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A METHOD FOR TRANSFORMING TEMPERATURE DISTRIBUTIONS TO NORMALITY

Morris S. Webb, Jr.
Weather Service Forecast Office
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UNITED STATES DEPARTMENT OF COMMERCE
Juanita M. Kreps, Secretary

NATIONAL OCEANIC AND ATMOSPHERIC ADMINISTRATION
Robert M. White, Administrator

NATIONAL WEATHER SERVICE
George P. Cressman, Director

CONTENTS

FIGURES

TABLES

LIST OF SYMBOLS

Symbols Used In Text	Equivalent Symbols Used In Appendix	
A	A	A term equal to $\ln(\bar{T}') - \frac{\Sigma \ln(Ti')}{N}$.
$\hat{a}$	$\hat{a}$	A term in the gamma distribution equal to $\frac{1 + \sqrt{1 + (4/3)A}}{4A}$.
$\hat{B}$	$\hat{B}$	A term in the gamma distribution equal to $\frac{\bar{T}'}{\hat{a}}$.
N	N	Number of elements in the distribution.
P(T)	P(x)	The cumulative frequency of T.
Q(T)	Q(x)	A term equal to 1 - P(T).
S	S	Standard deviation.
T	x	Temperature.
$\bar{T}$	$\bar{x}$	Mean temperature.
T'	x'	The absolute value of $T - T_{BASE}$.
T_{BASE}	x_{BASE}	Base temperature.
T_i	x_i	A sample temperature of the distribution where $i = 1,2,\cdots,N$.
T_i'	x_i'	The absolute value of $T_i - T_{BASE}$.
$\bar{T}'$	$\bar{x}'$	The mean of the T' distribution.
T_M	x_{MEDIAN}	Median temperature.
T_{MODE}	--	Mode.
T_X	$x_{MAX/MIN}$	The distribution's highest or lowest temperature, depending on the sign of the skew.
$Z_{N/(N+1)}$	$Z_{N/(N+1)}$	A point where the area under the standard normal curve from $-\infty$ to z is $\frac{N}{N+1} \cdot 100$ percent of the curve's total area.
3.891	$Z_{0.99995}$	The value of $Z_{0.99995}$
$\Gamma(\hat{a})$	$\Gamma(\hat{a})$	The gamma function of a.
$\gamma(\hat{a},T'/\hat{B})$	$\gamma(\hat{a},x'/\hat{B})$	The incomplete gamma function of $(\hat{a},T'/\hat{B})$.

A METHOD FOR TRANSFORMING TEMPERATURE DISTRIBUTIONS TO NORMALITY

Morris S. Webb, Jr.
Weather Service Forecast Office
National Weather Service Western Region, NOAA
San Francisco, California

ABSTRACT. A method of transforming and fitting temperature distributions to the normal curve using the gamma distribution is presented. At the same time, the worth of the programmable calculator is once again demonstrated. The concept of "base temperature" is introduced in the presentation: a concept which permits the gamma distribution to be used with non-positive as well as positive values. An actual temperature distribution is transformed to normality using steps contained in a program designed to normalize meteorological variates. The resultant normal curve is then used to determine cumulative temperature probabilities.

A METHOD FOR TRANSFORMING TEMPERATURE DISTRIBUTIONS TO NORMALITY

Most meteorological variates are not normally distributed. This is true in the case of temperature, whose distributions are often skewed.

It is desirable to transform and fit skewed distributions, such as temperature, to the normal curve. This is because many useful statistical tests such as analysis of variance, hypothesis testing about the mean, etc., are valid only for normal distributions. In addition, the temperature distribution once transformed can be represented as a smooth curve on probability graph paper, thus allowing easy determination of temperature frequency.

How can temperature distributions be transformed to normality? One way is to use the gamma distribution to fit and transform the temperature distribution (Panofsky and Brier 1958). Unfortunately, the transforming process is usually tedious, involves complicated equations, and hard-to-read tables.

Enter the programmable calculator! These electronic marvels make the process of "normalizing" temperatures virtually painless, especially if the calculator is designed to accept magnetic program cards.

However, the gamma distribution can be used only if the distribution to be transformed consists of positive numbers. Obviously, temperatures expressed in (^{o}F) or (^{o}C) can assume values of zero or below.

One way of solving this problem is to select a "base temperature" which, when added to each temperature in the distribution, will make all the temperatures positive. The base temperature should be a few degrees lower than the distribution's lowest temperature if the distribution has positive skew, or a few degrees higher than the distribution's highest temperature if the distribution has negative skew 1/.

The author's formula for objectively determining the base temperature is as follows:

$$T_{BASE} = \sqrt{\frac{3.891}{Z\left[\frac{N}{N+1}\right]}} \cdot (T_X - T_M) + T_M$$

Values of $Z[N/(N+1)]$ can be obtained from Figure 1.

The program to normalize meteorological distributions consists of 435 steps 2/. It is designed to run on a Texas Instruments SR-52 calculator. The program flowchart is shown in Figure 2.

For purposes of illustration, the author will show how January minimum temperatures at Laramie, Wyoming, are transformed to normality using steps outlined in the program 3/. Laramie's winter temperature distributions show large skew and wide extremes--in other words, a good test for the program.

The sample temperatures are selected and the resultant distribution checked for the sign of its skew. In this case, a random sample of 100 elements is selected from a population of 620 January minimum temperatures at Laramie. The sample temperatures are then arrayed, as in Table 1. Since $T_M = 12^{o}$F, $T_{MODE} = 14^{o}$F, and $T_X = 33^{o}$F (maximum T_i) and -46^{o}F (minimum T_i), the sample distribution is assumed to have negative skew.

T_{BASE} needs to be determined, because the distribution has negative skew and contains temperatures $\leq 0^{o}$F. Since N = 100, N/(N+1) = 100/101 = 0.9900990099. Using Figure 1, $Z_{100/101}$ is found to be equal to

1/ $\Sigma|T_i - T_{BASE}|$ should be kept as small as possible. Otherwise $\Gamma(\hat{a})$, a term influenced by the size of $\Sigma|T_i - T_{BASE}|$, will become too large for the calculator to handle.

2/ See appendix.

3/ The author had previously normalized Laramie's temperatures, while stationed at WSFO Cheyenne.

2.33. Substituting 12°F for T_M, 33°F for T_X, and 2.33 for $Z_{N/(N+1)}$ yields a T_{BASE} of 39.13763204°F. Pressing the keys FIX , 0 , and RUN truncates the fractional part of the number and places the integer 39 in the data register reserved for T_{BASE}.

The next steps involve the computation of sums which are used to solve later equations. Each T_i is entered into the program by successive keystrokes, with sums for T_i, T_i^2, $T_i' = |T_i - T_{BASE}|$, $\ln(T_i')$, and i accumulated in appropriate data registers. After all 100 temperatures are entered, the following sums are obtained:

$$\sum T_i = 997$$

$$\sum T_i^2 = 28,953$$

$$\sum T_i' = 2,903$$

$$\sum \ln(T_i') = 325.5576843\cdot\cdot \ ^{4}$$

The above sums are used to solve equations for the distribution's mean, standard deviation and skew, and the terms A, â, and $\hat{B}$ which are used to compute $\Gamma(\hat{a})$ and values of $\gamma(\hat{a},T'/\hat{B})$. Substituting the appropriate data register contents into the equations yield the following:

$$\bar{T} = \Sigma T_i/N = 997/100 = \underline{\underline{9.97^{o}F}}.$$

$$S = \sqrt{(\Sigma T_i^2 - N\bar{T}^2)/(N-1)} = \sqrt{(28,953 - 100(9.97)^2)/(100-1)}$$

$$= \underline{\underline{13.86^{o}F}}$$

$$SKEW = 3(\bar{T} - T_M)/S = 3(9.97 - 12)/13.86 = \underline{\underline{-0.439}}$$

$$A = \ln(\Sigma T_i'/N) - \Sigma \ln(T_i')/N = \ln(2,903/100) - 325.5576843\cdot\cdot/100$$

$$= \underline{\underline{0.1127529346\cdot\cdot}}$$

$$\hat{a} = (1 + \sqrt{1 + (4/3)A})/4A = \frac{(1 + \sqrt{1 + (4/3)\cdot(0.1127529346\cdot\cdot)})}{4(0.1127529346\cdot\cdot)}$$

$$= \underline{\underline{4.595307802\cdot\cdot}}$$

$$\hat{B} = (\Sigma T_i'/N)/\hat{a} = (2,903/100)/4.595307802\cdot\cdot = \underline{\underline{6.317313496\cdot\cdot}}$$

$$\Gamma(\hat{a}) = \int_0^\infty t^{\hat{a}-1} e^{-t} dt$$

$$\cong \frac{\sqrt{\frac{2\pi}{\hat{a}+5}}\cdot(\hat{a}+5)^{(\hat{a}+5)} e^{-\left[(\hat{a}+5) - \frac{1}{12(\hat{a}+5)} + \frac{1}{360(\hat{a}+5)^3}\right]}}{(\hat{a}+4)\cdot(\hat{a}+3)\cdot(\hat{a}+2)\cdot(\hat{a}+1)\cdot\hat{a}}$$

$$= \underline{\underline{13.29286832\cdot\cdot}}, \text{ with error}/\Gamma(\hat{a}) < 2(10^{-7})$$

$$\text{Thus, error} < \underline{\underline{2.7(10^{-6})}}$$

4/ The two dots preceding the superscript indicate the number is accurate to 12 significant digits in the data register. However, the display register in the SR-52 is capable of showing only 10 of these digits.

The solutions for $\hat{a}$, $\hat{B}$, and $\Gamma(\hat{a})$ are stored in data registers, so they can be recalled for subsequent computations.

By entering a temperature T into the program at this point, a value of the incomplete gamma function corresponding to T' can be obtained. The formula for the incomplete gamma function is:

$$\gamma(\hat{a},T'/\hat{B}) = \int_0^{T'/\hat{B}} t^{\hat{a}-1} e^{-t} dt$$
$$= (T'/B)^{a} \cdot e^{-(T'/B)} \sum_{n=0}^{\infty} \frac{(T'/\hat{B})^n}{\hat{a}(\hat{a}+1)\cdots(\hat{a}+n)} .$$

An iterative technique is used to solve the above formula. The iterative process continues until the series portion of the formula achieves a predetermined accuracy--in this case, accuracy to seven significant figures. The series is then multiplied by the terms preceding it, yielding a value of the incomplete gamma function.

Dividing $\gamma(\hat{a},T'/\hat{B})$ by $\Gamma(a)$ yields the cumulative frequencies P(T) and Q(T) = [1 - P(T)] for a temperature T. Values of $\gamma(\hat{a},T'/\hat{B})$, P(T), and Q(T) for selected January minimum temperatures at Laramie are listed in Table 2.

Information concerning temperature which was previously unknown or hard to determine can be easily interpolated from fitted gamma distributions such as the one in Figure 3. The January minimum temperature at Laramie should be $\geq$28°F in only about 5 percent of the observations; 21 percent of the observed minimums should be $\leq$0°F; 80 percent of the observations should lie between -8°F and 25°F, with 36 percent of the observations being between 16°F and 32°F. In other words, the gamma curve makes it easy to express temperature data in probabilistic terms.

Thus, temperatures as well as other meteorological distributions can be transformed to normality by using the gamma distribution as a transforming agent. The process can be hastened considerably and actually made easy by letting a programmable calculator do the dirty work. Hopefully, this presentation will prove useful to those who wish to try the method out at their own station, or on other meteorological variates.

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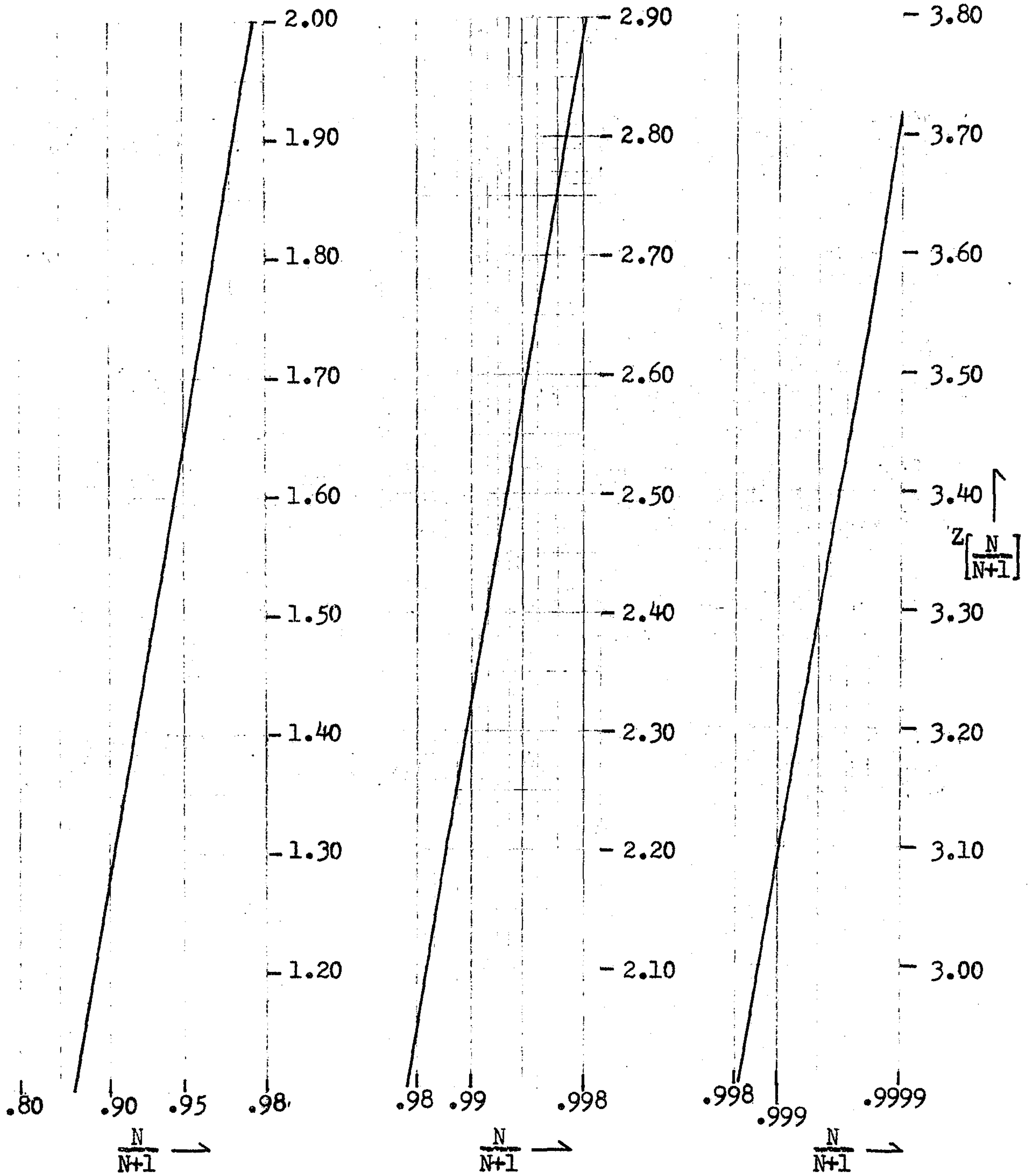

Figure 1. Graph Depicting N/(N+1) vs. $Z_{N/(N+1)}$.

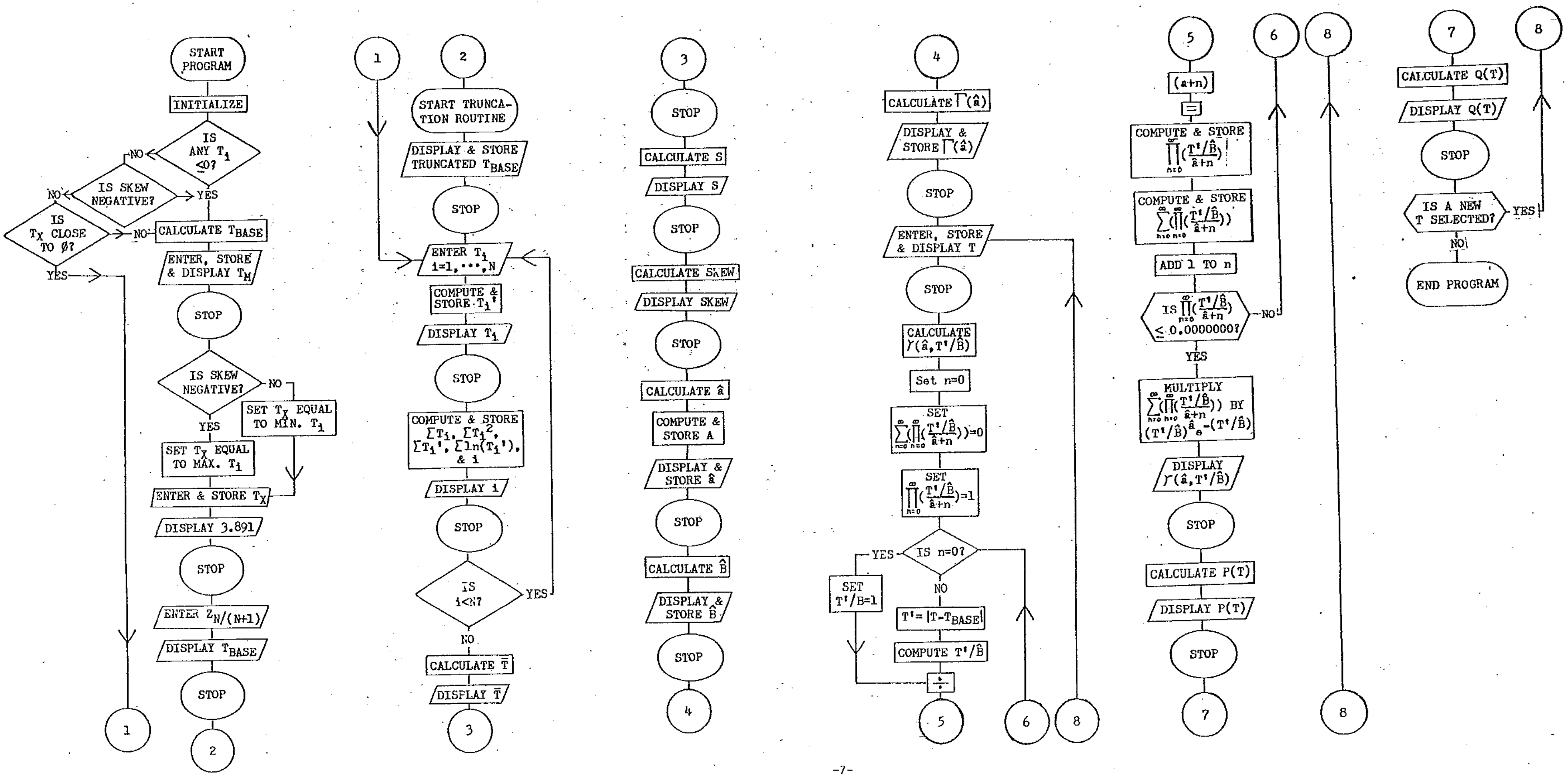

Figure 2. Flowchart of the Program Which Transforms Temperature Distributions to Normality

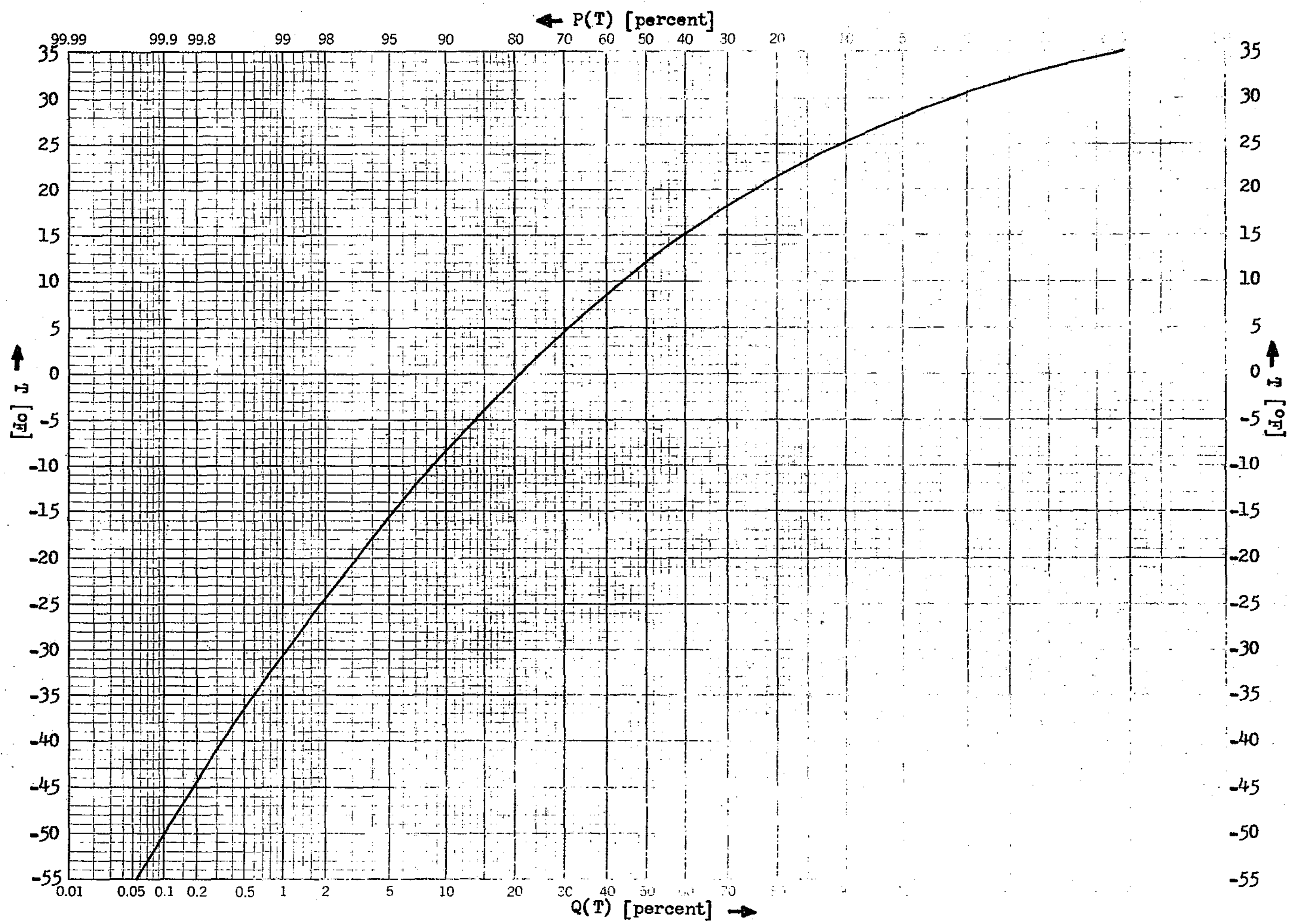

Figure 3. Graph Depicting the January Minimum Temperature at Laramie, Wyoming, (T) vs. the Cumulative Frequency of Temperature [P(T) and Q(T)].

TABLE 1

SAMPLE OF 100 JANUARY MINIMUM TEMPERATURES AT LARAMIE, WYOMING
USED TO TEST THE PROGRAM
(TEMPERATURES ARRANGED FROM HIGHEST TO LOWEST VALUE)

i	T_i(°F)	i	T_i(°F)	i	T_i(°F)
1	33	41	14	81	1
2	33	42	14	82	0
3	31	43	14	83	0
4	30	44	14	84	0
5	29	45	14	85	-1
6	27	46	14	86	-1
7	27	47	14	87	-2
8	26	48	13	88	-4
9	25	49	13	89	-5
10	25	50	12	90	-7
11	25	51	12	91	-8
12	24	52	11	92	-13
13	24	53	11	93	-13
14	23	54	10	94	-15
15	23	55	10	95	-16
16	23	56	10	96	-17
17	22	57	10	97	-18
18	22	58	9	98	-22
19	22	59	9	99	-33
20	22	60	9	100	-46
21	21	61	9		
22	21	62	8		
23	20	63	8		
24	20	64	8		
25	20	65	7		
26	19	66	7		
27	19	67	6		
28	18	68	6		
29	18	69	5		
30	18	70	5		
31	18	71	5		
32	18	72	5		
33	17	73	5		
34	17	74	4		
35	17	75	3		
36	17	76	3		
37	17	77	2		
38	16	78	2		
39	16	79	2		
40	15	80	2		

TABLE 2

VALUES OF $\gamma(a, T'/B)$, P(T), AND Q(T) FOR SELECTED JANUARY MINIMUM TEMPERATURES AT LARAMIE, WYOMING

T(°F)	$\gamma(\hat{a}, T'/\hat{B})$	P(T)	Q(T)
37	0.00085	0.00006	0.99994
35	0.01592	0.00120	0.99880
30	0.35198	0.02648	0.97352
25	1.45297	0.10930	0.89070
20	3.26784	0.24583	0.75417
15	5.40220	0.40640	0.59360
12	6.66542	0.50143	0.49857
10	7.46000	0.56120	0.43880
5	9.20132	0.69220	0.30780
0	10.54511	0.79329	0.20671
-5	11.51347	0.86614	0.13386
-10	12.17533	0.91593	0.08407
-15	12.60905	0.94856	0.05144
-20	12.88370	0.96922	0.03078
-25	13.05274	0.98194	0.01806
-30	13.15430	0.98958	0.01042
-35	13.21409	0.99407	0.00593
-40	13.24867	0.99668	0.00332
-45	13.26836	0.99816	0.00184
-50	13.27942	0.99899	0.00101
-55	13.28556	0.99956	0.00055
-60	13.28893	0.99970	0.00030

APPENDIX

ACTUAL SR-52 PROGRAM

USED TO NORMALIZE METEOROLOGICAL VARIATES

(X replaces T in most of the symbology, since the program is used to normalize meteorological variates other than temperature).

SR-52 User Instructions

TITLE: MEAN, STANDARD DEVIATION, SKEW, $\hat{a}$, AND $\hat{B}$ PAGE 1 OF 3

◄A◄				
X_{BASE}	$\bar{X}$	S	SKEW	INIT.
X_i	i	$\hat{a}$	$\hat{B}$	MEDIAN

◄B◄				

STEP	PROCEDURE	ENTER	PRESS			DISPLAY
1	LOAD PROGRAM CARD					
2	CLEAR REGISTERS		*E′			Ø
3	ENTER MEDIAN	X_{MEDIAN}	E			X_{MEDIAN}
4	CALCULATE X_{BASE}	$X_{MAX/MIN}$	*A′			3.891
	$=\sqrt{\frac{Z_{0.99995}}{Z_{[N/(N+1)]}}}\cdot[X_{MAX/MIN}-X_{MEDIAN}]$	$Z_{[N/(N+1)]}$		RUN		X_{BASE}
	$+X_{MEDIAN}$	*FIX m, m=Ø,…,9		RUN		X_{BASE} (TRUNCATED TO m DECIMAL PLACES)
	NOTE: USE STEPS 3 AND 4 WHEN ANY OF THE X_iS ARE ≤ Ø, AND/OR WHEN THE DISTRIBUTION HAS A NEGATIVE SKEW.					
5	INDEPENDENT VARIABLE X_i	X_i	A			X_i
				B		i, i=1,…,N
	REPEAT STEP 5 FOR ALL X_iS IN THE DISTRIBUTION					
6	COMPUTE THE MEAN		*B′			$\bar{X}$
7	COMPUTE THE STANDARD DEVIATION		*C′			S
8	COMPUTE THE SKEW		*D′			SKEW
9	COMPUTE $\hat{a}$		C			$\hat{a}$
1Ø	COMPUTE $\hat{B}$		D			$\hat{B}$
	$\bar{X}=\sum_{i=1}^{N}X_i/N$					
	$S=\sqrt{\frac{1}{N-1}\left(\sum_{i=1}^{N}X_i^2-N\bar{X}^2\right)}$					
	$SKEW=\frac{3(X-X_{MEDIAN})}{S}$					
	$\hat{a}=[1+\sqrt{1+4A/3}]/4A$ WHERE $A=\ln\bar{X}'-\frac{1}{N}\sum_{i=1}^{N}\ln X_i'$, $X_i'=\lvert X_i-X_{BASE}\rvert$					
	$\hat{B}=\bar{X}'/\hat{a}$ WHERE $\bar{X}'=\left[\sum_{i=1}^{N}\lvert X_i-X_{BASE}\rvert\right]/N$					

SR-52 Coding Form

TITLE: $\bar{X}$, S, SKEW, $\hat{a}$, AND $\hat{B}$ PAGE 2 OF 3
PROGRAMMER: MORRIS S. WEBB, JR. DATE 4/16/77

LOC	CODE	KEY	COMMENTS
000	46	*LBL	
	17	*B′	
	53	(	
	43	RCL	
	ØØ	Ø	COMPUTE
005	Ø6	6	$\bar{X}$
	55	÷	
	43	RCL	
	Ø1	1	
	ØØ	Ø	
010	54	)	
	56	*RTN	
	46	*LBL	
	18	*C′	
	53	(	
015	53	(	
	53	(	
	43	RCL	
	ØØ	0	
	Ø7	7	
020	75	—	
	43	RCL	
	Ø1	1	
	ØØ	Ø	
	65	X	
025	17	*B′	COMPUTE
	4Ø	$*X^2$	S
	54	)	
	55	÷	
	53	(	
030	43	RCL	
	Ø1	1	
	ØØ	Ø	
	75	—	
	Ø1	1	
035	54	)	
	54	)	
	3Ø	*√	

LOC	CODE	KEY	COMMENTS
	54	)	
	56	*RTN	
040	46	*LBL	
	11	A	
	53	(	
	42	STO	
	ØØ	Ø	
045	Ø4	4	
	75	—	
	43	RCL	COMPUTE
	ØØ	Ø	X_i'
	Ø3	3	$=\lvert X_i-X_{BASE}\rvert$
050	54	)	AND DIS-
	4Ø	$*X^2$	PLAY X_i
	3Ø	*√	
	42	STO	
	ØØ	Ø	
055	Ø5	5	
	43	RCL	
	ØØ	Ø	
	Ø4	4	
	81	HLT	
060	46	*LBL	COMPUTE
	12	B	SUMS
	43	RCL	
	ØØ	Ø	
	Ø4	4	
065	44	SUM	
	ØØ	Ø	$\sum_{i=1}^{N}X_i$
	Ø6	6	
	4Ø	$*X^2$	
	44	SUM	
070	ØØ	Ø	$\sum_{i=1}^{N}X_i^2$
	Ø7	7	
	43	RCL	
	ØØ	Ø	
	Ø5	5	
075	44	SUM	

LOC	CODE	KEY	COMMENTS
	ØØ	Ø	$\sum_{i=1}^{N}X_i'$
	Ø8	8	
	23	LNX	
	44	SUM	
080	ØØ	Ø	$\sum_{i=1}^{N}\ln X_i'$
	Ø9	9	
	Ø1	1	
	44	SUM	
	Ø1	1	N
085	ØØ	Ø	
	43	RCL	
	Ø1	1	DISPLAY
	ØØ	Ø	N
	81	HLT	
090	46	*LBL	
	13	C	COMPUTE
	53	(	$\hat{a}$
	53	(	AND STORE
	43	RCL	IN R_{12}
095	ØØ	Ø	
	Ø8	8	
	55	÷	
	43	RCL	
	Ø1	1	
100	ØØ	Ø	
	54	)	(COMPUTE
	23	LNX	A AND
	75	—	STORE
	43	RCL	IN R_{11})
105	ØØ	Ø	
	Ø9	9	
	55	÷	
	43	RCL	
	Ø1	1	
110	ØØ	Ø	
	54	)	

LABELS	
A	X_i
B	i
C	$\hat{a}$
D	$\hat{B}$
E	X_{MEDIAN}
A′	X_{BASE}
B′	$\bar{X}$
C′	S
D′	SKEW
E′	INIT.

REGISTERS	
00	
01	MEDIAN
02	$X_{MAX/MIN}$
03	X_{BASE}
04	X_i
05	X_i'
06	$\sum X$
07	$\sum X^2$
08	$\sum X'$
09	$\sum \ln X'$
10	N
11	A
12	$\hat{a}$
13	$\hat{B}$
14	
15	
16	
17	
18	
19	

FLAGS	
0	
1	
2	
3	
4	

TITLE X, S, SKEW, â, AND B̂ PAGE 3 OF 3
PROGRAMMER MORRIS S. WEBB, JR. DATE 4/16/77

SR-52 Coding Form

LOC	CODE	KEY	COMMENTS
112	42	STO	
	Ø1	1	
	Ø1	1	
	53	(	
	53	(	
117	Ø4	4	
	55	÷	
	Ø3	3	
	65	X	
	43	RCL	
122	Ø1	1	COMPUTE
	Ø1	1	â
	85	+	AND STORE
	Ø1	1	IN R_{12}
	54	)	
127	3Ø	*√	
	85	+	
	Ø1	1	
	54	)	
	55	÷	
132	Ø4	4	
	55	÷	
	43	RCL	
	Ø1	1	
	Ø1	1	
137	95	=	
	42	STO	
	Ø1	1	
	Ø2	2	
	81	HLT	
142	46	*LBL	
	14	D	
	43	RCL	
	ØØ	Ø	
	Ø8	8	
147	55	÷	COMPUTE
	43	RCL	B̂ AND
	Ø1	1	STORE IN

LOC	CODE	KEY	COMMENTS
	ØØ	Ø	R_{13}
	55	÷	
152	43	RCL	
	Ø1	1	
	Ø2	2	
	95	=	
	42	STO	
157	Ø1	1	
	Ø3	3	
	81	HLT	
	46	*LBL	
	15	E	STORE
162	42	STO	MEDIAN
	ØØ	Ø	IN
	Ø1	1	$R_{Ø1}$
	81	HLT	
	46	*LBL	
167	16	*A'	COMPUTE
	42	STO	X_{BASE}
	ØØ	Ø	
	Ø2	2	
	75	−	
172	43	RCL	
	ØØ	Ø	
	Ø1	1	
	95	=	
	65	X	
177	53	(	
	Ø3	3	
	93	.	
	Ø8	8	($Z_{Ø.99995}$)
	Ø9	9	
182	Ø1	1	
	55	÷	
	81	HLT	(INSERT $Z_{[N/(N+1)]}$)
	54	)	
	3Ø	*√	
187	85	+	

LOC	CODE	KEY	COMMENTS
	43	RCL	
	ØØ	Ø	
	Ø1	1	
	95	=	
192	81	HLT	(INSERT *FIX m = Ø,…,9)
	52	EE	
	22	INV	(TRUNCATION
	52	EE	ROUTINE)
	22	INV	
197	57	*FIX	
	42	STO	(DISPLAY
	ØØ	Ø	TRUNCATED
	Ø3	3	X_{BASE})
	81	HLT	
202	46	*LBL	
	19	*D'	
	53	(	
	17	*B'	
	75	−	
207	43	RCL	COMPUTE
	ØØ	Ø	SKEW
	Ø1	1	
	54	)	
	65	X	
212	Ø3	3	
	55	÷	
	18	*C'	
	95	=	
	81	HLT	
217	46	*LBL	
	1Ø	*E'	CLEAR
	47	*CMS	REGISTERS
	25	CLR	
	81	HLT	
222			

LABELS	
A	X_i
B	i
C	â
D	B̂
E	X_{MEDIAN}
A'	X_{BASE}
B'	$\bar{X}$
C'	S
D'	SKEW
E'	INIT.
REGISTERS	
00	
01	MEDIAN
02	$X_{MAX/MIN}$
03	X_{BASE}
04	X_i
05	X'_i
06	$\sum X$
07	$\sum X^2$
08	$\sum X'$
09	$\sum \ln X'$
10	N
11	A
12	â
13	B̂
14	
15	
16	
17	
18	
19	
FLAGS	
0	
1	
2	
3	
4	

SR-52 User Instructions

TITLE GAMMA FUNCTION AND INCOMPLETE GAMMA FUNCTION PAGE 1 OF 3

A	A'			
(â+5)	X'/B̂			
Γ(â)	X	γ(â, X'/B̂)	P(x)	Q(x)

B				

STEP	PROCEDURE	ENTER	PRESS	DISPLAY
	THE "MEAN, STANDARD DEVIATION, ⋯, B̂" PROGRAM IS TO BE USED PRIOR TO STEP 1.			
1	LOAD PROGRAM CARD			
2	CALCULATE Γ(â)		A	Γ(â)
3	INDEPENDENT VARIABLE X	X	B	X
4	CALCULATE γ(â, X'/B̂)		C	γ(â, X'/B̂) (UNROUNDED).
5	CALCULATE P(x)		D	P(x) (TO FIVE DECIMAL PLACES).
6	CALCULATE Q(x)		E	Q(x) (TO FIVE DECIMAL PLACES).
	REPEAT STEPS 3-6 FOR EACH X.			
	TO SEE (â+5)		*A'	
	TO SEE X'/B̂		*B'	

$$\Gamma(\hat{a}) = \int_0^\infty t^{\hat{a}-1} e^{-t}\,dt \cong \frac{\sqrt{\frac{2\pi}{(\hat{a}+5)}}\cdot(\hat{a}+5)^{(\hat{a}+5)}\, e^{-\left\{(\hat{a}+5) - \frac{1}{12(\hat{a}+5)} + \frac{1}{360(\hat{a}+5)^3}\right\}}}{(\hat{a}+4)\cdot(\hat{a}+3)\cdot(\hat{a}+2)\cdot(\hat{a}+1)\cdot\hat{a}}, \quad \frac{\text{ERROR}}{\Gamma(\hat{a})} < 2(10^{-7})$$

$$\gamma(\hat{a}, X'/\hat{B}) = \int_0^{X'/\hat{B}} t^{\hat{a}-1} e^{-t}\,dt \cong (X'/\hat{B})^{\hat{a}} e^{-(X'/\hat{B})} \sum_{n=0}^{\infty} \frac{(X'/\hat{B})^n}{\hat{a}(\hat{a}+1)\cdots(\hat{a}+n)}$$

$$P(x) = \frac{\gamma(\hat{a}, X'/\hat{B})}{\Gamma(\hat{a})}$$

$$Q(x) = 1 - P(x)$$

Part No. 1220479

TITLE *GAMMA/INCOMPLETE GAMMA FUNCTION* PAGE *2* OF *3*
PROGRAMMER *MORRIS S. WEBB, JR.* DATE *5/29/77*

SR-52 Coding Form

LOC	CODE	KEY	COMMENTS	LOC	CODE	KEY	COMMENTS	LOC	CODE	KEY	COMMENTS	LABELS
000	46	*LBL			Ø4	4	STEP 45)		52	EE	IF NOT,	A $\Gamma(\hat{a})$
	17	*B'			Ø5	5			22	INV	GO TO	B X
	53	(		040	17	*B'	(IF n≠Ø,		9Ø	*IFZRO	STEP Ø82)	C $\gamma(\hat{a}, x'/\hat{\beta})$
	53	(			41	GTO	CALCULATE		ØØ	Ø		D $P(x)$
	43	RCL			ØØ	Ø	$x'/\hat{\beta}$ AND	080	Ø3	3		E $Q(x)$
005	ØØ	Ø			Ø4	4	GO TO STEP		Ø3	3		A' $(\hat{a}+5)$
	Ø4	4			Ø6	6	46)		43	RCL		B' $x'/\hat{\beta}$
	75	−	COMPUTE	045	Ø1	1			Ø1	1		C'
	43	RCL	AND		55	÷			Ø7	7		D'
	ØØ	Ø	DISPLAY		53	(		085	65	×		E'
010	Ø3	3	$x'/\hat{\beta}$		43	RCL			17	*B'		REGISTERS
	54	)			Ø1	1	$\left[\frac{x'/\hat{\beta}}{\hat{a}+n}\right]$		45	y^x		00
	4Ø	$*x^2$		050	Ø2	2			43	RCL		01 MEDIAN
	3Ø	*√			85	+			Ø1	1		02 $X_{MAX/MIN}$
	55	÷			43	RCL		090	Ø2	2		03 X_{BASE}
015	43	RCL			Ø1	1			65	×		04 X OR X_i
	Ø1	1			Ø5	5			17	*B'		05 X_i'
	Ø3	3		055	95	=			94	+/−		06 $\sum x$
	54	)			49	*PROD	$\prod_{n=0}^{\infty}\left[\frac{x/\hat{\beta}}{\hat{a}+n}\right]$		22	INV		07 $\sum x^2$
	56	*RTN			Ø1	1	$= R_{16}$	095	23	LNX		08 $\sum x'$
020	46	*LBL	COMPUTE		Ø6	6			95	=		09 $\sum \ln x'$
	13	C	$\gamma(\hat{a}, x'/\hat{\beta})$		43	RCL	(RECALL		22	INV		10 N
	ØØ	Ø		060	Ø1	1	R_{16})		57	*FIX		11 A
	42	STO	(STORE		Ø6	6			42	STO	(STORE	12 $\hat{a}$
	Ø1	1	ZERO IN		44	SUM	$\sum_{n=0}^{\infty} R_{16}$	100	Ø1	1	$\gamma(\hat{a}, x'/\hat{\beta})$	13 $\hat{\beta}$
025	Ø5	5	R_{15} AND		Ø1	1	$= R_{17}$		Ø8	8	IN R_{18})	14 $\Gamma(\hat{a})$
	42	STO	R_{17})		Ø7	7			81	HLT		15 n
	Ø1	1		065	Ø1	1			46	*LBL		16 $\prod_{n=0}^{\infty}\left[\frac{x/\hat{\beta}}{\hat{a}+n}\right]$
	Ø7	7			44	SUM	(LOOP		16	*A'		17 $\sum_{n=0}^{\infty} R_{16}$
	Ø1	1			Ø1	1	COUNTER)	105	53	(	COMPUTE	18 $\gamma(\hat{a}, x'/\hat{\beta})$
030	42	STO	(STORE 1		Ø5	5			43	RCL	AND	19
	Ø1	1	IN R_{16})		43	RCL			Ø1	1	DISPLAY	FLAGS
	Ø6	6		070	Ø1	1			Ø2	2	$(\hat{a}+5)$	0
	43	RCL	(RECALL		Ø6	6			85	+		1
	Ø1	1	n)		57	*FIX	(IF R_{16} >	110	Ø5	5		2
035	Ø5	5			Ø7	7	0.0000000		54	)		3
	9Ø	*IFZRO	(IF n=Ø,		52	EE	GO TO					4
	ØØ	Ø	GO TO	075	22	INV	STEP Ø33;					

TITLE *GAMMA/INCOMPLETE GAMMA FUNCTION* PAGE *3* OF *3*
PROGRAMMER *MORRIS S. WEBB, JR.* DATE *5/29/77*

SR-52 Coding Form

LOC	CODE	KEY	COMMENTS	LOC	CODE	KEY	COMMENTS	LOC	CODE	KEY	COMMENTS	LABELS
112	56	*RTN			75	−			54	)		A $\Gamma(\hat{a})$
	46	*LBL			53	(			55	÷		B X
	14	D		152	Ø1	1			53	(		C $\gamma(\hat{a}, x'/\hat{\beta})$
	53	(			Ø2	2			16	*A'		D $P(x)$
	43	RCL			65	×		192	75	−		E $Q(x)$
117	Ø1	1			16	*A'			Ø2	2		A' $(\hat{a}+5)$
	Ø8	8	COMPUTE		54	)			54	)		B' $x'/\hat{\beta}$
	55	÷	P(x)	157	2Ø	*1/x			55	—		C'
	43	RCL			85	+			53	(		D'
	Ø1	1			53	(		197	16	*A'		E'
122	Ø4	4			Ø3	3			75	−		REGISTERS
	54	)			Ø6	6			Ø1	1		00
	57	*FIX		162	ØØ	Ø			95	=		01 MEDIAN
	Ø5	5			65	×			42	STO		02 $X_{MAX/MIN}$
	56	*RTN			16	*A'		202	Ø1	1		03 X_{BASE}
127	43	*LBL			45	y^x			Ø4	4		04 X OR X_i
	15	E			Ø3	3			81	HLT		05 X_i'
	Ø1	1	COMPUTE	167	54	)			46	*LBL		06 $\sum x$
	75	−	Q(x)		2Ø	*1/x			12	B		07 $\sum x^2$
	14	D			54	)		207	22	INV	ENTER	08 $\sum x'$
132	95	=			94	+/−			57	*FIX	AND DIS-	09 $\sum \ln x'$
	81	HLT			22	INV			42	STO	PLAY X	10 N
	46	*LBL		172	23	LNX			ØØ	Ø		11 A
	11	A			55	÷			Ø4	4		12 $\hat{a}$
	Ø2	2			43	RCL		212	81	HLT		13 $\hat{\beta}$
137	65	×			Ø1	1						14 $\Gamma(\hat{a})$
	59	*π			Ø2	2						15 n
	55	÷		177	55	÷						16 $\prod_{n=0}^{\infty}\left[\frac{x/\hat{\beta}}{\hat{a}+n}\right]$
	16	*A'	COMPUTE		53	(						17 $\sum_{n=0}^{\infty} R_{16}$
	95	=	$\Gamma(\hat{a})$		16	*A'		217				18 $\gamma(\hat{a}, x'/\hat{\beta})$
142	3Ø	*√	AND STORE		75	−						19
	65	×	IN R_{14}		Ø4	4						FLAGS
	16	*A'		182	54	)						0
	45	y^x			55	÷						1
	16	*A'			53	(		222				2
147	65	×			16	*A'						3
	53	(			75	−						4
	16	*A'		187	Ø3	3						