Method analyzes analogue plots of paperboard stress-strain data*

ABSTRACT
One of the most important properties of paperboard is characterized by the analogue plot of stress vs. strain obtained from an edgewise compression loading test. A method is given for determining the average stress–strain curve for a group of curves by the use of representative models. An approximate method also allows data to be read from a stress–strain curve so the analysis can be performed with a hand calculator.

KEYWORDS
Stress–strain properties
Paperboards
Compression tests

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Typically, the type of information obtained in an experiment is expressed as a discrete number, i.e., weight of an object, size, density, maximum strength, and so forth. Replicate tests are characterized by averaging the discrete values and applying statistical techniques to determine confidence intervals. However, if the data obtained are in the form of a continuous plot of many data points, such as a load–deformation curve obtained in a compression, tension, or shear test, the problem arises as to how to average a set of curves.

A method for analyzing continuous trace data (curves) is presented in this paper. To do this the curve is reduced to a two-parameter model. Individual fits between the model and each trace are characterized by the parameter estimates obtained from a regression analysis. Ultimately it is possible to obtain a single pair of parameter estimates representative of all the traces. The accuracy of the results is represented by a confidence region surrounding the estimates.

Increasingly more information is being published relating stress–strain behavior of paperboard to the structural performance of corrugated fiberboard (I). Converting processes and raw materials affect paperboard strength and stiffness (2). To analyze stress–strain data and identify factors affecting the curve shape and ultimately box performance, this report provides guideline information so the experimenter can objectively deduce an average curve for a set of curves.

Stress-strain model

One of the earliest procedures for characterizing stress–strain curves was given by Ramberg and Osgood (*3*). They used a three-parameter model to describe arbitrary materials. Their procedure uses logarithm graph paper and is sensitive to the plotting skills of the user. The procedure given in this paper uses a computer or a hand calculator and is more objective. Moreover, a specific stress–strain curve is given that works very well in a computer program for calculating the edgewise compressive strength of corrugated fiberboard (*1*).

The behavior of paperboard in edgewise compression, with stress, a, versus strain, ε, was found empirically to be represented by the model

$$\sigma(\epsilon) = c_1 \tanh\left[c_2(\epsilon + \delta)/c_1\right] \qquad (1)$$

This model is more accurate than the similar form used by Johnson *et al.* (*1*). The function is nearly linear for small strains, and then shows increasing curvature as strain increases (Fig. 1). The slope of the curve at zero stress is given by c_2; c_1 is a horizontal asymptote the curve approaches if extrapolated beyond the maximum stress. To accommodate some unknown prestrain, the parameter 6 shifts the function left of the origin. Only parameters c_1 and c_2 are of interest to describe material behavior. Once the best parameter estimates are obtained from the data, the function is used with 6 = 0 in the engineering calculations of Johnson *et* al. (1). The ever decreasing slope of the curve makes possible the buckling analysis of corrugated fiberboard by the iteration method used (1).

Individual data set analysis

The stress–strain data digitized from the six load–deformation curves are tabulated in Table I. Stress is calculated as the applied load divided by the original cross-sectional area, and strain as a deformation per undeformed specimen gage length. Units of deformation were measured relative to an assumed origin obtained by extrapolating the initial part of the curve to a zero load reference (δ = 0). For usual applications theorigin should be determined from δ rather than by extrapolation. However, later in this paper the same data are applied to the approximate method of analysis where extrapolation is required.

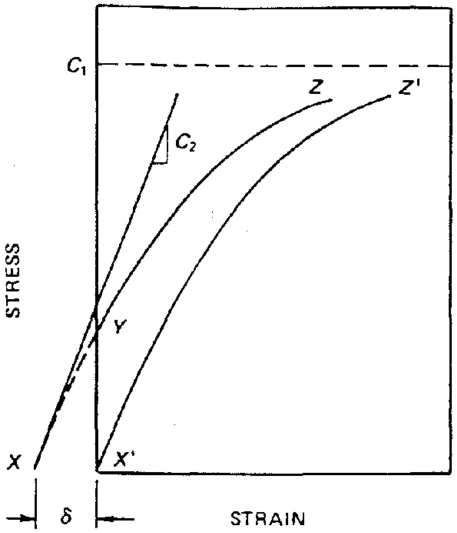
The parameters to Eq. 1 were estimated for each data set by the nonlinear regression subroutine NREG (4). The method for obtaining these estimates is explained in the next section. Results are given in Table II. Because the data in Table I are already adjusted for a prestrain, the best estimates of 6 were essentially equal to zero and thus check the accuracy of the extrapolations.

To check the accuracy of the model,

*See this month's "Calculator Corner" in the Workshop for an SR-52 calculator program relating to this method.

I. Stress-strain data

Curve	Thickness, mm	Strain %	Stress, MPa
1	0.203	0.0226	1.08
		0.0977	4.65
		0.1729	8.01
		0.2481	11.20
		0.3459	14.65
		0.4210	17.11
		0.4962	18.96
		0.5113	19.13
2	0.203	0.0301	1.42
		0.1053	5.30
		0.1804	8.74
		0.2556	12.07
		0.3308	15.08
		0.4067	17.66
		0.4812	19.56
		0.5564	20.46
3	0.198	0.0150	0.88
		0.0902	4.86
		0.1654	8.66
		0.2406	12.02
		0.3158	15.29
		0.3910	17.90
		0.4662	19.89
		0.5113	20.73
4	0.198	0.0376	2.21
		0.1128	5.79
		0.1880	9.28
		0.2632	12.42
		0.3383	15.42
		0.4135	17.77
		0.4887	19.45
		0.5639	20.33
5	0.198	0.0226	1.28
		0.0977	4.95
		0.1730	8.27
		0.2480	11.24
		0.3230	14.13
		0.3980	16.41
		0.4740	18.27
		0.5490	19.31
6	0.198	0.0301	1.32
		0.1050	4.86
		0.1800	8.20
		0.2560	11.24
		0.3310	14.13
		0.4060	16.62
		0.4810	18.55
		0.5110	18.96



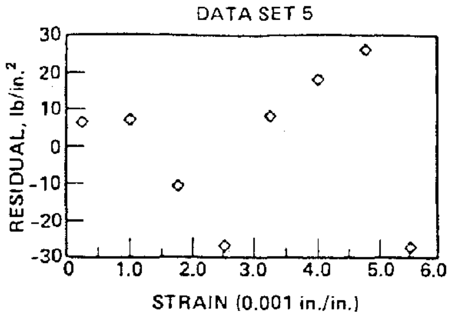
1. Stress-strain relation given by $\sigma(\varepsilon) = c_1 \tanh [c_2 (\varepsilon + \delta)/ c_1]$. Data are collected along segment *yz*. Segment *xy* is determined either by extrapolation or by regression analysis of δ. With δ = 0 the curve is shifted to *x' z'*.

the predicted stress was subtracted from the observed stress at each measured strain; the differences, called residuals, were plotted (Fig. 2). The relative magnitudes of the residuals are small, thus indicating the model is adequate.

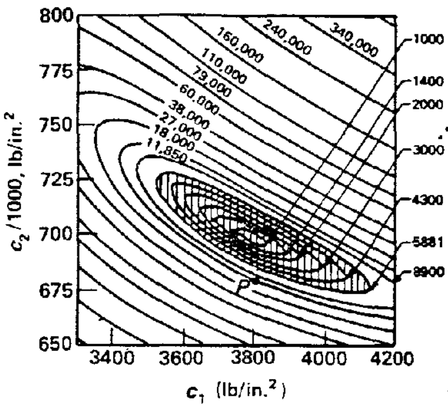
Determining best parameter estimates

According to the best parameter estimates of Curve 1 from Table II, c_1 = 26.03 MPa and c_2 = 4.832 GPa. Corresponding to each stress a residual R is calculated equal to the observation minus the prediction. A residual sum of squares S is obtained by summing the squares of all the residuals, i.e., ΣR^2. For the best estimates of c_1 and c_2, a minimized residual sum of squares S_R is desired, and in this instance S_R = 0.0338 MPa² (712 lb²/in.⁴) at c_1 = 26.03 MPa and c_2 = 4.832 GPa.

If the parameter estimates were arbitrarily selected at perhaps c_1 =



2. Typical residual plot of observed minus predicted stress obtained from Curve 5.



3. Residual sum of squares (lb²/in.⁴) contour for Curve 1. Point P represents an arbitrary selection of c_1 and c_2. The shaded area gives the 99% joint confidence region for the best c_1 and c_2 estimates.
1 lb/in.² = 6.89 kPa.

26.20 MPa (3800 lb/in.²) and c_2 = 4.688 GPa (680,000 lb/in.²) the sum of squares S = 0.5633 MPa² (11,850 lb²/in.⁴) would be obtained. Because S = 0.5633 MPa² is greater than S, = 0.0338 MPa², the model given by these estimates is thus not as good. One could sequentially select c_1 and c_2 and recalculate S to compare other parameter estimates.

In Fig. 3, contour levels are plotted relative to the S obtained from different parameter estimates of c_1 and c_2. This plot is analogous to a topographic map where each contour is a constant elevation above a valley. Point P is identified in Fig. 3 at the arbitrary coordinates (26.20 MPa, 4.688 GPa). Following the contour that P is on shows that many c_1,c_2 pairs yield an S = 0.5633 MPa². An S_R = 0.0338 MPa² is, however, obtained at only one pair of coordinates (26.03 MPa, 4.832 GPa). Hence, this minimized residual sum of squares S_R locates the best parameter estimates to the stress–strain model. In essence, various c_1 and c_2 values are selected till S is minimized.

Joint confidence region

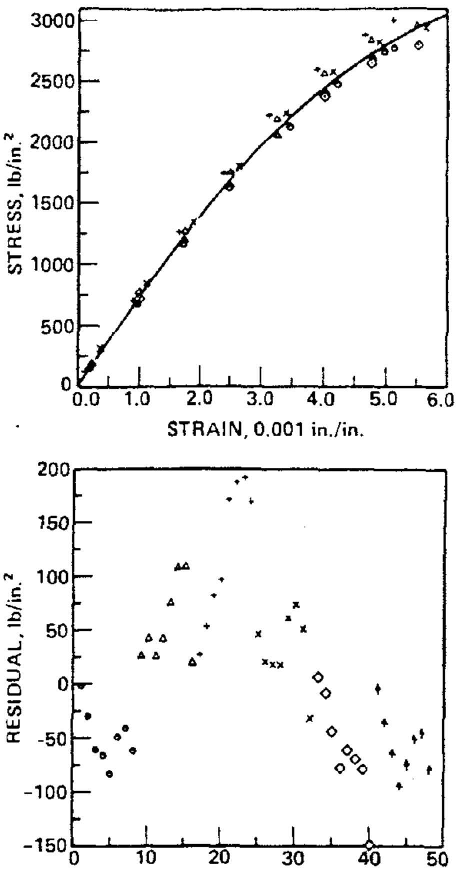
For a single curve

The accuracy of the c_1 and c_2 estimates depends on both the accuracy of the stress–strain data and the accuracy of

II. Parameter estimates to stress-strain model

	Regression solution					
	Parameter			Minimized residual sum of squares	Approximate method	
Curve	c_1, MPa	c_2, GPa	d, %	(S_R), MPa²	c_1, MPa	C2, GPa
1	26.03	4.832	–6.063 10⁻⁴	0.0338	25.59	4.843
2	25.28	5.327	–51.51 ·10⁻⁴	0.3019	25.15	5.143
3	26.72	5.480	–5.531·10⁻⁴	0.0867	26.88	5.378
4	24.67	5.270	–2.605·10⁻⁴	0.3265	24.33	5.206
5	23.85	4.972	23.10 ·10⁻⁴	0.1321	23.68	4.926
6	25.97	4.823	–32.43 ·10⁻⁴	0.0824	26.19	4.694
Average	25.20[a]	5.116[a]	0.0008605[a]	14.04[a]	25.30[b]	5.032[b]

[a]Obtained from regression analysis of all stress–strain data combined [b]obtamed by averaging individual estimates



4. Combined data plots and characteristic model (upper). Residual plot of observed minus predicted stress obtained from all data (lower). Legend: . , Curve 1; Δ, Curve 2; +, Curve 3; X, Curve 4; ◊, Curve 5;# ,Curve 6.

the model. An approximate measure of the parameter accuracy is determinable from the S contour levels.

Of particular interest in Fig. 3, for example, is the contour level S = 0.2796 MPa² (5881 lb²/in.⁴) derived from the following:

$$S = S_R \left[1 + \frac{p}{n-p} F_{0.01}(p, n-p) \right]$$

The number of parameters in the model is p; n is the number of data points; $F_{0.01}(p, n-p)$ is the value of a random variable having the F distribution with p adn n-p degrees of freedom at the 0.01 confidence level for $n > p$. F or a single curve with 8 data points, $F_{0.01}(3,5)$ = 12.10. If the experiment were repeated many times on identical paperboard specimens, assuming that the model form is adequate, approximately 99% of the best parameter estimates (1-0.01) would fall within the area, called the joint confidence region, bounded by this contour. The model given by point P is therefore different by at least approximately 99% certainty. For a smaller joint confidence region, the parameter estimates are hence more accurate.

For combined curves

According to the same method of analysis, a single set of parameter estimates and a joint confidence region were obtained by pooling all the data. To do this, the individually determined values of δ were first added to the strain values for each curve prior to combining all the data. This forced all curves to share a common origin. Thus, the best fitting model for all six samples is characterized by c_1 = 25.20 MPa (3655 lb/in.²) and c_2 = 5.116 GPa (742,000 lb/in.²).

The final model and the consecutive residuals are plotted with all the data in Fig. 4. The first eight residuals belong to the first data set, the second eight belong to the second data set, etc. Consecutive groups of eight residuals are displaced from each other, but residuals within groups are fairly close. The 99% confidence region is shown in Fig. 5 and thus depicts the most accurate characterization of all available data (n = 48).

Approximate method of analysis

The results so far obtained have required use of a digital computer. For practical reasons this is not always possible. Although the most reliable parameter estimates to Eq. 1 are obtained from a regression analysis on a large sample of data, the model can still be fit to only two points from a single curve with a numerical solution on a hand calculator.

To do this, extrapolate the curve to the origin to make δ = 0. Pick two pairs of stress–strain coordinates (ε_1, σ_1) at about one-halfof the maximum strain, and (ε_2, σ_2) at the maximum strain. The model will fit exactly for any arbitrary pair of points; however, it is proven in the Appendix that points at about one-half the maximum strain and the maximum strain are most representative of the full curve. As a first estimate let $c_1 = \sigma_2$ and $c_2 = \sigma_1/\varepsilon_1$. Next, perform the following sequence of calculations:

$f = \sigma_1 - c_1 \tanh(c_2\varepsilon_1/c_1)$
$g = \sigma 2 - c1 \tanh(c_2\varepsilon_2/c_1)$
$f_x = c_2\varepsilon_1/c_1/\cosh^2(c_2\varepsilon_1/c_1) - \tanh(c_2\varepsilon_1/c_1)$
$f_y = -\varepsilon_1/\cosh^2(c_2\varepsilon_1/c_1)$
$g_x = c_2 e_2/c_1/\cosh^2(c_2\varepsilon_2/c_1) - \tanh(c_2\varepsilon_2/c_1)$
$g_y = -\varepsilon_2/\cosh^2(c_2 e_2/c_1)$
$J = f_x \cdot g_y - g_x \cdot f_y$
New $c_1 = c_1 - (f \cdot g_y - g \cdot f_y)/J$
New $c_2 = c_2 - (g \cdot f_x - f \cdot g_x)/J$

Then repeat the iteration until the solution converges.

The Table I data are reanalyzed according to the approximate method by choosing the optimum stress–strain pairs, one at the maximum recorded strain and the other at the strain closest to one-half of this maximum. The results are given in Table II. A single representative curve is obtained by averaging the individual parameters. By the approximate method, c_1 = 25.30 MPa (3670 lb/in.²) and c_2 = 5.032 GPa (729,800 lb/in.²). This compares closely with c_1 = 25.20 MPa and c_2 = 5.115 GPa obtained from the regression solution.

Individual confidence intervals

Because it is not possible to determine a joint confidence region using the approximate method, individual confidence intervals were determined for c_1 and c_2, and the following method was used to check the accuracy of these parameters and compare them with the regression solution. These intervals can be determined by solving the equations derived in (*4*) which follow.

III. Standard error analysis by approximate method

Curve	Strain (ε_i), %	Stress: Observed (σ_i), MPa	Stress: Predicted (s), MPa	R, MPa	f_1	f_2
1	0.2481	11.20	11.56	−0.3568	0.06650	0.001963
	0.5113	19.13	19.45	−0.3140	0.3523	0.002093
2	0.2556	12.07	11.86	0.2085	0.07194	0.001995
	0.5564	20.46	20.31	0.1498	0.4094	0.001978
3	0.2406	12.02	11.26	0.7646	0.06130	0.001930
	0.5113	20.73	19.45	1.279	0.3523	0.002093
4	0.2632	12.42	12.16	0.2640	0.07767	0.002025
	0.5639	20.23	20.44	−0.1138	0.4188	0.001957
5	0.2480	11.24	11.56	−0.3183	0.06643	0.001963
	0.5490	19.31	20.18	−0.8742	0.4001	0.001998
6	0.2560	11.24	11.87	−0.6346	0.07223	0.001996
	0.5110	18.96	19.44	−0.4801	0.3520	0.002094
Sum of squares				4.092 MPa²	0.9043	0.00004837
Sum of $f_1 \cdot f_2$ = 0.005465						

List then stress–strain data pairs (ε_i, σ_i, i = 1, n) used to obtain an average c_1 and c_2. For each strain ε_i calculate the following:

$$\sigma = c_1 \tanh (c_2\varepsilon_i/c_1)$$
$$R = \sigma_i - \sigma$$
$$f_2 = \varepsilon_i/\cosh^2 (c_2\varepsilon_i/c_1)$$
$$f_1 = (\sigma - c_2 f_2)/c_1$$

Then calculate the summation of the following products for i = 1, n.

$$S_R = \Sigma R^2,\ f_{11} = \Sigma f_1^{\,2},\ f_{12} = \Sigma f_1 f_2,\ f_{22} = \Sigma f_2^{\,2}$$

A standard error is now obtained for each parameter.

For c_1,

$$e_1 = \sqrt{f_{22} \cdot S_R/(f_{11} \cdot f_{22} - f_{12}^2)/(n-2)}$$

For c2,

$$e_2 = \sqrt{f_{11} \cdot S_R/(f_{11} \cdot f_{22} - f_{12}^2)/(n-2)}$$

The stress–strain data used to obtain the approximate c_1 and c_2 are listed in Table III, and all the calculations are carried out. It is found that e_1 = 1.194 MPa and e_2 = 163.3 MPa. An interval can be obtained for each parameter over which the best estimate would occur from a large sample of data. For example, at 99% certainty the interval for parameter c_1 is

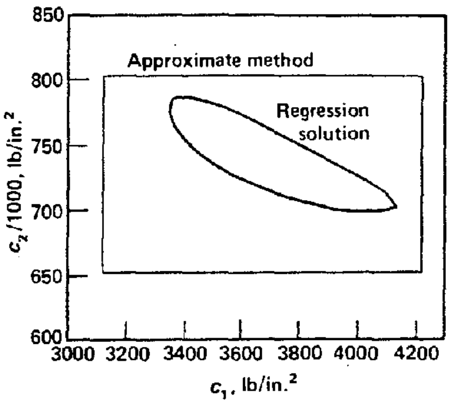
$$c1 \pm t_{0.01/2}(n-2)\, e_1$$

where $t_{0.01/2}$ (n - 2) is the value of a random variable having the Student-t distribution with $n-2$ degrees of freedom at the 0.01/2 confidence level. For 6 curves with 12 data points, $t_{0.005}(10)$ = 3.169. The interval for c_2 is likewise calculated using e_2. The 99% confidence intervals for the Table III data are therefore

21.52 MPa (3121 lb/in.^{2}) $\leq c_1 \leq$ 29.09 MPa(4219 lb/in.^{2})

4.514 GPa (654,700 lb/in.^{2}) $\leq c_2 \leq$ 5.550 GPa (804,900 lb/in.^{2})

The area bounded by these individual intervals is shown in Fig. 5. It compares very well with the joint confidence region obtained by the regression solution.



5. Confidence regions obtained at the 99% level for stress-strain parameters determined for all the data. The regression solution is more specific, but the approximate method gives a general answer.

Conclusions

A method is given for obtaining a single representative curve and a measure of parameter accuracy for a group of continuous stress–strain curves. A single confidence region characterizes a sample of paperboard stress–strain data. An approximate method of analysis is given for use with a hand calculator. Only two stress-strain coordinate pairs need to be read from each curve. The approximate method agrees most closely with the results of the regression solution if the stress–strain data are read at the point of maximum strain and at a point relative to about one-half the maximum strain.

Experimental

The load–deformation relation was determined for corrugating medium from edgewise compression tests on paper samples. The material was a nominal 126-g/m^2 (26-lb/1000 ft^2), commercially made medium used in a previous study (*5*) of its instability in a corrugated structure. Six 25.4 × 102-mm specimens were cut in the machine direction from a larger 203 × 254-mm sheet of medium. Each specimen was measured for thickness with a stylus apparatus (*6, 7*). This approach yields a smaller value than does the TAPPI method, but it is more representative of the effective sheet thickness.

The specimens were tested in edgewise compression in the machine direction using the lateral support device described by Jackson *et al.* (*8*). A universal testing machine having a movable frame with adjustable loading speeds and equipped with an electronic load cell was used to apply the load to and measure the load on the specimen. To record displacement, x–y plotting paper was guided by an apparatus with a speed mechanically proportional to the speed of the movable frame. A preload was applied to each specimen and deformation recorded relative to the zero reference at this load. The load and deformation were measured as the specimen was compressed at 8.47 μm/s. The point at which the load began to decrease with increasing deformation was accepted as the maximum load attainable for the specimen and the test was terminated. The specimen gage length, 33.8 mm, yielded a strain rate of 0.025%/s. The load–deformation trace was manually digitized for analysis.

All specimens were preconditioned in an environment below 30% RH prior to conditioning and testing at 73°F (22.8°C) and 50% RH.

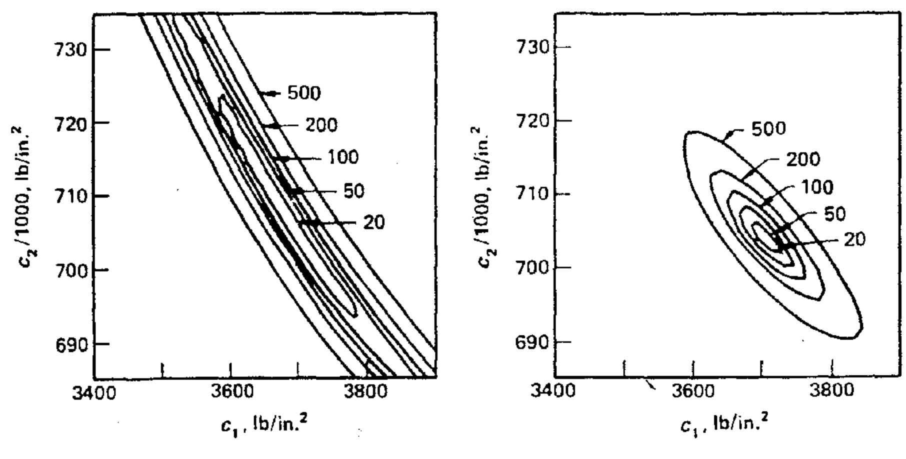
Appendix

If the stress–strain model is fit by the approximate method to the first and the eighth points (pair 1, 8) of curve 1 [(0.0226, 1.08), (0.5113, 19.13)], the results are c_1 = 26.06 MPa, c_2 = 4.762 GPa. If, however, pair (4,8) is analyzed [(0.2481, 11.20),(0.5113, 19.13)], then c_1 = 25.59 MPa, c_2 = 4.843 GPa. The difference can be explained by the degree of certainty with which the data support the model.

Figure 6 shows S contour plots derived from the model fit to pairs (1,8) and (4,8). In both cases S_R = 0 since the model fits exactly. Additional information is, however, contained in the size of the area bounded by a given contour, for example, S = 9.507 · 10^{-4} MPa^2 (20 lb^2/in.^{4}). That area is much smaller for pair (4,8) than for pair (1,8). Hence, fewer (c_1, c_2) combinations will satisfy the model for this level of accuracy. One is, therefore, more certain that the results obtained from pair (4,8) are indeed representative of the data.

It is of interest to investigate if other stress–strain pairs further minimize the area bounded by some contour. Box and Lucas (*9*) describe a graphical approach for determining the optimum points. The results only are summarized here.

First, determine partial differentials $\partial\sigma/\partial c_1$, and $\partial\sigma/\partial c_2$ from $\sigma(\varepsilon)$ when $\delta = o$. These are already given by f_1 and f_2, respectively. Next, plot f_1 vs. ε and f_2 vs. ε (Fig. 7). In the calculations, c_1 = 25.20 MPa and c_2 = 5.116 GPa. An outer boundary to the Box-Lucas design locus is then obtained from f_1 vs. f_2. A triangle is



6. Residual sum of squares (lb^2/in.^{4}) contour for Curve 1 obtained from first and eighth points (left) and fourth and eighth points (right).

inscribed within this boundary with one point at the origin and two points on the curve so as to maximize its area. It has been shown by Box and Lucas (9) that the area of this triangle is inversely proportional to the area of an *S* contour for c_1, c_2. The f_1, f_2 coordinates of the triangle occur at e equal to 0.245% and 0.5%. The design locus shows the optimum choice of data for the most reliable parameter estimates to occur when stress is measured at (a) the maximum point on the stress–strain curve, and (b) at a point relative to about one-half of the maximum strain.

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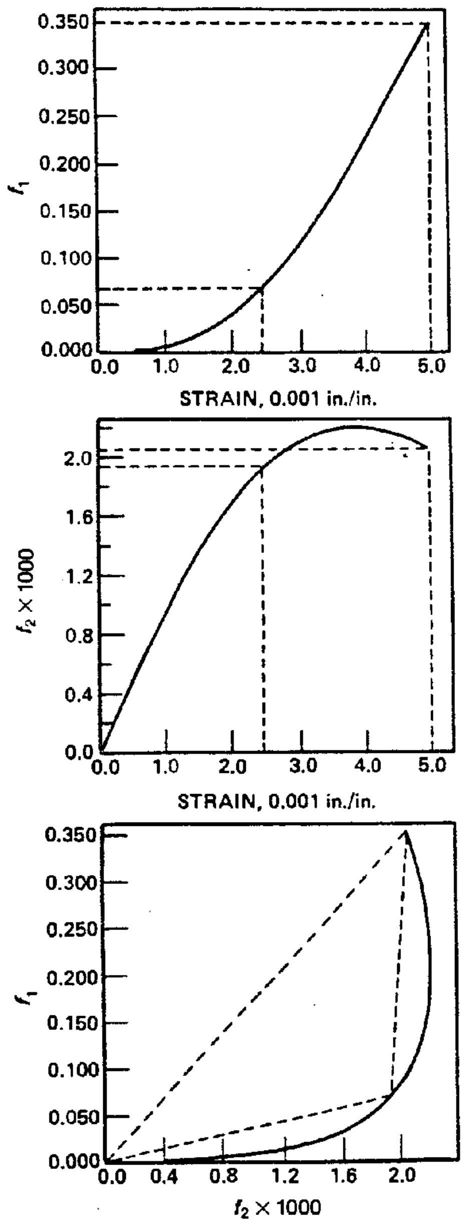

7. Graphical analysis of optimum stress–strain points. Top: f_1 vs. strain; center: f_2 vs. strain; bottom: Box-Lucas (*9*) design locus.

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THE CALCULATOR CORNER

Stress - strain relation for paperboard in edgewise compression*

Program for SR-52 calculator

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This program is useful to papermakers for analyzing the edgewise compressive behavior of paperboard. It determines the coefficients to a formula that characterizes the stress–strain relation of paperboard tested in edgewise compression. The Forest Products Laboratory uses the results of this program to calculate the theoretical effects of different paperboards on corrugated box top-to-bottom compressive strength[1].

Equation

Determine parameter values c_1 and c_2 to fit the equation

$$\sigma(e) = c_1 \tanh(c_2 \varepsilon / c_1)$$

to (ε, σ) data read from a continuous edgewise compression stress–strain trace where

ε = strain value along curve
σ = stress value corresponding to E
c_1, c_2 = parameters to be determined

Input

The following input are read directly from a continuous stress - strain trace:

ε_1 = strain at approximately one-half the ultimate strain
ε_2 = ultimate strain
σ_1 = stress measured at ε_1
σ_2 = stress measured a ε_2

*For a background report on this method. see the article "Method Analyzes Analogue Plots of Paperboard Stress–Strain Data" by T. J. Urbanik in this issue.

[1]Urbanik, T. J.. "Effect of Paperboard Stress–Strain Characteristics on Strength of Singlewall Corrugated Fiberboard: A Theoretical Approach." USDA For. Serv. Res. Pap. 401, Forest Products Laboratory, Madison, Wis., 1981.

Please see next page for program and continuation of text

User instructions

STEP	PROCEDURE	ENTER	PRESS			DISPLAY
1	Store ε_1 in 03.	ε_1	ST0	0	3	ε_1
2	Store ε_2 in 04.	ε_2	ST0	0	4	ε_2
3	Store σ_1 in 05.	σ_1	ST0	0	5	σ_1
4	Store σ_2 in 06.	σ_2	ST0	0	6	σ_2
5	Calculate initial		C			c_1
	estimate of c_1.					
6	Calculate improved		A			New c_1
	estimate of c_1.					
7	If New c_1 ^{1} c_1,					
	let c_1 = New c_1					
	and go to step 6.					
	If New c_1 = c_1,					
	go to step 8.					
8	Display New c_2.		RUN			New c_2

Sample problem and solution

Problem

ε_1 = 0.003 in./in
ε_2 = 0.006 in./in.
σ_1 = 1500 lb/in.^{2}
σ_2 = 2500 lb/in.^{2}

Answer

Successive iterations converge to the exact answers for c_1 and c_2.

c_1 (lb/in.^{2})	c_2 (lb/in.^{2})
3253	
3352	
3354	
3354	538,011

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SR-52 Program Form

LOC	CODE	KEY	COMMENTS
000 112	46	LBL	
	13	C	
	43	RCL	
	00	0	
	06	6	
005 117	42	STO	
	00	0	
	01	1	C_1
	43	RCL	
	00	0	
010 122	05	5	
	55	÷	
	43	RCL	
	00	0	
	03	3	
015 127	95	=	
	42	STO	
	00	0	
	02	2	C_2
	46	LBL	
020 132	11	A	
	43	RCL	
	00	0	
	03	3	ε_1
	51	SBR	
025 137	12	B	
	43	RCL	
	00	0	
	05	5	
	95	=	
030 142	42	STO	
	01	1	
	03	3	-f
	43	RCL	
	00	0	
035 147	03	3	
	65	x	
	43	RCL	

LOC	CODE	KEY	COMMENTS
	01	1	
	01	1	
040 152	95	=	
	42	STO	
	01	1	
	07	7	$-f_y$
	43	RCL	
045 157	01	1	
	06	6	
	48	EXC	
	01	1	
	05	5	f_x
050 162	43	RCL	
	00	0	
	04	4	ε_2
	51	SBR	
	12	B	
055 167	43	RCL	
	00	0	
	06	6	
	95	=	
	42	STO	
060 172	01	1	
	04	4	-g
	43	RCL	
	00	0	
	04	4	
065 177	65	x	
	43	RCL	
	01	1	
	01	1	
	95	=	
070 182	42	STO	
	01	1	
	08	8	$-g_y$
	65	x	
	43	RCL	
075 187	01	1	

LOC	CODE	KEY	COMMENTS
	05	5	f_x
	75	−	
	43	RCL	
	01	1	
080 192	06	6	g_x
	65	x	
	43	RCL	
	01	1	
	07	7	$-f_y$
085 197	95	=	
	20	1/x	
	42	STO	
	01	1	
	09	9	-1/J
090 202	65	x	
	53	(	
	43	RCL	
	01	1	
	03	3	-f
095 207	65	x	
	43	RCL	
	01	1	
	08	8	$-g_y$
	75	−	
100 212	43	RCL	
	01	1	
	04	4	-g
	65	x	
	43	RCL	
105 217	01	1	
	07	7	$-f_y$
	54	)	
	95	=	
	44	SUM	
110 222	00	0	
	01	1	New C_1

LABELS
A
B
C
D
E
A'
B'
C'
D'
E'
REGISTERS
00
01
02
03
04
05
06
07
08
09
10
11
12
13
14
15
16
17
18
19
FLAGS
0
1
2
3
4

Program continued on next page

LOC	CODE	KEY	COMMENTS	LOC	CODE	KEY	COMMENTS	LOC	CODE	KEY	COMMENTS	LABELS
000 112	43	RCL			02	2	C_2		48	EXC		A
	01	1			55	÷			00	0		B
	09	9	-1/J	040 152	43	RCL			09	9	tanh	C
	65	x			00	0			40	x^2		D
	53	(			01	1	C_1	080 192	55	÷		E
005 117	43	RCL			95	=			04	4		A'
	01	1			42	STO			95	=		B'
	03	3	-f	045 157	00	0			20	1/x		C'
	65	x			07	7	$C_2\varepsilon/C_1$		42	STO		D'
	43	RCL			27	INV		085 197	01	1		E'
010 122	01	1			23	ln			01	1	$1/\cosh^2$	REGISTERS
	06	6	g_x		75	-			65	x		00
	75	-		050 162	43	RCL			43	RCL		01
	43	RCL			00	0			00	0		02
	01	1			07	7		090 202	07	7		03
015 127	04	4	-g		94	+/-			75	-		04
	65	x			27	INV			43	RCL		05
	43	RCL		055 167	23	ln			00	0		06
	01	1			95	=			09	9		07
	05	5	f_x		55	÷		095 207	95	=		08
020 132	54	)			53	(			42	STO		09
	95	=			43	RCL			01	1		10
	44	SUM		060 172	00	0			06	6	f_x, g_x	11
	00	0			07	7			43	RCL		12
	02	2	New C_2		27	INV		100 212	00	0		13
025 137	43	RCL			23	ln			01	1		14
	00	0			85	+			65	x		15
	01	1		065 177	43	RCL			43	RCL		16
	81	HLT			00	0			00	0		17
	43	RCL			07	7		105 217	09	9		18
030 142	00	0			94	+/-			75	-		19
	02	2			27	INV			56	rtn		FLAGS
	81	HLT		070 182	23	ln						0
	46	LBL			54	)						1
	12	B			42	STO		110 222				2
035 147	65	x			00	0						3
	43	RCL			09	9						4
	00	0		075 187	95	=						

Readers are invited to send their comments and programs to the Tappi Workshop editor, One Dunwoody Park, Atlanta, Ga. 30338.