Present Value Formulas for Calculating
Maximum Bid Prices for Land with
Applications for the TI-59
Hand Held Computer\*\*

by

Ghanbar Kooti and Lindon J. Robison\*

- \* The authors are former graduate student and assistant professor respectively in the Department of Agricultural Economics at Michigan State University. They wish to thank Bud Search for preparing Appendix A of this report.
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## Introduction

Whether or not to purchase farmland can be one of the most difficult investment decisions confronting farm operators. Compared wth other production inputs, land is purchased infrequently, usually in discrete units and involves a longterm financial obligation. The decision to purchase a parcel of farmland is crucial since only about 3 percent of all the farmland in the U.S. is transferred from one owner to another each year (Scott).

Because land is traded infrequently and each parcel has a locational monopoly, an opportunity to purchase a particular tract may come along only once in a lifetime. So determining the maximum bid price one can offer for a parcel of land is critical. If a decision maker's bid price is significantly below the asking price, then he might lose the opportunity to purchase. On the other hand, if it is significantly above the true value, his offered price might put him in a difficult financial position. Therefore, finding accurate ways of estimating land values is important for those wishing to purchase land.

## Factors Affecting Land Values

The maximum bid price farmers can pay for land and still "break even" is affected by several factors, the most important being net returns and increases in net returns expected over the planning horizon.

Net returns may increase over time because of inflation, or persistent price increases. People are willing to pay extra for land if they expect continued inflation. The extent of inflationary impacts in the economy will then be reflected by the equilibrium price agreed on by buyers and sellers.

There are, of course, other factors affecting land values. One such factor influencing land prices is job security. A farmer buying land assures himself of longer tenure than he could if he were renting. Another factor is the pride of ownership. Many derive satisfaction from owning farmland and this satisfaction in owning farmland is certainly not confined to rural people (Scott).

Excess machinery and labor within a farm operation can also increase the amount a buyer can afford to pay for land. When technology creates a situation in which a farm operation becomes land deficient in relation to other inputs, the farmer needs to increase the land input. Thus, it may be economically feasible to pay a higher price for land in order to increase the total land input and spread his fixed costs over a larger land area.

Availability and cost of credit also influences the amount a farmer can pay for a parcel of land and still break even. As credit becomes easier to obtain the number of potential buyers for a tract of land increases. As a result, land prices may increase. The cost of credit adds to the cost of land purchase; therefore, if a buyer pays less for credit, he may be able to pay more for the farmland.

Government programs also influence farmland prices. Commodity price support programs insure farm owners a minimum price for their crops. With the price uncertainty reduced, farmers in many cases, respond with increased output and possibly higher returns which are then capitalized into higher land prices.

#### Objectives

After having reviewed some of the factors which may influence land prices we can construct analytic models which will include some of the factors discussed. The models focus on the relationship between timing and certainty of returns and the present value of land. Present value models which convert future income to present value equivalents were used to find maximum bid prices for land purchasers. These present value formulas will be introduced in increasing complexity—in the end exhausting our ability to deduce understandable formulas. At that point we will introduce a computer model which was constructed by Lee and Rask and adapted for the TI-59 hand held computer by Kooti. Instructions for the implementation of this computer model will also be discussed. It is our object, then, to establish both the theoretical understanding for determining land value as well as providing the computing capacity to make the calculations in applied settings.

### Present Value Formulas

### The Role of Time

An-investment such as farmland generates a stream of income over future time periods. The value of these future returns may be converted to current dollar equivalents through discounting. Once future incomes have been converted to current dollar equivalents and summed, the sum can be compared to the acquisition cost of land expressed in current dollars to determine if the investment is profitable.

The discount rate is composed of two parts: the time preference rate denoted r and the inflation rate denoted i. The rate of return required

to induce savers to postpone consumption, assuming constant prices and certain knowledge is the time preference rate, sometimes referred to as the real rate of return. The inflation rate is a rate of return savers must receive in addition to the time preference rate to compensate them for purchasing power losses due to increased prices. Finally a risk premium may be subtracted from expected income to convert it to its certainty equivalence in order to compare it to the certain outlay required to purchase land. 1/

To begin, however, we consider a world without inflation, certain prices and an n period income stream produced by the investment. This simplified model is referred to as the Basic Capital Budgeting Model (BCB) and provides a point of departure for later models.

## The Basic Capital Budgeting Model (BCB)

Consider an investment (in land) that generates a return R for the next n time periods. In addition, assume that the opportunity cost of capital is r and denote the land's acquisition and salvage value as V.

The present value of the investment is the present value of income expected from land plus the discounted price received when the land is sold.

Under these assumptions, the present value of land is equal to:

(1) 
$$V = S + V/(1+r)^n$$

where S is an annuity of R dollars over n periods equal to

(2) 
$$S = R(1 - (1+r)^{-n})/r$$

Substituting for S in (1) and solving for V we obtain:

<sup>1/</sup> In this paper risk and uncertainty will be used interchangeably.

(3) 
$$V = R/r$$

which is our BCB model. To illustrate, if constant net returns from land R were \$50 and the opportunity cost or cost of borrowing were 5 percent, the maximum bid price would equal \$50/.05 or \$1,000.

## The BCB Model With Inflation

The BCB model assumed no inflation; that is, the discount rate included no premium for savers to offset the reduced purchasing power brought on by inflation. The increasing importance of inflation forces us to rethink our BCB model and allow both returns, land values and discount rates to be influenced by what is assumed to be a constant level of inflation i.

Inflation may be introduced into the BCB model by assuming that expected net returns to land and the discount rate increase by the same inflation rate. If the net returns to land increase by the rate of inflation, then the net income to land in the first period becomes R(1+i) where i is the inflation rate, and in the n-th period becomes R(1+i)<sup>n</sup>. Meanwhile, the discount rate which also is increased by the inflation rate equals (1+r)(1+i) in the first period and in the n-th period equals (1+r)<sup>n</sup> (1+i)<sup>n</sup>. Thus, the value of an asset V with inflation rate i can be written as:

(4) 
$$V = \frac{R(1+i)}{(1+i)(1+r)} + \dots + \frac{R(1+i)^n}{(1+i)^n(1+r)^n} + \frac{V(1+i)^n}{(1+i)^n(1+r)^n}$$

But in (4) the impact of inflation on the discount rate is exactly offset by the impact of inflation on returns. After cancelling the inflation terms in (4) we are left with the BCB in (3) with one difference: the income stream R increases over time by the compound rate i. So in the initial period, period zero, the present value of land  $V_0$  is:

(5) 
$$V_0 = R/r$$

But t periods later land price is:

(6) 
$$V_{t} = R(1+i)^{t}/r$$

## A Generalized Inflation Model

The BCB model with inflation assumed equal and constant inflationary impacts on both income and the discount rate. There is evidence that inflationary pressures are not always so uniform in their impact (Lins and Duncan). To consider such a model, continue to let the inflation increase the discount rate by i + ir so that the discount rate r\* equals i+r+ir. Then let income R increase at a compound rate g. This model can be written as:

(7) 
$$V_0 = R(1+g)/(1+r*) + ... + R(1+g)/(1+r*) + V(1+g)/(1+r*)^n$$

The income portion of the right hand side of (7) represents a geometric series whose sum G can be written as: $\frac{2}{}$ 

(8) 
$$G = R[1-(1+g)^n(1+r*)^{-n}] (1+g)/(r*-g)$$

Then substituting (8) into (7) and solving for V we obtain:

(9) 
$$V_0 = R(1+g)/(r*-g)$$

The interesting feature of this model is that inflation is not neutral even in the initial period unless, of course, g equals i. $\frac{3}{}$  Also of interest

Then multiply both sides above by (1+r\*)/(1+g) to obtain:

$$(1+r*)G/(1+g) = R + ... + R(1+g)^n/(1+r*)^n$$

Finally, subtract from the second expression the first and solve for G.

<sup>2</sup>/ The solution to a geometric sum can be easily illustrated. Let  $G = R(1+g)/(1+r^*) + ... + R(1+g)^n/(1+r^*)^n$ 

<sup>3/</sup> In such an event equation (9) can be written as  $V_0 = R(1+i)/(r+i+ir-i) = R/r$ 

is the rate of increase in V, over time, equal to the compound rate g, so that in the t-th period land price  $V_{+}$  is:

(10) 
$$V_r = R(1+g)^t/(r*-i)$$

One aspect of equation (10) is particularly revealing: the sensitivity of  $V_t$  to differences in g and i. Starting with g and i equal, a one percent increase in g, for example, increases  $V_t$  by 25 percent. On the other hand, if i and g are equal and i increases by one percent,  $V_t$  decreases by 17 percent (see Table 1). That such wide variations in land prices have actually occurred is demonstrated in Table 1.

## Basic Capital Budgeting Model With Taxes

We now address a still different concern of investors: taxes. We will begin by introducing taxes into the BCB model and then combine them with inflation. The important concept to be understood about the introduction of taxes into the BCB model is that it affects both the discount rate and income. Income, of course, must be adjusted to an after-tax basis because it is being compared to an after-tax outlay of funds for land's purchase. Income, however, is adjusted to a present value by discounting it with a rate reflecting the opportunity cost rate of return in the next best investment opportunity or the cost of borrowing. But taxes are involved in alternative investments and also affect the actual interest cost incurred by borrowing.

To include such features into the model we write V equal to the present value of an income stream adjusted for taxes. Letting  $t_p$  be the personal income tax rate we write:

Table 1. The Effects of Inflation and Increases in Net Return to Land on the Percentage Change in Land Values Assuming a Time Preference Rate of Five Percent.

Expected Rate of Inflation	•		Percentage In C	Increase E ash Rents	Expected		
		0	1	2	3	4	
				Percent			
0		0	. 25	65	140	333	
1		-17	0	24	63	136	
2		-29	-17	0	24	63	
3		-38	-28	-16	0	24	
4		-44	-37	-28	-16	0	

If g equals the rate of increase in income, R equals last periods returns, r equals the time preference rate, and i equals inflation, then the value of land V\* = R(1 + g) - (i + r + ir - g). Dividing V\* by R/r after subtracting 1 produces the numbers in Table 1.

Source: Robison, Lindon J. "Income from Land and Land Values: Is
There a Connection"? Michigan Farm Economics, No. 439, June
1980.

Table 2

Percentage Change in Land Values, Cash Rents, and the Consumer Price Index, 1968-1980

Year	Annual Actual Change in Land Values (Percent)	Annual Change in Cash Rents (Percent)	Annual Change in the CPI (Percent)
1968	14.2	-9.9	4.2
1969	.3	3.6	5.4
1970	.3	-6.1	5.9
1971	1.0	12.3	4.3
1972	16.6	-1.8	
1973	7.8	14.7	3.3
1974	16.7	15.2	6.2
1975 .	3.0	8.7	11.0
1976	22.4	9.4	9.1
1977	34.6	20.3	5.8
1978	3.5		6.5
1979	7.8	1.3	7.7
1980	13.2	5.3	11.3
Average		16.0	13.1
Annual Change	10.2	6.5	7.2

Source: Espel, T., The Theoretical Basis for Estimating Land Value: A Market Equilibrium Approach, p. 85.

(11) 
$$V = \frac{R(1-t_p)}{[1+r(1-t_p)]} + \dots + \frac{R(1-t_p)}{[1+r(1-t_p)]^n} + \frac{V}{[1+r(1-t_p)]^n}$$

Since inflation is absent from the model and V does not inflate (deflate) we need not concern ourselves, for the moment, with capital gains taxes. Equation (11) can be easily solved if we let  $R(1-t_p)$  be  $\hat{R}$  and  $r(1-t_p)$  be  $\hat{r}$ . Expressed as a relationship between V,  $\hat{R}$ , and  $\hat{r}$ , equation (11) turns out to be nothing more than the BCB model which can be written as:

(12) 
$$V = \hat{R}/\hat{r}$$
  
=  $R(1-t_p)/r(1-t_p)$ 

which after cancelling produces again the BCB model results of equation (3). The implication of this model is that taxes have no impact on the maximum bid price for land. But a conclusion quite different from this results is obtained once inflation is introduced. Such a model has been deduced by Baker which we now explore.

### Taxes, Generalized Inflation, and the BCB model

Letting inflation be the generalized type described in equation (7) and continuing to let the personal income tax rate be reflected by tp we are prepared to deduce the analytic model. But before doing so, however, two simplifications in our solution procedures are required. First, if income is inflating the progressive personal income tax rate tp would also increase. With indexation, however, it remains constant. We incorporate the indexation feature into this model by letting tp be constant over the entire planning period.

The second simplification involves capital gains. We assume in this model that the land purchaser intends to hold the land for n periods at which time he expects to sell his land for an inflated price  $V_0(1+g)^n$  and pay a capital gains tax. The difference between the inflated price and the original purchase price  $V_0$  is the amount subject to a capital gains tax of  $.4t_p$ . To solve the model with this capital gains tax provision produces results of uninteresting complexity—a solution best reserved for the computer. We approximate the solution instead by adopting a logical alternative. The price received by the first owner depends on the income expected from the land by the second buyer and the sale price received by the second buyer depends on the income expected from the land by the third buyer etc. So our alternative to solving the model with capital gains and capital gains tax included is to solve it with an infinitely long income stream.  $\frac{4}{}$ 

Our infinitely long income stream from land can be written as:

(13) 
$$V_o = \frac{R(1+g)(1-t_p)}{1+r*(1-t_p)} + \dots + \frac{R(1+g)^n(1-t_p)}{[1+r*(1-t_p)]^n}$$

where r\* is i+r+ir as before and n is allowed to approach an infinitely large number. The geometric sum of the right hand side of (13) can be expressed as:

(14) 
$$V_o = \left[\frac{R(1-t_p)(1+g)}{r*(1-t_p)-g}\right] \left[1 - \frac{(1+g)^n}{(1+r*(1-t_p))^n}\right]$$

<sup>4/</sup> As Baker has demonstrated the solution isn't exact since capital gains tax is not included explicitly. The solution improves in accuracy the longer the first buyer holds the land before selling to buyer two.

<sup>5</sup>/ The model converges just in case g ( r\*(1-t<sub>p</sub>).

Then taking the limit of (14) by letting n become very large we obtain the results in the equation below:

(15) 
$$\lim_{n\to\infty} V_o = \frac{R(1-t_p)(1+g)}{r*(1-t_p)-g}$$

The rather surprising result from this model is that  $V_0$  is no longer invariant with respect to changes in the personal income tax rate  $\frac{6}{}$ . In fact, for increases in  $t_p$ ,  $V_0$  increases. The conclusion is that the higher the tax rate, the larger the maximum bid price other things being equal opportunity costs which are reduced by taxes more than offset the reduction in after tax income. In addition, inflation produces capital gains which are sheltered from taxes until the investment is sold—and then is taxed at a lower rate. The higher the tax rate, the more important is the tax shelter for investors. Moreover, they earn a return on the capital gains. It should also be apparent that the larger the inflation rate g, the larger will be the capital gains sheltered, distinguishing even more the differences in the maximum bid price resulting from income tax rate differences.

## Inflation, Taxes, and Risk

Our final extension of the BCB model is to include risk. Thus far we have assumed perfect knowledge about the future. Net income to land in each period has been assumed known with certainty. However, the value of the future net income which determines land prices is rarely known with certainty. Because the net return to land is a function of the price

$$\frac{d(\text{limit } V_o)}{dt_o} = \frac{R(1+g)g}{[r*(1-t_p)-g]^2} > 0$$

Unambiguous results are not obtained, however, when capital gains are included as Baker has shown, but for most reasonable values, the results above hold.

 $<sup>\</sup>underline{6}/$  This result is demonstrated by taking the derivitive of V with respect to t<sub>p</sub>. This can be expressed as:

of the agricultural output, the level of output, and the cost of agricultural input, all of which are uncertain.

The guiding principle, we believe, for including risk into the model is what we refer to as the "homogeneity of measurements" rule. That is, the income stream which is discounted and summed to obtain the maximum bid price for land must reflect the same kind of measure used to identify the land's acquisition prices. If the price of land is an after-tax price, then the income stream must be measured in after-tax discounted dollars. If the price V reflects a certain outlay of dollars, then the income stream must be converted to its certainty equivalent value. This requires no adjustment in the discount rate because r and i were both assumed certain. If the returns R, however, is what returns are expected to be on average, then a risk averse decision maker would be willing to pay some risk premium π to obtain R with certainty. The average or expected minus the risk premium  $\pi$ is called a certainty-equivalent income of R, CE(R). equivalent income then is the income which if received with certainty produces the same level of satisfaction as would the rights to the uncertain income R.

Pratt obtained an explicit measure of  $\pi$  as:

(16) 
$$\pi \approx \lambda \sigma^{2}/2$$

where  $\sigma^2$  is the variance of the returns and  $\lambda$  is one-half of the preferred trade-off between expected income and variance or risk.

<sup>7/</sup> A risk averse decision maker is defined as one whose marginal utility of income is diminishing, or one whose utility function is concave (see Pratt).

<sup>8</sup>/ To show this we first recognize that  $\pi$  is the difference between expected returns R and certainty equivalent income CE(R). We can replace  $\pi$  with R-CE(R) in (16) to obtain the expression:

CE(R) = R -  $\lambda\sigma^2/2$ Then taking the total derivitive with respect to R and  $\sigma^2$  while holding the certainty equivalent constant, we find the optimal trade-off between expected income and variance (risk)  $\sigma^2$ . The result is:  $dR/d\sigma^2 = \lambda/2$ 

To solve our analytic model we replace R, now considered the average income in (13) and solve.

(17) 
$$V_o = \frac{(\bar{R} - \lambda \sigma^2 / 2)(1+g)(1-t_p)}{1+r*(1-t_p)} + ... + \frac{(\bar{R} - \lambda \sigma^2 / 2)(1+g)^n(1-t_p)}{[1+r*(1-t_p)]^n}$$

The simplifying assumptions of (17) are employed, namely that the land is never sold. This assumption allows us to approximate land's value with an infinitely long income stream. We also assume that the inflation rates i and g are known with certainty. Equation (17), after replacing  $(R-\lambda\sigma^2/2)$  with its certainty equivalent CE(r), can be solved in the same manner as equation (13). The result is:

(18) 
$$V_o = \frac{CE(R)(1-t_p)(1+g)}{r*(1-t_p)-g}$$

## Financial Considerations in the BCB Model

We have now proceeded as far as our analytic methods allow us while still producing relatively simple models. But, so far financial considerations have not been included.

Land purchases are usually financed with borrowed money. A down payment from 10 to 50 percent of the purchase price is usually required with the remaining amount paid over a number of years. Financial arrangements, such as interest rates, down payments, and the length of the loan amortization period, should be considered in evaluating agricultural land values, along with the other factors which have been considered.

#### The Lee and Rask Model

To overcome the limitations of simple analytic models, Lee and Rask constructed a computer model to calculate the maximum bid price for land.

They included as this paper has, net income in the first period, the rate of increase in income, the discount rate, the length of the investment period and the income tax rate. In addition, their model considers the impact of financial arrangements on the maximum bid price which our models have not. They include such variables in their model as:

- (a) the proportion of the purchase price paid initially (the down payment);
- (b) the mortgage interest rate; and
- (c) the amortization period for the loan

A curious feature of the Lee and Rask model, however, is their treatment of capital gains. In our models, land's value cannot increase independent of it's increase in land's value. But in the Lee and Rask model it can. The user can specify any rate of increase in land values desired. But in order to bound capital gains, the initial land value or value of comparable tracts must be inputed by the user.

We caution the users of this program that land values cannot theoretically change at rates different from rates of expected changes in lands earnings.

While there are short run aberrations, the longer run has shown quite a direct relationship between land value and earnings from land. So unless there are overriding factors such as outside pressure for land, the rate of increase in income should equal the rate of in land's value.

#### Hand Held Computers

The Lee and Rask model, despite its many advantages is somewhat inaccessible. It is available on many large computer systems but many decision makers simply do not have access to them. So there is a need,

to increase its availability and reduce its application cost. Recent developments in computer technology have led to the development of the hand-held computer which has provided a powerful computing capacity that can solve problems that formerly could be solved only by large computers. The Lee and Rask model is an example. These programmable calculators are currently available at reasonable prices which seem to be decreasing as technology advances promising to reduce the application costs.

The hand-held programmable calculator, like any computer, can carry out the following:

- (1) Read in both data and instructions.
- (2) Store the data and instructions in a memory.
- (3) Perform calculations in manner prescribed by the instructions.
- (4) Read out the results.
- (5) Control all aspects involved in getting an answer.

The advantages of these hand-held programmable calculators to a large number of decision makers and professionals are clear-cut. Its use helps speed up business decisions and eliminates manual calculations.

Many of the principles of programming are common to large computers and programmable calculators of all manufacturers. However, each manufacturer's equipment requires the user to follow some specific rules and conventions that are unique to that particular line. Since the Texas Instruments-59 programmable calculator was used to solve for land values in this paper, some of its features will be discussed briefly.

The TI-59 is one of the recent programmable calculators made by

Texas Instruments and capable of handling problems that formerly could be
solved only by large computers. The most striking feature of the TI-59
is the use of removable solid-state modules for the storage and execution
of library programs.

Program steps are entered into the memory of the calculator by pressing keys on the keyboard. The program will be stored in the memory and can be used repeatedly with different data. If a given program is to be used only once, it can be erased from the program memory when the power is turned off. However, if needed again, the same program can be saved by recording it on a magnetic card. Then, when it is needed, the card containing the program can be read into the calculator memory and the program reused.

The program to determine the maximum bid price and cash flow using the Lee and Rask model was programmed by Kooti for the TI-59 and is listed in Appendix B. This program estimates the maximum bid price for land, annual loan payment, unpaid balance remaining on loan in any year, net cash flow in any period, market value of the land and equity, given the variables listed earlier under the Lee and Rask model. The input form for the model is listed in appendix A.

To test the sensitivity of the program, a sample problem was first solved with input data equal to:

- Income growth rate g of 8 percent.
- (2) Before-tax opportunity cost of capital, i + r + ir or r\* of 11 percent.
- (3) Certainty equivalent income R equal to \$50/acre.
- (4) Marginal tax rate of 25 percent
- (5) Expected rate of inflation on land values of 10 percent.
- (6) The market value of comparable land is \$1,000/acre.
- (7) The capital gain income tax rate of 10 percent (40 percent of 25 percent).
- (8) Down payment of 25 percent.
- (9) Interest rate on mortgage loan of 10 percent per annum.
- (10) Planning horizon, 20 years.
- (11) Amortization period on the loan, 20 years.

The above values are stored in accordance with the input format given in appendix A. The resulting maximum bid price by pressing key A equals \$2,105. The solution for the base case is the point of departure to examine the sensitivity of the maximum bid price to changes in the input variables.

The sensitivity to the maximum bid price was tested by altering the input variables one at a time from the base solution. Each variable was examined over a range. In every case the values for all variables, other than the one being tested, were fixed as specified in the original case.

The results of the sensitivity analysis from the base case are summarized below:

- (1) An increase in the mortgage loan interest rate from 10 to 14 percent reduces the maximum bid price for land from \$2,105 to \$1,772.
- (2) Increasing the percent of loan paid as a down payment from 25 percent to 50 percent decreases the maximum bid price for land from \$2,105 to \$2,073.
- (3) An increase in the before-tax opportunity cost of capital from 11 percent to 15 percent reduces the maximum bid price from \$2,105 to \$1,577.
- (4) An increase in average price of comparable tract of land from \$1,000 to \$1,500 increases the maximum bid price from \$2,105 to \$2,767.
- (5) Increase in the expected rate of inflation from 10 percent to 15 percent increases the maximum bid price from \$2,105 to \$4,001.
- (6) If the expected net income to land increases from \$50 to \$100, the maximum bid price increases from \$2,105 to \$2,886.
- (7) Income growth rate of 10 percent instead of 8 percent increases the maximum bid price from \$2,105 to \$2,275.
- (8) An increase in the income tax rate from 25 to 50 percent and capital gain tax rate from 10 to 20 percent increases the maximum bid price from \$2,105 to \$2,773. This result occurs because reduction in the expected annual net income per acre, due to income taxes, is more than offset by the tax deductible interest payments and the decrease in after-tax opportunity cost of capital.
- (9) An increase in loan amortization and planning horizon from 20 to 30 years increases the maximum bid price from \$2,105 to \$2,712.

#### The Cash Flow Statement

The program is not only capable of determining the maximum bid price but can determine the annual loan payment, the unpaid balance remaining on the loan in any one year, the net cash flow in each period, the market price of land in each period and the equity. The procedures used to produce the cash flow statement are listed in appendix A.

## How To Calculate Income

Central to our efforts to calculate a maximum bid price for land was the determination of the net return attributable to land. We conclude this paper by discussing two methods for calculating the returns attributable to land. These two approaches are: (1) the landlord method, and (2) the residual method.

The landlord method involves an estimation of the income stream (R) to farmland based on the net rental payments received by the landlord for the use of his farmland. Where land is cash rented and the rental fee is known, as well as the costs associated with land ownership (such as taxes), the net income stream to the landlord is also the return on land and is relatively certain.

The residual approach is best illustrated with an example. Consider Table 2 which illustrates net income for a typical corn grain farm which yields an average of 85 bushels an acre. The income from the land is the income earned from the sale of the corn grain or its equivalent value if the grain is used on the farm. From this gross income, we subtract all the operating expenses associated with growing the corn, including seeds, fertilizer, fuel for machines, labor, interest charged on short-term debt, herbicides, insecticides, and taxes. The difference between the gross income and farm operating expenses equals net income—the income expected from the land purchase. This income may then be adjusted to its certainty equivalence by the decision maker.

TABLE 2: Enterprise Budget for One Acre of Medium-Yield Corn Grain

GROSS INCOME (100 bu. X \$2.00 )		\$200.00
EXPENSES:		
Labor (6.1 hrs. X \$5.00)	\$ 30.50	
Repairs and Maintenance	9.80	
Seeds	11.33	
Fertilizer	38.25	
Insecticides and Herbicides	12.40	
Fuel	6.00	
Utilities	2.30	
Harvesting, Trucking	6.20	
Corn Drying	14.00	
Other Expenses (including interest on operating debt)	\$ 7.53	Y
	\$138.31	
NET INCOME (Gross Income - Expen	ses)	\$ 61.69

Source: Robison, Lindon J. and John R. Brake.

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## Summary and Conclusions

Calculating the maximum bid price one can offer for land is important for applied decision makers. Making such a calculation may be complicated because of the many factors affecting the returns attributable to land.

These may include inflation, taxes, uncertainty, and financial arrangements.

In this paper, present value models of increasing complexity were introduced to demonstrate how maximum bid prices are calculated. Finally, hand held programable computers were introduced to solve the model which included all the considerations discussed in this paper. The program and an input and output format were described in the paper and listed in Appendices A and B. Those wishing to check their models were provided a solved example.

What the paper has provided, then, is a practical aid for those who make land investment decisions. But it is important to understand that it is only an aid. A successful decision maker will continue to find no substitute for good judgment.

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## Appendix A

Instructions for Using the Lee and Rask Program on the TI-59 Appendix A. Instructions for using the Lee and Rask Program on the TI-59.

Objective: To determine: (1) The maximum amount one can afford to pay for one acre of land, (2) Annual loan payment, (3) Unpaid balance remaining on loan at year j; (4) Netcash flow at period j, (5) Market price of land at period j, (6) Equity at year j; j=1 . . . m, where m is the amortization period of the loan.

STI	EP INPUT DESCRIPTION INPUT VALUE	PRESS
1.	Turn calculator off, and back on, to clear program and memory.	
2.	Partition memory (Note 639.39 should appear on the screen. If not, return to step 1.)	(4) (2nd) (op) (17)
3.	Clear Display	(CLR)
4.	Insert side 1 of the card containing the program (A:1). If the calculator has read the card successfully, a "1" will appear and remain stationary. If a flashing "0" appears, repeat step 3 and 4.	
5.	Clear Display	(CLR)
6.	Insert side 2 of the card. If the calculator reads side	

2 successfully, a "2" will

# Appendix A. (continued)

STEP	INPUT DESCRIPTION INPUT VALUE	PRESS
	appear and remain stationary.  If a "0" appear, repeat steps 5 and 6.	
7.	Clear Display	(CLR)
8.	Insert side 3 of the card containing the program.  If the calculator has read the card successfully, a "3" will appear and remain stationary. If a "0" appears, repeat steps 7 and 8.	
9.	Clear Display	(CLR)
10.	Insert side 4 of cards contin- ing the program. If the calcu- lator has read the card success- fully, a "4" should appear and remain stationary. If "0" flashes on the display after the card : has been read, steps 9 and 10 should be repeated.	
11.	Clear Display	(CLR)
12.	Growth rate of annual net income to land, % annum	(STO) 10
13.	Before tax opporunity cost of capital, % annum	(STO) 11
14.	Annual Net Income to land; \$ per acre.	(STO) 12
15.	Marginal tax rate on annual income, %.	(STO) 13
16.	Expected rate of inflation	(STO) 14

Appendix A. (continued)

STEP	INPUT DESCRIPTION	INPUT VA	LUE	PRESS
17.	Price of comarable tract, \$ per acre.	-	(:	STO) 15
18.	Capital gain tax rate, %		(:	STO) 16
19.	Down payment, %		(:	STO) 17
20.	Interest rate, % annum.		(:	STO) 18
21.	Planning horizon, years.		(:	STO) 19
22.	Amortization period, years.		(2	STO) 20
	OUTPUT			
STEP	OUTPUT DESCRIPTION	PRESS	VALUE	RESULTS
1.	The maximum bid price \$/ac.	A		
	INPUT DESCRIPTION			
2.	Enter the price \$/acre that will be used in the cash flow analysis.  OUTPUT DESCRIPTION	(STO) 21		
3.	Annual loan payment (principal and interest).	В		
	CASH FLOW ANALYSIS			3

Note: To prepare an annual cash flow chart, enter the year you want to examine in (STO) 22. Then press (C) to get the unpaid balance at the end of that year. Press (D) and you will see the taxable income. Press (E) for income tax paid and (2nd) A for the net cash flow that year. Press (2nd) B for the inflated investment (market price) and press (2nd)cfor the equity (cost less principal paid plus inflation) use the chart as shown in the next page to record your data.

Appendix A. (continued)

# CASH FLOW CHART

Year	Unpaid Balance	Taxable Income.	Income Tax.	Net Cash Flow	Market Price	Equity
		4				P.

## Appendix B

A Listing of the Lee and Rask Maximum Bid Price and Cash Flow Program

Appendix B. A Listing of the Lee and Rask Maximum Bid Price and Cash Flow Program

		rogram						
1.	is Line	No.	2. is	Key	Code	3.	is Key	
1	.2 ,3		1 2	3		1.	2 3	
000 001 002 003 004 005 006 007 009 011 013 014 015 016 017 018 021 022 023 024 025 027 028 029 030 031 032 033 035 036 037 038	91 R LBI 76 LBI 11 R CL 20 42 STU 42 STU 42 STU 43 R CL 43 R CL 43 STU 43 STU 43 R CL 43 STU 43 R CL 43 R CL 45 R CL 4	04 04 04 04 04 04 04 04 04 04 05 05 05 05 05 06 06 06 06 06 06 06	0123456789012345678901234567890123456 0123456789012345678901234567890123456	RCL9		107 108 109 110 111 112 113 114 115	19 + ( ( 1 + L1)	

Appendix B. (continued)

1 117 118 119 120 121 122 123 124 125 126 127 128 129 130 131 133 134 135 137 138 139 140 141 142 143 144 145 146 147 148 149 151 152 153 154 155 155 156 157 157 157 157 157 157 157 157 157 157	701455334553315331545 0653315331545	3 (1 - L17) x < < < (1 + L1) x CL0 RC17	1 157 157 158 159 161 163 164 165 167 168 169 177 177 178 179 179 179 179 179 179 179 179 179 179	2 4444533338533153348453331533315333451444	3 ))) X ( (L8 X ( (1 + L8 ) X L0 ) + ( ( (1 + L8 ) X L0 ) + ( ( (1 + L8 ) X L0 ) + ( ( ( (1 + L8 ) X L0 ) + ( ( ( ( ( ( ( ( ( ( ( ( ( ( ( ( ( (	1 195678990123456789901232222222222222222222222222222222222	2 524315374533533533353153345304533331533	3 = 00 (1 - LL7 ) x LL3 x CL8 x (1 + LL8 ) x CL2 ) + ( (1 + LL8 ) x CL2 ) + ( (1 + LL8 ) x CL2 ) + ( (1 + LL8 ) x CL2 ) + ( (1 + LL8 ) x CL2 ) + ( (1 + LL8 ) x CL2 ) + ( (1 + LL8 ) x CL2 ) + ( (1 + LL8 ) x CL2 ) + ( (1 + LL8 ) x CL2 ) + ( (1 + LL8 ) x CL2 ) + ( (1 + LL8 ) x CL2 ) + ( (1 + LL8 ) x CL2 ) + ( (1 + LL8 ) x CL2 ) + ( (1 + LL8 ) x CL2 ) + ( (1 + LL8 ) x CL2 ) + ( (1 + LL8 ) x CL2 ) + ( (1 + LL8 ) x CL2 ) + ( (1 + LL8 ) x CL2 ) + ( (1 + LL8 ) x CL2 ) + ( (1 + LL8 ) x CL2 ) + ( (1 + LL8 ) x CL2 ) + ( (1 + LL8 ) x CL2 ) + ( (1 + LL8 ) x CL2 ) + ( (1 + LL8 ) x CL2 ) + ( (1 + LL8 ) x CL2 ) + ( (1 + LL8 ) x CL2 ) + ( (1 + LL8 ) x CL2 ) + ( (1 + LL8 ) x CL2 ) + ( (1 + LL8 ) x CL2 ) + ( (1 + LL8 ) x CL2 ) + ( (1 + LL8 ) x CL2 ) + ( (1 + LL8 ) x CL2 ) + ( (1 + LL8 ) x CL2 ) + ( (1 + LL8 ) x CL2 ) + ( (1 + LL8 ) x CL2 ) + ( (1 + LL8 ) x CL2 ) + ( (1 + LL8 ) x CL2 ) + ( (1 + LL8 ) x CL2 ) + ( (1 + LL8 ) x CL2 ) + ( (1 + LL8 ) x CL2 ) + ( (1 + LL8 ) x CL2 ) + ( (1 + LL8 ) x CL2 ) + ( (1 + LL8 ) x CL2 ) + ( (1 + LL8 ) x CL2 ) + ( (1 + LL8 ) x CL2 ) + ( (1 + LL8 ) x CL2 ) + ( (1 + LL8 ) x CL2 ) + ( (1 + LL8 ) x CL2 ) + ( (1 + LL8 ) x CL2 ) + ( (1 + LL8 ) x CL2 ) + ( (1 + LL8 ) x CL2 ) + ( (1 + LL8 ) x CL2 ) + ( (1 + LL8 ) x CL2 ) + ( (1 + LL8 ) x CL2 ) + ( (1 + LL8 ) x CL2 ) + ( (1 + LL8 ) x CL2 ) + ( (1 + LL8 ) x CL2 ) + ( (1 + LL8 ) x CL2 ) + ( (1 + LL8 ) x CL2 ) + ( (1 + LL8 ) x CL2 ) + ( (1 + LL8 ) x CL2 ) + ( (1 + LL8 ) x CL2 ) + ( (1 + LL8 ) x CL2 ) + ( (1 + LL8 ) x CL2 ) + ( (1 + LL8 ) x CL2 ) + ( (1 + LL8 ) x CL2 ) + ( (1 + LL8 ) x CL2 ) + ( (1 + LL8 ) x CL2 ) + ( (1 + LL8 ) x CL2 ) + ( (1 + LL8 ) x CL2 ) + ( (1 + LL8 ) x CL2 ) + ( (1 + LL8 ) x CL2 ) + ( (1 + LL8 ) x CL2 ) + ( (1 + LL8 ) x CL2 ) + ( (1 + LL8 ) x CL2 ) + ( (1 + LL8 ) x CL2 ) + ( (1 + LL8 ) x CL2 ) + ( (1 + LL8 ) x CL2 ) + ( (1 + LL8 ) x CL2 ) + ( (1 + LL8 ) x CL2 ) + ( (1 + LL8 ) x CL2 ) + ( (1 + LL8 ) x CL2 ) + ( (1 + LL8 ) x CL2 ) + ( (1 + LL8 ) x CL2 ) + ( (1 + LL8 ) x CL2 ) + ( (1 + LL8 ) x CL2 ) + ( (1 + LL8 ) x CL2 ) + ( (1 + LL8 ) x	

Appendix B. (continued)

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Appendix B. (continued)

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Appendix B. (continued)

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Appendix B. (continued)

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