscillatory mode was:

$$WR = 0.00001423\,\epsilon^{(0.1021\,L + 0.004504\,V - 0.00006866\,LV)} \tag{9}$$

where WR = wear rate (grams per minute)
L = load (MPa)
V = average velocity (millimeters per second)

The wear rate surface representation has been presented in Figure 8 in which the vertical height of the surface represents the wear rate.

Reflection on the negative value of D, the coefficient for the combined effect of load and velocity, revealed that the effect of the combined load and velocity was not really a reduction of wear rate because the values of load and velocity were increased. Instead, the rate of increase in the combined effect was merely reduced. The high-load, high-velocity portion of the wear rate surface still has a higher rate of wear than the portion represented by the lower values of load and velocity, as evidenced by the upward twist or warp of the surface, shown in Figure 8.

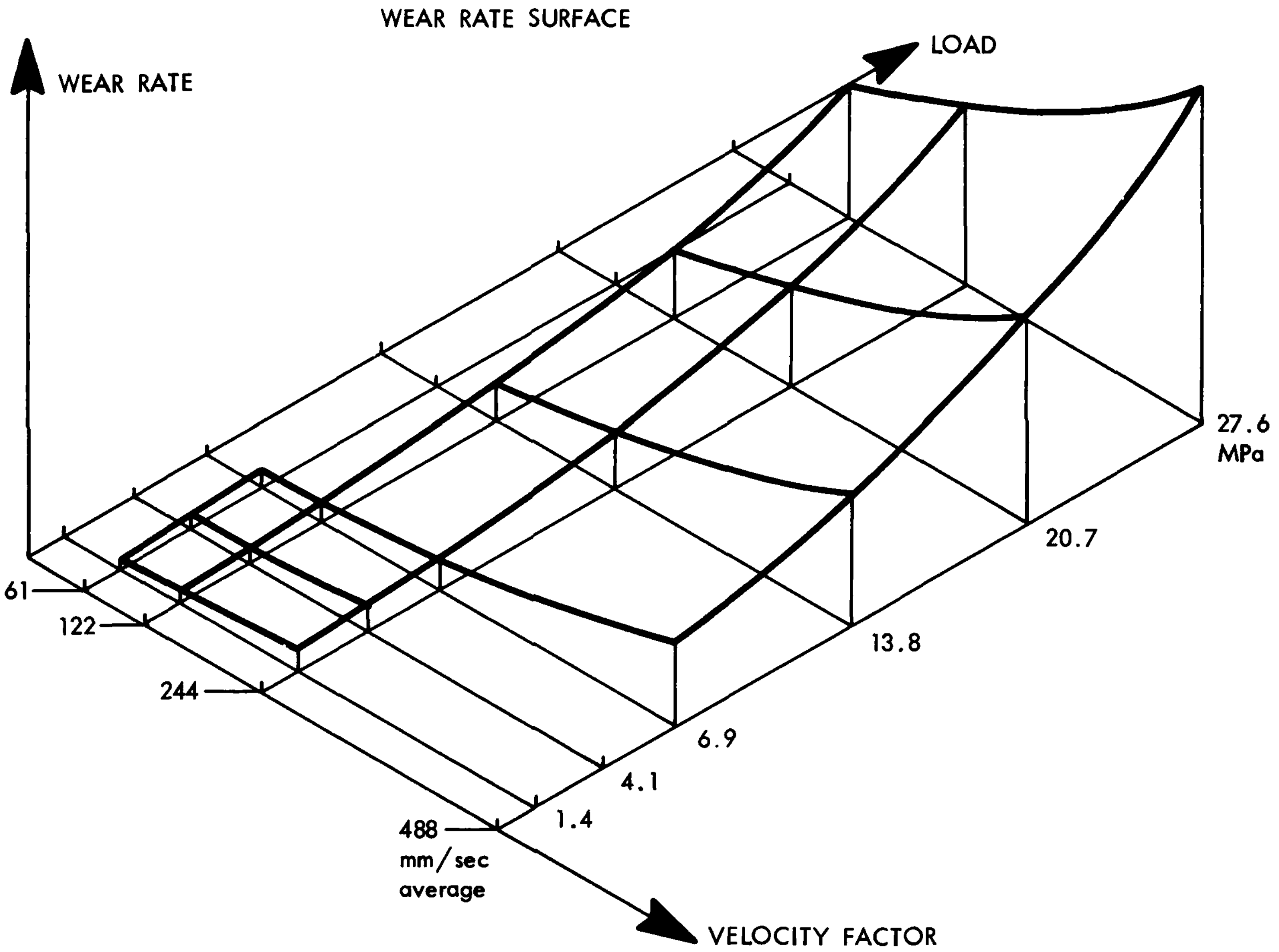


Figure 8 - Wear Rate Surface, Oscillatory Mode, from the Surface Equation

The values that were determined for the unidirectional mode of operation were:

$$
\begin{aligned}
A &= 0.0001501 \\
B &= -0.2009 \\
C &= -0.002556 \\
D &= 0.006221
\end{aligned}
$$

The resulting wear rate surface equation for the unidirectional mode was

$$WR = 0.0001501\epsilon^{(-0.2009\ L\ -\ 0.002556\ V\ +\ 0.006221\ LV)} \qquad (10)$$

where WR = wear rate (grams per minute)
L = load (MPa)
V = velocity (meters per minute)

The wear rate surface representation has been presented in Figure 9 in which the vertical height of the surface represents the wear rate. The scale of the vertical height of Figure 9 is not the same as the scale used for Figure 8.

Reflection on the values for the unidirectional mode revealed that the negative values for B and C could be interpreted as a reduction in wear rate as either the load or velocity were increased. In reality, the negative coefficients were the mathematical way of keeping the wear rate surface relatively flat at the low-load and low-velocity condition while allowing the surface to warp greatly in the high-load, high-velocity portion. It might be interpreted that the combined high-load and high-velocity operation generated more energy at the rubbing interface than the material could adequately dissipate. However, high temperatures were not recorded for any of these operating conditions. It might also be interpreted that the combined high-load and high-velocity operation allowed the retention of the wear debris to a greater extent than at the lower conditions of operation, in turn generating more debris by the abrasive action of the unremoved debris. Whatever the cause, the developed equation that was fitted to the experimental data would predict that combined high-load, high-velocity operation in the unidirectional operation would not be advisable.

The numerical values of A, B, C, and D were used in a computational program to verify the correctness and the accuracy of the values for each surface equation. Each of the intersection wear rate values was calculated and was found to agree with the values from the last iteration for each of the two fields of data.

Representations of the surfaces derived from the mathematical equations, presented in Figures 8 and 9, have also been presented in Figures 10 and 11 with the experimentally determined wear rates and data scatter included. These figures have been presented to allow visualization of the original wear rate data and the data scatter in comparison to the final wear rate surface equations and the mathematically determined wear rates. As can be seem from Figure 10 for the oscillatory mode, this surface was the one used in Figures 4 through 7 in the surface development explanation.

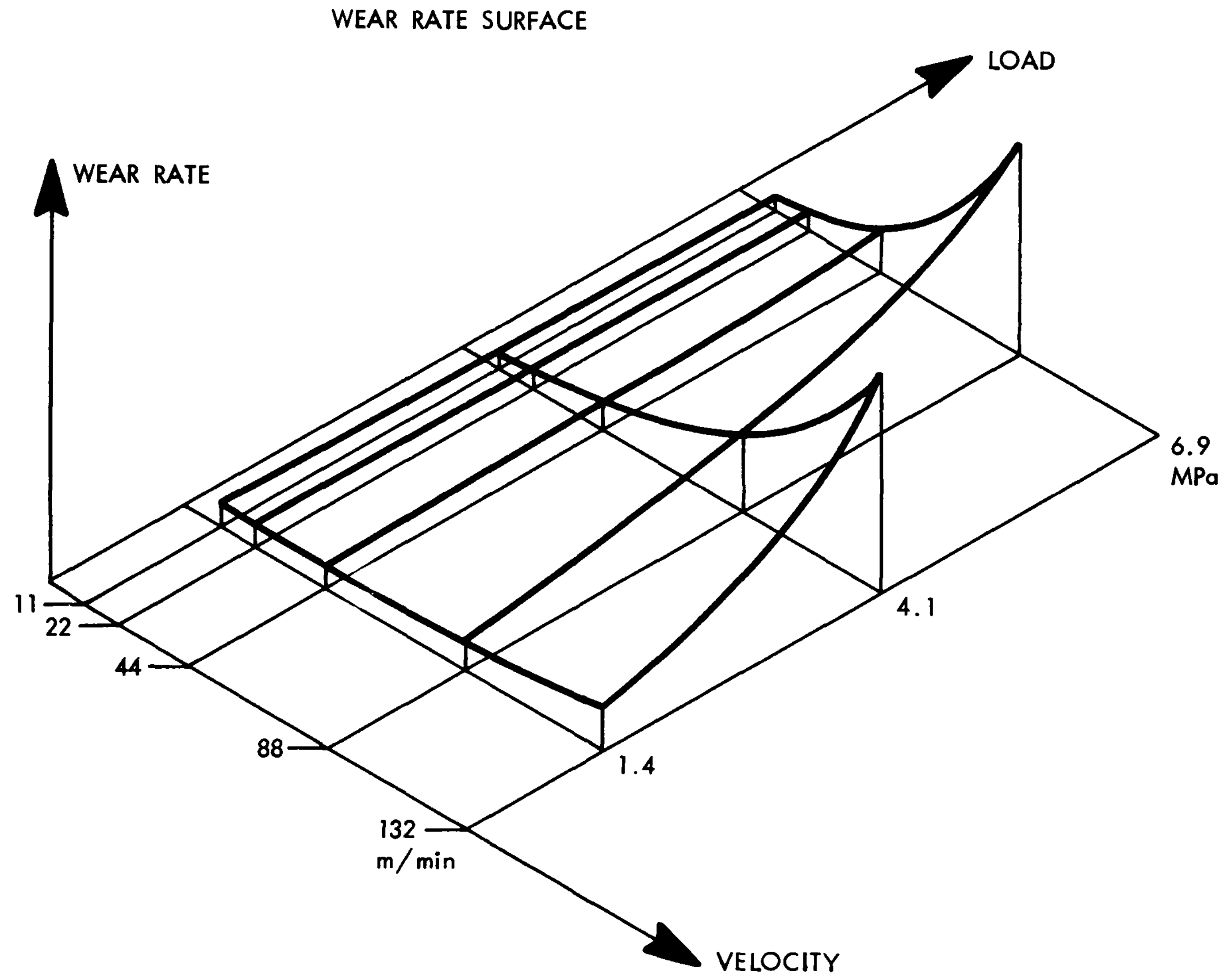


Figure 9 - Wear Rate Surface, Unidirectional Mode, from the Surface Equation

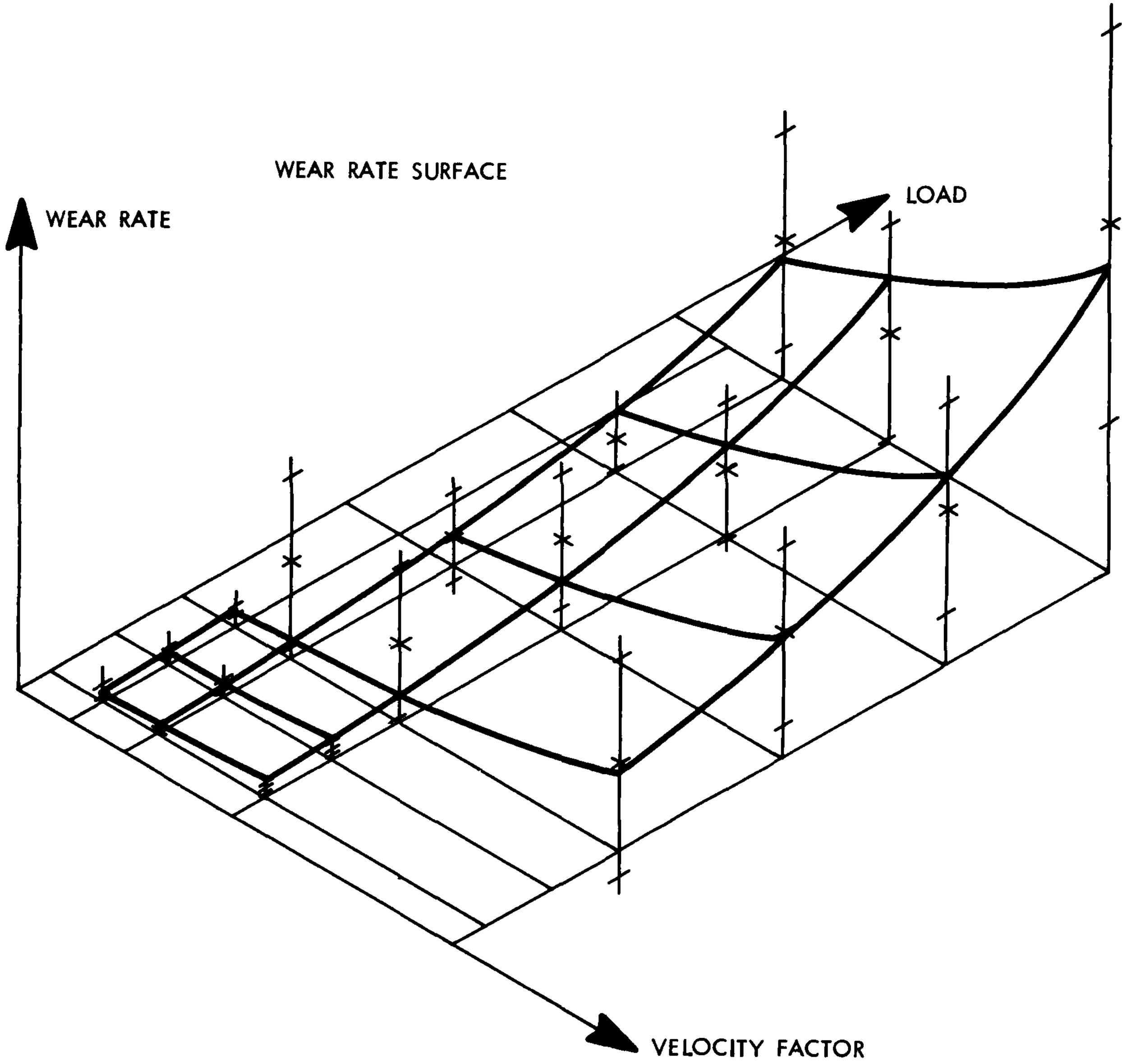


Figure 10 - Wear Rate Surface, Oscillatory Mode, Showing Experimental Wear Rates and Data Scatter

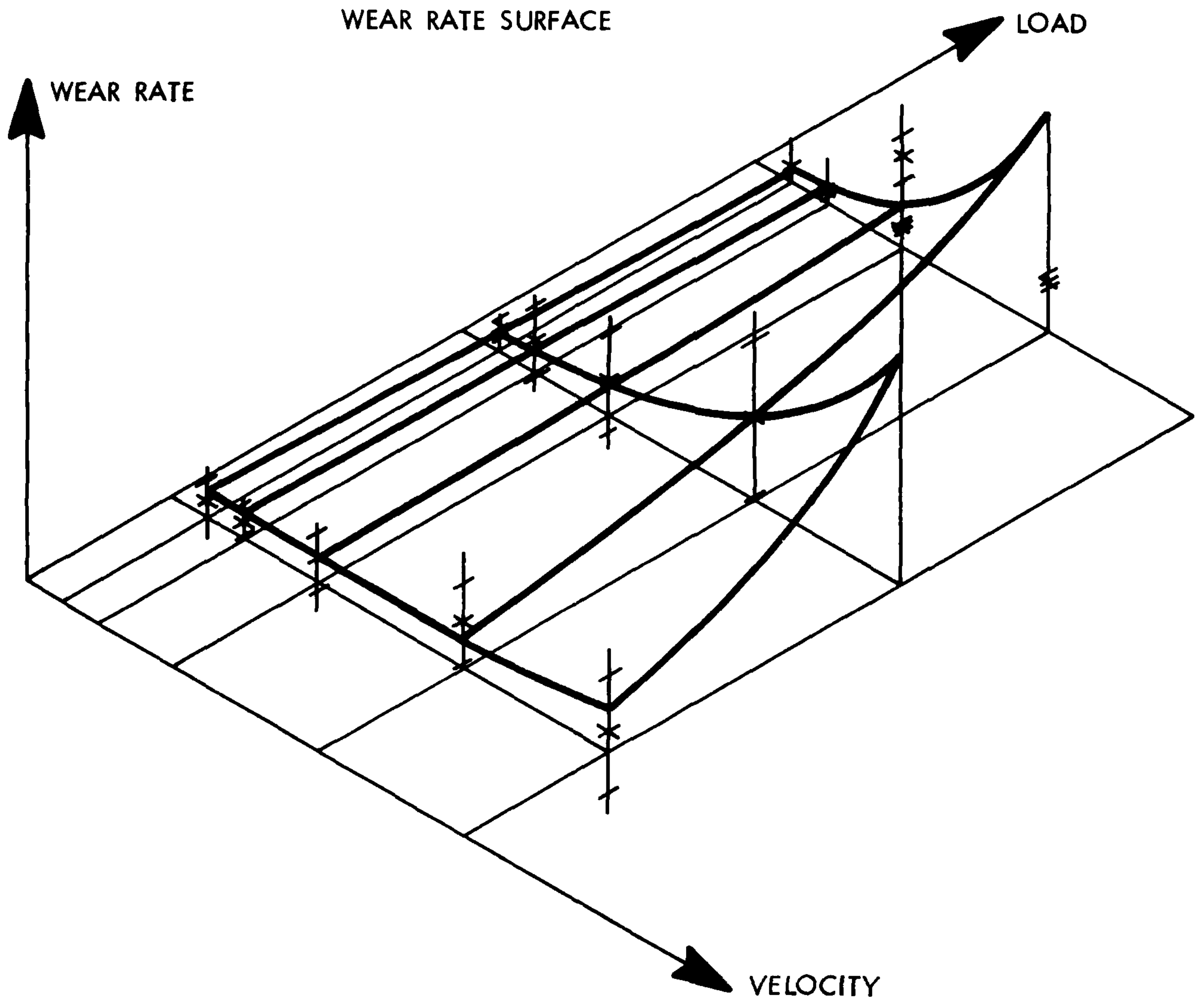


Figure 11 - Wear Rate Surface, Unidirectional Mode, Showing Experimental Wear Rates and Data Scatter

The experimental oscillatory wear rate data, shown graphically in Figure 10, exhibited a large degree of data scatter in the mid-range of operation. As was reported earlier, the large scatter resulted from the data determined in the first part of the experimental work. As the operating procedure and handling techniques were improved, the "quality" of the data improved. These later experiments were conducted in the lower ranges of load and velocity and the improvement in data quality has been demonstrated by the reduced bands of data scatter in that region.

The experimental unidirectional wear rate data, shown graphically in Figure 11, presented several conditions with very small data scatter bands. Some of these extremely small data bands, especially in the high-load portion, were misleading. Testing at the higher load and higher velocity was limited due to the lack of specimens, and some of the data presented represent only one test with the variation shown being only the difference between the left and right shoe. Special allowances in the weighting value for these conditions were made during the curve-fitting operation so that single-point (two-value) experimental results were not emphasized as much as strict adherence to the system would have required.

IV. ALTERNATE EQUATION FORMS

A. Curve-Fitting

The form of the wear rate surface equation was presented in the previous section and was not accompanied by the rationale for the use of that specific form. In this part of the report, various other forms that were tried have been presented and discussed, giving adequate reasons for their nonuse in this work.

Weight loss data for three lubricant compact materials previously studied (Reference 3) were subjected to various analyses to determine a satisfactory form for the wear rate surface equation. In the work of Reference 3, the basic form of the curve-fitting equation had been determined. Two of the materials were best satisfied by the exponential curve form for the curve-fitting, which was also found to be the best fit for this work with the Thermid 600 polyimide material. The third material in the previous work reported in Reference 3 was best satisfied by the linear form of the equation (the equation of a straight line). The analysis work done on the previous data for this program was the effort described herein as curve-fitting, in which the wear rate data values were modified until the same numbers were obtained for the intersecting points from both sets of curves, one set from load variation and one set from velocity variation. From that point (of fitted curves) to the surface equation form was a matter of study, trial-and-error, and experimentation with various mathematical ideas in field theory. One set of data was used in some of these "hunt and seek" efforts. When the procedure was found and the recalculated wear rates were found to be the same as those used in generating the equation, the technique was applied to another set of data on a second material. The concept was verified.

The same curve-fitting and coefficient valuation were performed on the third material, the material whose curve-fitting equation format was best fit by a straight line relationship. The results were surprisingly similar to those of the curve-fitting and the coefficient determination, but the form of the surface equation was somewhat different. The format for the surface equation for the first two materials was

$$WR = A\epsilon^{(B + DV)L + (C + DL)V} \tag{11}$$

derived from curve-fitting equations of the form

$$y = a\epsilon^{bx} \tag{12}$$

The format for the surface equation for the third material was

$$WR = A + (B + DV)L + (C + DL)V \qquad (13)$$

derived from the curve-fitting equations of the form

$$y = mx + b. \qquad (14)$$

Since this work concerns only the form of the equation, the relationship between the A, B, C, and D from one equation type to another has not been established, defined, or otherwise explained. The WR, V, and L terms have been previously defined as wear rate, velocity, and load, with the units not specified for this explanation and discussion.

It will be noted that the zero-zero intercept was still A, as described in Section III; the concept of the zero-zero intercept applied regardless of the form of the surface equation. The term, (B + DV)L + (C + DL)V, appeared in both forms. This term was reduced to (BL + CV + DLV) by simply rearranging terms. It will be noted that the value of D was not the same for both forms of the term. The first D was one-half the numerical value of the second D. The reduced form was used for the surface representation in Section III.

Evaluation of the coefficients B, C, and D was handled in the same manner as explained in Section III, including the seemingly unusual crossover of the B and C values (from how they were determined to how they were applied). It may be recalled that B, the coefficient used in conjunction with the load value, came from the slope of the line based upon the "b" values from the velocity set of curves. Likewise, C, the coefficient used in conjunction with the velocity value, came from the slope of the line based upon the "b" values from the load set of curves.

The transition from the curve-fitting equations to the surface equation was thus not as great a transition as might have been believed. The basic accuracy of the wear rate surface equation depended upon the selection of the form of the curve-fitting equation.

The computer/calculator used for this work was a TI-59, for which several "solid state" soft-ware libraries were available. One of these libraries was entitled "Real Estate/Investment." In this library were two programs, RE-10 and RE-11, that dealt with curve-fitting. Program RE-10 dealt with forecasting, based upon a user selection of one of four equation formats. Program RE-11 dealt with the automatic selection of the best fit of the inputted data to one of the four types of curves. The operator simply placed the data into the computer program and, when the data input was completed, the form of the best fit curve was determined by the program and the coefficients were calculated. The four curve forms were:

$$y = a + bx \qquad \text{(linear)} \tag{15}$$

$$y = a\,x^{b} \qquad \text{(power)} \tag{16}$$

$$y = a\,\epsilon^{bx} \qquad \text{(exponential)} \tag{12}$$

$$y = ab^{x} \qquad \text{(logarithmic)} \tag{17}$$

The data for this work were inputted and, although not every condition indicated that the exponential form was the best form (some indications of linear form being the best fit were made when some conditions of only two points per curve were inputted), the exponential form was indicated as the best fit for over 85% of the cases. Thus, it was decided that the use of the exponential form of the line equation for the curve-fitting operation was justified.

B. Wear Equations

Wear has been handled in works too numerous to mention or attempt to reference. One of the forms for presenting wear data has been referred to as the Archard equation:

$$W = A\,P\,V\,t \tag{18}$$

where W = weight loss

P = pressure

V = velocity

t = time

A = Archard wear coefficient (sometimes referred to as "K").

For the following formula comparison work, the values for W, P, V, and t were a mixture of units, basically English, that were used in the working description of the experimental conditions, such as W in terms of grams, P in terms of pressure in psi, V in terms of cpm or rpm, and t in terms of minutes. The units on the developed surface equations are not the same and have been specified with the formulas to reduce the confusion.

The coefficient A for the basic Archard equation was determined by three different methods, the first a simple averaging of the A_i, the second by determining the quotient of the summation of W divided by the summation of the PVt products, and the third an average of the logarithm of A_i.

For the first method, the coefficient A was the number that made the equation:

$$W = A\ P\ V\ t \tag{18}$$

correct for each test condition. (For the oscillatory data set, there were 127 different conditions used.) Calculations of A were based upon:

$$A = \frac{W}{PVt} \tag{19}$$

and an average A was calculated simply by summing all the A values and dividing by the number of terms. For the calculated results for the 127 oscillatory conditions, A was found to be 4.093×10^{-10}.

The second calculation of A computed the average in a slightly different manner, as given by the expression:

$$A = \frac{\Sigma W}{\Sigma(PVt)} \tag{20}$$

For the calculated results for the 127 oscillatory test conditions, A from this format was found to be 2.772×10^{-10}.

For the third method, the calculation of A was based upon a log-average computation in which the logarithm of each factor (L, V, T, and W) was used to calculate the logarithm of A and then the average of the logarithm was determined, from the formula:

$$\ln \bar{A} = \frac{\Sigma \ln W - \Sigma \ln P - \Sigma \ln V - \Sigma \ln t}{n} \tag{21}$$

For the calculated results for the 127 oscillatory test conditions, A was found to be 2.477×10^{-10}.

Since the "intercept" concept for the wear rate surface equation was found to be effective in determining the constants (although not physically explainable), another form for the basic wear equation was tried. This form of the equation was a linear regression modification of the basic Archard equation:

$$W = K + A\ P\ V\ t \tag{22}$$

where K, a constant, was separated from the basic Archard product of conditions. The calculated results for the oscillatory test conditions were:

$$K = 6.496 \times 10^{-3}$$

$$A = 1.793 \times 10^{-10}$$

with a correlation coefficient of only 0.5004.

Another of the forms for presenting wear data has been the modified Archard equation, also referred to as the Rhee equation:

$$W = A\ P^a V^b t^c \tag{23}$$

with the same notation as above with the a, b, and c coefficients being power exponents on their respective variables.

The mathematical treatment of the modified Archard equation was somewhat more involved and, since this equation was supposed to provide excellent results, more analysis work was applied to determine the coefficients and check the applicability of the resulting wear equations.

The determination of A, a, b, and c required restating the basic equation in terms of logarithms, manipulation of the variables into four equations with four unknowns, and solving for the four unknowns. The mathematical techniques have been presented in Appendix C, again for the 127 oscillatory test conditions. This appendix also contains the complete computer program used for this work. The resulting Rhee or modified Archard equation was:

$$W = 3.73 \times 10^{-8}\ P^{0.584}\ V^{0.703}\ t^{0.915} \tag{24}$$

for the same type of units being used in this section of the report.

In order to compare the Rhee equation, above, to the wear rate surface equation, wear rates were determined. Since the time for the tests varied with each set of conditions, the wear rates were calculated for the high and low time periods for each set of conditions. These values have been superimposed on the wear rate surface equation values (with data scatter), and the results are shown in Figure 12. As presented in the graph, the wear rate intercepts for the lower load and velocity conditions were shown to be

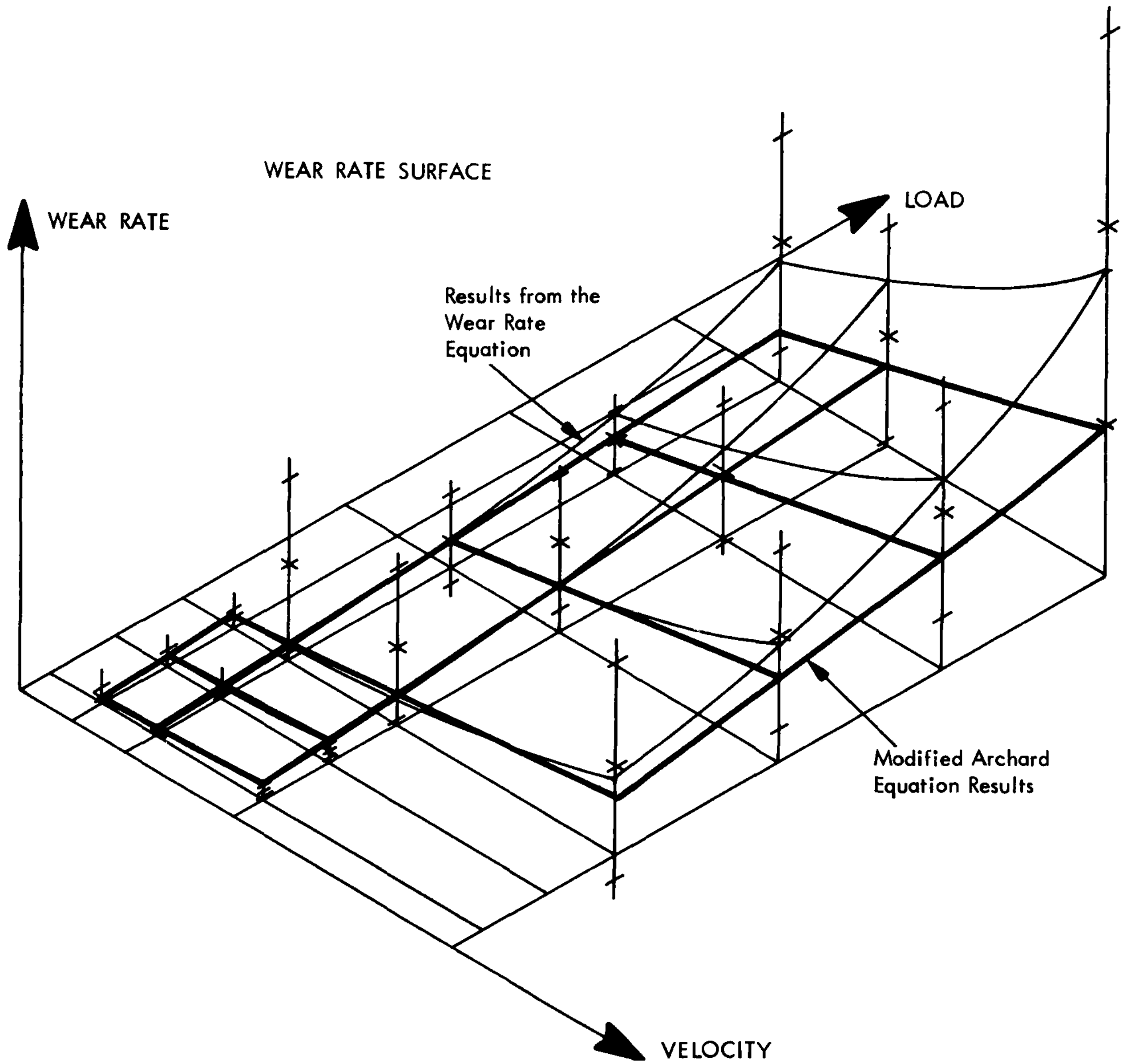


Figure 12 - Wear Rate Surface, Oscillatory Mode, with the Surface from the Modified Archard Equation Superimposed

greater than those of the wear rate surface equation. The intercepts for the higher load and velocity conditions were approximately half of the wear rate surface equation values and were basically well below the actual experimental results.

The basic form of the modified Archard equation has no ability to adequately increase the values of the wear rate for the combined higher load and higher velocity conditions.

The modified Archard equation contained more refinement than the basic Archard equation; hence, the results from the inspection of the coefficients a, b, and c on load, velocity, and time for the modified equation can be applied equally to the basic equation. The prime point of the inspection of these coefficients was that the coefficient on time was 0.915, meaning that the effect on wear by time, for this material under these operating conditions, was essentially a direct relationship. The value of the time coefficient was used previously in the discussion on Test Results (Section II-D), Mathematical Techniques (Section III), and Scattering Coefficient (Section III-B). This time coefficient value formed the basis for the justification of the use of wear rate instead of wear for this work.

All of these equations were used in an attempt to determine a better fit for the data. The previous equations, with their various values of A, were all found to be deficient, especially in the operating regime most likely to be encountered when the equation for this material would be used for engine-bearing loads.

V. CONCLUSIONS

1. One of the objectives of this work was to develop a technique for reducing weight loss information on a lubricant compact material to a single equation of wear or wear rate as a function of load, velocity, and, possibly, time. This objective was realized as the technique was developed and has been presented herein.

2. The second objective was to determine the wear rate equation for the Thermid 600 polyimide material. Two wear rate equations were developed, one for the oscillatory mode and one for the unidirectional mode.

3. The developed procedure was found to work on more than one material, as in reality five separate data sets were used in the analytical procedure.

4. The handling techniques (the computer programs, iterative procedures, and application of coefficient evaluations) were found to be satisfactory and not too cumbersome or tedious to produce accurate results in a timely manner.

5. The Thermid 600 polyimide material showed good operating characteristics, satisfactorily sustaining 27.6 MPa (4,000 psi) loadings at various velocities, while operating at 315°C (600°F).

6. The wear rate equation developed for this work on the Thermid 600 polyimide material provided an accurate representation of the actual wear rate within the conditions studied and within the accuracy of the experimentally determined wear rate.

7. The coefficients of the wear rate equation must incorporate material property information because these values vary for different materials and operational modes.

8. The consistency of the experimentally determined weight loss measurements revealed that the quality control of the material studied could be improved.

APPENDIX A

COMPUTER PROGRAM FOR WEIGHT LOSS DATA REDUCTION

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COMPUTER PROGRAM FOR WEIGHT LOSS DATA REDUCTION

In order to handle the weight loss data in a precise manner, a computer program for the wear rate determinations was generated. This program started with the basic data of time of test, weight loss for the left shoe, and weight loss for the right shoe. When all the data for the same conditions of load and speed had been entered, the answers were requested. The output information contained 37 different items relating to wear rate and statistical significance of the data. These items will be explained in this appendix as the computer program is presented.

The computer program was established for the Texas Instruments TI-59 computer/calculator. As such, the designations of "Code" and "Key" may not apply to other calculators. However, the concept and data handling techniques are not restricted to any one type of machine. The following list is the program used for the data reduction:

Step	Code	Key	Comments
000	76	LBL	By pressing "A" on the computer, an initializa-
	11	A	tion process is started.
	36	PGM	Program 01 is called from any of the library
	01	01	modules that might be used. The subroutine
	71	SBR	"CLR" clears the statistical memories.
	25	CLR	
	47	CMS	Clear data storage memories.
	04		
	09		
	42	STO	Store 49 in Register 01.
010	01	01	(This will be an indirect address location for inputting time values.)
	02		

Step	Code	Key	Comments
	03		
	42	STO	Store 23 in Register 02.
	02	02	(This will be an indirect address location for inputting weight losses for the left shoe.)
	03		
	06		
	42	STO	Store 36 in Register 03.
	03	03	(This will be an indirect address location for inputting weight losses for the right shoe.)
	91	R/S	Stop.
020	76	LBL	By entering the value of time, in minutes, and
	12	B	pressing "B," the time values are stored in
	72	ST*	the location specified in Register 01. The
	01	01	first location is Register 49.
	01		
	44	SUM	Add 1 to Register 01.
	01	01	(The next time input will go to Register 50, etc.)
	91	R/S	Stop.
	76	LBL	By entering the value of the weight loss for the
	13	C	left shoe, in grams, and pressing "C," the
030	72	ST*	values are stored in the location specified
	02	02	in Register 02. The first location is Register 23.
	01		

Step	Code	Key	Comments
	44	SUM	Add 1 to Register 02.
	02	02	(The next weight loss for the left shoe will go to Register 24, etc.)
	91	R/S	Stop.
	76	LBL	By entering the value of the weight loss for the
	14	D	right shoe, in grams, and pressing "D," the
	72	ST*	values are stored in the location specified in
	03	03	Register 03. The first location is Register 36.
040	01		
	44	SUM	Add 1 to Register 03.
	03	03	(The next weight loss for the right shoe will go to Register 37, etc.)
	91	R/S	Stop.
	76	LBL	When all the data have been entered (maximum of
	15	E	5 points), the program is started by pressing "E."
	53	(	Start a calculation.
	43	RCL	Recall the last address location for left side weight loss.
	02	02	
	75	-	Subtract 23 from that value.
050	02		
	03		

Step	Code	Key	Comments
	54	)	Complete the calculation.
	42	STO	Store the number of values in Register 04.
	04	04	(This is the number of data points for the left side.)
	53	(	Start the calculation.
	43	RCL	Recall the last address location for the right side weight loss.
	03	03	
	75	-	Subtract 36 from that value.
	03		
060	06		
	54	)	Complete the calculation.
	42	STO	Store the number of values in Register 05.
	05	05	(This is the number of data points for the right side.)
	04		
	09		
	42	STO	Store 49 in Register 01.
	01	01	(This resets the indirect address location starting point.)
	02		
	03		
070	42	STO	Store 23 in Register 02.
	02	02	(This resets the indirect address location starting point.)

Step	Code	Key	Comments
	03		
	06		
	42	STO	Store 36 in Register 03.
	03	03	(This resets the indirect address location starting point.)
	02		
	08		
	42	STO	Store 28 in Register 06.
	06	06	
080	04		
	01		
	42	STO	Store 41 in Register 07.
	07	07	
	76	LBL	Establish an internal point for return for sub-
	38	SIN	routine; the label is "SIN."
	73	RC*	Recall the data from the register specified in
	02	02	Register 02. (Starts at Register 23, left
			shoe losses.)
	67	EQ	Compare this value to the value in the T
			register
	60	DEG	If the value is zero, go to "DEG."
090	44	SUM	Add the weight loss to Register 08.
	08	08	(We are forming a summation of left shoe losses.)

Step	Code	Key	Comments
	44	SUM	Add the weight loss to Register 10.
	10	10	(We are forming a summation of all weight losses).
	55	÷	Divide this weight loss value by:
	73	RC*	Recall the data from the register specified in
	01	01	Register 01. (Starts at Register 49, time values.)
	95	=	Determine the quotient, the wear rate.
	72	ST*	Store this value in the register specified in
	06	06	Register 06. (Starts at Register 28.)
100	73	RC*	Recall the data from the register specified in
	01	01	Register 01. (Starts at Register 49, time values.)
	44	SUM	Add the time to Register 11.
	11	11	(We are forming a summation of time for the left shoe weight losses.)
	44	SUM	Add the time to Register 13.
	13	13	(We are forming a summation of all time for weight losses.)
	01		
	44	SUM	Add 1 to Register 14.
	14	14	(We are counting the left shoe losses.)
	44	SUM	Add 1 to Register 16.
110	16	16	(We are counting all the shoe losses.)

Step	Code	Key	Comments
	76	LBL	The label "DEG" is established. If there was no weight loss, the program skipped from Step 089
	60	DEG	to here.
	01		
	44	SUM	Add 1 to Register 01.
	01	01	
	44	SUM	Add 1 to Register 02.
	02	02	
	44	SUM	Add 1 to Register 06.
	06	06	
120	97	DSZ	Decrease Register 04 by 1 and skip the next
	04	04	instruction if the register is zero.
	38	SIN	If we still have left shoe data, the program repeats to Step 084.
	04		
	09		Ready to start the right side.
	42	STO	Store 49 in Register 01.
	01	01	(This resets the indirect address location starting point.)
	76	LBL	Establish an internal point for return for sub-
	39	COS	routine; the label is "COS."
	73	RC*	Recall the data from the register specified in
130	03	03	Register 03. (Starts at Register 36, right
			shoe losses.)
	67	EQ	Compare this value to zero.

Step	Code	Key	Comments
	70	RAD	If the value is zero, go to "RAD."
	44	SUM	Add the weight loss to Register 09.
	09	09	(We are forming a summation of right shoe losses.)
	44	SUM	Add the weight loss to Register 10, where we have
	10	10	the summation of all weight losses.
	55	÷	Divide this weight loss value by:
	73	RC*	Recall the data from the register specified in
	01	01	Register 01. (Starts at Register 49, time values.)
140	95	=	Determine the quotient, the wear rate.
	72	ST*	Store this value in the register specified in
	07	07	Register 07. (Starts at Register 41.)
	73	RC*	Recall the data from the register specified in
	01	01	Register 01. (Starts at Register 49, time values.)
	44	SUM	Add the time to Register 12.
	12	12	(We are forming a summation of time for the right shoe weight losses.)
	44	SUM	Add the time to Register 13, where we have a
	13	13	summation of all time for weight losses.
	01		
150	44	SUM	Add 1 to Register 15.
	15	15	(We are counting the right shoe losses.)
	44	SUM	Add 1 to Register 16.

Step	Code	Key	Comments
	16	16	(We continue to count all the shoe losses.)
	76	LBL	The label "RAD" is established. If there was no weight loss, the program skipped from Step 132 to here.
	70	RAD	
	01		
	44	SUM	Add 1 to Register 01.
	01	01	
	44	SUM	Add 1 to Register 03.
160	03	03	
	44	SUM	Add 1 to Register 07.
	07	07	
	97	DSZ	Decrease Register 05 by 1 and skip the next
	05	05	instruction if the register is zero.
	39	COS	If we still have right shoe data, the program repeats to Step 127. Calculations begin.
	43	RCL	Recall the contents of Register 08.
	08	08	(This is the total left side weight losses.)
	55	÷	Divide the value by:
	43	RCL	Recall the contents of Register 11.
170	11	11	(This is the total left side weight losses.)
	95	=	Determine the quotient, the left side average wear rate.
	42	STO	Store this value in Register 33.
	33	33	

Step	Code	Key	Comments
	43	RCL	Recall the contents of Register 09.
	09	09	(This is the total right side weight losses.)
	55	÷	Divide this value by:
	43	RCL	Recall the contents of Register 12.
	12	12	(This is the total time for right side losses.)
	95	=	Determine the quotient, the right side average wear rate.
180	42	STO	Store the value in Register 46.
	46	46	
	43	RCL	Recall the contents of Register 10.
	10	10	(This is the total weight loss.)
	55	÷	Divide this value by:
	43	RCL	Recall the contents of Register 13.
	13	13	(This is the total time for all losses.)
	95	=	Determine the quotient, the average wear rate.
	42	STO	Store this value in Register 55.
	55	55	
190	02		
	08		
	42	STO	Store 28 in Register 06.
	06	06	(This resets the counting register for left side wear rates.)
	05		
	42	STO	Store 5 in Register 04.

Step	Code	Key	Comments
	04	04	(This will be a counting register.)
	76	LBL	The label "1/x" is established.
	35	1/x	
	73	RC*	Recall the data from the register specified in Register 06. (Starts at Register 28, left side wear rates.)
200	06	06	
	67	EQ	Compare this value to zero.
	85	+	If the value is zero, go to "+".
	53	(	Start the calculation.
	73	RC*	Recall the data from the register specified in Register 06. (Starts at Register 28, left side wear rates.)
	06	06	
	75	-	Subtract:
	43	RCL	Recall the contents of Register 33.
	33	33	(This is the left side average wear rate.)
	54	)	Complete this calculation.
210	33	x^2	Square the value.
	44	SUM	Add this value to Register 17.
	17	17	(We are forming a left side summation for statistics.)
	76	LBL	Establish an internal point for a skip operation, Step 202.
	85	+	If there was no wear rate, we do not form a difference^2.
	01		
	44	SUM	Add 1 to Register 06.

Step	Code	Key	Comments
	06	06	
	97	DSZ	Decrease Register 04 by 1 and skip the next
	04	04	instruction if the register is zero.
220	35	1/x	If we still have left side data, we return to Step 198.
	53	(	When all the data for the left side are complete,
	43	RCL	start the calculation. Recall the contents
	17	17	of Register 17.
	55	÷	Divide this value by:
	53	(	Start an intermediate calculation.
	43	RCL	Recall the contents of Register 14.
	14	14	(This is the number of left side weight loss values.)
	75	-	Subtract 1 from this value.
	01		
230	54	)	Complete the N-1 calculation.
	54	)	Complete the calculation for the statistical operation.
	34	$\sqrt{x}$	Take the square root of this value.
	42	STO	Store this value, the left side scattering co-
	34	34	efficient, in Register 34.
	04		
	01		

Step	Code	Key	Comments
	42	STO	Store 41 in Register 07.
	07	07	(This resets the counting register for right side wear rates.)
	05		
240	42	STO	Store 5 in Register 05.
	05	05	(This will be a counting register.)
	76	LBL	The label "x^2" is established.
	33	x^2	
	73	RC*	Recall the data from the register specified in Register 07. (Starts at Register 41, right side wear rates.)
	07	07	
	67	EQ	Compare this value to zero.
	75	-	If the value is zero, go to "-".
	53	(	Start the calculation.
	73	RC*	Recall the data from the register specified in Register 07. (Starts at Register 41, right side wear rates.)
250	07	07	
	75	-	Subtract:
	43	RCL	Recall the contents of Register 46.
	46	46	(This is the right side average wear rate.)
	54	)	Complete this calculation.
	33	x^2	Square the value.
	44	SUM	Add this value to Register 18.
	18	18	(We are forming a right side summation for statistics.)
	76	LBL	Establish an internal point for a skip operation, Step 247.

Step	Code	Key	Comments
	75	-	If there was no wear rate, we do not form a difference^2.
260	01		
	44	SUM	Add 1 to Register 07.
	07	07	
	97	DSZ	Decrease Register 05 by 1 and skip the next
	05	05	instruction if the register is zero.
	33	x^2	If we still have right side data, we return to Step 242.
	53	(	When all the data for the right side is complete,
	43	RCL	start another calculation. Recall the contents
	18	18	of Register 18.
	55	÷	Divide this value by:
270	53	(	Start an intermediate calculation.
	43	RCL	Recall the contents of Register 15.
	15	15	(This is the number of right side weight loss values.)
	75	-	Subtract 1 from this value.
	01		
	54	)	Complete the N-1 calculation.
	54	)	Complete the calculation for the statistical operation.
	34	$\sqrt{x}$	Take the square root of this value.

Step	Code	Key	Comments
	42	STO	Store this value, the right side scattering
	47	47	coefficient, in Register 47.
280	02		
	08		
	42	STO	Store 28 in Register 06.
	06	06	(This resets the counting register for the left side wear rates.)
	04		
	01		
	42	STO	Store 41 in Register 07.
	07	07	(This resets the counting register for the right side wear rates.)
	05		
	42	STO	Store in Register 04.
290	04	04	(This will be a counting register.)
	42	STO	Store 5 in Register 05.
	05	05	(This will be a counting register.)
	76	LBL	The label "y^x" is established.
	45	y^x	
	73	RC*	Recall the data from the register specified in Register 06. (Starts at Register 28, left side wear rates.)
	06	06	
	67	EQ	Compare this value to zero.
	65	X	If the value is zero, go to "X".
	53	(	Start the calculation

Step	Code	Key	Comments
300	73	RC*	Recall the data from the register specified in Register 06. (Starts at Register 28, left side wear rates.)
	06	06	
	75	-	Subtract:
	43	RCL	Recall the contents of Register 55.
	55	55	(This is the average wear rate.)
	54	)	Complete this calculation.
	33	x^2	Square the value.
	44	SUM	Add this value to Register 19.
	19	19	(We are forming a total summation for statistics.)
	76	LBL	Establish an internal point for a skip operation, Step 298.
310	65	X	If there was no wear rate, we do not form a difference^2.
	01		
	44	SUM	Add 1 to Register 06.
	06	06	
	97	DSZ	Decrease Register 04 by 1 and skip the next instruction if the register is zero.
	04	04	
	45	y^x	If we still have left side data, return to Step 293.
	76	LBL	The label "[x]" is established.
	50	[x]	
	73	RC*	Recall the data from the register specified in

Step	Code	Key	Comments
320	07	07	Register 07. (Starts at Register 41, right side wear rates.)
	67	EQ	Compare this value to zero.
	55	÷	If the value is zero, go to "÷".
	53	(	Start the calculation.
	73	RC*	Recall the data from the register specified in
	07	07	Register 07. (Starts at Register 41, right side wear rates.)
	75	-	Subtract:
	43	RCL	Recall the contents of Register 55.
	55	55	(This is the average wear rate.)
	54	)	Complete this calculation.
330	33	x^2	Square the value.
	44	SUM	Add this value to Register 19.
	19	19	(We continue the total summation for statistics.)
	76	LBL	Establish an internal point for a skip operation, Step 322.
	55	÷	If there was no wear rate, we do not form a difference^2.
	01		
	44	SUM	Add 1 to Register 07.
	07	07	
	97	DSZ	Decrease Register 05 by 1 and skip the next
	05	05	instruction if the register is zero.
340	50	[x]	If we still have right side data, we return to Step 317.

Step	Code	Key	Comments
	53	(	Start the calculation.
	43	RCL	Recall the contents of Register 19.
	19	19	
	55	÷	Divide this value by:
	53	(	Start an intermediate calculation.
	43	RCL	Recall the contents of Register 16.
	16	16	(This is the total number of weight loss values.)
	75	-	Subtract 1 from this value.
	01		
350	54	)	Complete the N-1 calculation.
	54	)	Complete the calculation for the statistical operation.
	34	$\sqrt{X}$	Take the square root of this value.
	42	STO	Store this value, the scattering coefficient,
	56	56	in Register 56.
	53	(	Start the calculation.
	43	RCL	Recall the contents of Register 55.
	55	55	(This is the average wear rate.)
	85	+	Add the following:
	43	RCL	Recall the contents of Register 56.
360	56	56	(This is the scattering coefficient.)
	54	)	Complete the calculation.
	42	STO	Store this sum in Register 57.

Step	Code	Key	Comments
	57	57	(This is the average wear rate plus scattering coefficient.)
	53	(	Start the calculation.
	43	RCL	Recall the contents of Register 55.
	55	55	(This is the average wear rate.)
	75	-	Subtract the following:
	43	RCL	Recall the contents of Register 56.
	56	56	(This is the scattering coefficient.)
370	54	)	Complete the calculation.
	42	STO	Store this difference in Register 58.
	58	58	(This is the average wear rate minus scattering coefficient.)
	53	(	Start the calculation.
	43	RCL	Recall the contents of Register 55.
	55	55	(This is the average wear rate.)
	55	÷	Divide by:
	43	RCL	Recall the contents of Register 56.
	56	56	(This is the scattering coefficient.)
	54	)	Complete the calculation.
380	42	STO	Store this quotient in Register 59.
	59	59	(This is the weighting factor.)
	02		Set 23 as the first register to be printed.
	03		
	22	INV	List the contents of all registers, beginning

Step	Code	Key	Comments
	90	LST	with 23 and going to 59 (the last).
	98	ADV	Advance the paper for five blank lines.
	98	ADV	(This allows all the data to be clear of
	98	ADV	the machine and ready for removal.)
	98	ADV	
390	98	ADV	
391	91	R/S	Stop. End.

General Comments

If the weight loss data for any one shoe is not present, a value of zero is entered for the weight loss for that particular test. The program is written so that when the sum of all the weight losses is divided by the sum of all the test times, only those times that have weight losses will be used. If the weight loss for a particular test does not exist, that test time is skipped.

The original inputted data are also printed on the outputted answers so that the values can be verified. If any mistakes are present, the entire sequence must be rerun.

The output information looks like the following sample, from the data for the 4,000-psi loading and 800-cpm tests:

```
0.0119  -  Weight Loss, Left Side, Shoe #1

0.0001  -                                2

0.049   -                                3

0.0611  -                                4

0.0185  -                                5
```

.0004576923 - Wear Rate, Left Side, Shoe #1

.0000166667 - 2

.0009683794 - 3

.0010930233 - 4

.000616667 - 5

.0008344214 - Average Wear Rate, Left Side

.0004855112 - Scattering Coefficient, Left Side

0. - Blank

0. - Weight Loss, Right Side, Shoe #1

0. - 2

0. - 3

0.0854 - 4

0.018 - 5

0. - Wear Rate, Right Side, Shoe #1

0. - 2

0. - 3

.0015277281 - 4

0.0006 - 5

.0012037253 - Average Wear Rate, Right Side

0.000685173 - Scattering Coefficient, Right Side

0. - Blank

26. - Time for Test #1

6. - 2

50.6 - 3

55.9 - 4

30. - 5

0. - Blank

.0009591195 - Average Wear Rate, WR

0.000536535 - Scattering Coefficient, SC

.0014956545 - WR + SC, Expected High

.0004225845 - WR - SC, Expected Low

1.787617675 - Weighting Factor

APPENDIX B

WEIGHTED CURVE-FITTING PROGRAM

APPENDIX B

WEIGHTED CURVE-FITTING PROGRAM

After the data have been reduced to the form of average wear rate, with the appropriate scattering coefficient and weighting factor, the information must be used in a curve-fitting operation. This linear regression analysis can be used to fit any type of curve, but the one most likely to fit the data has the form:

$$y = a\epsilon^{bx} \tag{12}$$

which, for this work, takes the form:

$$WR = a\epsilon^{bx} \tag{5}$$

where WR = wear rate

ϵ = natural base

x = either speed or load, whichever way is being analyzed

a,b = constants to be determined from the regression analysis.

To perform the analysis while giving more emphasis on the less-scattered data, a weighting value was used. The weighting value was defined to be 10 times the weighting factor, rounded to the nearest whole number. To give the proper emphasis, each point was entered into the linear regression analysis as many times as specified by the weighting value. Thus, a point that had less scatter would have more influence on the curve than a point that had more scatter.

The computer program for handling the data is given below, with the appropriate comments reflecting the reasons for each operation. The program is shown in an abbreviated style, both to avoid unnecessary repetition and because the analysis program itself is part of the calculator.

Step	Code	Key	Comments
[illegible]	76	LBL	Pressing "E" on the computer calls for an
	[illegible]	E	initialization process.
	[illegible]	PGM	Program 01 is called from any of the

Step	Code	Key	Comments
000	01	01	library modules that might be used.
	71	SBR	The subroutine "CLR" is used to clear the statistical memories.
	25	CLR	
	03		
	00		
	42	STO	Store 30 in Register 13.
	13	13	
010	02		
	00		
	42	STO	Store 20 in Register 14.
	14	14	
	91	R/S	Stop.
	76	LBL	The data are entered by pressing "A." The value of x is entered, then the wear rate, and then the weighting factor. The data are stored in the register specified in Register 13. (Starts with Register 30).
	11	A	
	72	ST*	
	13	13	
	01		
020	44	SUM	Add 1 to the contents of Register 13.
	13	13	
	91	R/S	Stop. This allows the next item of data to be entered.

Step	Code	Key	Comments
	76	LBL	Specific values for determining calculated
	12	B	wear rates are entered by pressing "B."
	72	ST*	Store these values (one at a time, up to 10) in
	14	14	the register specified in Register 14. (Starts
			with Register 20.)
	01		
	44	SUM	Add 1 to the contents of Register 14.
	14	14	
030	91	R/S	Stop.
	76	LBL	Pressing "C" on the computer starts the calculation process.
	13	C	
	53	(	Start a calculation.
	53	(	Start an intermediate calculation.
	43	RCL	Recall the contents of Register 13.
	13	13	(This is the last inputted data location.)
	75	-	Subtract 30 from that value.
	03		
	00		
040	54	)	Complete the determination of the number of inputs.
	55	÷	Divide this value by 3.
	03		

Step	Code	Key	Comments
	54	)	Determine the quotient, the number of data sets.
	42	STO	Store this value in Register 08.
	08	08	
	53	(	Start another calculation.
	43	RCL	Recall the contents of Register 14.
	14	14	(This is the location of the last value for
	75	-	calculated wear rates.) Subtract 20 from
			this value.
050	02		
	00		
	54	)	*Complete* the determination of the number of points for output calculations.
	42	STO	Store this value in Register 09.
	09	09	
	76	LBL	Establish an internal point for return from
	17	B'	subroutine; the label is "B^1."
	43	RCL	Recall the contents of Register 32.
	32	32	(This is the weighting factor for the data set.)
	42	STO	Store this value in Register 07.
060	07	07	
	76	LBL	Establish an internal point for return from
	16	A'	subroutine; the label is "A1."

Step	Code	Key	Comments
	43	RCL	Recall the contents of Register 30.
	30	30	(This is the x value for the equation.)
	99	PRT	Print this value.
	32	x-T	Exchange this value with the T register contents.
	43	RCL	Recall the contents of Register 31.
	31	31	(This is the wear rate value.)
	99	PRT	Print this value.
070	23	LNX	Take the natural log of the wear rate.
	78	$\Sigma +$	Add this value and the T register contents into memory.
	97	DSZ	Decrease Register 07 by 1 and skip the next
	07	07	instruction if the register is zero.
	16	A'	If Register 07 is not zero, return to A', Step 061.
	71	SBR	Depart from the main program to the subroutine
	33	x^2	labeled "x^2," the "data roll" process.
	97	DSZ	Decrease Register 08 by 1 and skip the next
	08	08	instruction if the register is zero.
	17	B'	If Register 08 is not zero, return to B', Step 055.
080	98	ADV	After all the data have been entered, skip a line.
	69	OP	This operational call tells the computer to cal-
	12	12	culate the linear line slope and intercept point.

Step	Code	Key	Comments
	22	INV	The first value is the y-intercept, a, which must
	23	LNX	be "inverse logarithmed."
	99	PRT	Print out the value of A.
	32	x-T	Exchange the contents with the T register.
	99	PRT	Print this value, the slope of the line, b.
	69	OP	This operational call tells the computer to
	13	13	determine the correlation coefficient.
090	99	PRT	Print the correlation coefficient.
	98	ADV	Now that the data have been analyzed, skip a line.
	02		
	00		
	42	STO	Store 20 in Register 14. (We are preparing to
	14	14	calculate wear rates from the determined equation.)
	76	LBL	Establish an internal point for return from
	18	C'	subroutine; the label is "C'."
	73	RC*	Recall the data from the register specified in
	14	14	Register 14. (Starts with Register 20.)
100	69	OP	This operational call tells the computer to
	14	14	calculate a new wear rate for the x in display.
	22	INV	This value must be "inverse logarithmed" to get
	23	LNX	the new wear rate.

Step	Code	Key	Comments
	99	PRT	Print the new wear rate.
	01		
	44	SUM	Add 1 to Register 14.
	14	14	
	97	DSZ	Decrease Register 09 by 1 and skip the next in-
	09	09	struction if the register is zero.
110	18	C'	If there are x values left, return to Step 096.
	98	ADV	Advance the paper for five blank lines.
	98	ADV	(This allows all the data to be clear of the
	98	ADV	machine and ready for removal.)
	98	ADV	
	98	ADV	
116	91	R/S	Stop.
117	76	LBL	Establish the label "x^2" for the start of the
	33	x^2	"data roll" subroutine.
	43	RCL	Recall the contents of Register 33.
	33	33	
	42	STO	Store the value in Register 30.
	30	30	
	43	RCL	Recall the contents of Register 34.
	34	34	

Step	Code	Key	Comments
	42	STO	Store the value in Register 31.
	31	31	
			Continue in this manner until each datum value has been moved to another register, numbered three less than its original location.
	43	RCL	Recall the contents of Register 59.
	59	59	
	42	STO	Store the value in Register 56.
	56	56	
	92	RTN	Return to the main program, Step 075.
	91	R/S	Stop. End.

General Comments

The present limit for data input is 10 points, each point consisting of three values: set point of either speed or load; wear rate for that condition; and the weighting factor. The memory must be repartitioned to accept more than the 10 points. After repartitioning, the "data roll" subroutine would have to be expanded to accept additional data sets.

At Step 023, the program makes allowance for determining the calculated value of wear rate for specific values of either speed or load. The specific values should be entered after the inputted data and before the "execute" command. The specific values are entered by pressing "B."

At Step 072, the computer decreases the weighting factor by one and enters the same data into the statistical memories again. When the data have been entered as many times as specified by the weighting factor, the program continues.

At Step 077, the computer decreases the number of data sets by one and prepares for the next data set. If there are no more sets, the program continues.

At Step 117, a "data roll" subroutine is specified. This subroutine moves each datum value to another register, starting with the contents of Register 33 being moved to Register 30. Next, the contents of Register 34 are moved to Register 31. The process continues until all the data have been moved and the first set of three data values has been discarded. This technique takes some time to perform. However, the counting registers used in the DSZ operation, 00 to 09, were all in use. Registers 00 to 06 are used for the statistical calculations; 07 stored the weighting factor; 08 contained the number of data sets; and 09 contained the number of points for calculating the wear rates from the determined equation.

If the basic equation for the linear regression analysis were changed to:

$$y = mx + b \tag{14}$$

certain modifications to this program would be required.

These changes are listed below:

Step	Code	Key	Comments
070	23	LNX	Delete.
083	22	INV	Delete.
084	23	LNX	Delete.
102	22	INV	Delete.
103	23	LNX	Delete.

APPENDIX C

RHEE EQUATION COEFFICIENT EVALUATION PROGRAM

APPENDIX C

RHEE EQUATION COEFFICIENT EVALUATION PROGRAM

An equation of the form,

$$W = A\ P^a V^b t^c \qquad (23)$$

where W = weight loss (grams)

P = loading factor, pressure, (psi)

V = speed factor, velocity or oscillatory rate, units per time

t = time, minutes

A,a,b,c, = coefficients to be experimentally determined

has been suggested as being a better, more advanced form of the basic Archard equation:

$$W = A\ P\ V\ t \qquad (18)$$

How do the data generated in this work fit equation (23) and how do the calculated results from the equation fit the wear rate surface previously generated are two significant questions that had to be answered. To answer the first question, the data had to be combined at one time into a single equation and the coefficients determined. To answer the second question, the basic set points had to be used to calculate the expected wear rates and these data compared to the original data.

This appendix presents the computer program and the technique for fitting all the original data into the format of equation (23).

The first step in performing the linear regression analysis of equation (23) is to convert the equation to a linear form, such as

$$W = A\ P^a V^b t^c \qquad (23)$$

$$\ln W = \ln A + a \ln P + b \ln V + c \ln t \qquad (25)$$

Now, the equation can be handled as a linear equation, one with four coefficients to be determined from the entire field of weight loss data.

To handle this equation for the determination of the four unknowns, there must be four different distinctive equations. The technique is certainly not new and can be found in college review booklets[4], where the topic is termed "Multiple Correlation."

The coefficient of ln A in equation (25) is unity, so all that has to be done is to add all the equations to get:

$$\Sigma \ln W = \Sigma \ln A + \Sigma\, a \ln P + \Sigma\, b \ln V + \Sigma\, c \ln t \qquad (26)$$

for which the constants can be removed from the summation and the summation of ln A becomes N ln A to produce:

$$\Sigma \ln W = N \ln A + a\, \Sigma \ln P + b\, \Sigma \ln V + c\, \Sigma \ln t \qquad (27)$$

The coefficient of "a" in equation (25) is ln P, so each term is multiplied by ln P and then summed to get:

$$\Sigma \ln W \ln P = \ln A\, \Sigma \ln P + a\, \Sigma\, (\ln P)^2 + b\, \Sigma\, (\ln P \ln V) + c\, \Sigma\, (\ln P \ln t) \qquad (28)$$

The coefficient of "b" in equation (25) is ln V, so each term is multiplied by ln V and summed to get:

$$\Sigma \ln W \ln V = \ln A\, \Sigma \ln V + a\, \Sigma\, (\ln P \ln V) + b\, \Sigma\, (\ln V)^2 + c\, \Sigma\, (\ln V \ln t) \qquad (29)$$

And, finally, the coefficient of "c" in equation (25) is ln t, so each term is multiplied by ln t and summed, to get:

$$\Sigma \ln W \ln t = \ln A\, \Sigma \ln t + a\, \Sigma\, (\ln P \ln t) + b\, \Sigma\, (\ln V \ln t + c\, \Sigma\, (\ln t)^2 \qquad (30)$$

Now, there are four equations in A, a, b, and c (equations 27, 28, 29, and 30) which need to be solved simultaneously for the unknowns.

As can be seen, when handling all the data (127 sets), only a computer can handle the information adequately. Several products and several summations are required. The following data handling program was written for the TI-59 to compile the required numbers:

Step	Code	Key	Comments
000	76	LBL	Initialize the computer.
	10	E'	
	47	CMS	Clear all memories.
	25	CLR	Clear the calculation stage.
	01		
	01		
	42	STO	Store 11 in Register 00.
	00	00	(This will be an indirect address location.)
	04		
	07		
010	42	STO	Store 47 in Register 20.
	20	20	(This will be an indirect address location for storing the answers.)
	04		
	42	STO	Store 4 in Register 01
	01	01	for counting the input terms.
	91	R/S	Stop.
	76	LBL	(This is an internal marker for a subroutine.)
	16	A'	
	91	R/S	Stop.
	76	LBL	(This is the data input method.)
020	11	A	
	99	PRT	Print the input.

Step	Code	Key	Comments
	23	LNX	Take the natural logarithm of the input.
	72	ST*	Store in the register specified in
	00	00	Register 00. (Starts with Register 11; see Step 006.)
	74	SM*	Sum into register specified in Register 20.
	20	20	(Starts with 47; see Step 010.)
	01		
	44	Sum	Add 1 to Register 00.
	00	00	
030	44	Sum	Add the same 1 to Register 20.
	20	20	
	97	DSZ	Decrease Register 01 by 1 and skip the next
	01	01	instruction if Register 01 is zero.
	16	A'	Since Register 01 started with 4 (see Step 012), we will have three returns to A' before going on. This allows the four elements of the data set (ln W, ln P, ln V, ln t) to be entered and stored in Registers 11, 12, 13, and 14 and added to Registers 47, 48, 49, and 50. We are now ready to start the calculations of these data.
	01		
	44	Sum	Add 1 to Register 46 for each set of data.
	46	46	(This counts N, the number of sets.)
	43	RCL	Recall the contents of Register 12 (ln P).
	12	12	

Step	Code	Key	Comments
040	33	X^2	Square this value.
	44	Sum	Add this value to Register 51.
	51	51	(This forms $\Sigma (\ln P)^2$ in Register 51.)
	43	RCL	Recall the contents of Register 13 (ln V).
	13	13	
	33	X^2	Square this value.
	44	Sum	Add this value to Register 52.
	52	52	(This forms $\Sigma (\ln V)^2$ in Register 52.)
	43	RCL	Recall the contents of Register 14 (ln t).
	14	14	
050	33	X^2	Square this value.
	44	Sum	Add this value to Register 53.
	53	53	(This forms $\Sigma (\ln t)^2$ in Register 53.)
	53	(	Start a calculation.
	43	RCL	Recall the contents of Register 11 (ln W).
	11	11	
	65	X	Multiply by:
	43	RCL	Recall the contents of Register 12 (ln P).
	12	12	
	54	)	Complete this calculation.
060	44	Sum	Add this value to Register 54.
	54	54	(This forms Σ ln W ln P in Register 54.)

Step	Code	Key	Comments
	53	(	Start another calculation.
	43	RCL	Recall the contents of Register 11 (ln W).
	11	11	
	65	X	Multiply by:
	43	RCL	Recall the contents of Register 13 (ln V).
	13	13	
	54	)	Complete this calculation.
	44	Sum	Add this value to Register 55.
070	55	55	(This forms Σ ln W ln V in Register 55.)
	53	(	Start another calculation.
	43	RCL	Recall the contents of Register 11 (ln W).
	11	11	
	65	X	Multiply by:
	43	RCL	Recall the contents of Register 14 (ln T).
	14	14	
	54	)	Complete the calculation.
	44	Sum	Add this value to Register 56.
	56	56	(This forms Σ ln W ln T in Register 56.)
080	53	(	Start another calculation.
	43	RCL	Recall the contents of Register 12 (ln P).
	12	12	

Step	Code	Key	Comments
	65	X	Multiply by:
	43	RCL	Recall the contents of Register 13 (ln V).
	13	13	
	54	)	Complete this calculation.
	44	Sum	Add this value to Register 57.
	57	57	(This forms Σ ln P ln V in Register 57.)
	53	(	Start another calculation.
090	43	RCL	Recall the contents of Register 12 (ln P).
	12	12	
	65	X	Multiply by:
	43	RCL	Recall the contents of Register 14 (ln t).
	14	14	
	54	)	Complete this calculation.
	44	Sum	Add this value to Register 58.
	58	58	(This forms Σ ln P ln t in Register 58.)
	53	(	Start another calculation.
	43	RCL	Recall the contents of Register 13 (ln V).
100	13	13	
	65	X	Multiply by:
	43	RCL	Recall the contents of Register 14 (ln t).
	14	14	
	54	)	Complete this calculation.
	44	Sum	Add this value to Register 59.

Step	Code	Key	Comments
	59	59	(This forms Σ ln V ln t in Register 59.)
	01		The data set has been processed.
	01		
	42	STO	Reset 11 in Register 00 for the
110	00	00	indirect address location.
	04		
	07		
	42	STO	Reset 47 in Register 20 for the next
	20	20	data set.
	04		
	42	STO	Reset 4 in Register 01 for the
	01	01	data set counting procedure.
	98	ADV	Skip a line on the printer, indicating that the calculations are complete and the machine is ready for the next set of data. This also separates the elements of the data set for review and verification.
	91	R/S	Stop.
120	76	LBL	When all the data have been entered, press
	15	E	"E" for the printout command for the answers.
	04		Set 46 as the first register to be
	06		printed.
	22	INV	List the contents of all registers, beginning
	90	LST	with 46 (N) and going to 59 (the last).

Step	Code	Key	Comments
	98	ADV	Advance the paper for five blank lines.
	98	ADV	(This allows all the data to be clear of
	98	ADV	the machine and ready for removal.)
	98	ADV	
130	98	ADV	
	91	R/S	Stop. (This is the end of the program.)
	76	LBL	If an error was made in posting the data,
	17	B'	this portion will remove the error (except
	91	R/S	for an input of zero, which requires restart).
	76	LBL	Use "B" to enter the wrong data.
	12	B	
	94	+/-	Change the sign.
	99	PRT	Print the input with the negative sign.
	94	+/-	Change the sign back again.
140	23	LNX	Take the natural logarithm of the input.
	72	ST*	Store in the register specified in Register 00.
	00	00	
	22	INV	Do the opposite of adding (subtracting) into
	74	SM*	the register specified in Register 20.
	20	20	
	01		

Step	Code	Key	Comments
	44	Sum	Add 1 to Register 00.
	00	00	
	44	Sum	Add 1 to Register 20.
150	20	20	
	97	DSZ	Decrease Register 01 by 1 and skip the next
	01	01	instruction if Register 01 is zero. Other-
	17	B'	wise, return to B' (Step 132).
	01		
	22	INV	Subtract 1 from Register 46 (N).
	44	Sum	
	46	46	
	43	RCL	Recall ln P used in the incorrect data set.
	12	12	
160	33	x^2	Square this value.
	22	INV	Subtract this value from Register 51.
	44	Sum	
	51	51	
	43	RCL	Recall ln V used in the incorrect data set.
	13	13	
	33	x^2	Square this value.
	22	INV	Subtract this value from Register 52.

Step	Code	Key	Comments
	44	Sum	
	52	52	
170	43	RCL	Recall ln t used in the incorrect data set.
	14	14	
	33	X^2	Square this value.
	22	INV	Subtract this value from Register 53.
	44	Sum	
	53	53	
	53	(	Start the calculation.
	43	RCL	Recall ln W.
	11	11	
	65	X	Multiply by:
180	43	RCL	Recall ln P.
	12	12	
	54	)	Complete this calculatio
	22	INV	Subtract from Σ ln W ln P, Register 54.
	44	Sum	
	54	54	
	53	(	Start another calculation.
	43	RCL	Recall ln W.
	11	11	
	65	X	Multiply by:
190	43	RCL	Recall ln V.

Step	Code	Key	Comments
	13	13	
	54	)	Complete this calculation.
	22	INV	Subtract from Σ ln W ln V, Register 55.
	44	Sum	
	55	55	
	53	(	Start another calculation.
	43	RCL	Recall ln W.
	11	11	
	65	X	Multiply by:
200	43	RCL	Recall ln t.
	14	14	
	54	)	Complete this calculation.
	22	INV	Subtract from Σ ln W ln t, Register 56.
	44	Sum	
	56	56	
	53	(	Start another calculation.
	43	RCL	Recall ln P.
	12	12	
	65	X	Multiply by:
210	43	RCL	Recall ln V.
	13	13	
	54	)	Complete this calculation.

Step	Code	Key	Comments
	22	INV	Subtract from Σ ln P ln V, Register 57.
	44	Sum	
	57	57	
	53	(	Start another calculation.
	43	RCL	Recall ln P.
	12	12	
	65	X	Multiply by:
220	43	RCL	Recall ln t.
	14	14	
	54	)	Complete this calculation.
	22	INV	Subtract from Σ ln P ln t, Register 58.
	44	Sum	
	58	58	
	53	(	Start another calculation.
	43	RCL	Recall ln V.
	13	13	
	65	X	Multiply by:
230	43	RCL	Recall ln t.
	14	14	
	54	)	Complete this calculation.
	22	INV	Subtract from Σ ln V ln t, Register 59.
	44	Sum	

Step	Code	Key	Comments
	59	59	
	61	GTO	Return to Step 107 in the routine, where the
	01		data set was completely processed and the
	07		resetting of the counting and address
239	91	R/S	locations was begun.
			Stop. End.

After this program has been completed, one is left with all the sums needed for the simultaneous solution of four equations with four unknowns. The coefficients are somewhat different from the normal "numbers," but the technique is much the same. Fortunately, there is a program in the Master Library of the TI-59 that handles this solution directly. All that has to be done is to identify the coefficients for the equations by noting in which register they are located. The following locations relate the registers and the coefficients of equations 27, 28, 29, and 30:

$$\left|\begin{array}{c} \text{Reg } 47 \\ \text{Reg } 54 \\ \text{Reg } 55 \\ \text{Reg } 56 \end{array}\right| = \left|\begin{array}{l} \text{Reg } 46 \cdot \ln A + \text{Reg } 48 \cdot a + \text{Reg } 49 \cdot b + \text{Reg } 50 \cdot c \\ \text{Reg } 48 \cdot \ln A + \text{Reg } 51 \cdot a + \text{Reg } 57 \cdot b + \text{Reg } 58 \cdot c \\ \text{Reg } 49 \cdot \ln A + \text{Reg } 57 \cdot a + \text{Reg } 52 \cdot b + \text{Reg } 59 \cdot c \\ \text{Reg } 50 \cdot \ln A + \text{Reg } 58 \cdot a + \text{Reg } 59 \cdot b + \text{Reg } 53 \cdot c \end{array}\right| \qquad \begin{array}{r} (27A) \\ (28A) \\ (29A) \\ (30A) \end{array}$$

By recalling the contents of each register, the numbers are entered into Program 02 of the Master Library and the coefficients are determined:

$$\begin{aligned} \ln A &= -17.10393233 \\ A &= .0000000373 \\ a &= .584(0567501) \\ b &= .703(3895051) \\ c &= .915(1222567) \end{aligned}$$

Thus, the best fit equation for the data would be:

$$W = .0000000373\ P^{.584}\ V^{.703}\ t^{.915} \qquad (31)$$

where W would be in grams, P in psi, V in cpm, and t in minutes.

To be able to compare this equation to the wear rate surface equation, a wear rate must be determined. Since the times for the tests varied with each set of conditions, the wear rates were calculated for the high and low time period for each set of conditions. This operation was also handled by computer. The following program shows how the calculations were performed:

Step	Code	Key	Comments
000	76	LBL	Initialize the computer.
	10	E'	
	47	CMS	Clear all memories.
	25	CLR	Clear the calculation stage.
	91	R/S	Stop. Place ln A in display, INV LN to get A, then R/S to enter.
	42	STO	
	21	21	Puts A in Register 21.
	91	R/S	Stop. Place a (0.584) in display, then R/S to enter.
	42	STO	
	22	22	Puts a in Register 22.
010	91	R/S	Stop. Place b (0.703) in display, then R/S to enter.
	42	STO	
	23	23	Puts b in Register 23.
	91	R/S	Stop. Place c (0.915) in display, then R/S to enter.
	42	STO	
	24	24	Puts c in Register 24.
	01		

Step	Code	Key	Comments
	01		
	42	STO	Store 11 in Register 10 for indirect address
	10	10	location.
020	03		
	42	STO	Store 3 in Register 01 for counting input
	01	01	terms.
	91	R/S	Stop.
	76	LBL	Internal marker for a subroutine.
	16	A'	
	91	R/S	Stop.
	76	LBL	This is the input data method. First enter P, then V, and then T, pressing "A" after each one
	11	A	
	99	PRT	Prints the input value.
030	72	ST*	Stores the input in the register specified in
	10	10	Register 10. (Starts with 11; see Step 018.)
	01		
	44	Sum	Add 1 to the contents of Register 10.
	10	10	
	97	DSZ	Decrease Register 01 by 1 and skip the next
	01	01	instruction if Register 01 is zero. Otherwise,
	16	A'	go to A', Step 24. After the three values have
			been entered, it is time to calculate.

Step	Code	Key	Comments
	53	(	Start the main calculation.
	53	(	Start another calculation.
040	43	RCL	Recall the contents of Register 21, A.
	21	21	
	65	X	Multiply by the quantity:
	53	(	Start another calculation.
	43	RCL	Recall the contents of Register 11, P.
	11	11	
	45	y^x	Determine P to the power:
	43	RCL	Recall the contents of Register 22, a.
	22	22	
	54	)	Complete the p^a determination.
050	65	X	Multiplying by the quantity:
	53	(	Start another calculation.
	43	RCL	Recall the contents of Register 12, V.
	12	12	
	45	y^x	Determine V to the power:
	43	RCL	Recall the contents of Register 23, b.
	23	23	
	54	)	Complete the V^b determination.
	65	X	Multiply by the quantity:
	53	(	Start another calculation.

Step	Code	Key	Comments
060	43	RCL	Recall the contents of Register 13, t.
	13	13	
	45	y^x	Determine t to the power:
	43	RCL	Recall the contents of Register 24, c.
	24	24	
	54	)	Complete the t^c determination.
	54	)	Complete the intermediate product $AP^aV^bt^c$.
	55	÷	Divide this value of the weight loss by:
	43	RCL	Recall the contents of Register 13, t.
	13	13	
070	54	)	Complete the wear rate determination for the
	98	ADV	time used. Advance a line on the printer.
	99	PRT	Print the answer.
	98	ADV	Advance a line on the printer.
	01		
	01		
	42	STO	Store 11 in Register 10 for the next input
	10	10	conditions. (No need to reenter A, a, b, and c.)
	03		
	42	STO	Store 3 in Register 01 for input counting.
080	01	01	
081	91	R/S	Stop. End.

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