Wind Dispersion: A Program for the Texas Instruments 59 Calculator

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With the introduction of magnetic tape program storage capabilities for the modern pocket calculator, equations once considered too lengthy or cumbersome for anything less than a computer are becoming available to many people in the air pollution control professions. One such equation, used to calculate ambient air pollution concentrations downwind from a point source, is an equation for atmospheric dispersion estimates.

On the basis of the gaussian properties of turbulence, Sutton[1] developed the following equation for gas concentration at any location downwind from a point source:

$$X = \frac{Q_m \exp(-y^2 C_y{}^2 x^{2-n})}{\P C_y C_z u x^{2-n}} \times \left\{\exp\left[-\frac{(z-h_e)^2}{C_z{}^2 x^{2-n}}\right] + \exp\left[-\frac{(z+h_e)^2}{C_z{}^2 x^{2-n}}\right]\right\}$$

A modification of this equation, substituting standard deviations for Sutton's dispersion parameters, has become a generally accepted model for estimating ambient air pollution concentrations (gas, or particulates less than 20 μm) from a continuous point source. The equation becomes:

$$X(x,y,z;H) = \frac{Q}{2\P\sigma_y\sigma_z U}\exp\left[\frac{-1}{2}\left(\frac{y}{\sigma_y}\right)^2\right] \times \left\{\exp\left[\frac{-1}{2}\left(\frac{z-H}{\sigma_z}\right)^2\right] + \exp\left[\frac{-1}{2}\left(\frac{z+H}{\sigma_z}\right)^2\right]\right\}$$

Where X = pollution concentration in g/m^3 (or μg/m^3)
Q = emission rate in g/sec
U = average wind speed in m/sec
σ_y = diffusion coefficients in y direction, m
σ_z = diffusion coefficients in z direction, m
H = effective height of pollution source, m

Utilizing the modified equation, a procedure for making dispersion estimates was suggested by Pasquill[2] and modified by Gifford[3] in 1961. The procedure considered the pollution source at or below the point of emission, with the x-axis extending horizontally with the mean wind direction. The y-axis is horizontally perpendicular to the x-axis, and the z-axis extends vertically. The plume spread has a gaussian distribution in both the horizontal and vertical planes, with standard deviations of plume concentration distribution in the horizontal and vertical of σ_y and σ_z, respectively. It is assumed that there is no diffusion in the x direction, i.e., the plume release is continuous, or that the duration of release is equal to or greater than the travel time from source to point of interest. The concept may be visualized as shown in Figure 1.

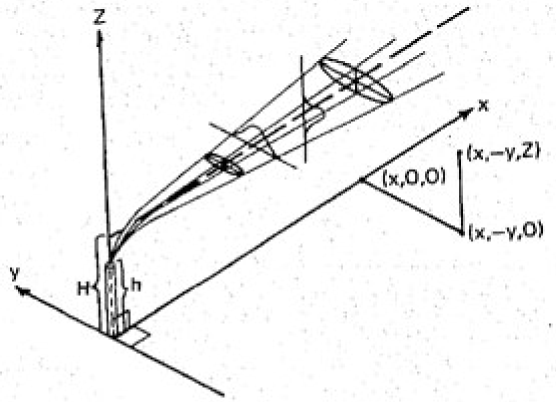

Figure 1. Coordinate system showing gaussian distributions in the horizontal and vertical.

H is the height of the plume centerline when it becomes essentially level, and is the sum of the physical stack height h and the plume rise above the stack. The mean wind speed affecting the plume is U; the uniform emissions rate of pollutants is Q; and total reflection of the plume occurs at the earth's surface, i.e., no deposition or reaction at ground level.

Turner[4] developed a key to stability categories for both night and day which permits the selection of the appropriate σ_y and σ_z for horizontal and vertical dispersion coefficients. Table I consists of five stability categories, A (most unstable) through F, (most stable) based on wind speed in m/sec^{-1} at an elevation above ground level of 10 m. Daytime categories are predicted upon subjective determinations of strong, moderate, and slight incoming solar radiation. Night time categories are divided into ≥ 4/8 cloud cover, and ≤ 3/8 cloud cover. A choice of the most appropriate stability category (A through F) is made, based on wind speed and night/day subdivisions.

Having selected the appropriate stability category, a numerical value for σ_y may be determined by locating the downwind distance of interest on the x-axis of Figure 2, tracing it vertically until it intersects with the appropriate stability category dotted line, then following a horizontal line from that point to the y-axis; σ_y may then be determined in meters. σ_z may be determined in a similar manner from Figure 3.

These methods give representative coefficient over open country or rural areas, but are less reliable for urban areas due to relative surface roughness and the heat island effects found in cities.

It is important to recognize that this model is only one of several possible models of gaussian behavior, but when *best estimate*, not infallible prediction, is sufficient, this model may be programmed for the Texas Instrument Programmable 59 and stored on magnetic tape for quick recall. The program is as follows:

Step	*Code*	*Key*	*Step*	*Code*	*Key*	*Step*	*Code*	*Key*
000	76	LBL	053	55	÷	105	43	RCL
001	11	A	054	43	RCL	106	11	11
002	42	STO	0.55	06	06	107	95	=
003	01	01	056	95	=	108	42	STO
004	91	R/S	057	33	X^2	109	14	14
005	76	LBL	058	55	÷	110	43	RCL
006	12	B	059	02	2	111	12	12
007	42	STO	060	94	+/−	112	65	x
008	02	02	061	95	=	113	53	(
009	91	R/S	062	42	STO	114	43	RCL
010	76	LBL	063	10	10	115	13	13
011	13	C	064	43	RCL	116	85	+
012	42	STO	065	03	03	117	43	RCL
013	03	03	066	75	−	118	14	14
014	91	R/S	067	43	RCL	119	54	(
015	76	LBL	068	04	04	120	95	=
016	14	D	069	95	=	121	42	STO
017	42	STO	070	55	÷	122	15	15
018	04	04	071	43	RCL	123	43	RCL
019	91	R/S	072	06	06	124	09	09
020	76	LBL	073	95	=	125	45	yx
021	15	E	074	33	X^2	126	43	RCL
022	42	STO	075	55	÷	127	15	15
023	05	05	076	02	2	128	95	=
024	91	R/S	077	94	+/−	129	42	STO
025	76	LBL	078	95	=	130	16	16
026	16	A′	079	42	STO	131	53	(
027	42	STO	080	11	11	132	43	RCL
028	06	06	081	43	RCL	133	08	08
029	91	R/S	082	02	02	134	55	÷
030	76	LBL	083	55	÷	135	53	(
031	17	B′	084	43	RCL	136	02	2
032	42	STO	085	05	05	137	65	×
033	07	07	086	95	=	138	89	¶
034	91	R/S	087	33	X^2	139	65	×
035	76	LBL	088	55	÷	140	43	RCL
036	18	C′	089	02	2	141	05	05
037	42	STO	090	94	+/−	142	65	×
038	08	08	091	95	=	143	43	RCL
039	91	R/S	092	42	STO	144	05	06
040	76	LBL	093	12	12	145	65	×
041	19	D′	094	43	RCL	146	43	RCL
042	42	STO	095	09	09	147	07	07
043	09	09	096	45	yx	148	54	)
044	91	R/S	097	43	RCL	149	54	)
045	76	LBL	098	10	10	150	65	×
046	10	E′	099	95	=	151	43	RCL
047	43	RCL	100	42	STO	152	16	16
048	03	03	101	13	13	153	95	=
049	85	+	102	43	RCL	154	91	R/S
050	43	RCL	103	09	09			
051	04	04	104	45	yx			
052	95	=						

To enter this program into the TI 59 calculator, press 2nd CP LRN, and enter the program from 000 76 LBL thru 154 91 R/S. Press LRN, which closes the program. By pressing 1 2nd Write and passing a magnetic tape through the calculator, the program is recorded on tape for future use.

To reprogram TI 59 at a later date for wind dispersion estimations, press CLR and pass the programmed tape thru the calculator. A sample problem would be entered as follows:

The stability category which best fits today's weather is D. The point of interest downwind on the x-axis is 1,000 m. STEP 1: enter 1,000; press A.

The perpendicular point of interest on the y-axis is 350 m.

Table 1. Key to stability categories.[a]

Surface wind speed (at 10 m), m sec^{-1}	Day			Night	
	Incoming solar radiation				
	Strong	Moderate	Slight	Thinly overcast or ≥4/8 low cloud	≤3/8 cloud
<2	A	A–B	B		
2–3	A–B	B	C	E	F
3–5	B	B–C	C	D	E
5–6	C	C–D	D	D	D
>6	C	D	D	D	D

[a] The neutral class, D, should be assumed for overcast conditions during day or night. Source: Turner, 1970.[4]

STEP 2: enter 350; press B.

The vertical point of interest on the z-axis is 175 m.

STEP 3: enter 175; press C.

The effective stack height, H, is 150 m.

STEP 4: enter 150; press D.

σ_y, based on stability category D and 1000 m downwind, is found to be 170 m on Figure 2.

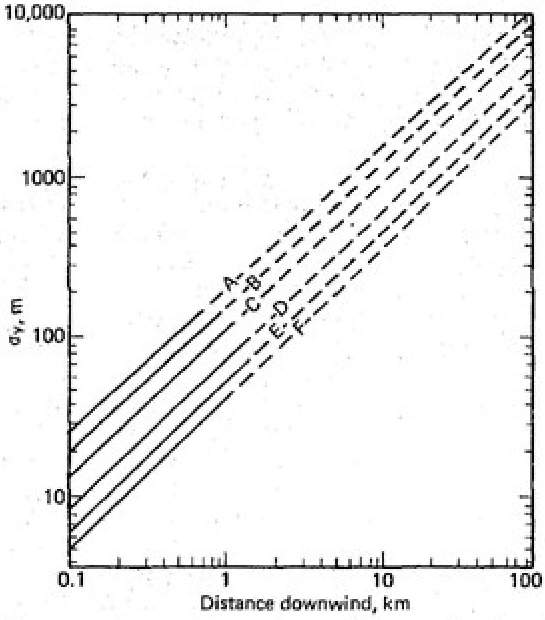

Figure 2. Horizontal dispersion coefficient as a function of downwind distance from the source.[4]

STEP 5: enter 170; press E.

σ_z, based on stability category D and 1000 m downwind, is found to be 32 m on Figure 3.

STEP 6: enter 32; press 2nd A′.

The appropriate value for U is the average wind speed in m/sec through the stack plume. More commonly, an estimate is made, based on an assumed velocity profile, i.e.,

$$U = U_1 \left(\frac{H}{z_1}\right)^n$$

where:

U_1 = wind speed at top of wind tower, m/sec
H = effective stack height, m
z_1 = height of wind tower used to measure U_1 m
N = 0.25 for unstable condition; N = 0.50 for stable condition

Wind speed for today measured from a 10 m tower is found to be 5 m/sec:

$$U = 5\,\frac{(150)^{0.37}}{10}$$

$$U = 13.6 \text{ m/sec}$$

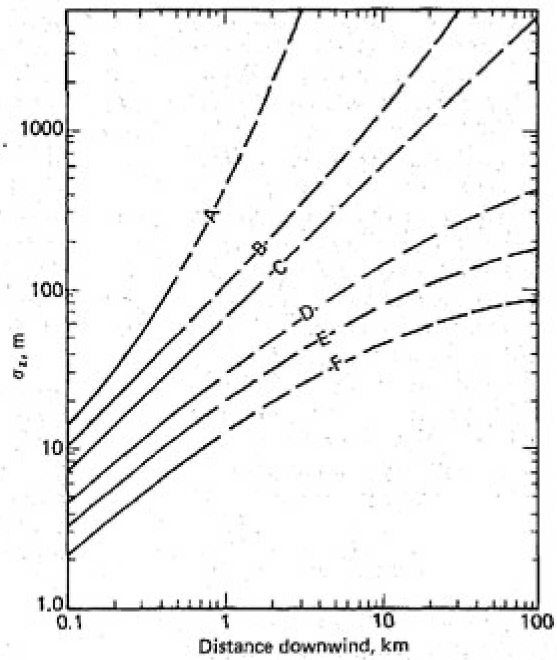

Figure 3. Vertical dispersion coefficient as a function of downwind distance from the source.[4]

STEP 7: enter 13.6; press 2nd B′.

Q, the pollutant concentration in μg/sec, is 1.075×10^9 μg/sec.

STEP 8: enter 1.075×10^9; press 2nd C′.

Exp = base of natural logarithm, 2.718281828

STEP 9: enter 1; press INV lnx 2nd D′.

STEP 10: press 2nd E; to run program.

The pollutant concentration, 484.99 μg/m^3, will appear at the end of the program run.

References

1. O. G. Sutton, "The theoretical distribution of air-borne pollution from factory chimneys," *Quart. J. Royal Meteorol. Soc.* **73**: 426 (1947).
2. F. Pasquill, "The estimation of the dispersion of windborne material," *Meteorol. Magazine* **90** (1063): 33 (1961).
3. F. A. Gifford, "Use of routine meteorological observations for estimating atmospheric dispersion," *Nuclear Safety* **2** (4): 47 (1961).
4. D. B. Turner, "Workbook of Atmospheric Dispersion Estimates," U.S. Public Health Publcation 999-AP-26, Revised, 1970.

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