Predict system dependability

with a pocket calculator. A programmable unit speeds computations of reliability and maintainability models.

A handheld programmable calculator can rapidly predict dependability of electronic systems. Carefully prepared, a calculator program, once loaded onto a program card, can provide calculations almost as precise as even a large-scale computer. Furthermore, the calculator is easier to use and less expensive than any computer, large or small.

Set up system models

Three subsystem models—Series, Parallel and Any-R-of-N configuration—have been programmed for a scientific pocket calculator (see Table 1). To use the models, you need to know the mean time between failure (MTBF) and mean time to repair (MTTR) for each system component. You can either rely on figures vendors supply or, if you have access to trouble reports for the components, you can easily compute MTBF and MTTR parameters. For MTBF, multiply the operating time period in hours by the number of units sampled, then divide this product by the total number of failures. To obtain MTTR, average the "out-of-service" times.

By applying the models, you can predict MTBF and MTTR for the total configuration, as well as for every underlying subsystem. Not only that, you can also determine availability, reliability and failure probability from MTBF and MTTR.

Availability, A, is defined as the probability of finding the subsystem working at any arbitrary future time. Reliability, R(t), is the probability of completely successful operation in time period, t. The probability of n failures occurring in t is designated P(n).

To generate a model, convert the hardware configuration into a reliability "bubble" diagram, which shows how individual elements affect a subsystem's over-all reliability. The bubble diagram is a network of connected circles, showing MTBF and MTTR figures within the bubbles. In Table 1, Example 1, a single element has an MTBF of 500 hours and an MTTR of 3 hours. To assemble single elements into first-level subsystems, use the three types of models: Series, Parallel and Any R of N.

In a Series connection, should any one element fail, the whole subsystem will cease to function. In a Parallel connection, the entire subsystem is considered operational if any one of the elements is working. However, an Any R-of-N connection is more versatile than the Series or Parallel configurations. In an N-element arrangement, the subsystem functions as long as R or more of its elements work.

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When R=1, the Any-R-of-N connection is equivalent to a Parallel connection. Only one shunted element is required to maintain an operational sybsystem. In Example 8, R=1 and the results approximate those for the Parallel connection in Example 7. When R=N, "Any" and Series correspond. Example 4 produces the same results as the Series connection in Example 3. A unique example of the "Any" arrangement, where R=2 and N=3, which can't be duplicated by either the Series or Parallel models, is illustrated by Example 9.

Proceeding in this manner, you join primary elements into first-level subsystems. Then, repeat this process, assembling these first-level subsystems in turn to form higher-level subsystems, until the reliability diagram comprises the total system. Continue the same Series, Parallel, and Any R-of-N reduction strategy through each phase of the system.

Realistic assumptions are needed

To derive equations you can solve, several realistic assumptions, which apply to most encountered systems, must be made for the three models:

1. Equipment is either up (operational) or down (failed)—No in-between condition is allowed. When any module malfunctions, repair or replace it.
2. The state of one unit is unaffected by the states of its adjacent elements.
3. Redundant elements can be switched into place as a failure occurs, before the over-all system is considered to have failed.
4. Exponential service times are assumed, as is standard in the derivation of many analytical models.

In this last case, the probability of MTBF and MTTR time periods is assumed to follow a negative exponential distribution. In other words, the number of failures and repairs occurring in a unit time obeys the Poisson distribution; therefore, the variance is equal to the mean.

Assume widely fluctuating failure and repair times, with variance commensurate with MTBF and MTTR.

Table 1. Examples of reliability models

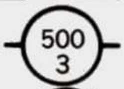
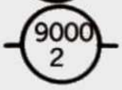
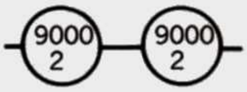
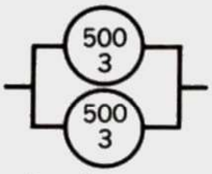
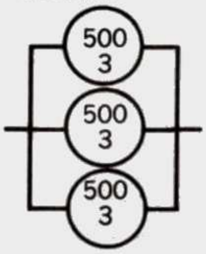
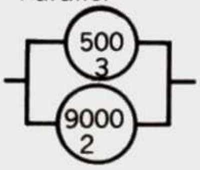
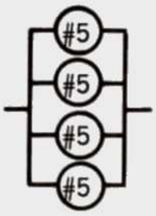
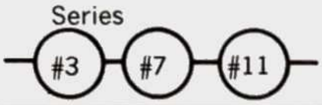
Example No.	Reliability models	MTBF	MTTR	A	R(8760)	P(1)	P(2)
1		500.00	3.00	0.994036	0.000000	0.000000	0.000004
2		9000.00	2.00	0.999778	0.377822	0.367746	0.178970
3	Series 	4500.00	2.00	0.999556	0.142749	0.277885	0.270475
4	Any 2 of 2 (same diagram as example 3)	4500.00	2.00	0.999556	0.142749	0.277885	0.270475
5	Series 	450.00	2.90	0.993594	0.000000	0.000000	0.000001
6	Parallel 	42,166.67	1.50	0.999964	0.812413	0.168776	0.017531
7	Parallel 	4,713,462.96	1.00	1.000000	0.998143	0.001855	0.000002
8	Any 1 of 3 (same diagram as example 7)	4,629,629.63	1.00	1.000000	0.998110	0.001889	0.000002
9	Any 2 of 3 (same diagram as example 7)	13,888.89	1.50	0.999892	0.532208	0.335675	0.105858
10	Parallel 	905,600.00	1.20	0.999999	0.990373	0.009580	0.000046
11	Any 3 of 4 	5,818.97	1.45	0.999751	0.221924	0.334089	0.251472
12	Series 	2,536.23	1.76	0.999307	0.031621	0.109216	0.188613

Table 2. SR52 coding form

LOC	CODE	KEY	LOC	CODE	KEY	LOC	CODE	KEY	LOC	CODE	KEY	LOC	CODE	KEY	LOC	CODE	KEY
000	46	LBL	025	44	SUM	050	75	—		05	5		42	STO		95	=
	18	C′		00	0		04	4		17	B′		00	0		42	STO
	22	INV		02	2		17	B′	090	65	x	115	06	6	140	00	0
	57	fix		02	2		95	=		03	3		81	HLT		01	1
	04	4		17	B′		55	÷		17	B′		42	STO		81	HLT
005	46	LBL	030	20	1/x	055	04	4		95	=		00	0		65	x
	17	B′		42	STO		17	B′		42	STO		07	7		53	(
	42	STO		00	0		65	x	095	00	0	120	81	HLT	145	43	RCL
	01	1		03	3		03	3		03	3		57	fix		55	÷
	09	9		81	HLT		17	B′		81	HLT		02	2		02	2
010	36	IND	035	46	LBL	060	95	=		46	LBL		42	STO		17	B′
	43	RCL		12	B		81	HLT		16	A′		00	0		54	)
	01	1		85	+		46	LBL	150	45	y^x		85	+		95	=
	09	9		01	1		14	D		08	8		01	1		42	STO
	56	rtn		17	B′		55	÷		17	B′		17	B′		01	1
015	46	LBL	040	95	=	065	53	(		65	x		95	=	205	00	0
	11	A		55	÷		43	RCL		08	8	180	42	STO		94	+/−
	46	LBL		01	1		85	+	155	17	B′		00	0		22	INV
	13	C		17	B′		01	1		29	x!		04	4		23	ln x
	57	fix		95	=		17	B′		65	x		03	3		65	x
020	02	2	045	20	1/x	070	54	)		53	(		17	B′	210	43	RCL
	42	STO		49	PROD		94	+/−		06	6	185	81	HLT		01	1
	00	0		00	0		85	+	160	17	B′		46	LBL		00	0
	01	1		04	4		01	1		75	—		19	D′		45	y^x
	20	1/x		01	1		95	=		01	1		42	STO		00	0
075	49	PROD	100	47	CMs	125	02	2		54	)		00	0	215	17	B′
	00	0		01	1		55	÷		29	x!	190	09	9		55	÷
	05	5		42	STO		53	(	165	55	÷		00	0		00	0
	05	5		00	0		07	7		07	7		46	LBL		17	B′
	17	B′		04	4		17	B′		17	B′		10	E′		29	x!
080	75	—	105	42	STO	130	75	—		29	x!		42	STO	220	95	=
	01	1		00	0		06	6		95	=	195	00	0		22	INV
	95	=		05	5		17	B′	170	42	STO		00	0		57	fix
	94	+/−		86	rset		85	+		00	0		09	9		81	HLT
	92	STO		46	LBL		42	STO		03	3		17	B′			
085	00	0	110	15	E	135	00	0		55	÷		55	÷			
	04	4		57	fix		08	8		53	(	200	03	3			
	55	÷		00	0		01	1	175	43	RCL		17	B′			

These distributions help in the formula derivations because of their inherent memoryless property, since at any instant, the remaining time to failure or repair is independent of what has preceded.

To estimate total system dependability, apply the models repeatedly to primary elements, first-level subsystems, second-level subsystems, and soon. Repair and failure rates of all elements and subsystems are presumed to have Poisson distributions. But don't go overboard. Although this is a fairly good estimate, it is not always precisely the case, especially for high-level subsystems containing low levels of redundancy.

While your calculator might be accurate to 12 places, your assumptions and input data are probably not as accurate. Therefore, don't carry MTBF and MTTR figures to more than a few decimal places. Round them off.

Also, of course, while you rely on the three subsystem models, don't get carried away. Applying them demands thought and proficiency, and the resulting predictions require proper interpretation.

Equations for three models

The formulas for computing total system dependability can be solved on any scientific pocket calculator. But solving them on the SR52, or another programmable type, not only enables you to complete the analysis much faster, but also reduces the risk of error. If you use an SR52, the following material will serve as a background, since you need only to follow the "User Instructions" detailed in this article. For other programmable calculators, you have to recode the equations. Those with manual calculators will have to grind through the formulas step-by-step.

Consider the Series model first. The inverse of total MTBF is the summation of the inverses of unit MTBFs:

Table 3. SR52 user instructions

STEP	PROCEDURE	ENTER	PRESS	DISPLAY
1.0	Series Connection		START	1.
1.1	Enter MTBF & MTTR for each series element.	element MTBF	MTBFs	Cumulative subsystem MTBF
1.2	Repeat steps 1.1 & 1.2 for each series element. Cumulative result is displayed after each entry. After each element is added, you may interrupt the loop to compute the following:	element MTTR	MTTRs	Cumulative subsystem MTTR
1.3	Availability		A	Availability
1.4	Reliability	t	R(t)	Reliability
1.5	Probability of n failures. Step 1.4 must precede the first use of P(n)	n	P(n)	Probability of n failures
2.0	Series Connection for calculation of MTBF only. MTTRs are unknown.		START	1.
2.1	Enter MTBF for each series element. Repeat step 2.1 for each element. Cumulative results are displayed after each entry. After each MTBF is entered, you may request the following:	element MTBF	MTBFs	Cumulative subsystem MTBF
2.2		t	R(t)	Reliability
2.3	Step 2.2 must precede the first use of P(n)	n	P(n)	Probability of n failures

STEP	PROCEDURE	ENTER	PRESS	DISPLAY
3.0	Parallel Connection		START	1.
3.1	Enter MTTR & MTBF for each parallel element	element MTTR	$MTTR_p$	Cumulative subsystem MTTR
3.2	Repeat steps 3.1 & 3.2 for each parallel element. Cumulative results are displayed after each entry. After each element is added, you may interrupt the loop to compute the following:	element MTBF	$MTBF_p$	Cumulative subsystem MTBF
3.3	Availability		A	Availability
3.4	Reliability	t	R(t)	Reliability
3.5	Probability of n failures occuring in time period t	n	P(n)	Probability of n failures
4.0	ANY R-of-N Connection			
4.1	R elements required	R	ANY	
4.2	N elements available	N	RUN	
4.3	Assume all elements identical. Enter MTTR & MTBF only once.	element MTTR	RUN	Cumulative subsystem MTTR
4.4		element MTBF	RUN	Cumulative subsystem MTBF
4.5	Availability		A	Availability
4.6	Reliability	t	R(t)	Reliability
4.7	Step 4.6 must precede the first use of P(n)	n	P(n)	Probability of n failures

$$\frac{1}{MTBF_T} = \sum_{i=1}^{N} \frac{1}{MTBF_i}, \text{ for N units.}$$

Therefore,

$$MTBF_T = \frac{1}{\sum_{i=1}^{N} \frac{1}{MTBF_i}}.$$

Multiplying unit availabilities results in total availability:

$$A_T = \prod_{i=1}^{N} A_i.$$

And once $MTBF_T$ and A_T are determined, total MTTR can be found from

$$MTTR_T = \left(\frac{1-A_T}{A_T}\right) MTBF_T.$$

For a Parallel connection, adding the inverses of MTTR for each unit results in the inverse of total MTTR:

$$\frac{1}{MTTR_T} = \sum_{i=1}^{N} \frac{1}{MTTR_i}, \text{ for N units.}$$

Therefore,

$$MTTR_T = \frac{1}{\sum_{i=1}^{N} \frac{1}{MTTR_i}}.$$

Unavailability (U) is 1−A. Parallel total unavailability is the product of unit unavailabilities:

$$U_T = \prod_{i=1}^{N} U_i$$

$$(1 - A_T) = \prod_{i=1}^{N} (1 - A_i)$$

$$A_T = 1 - \prod_{i=1}^{N} (1 - A_i)$$

Once you know $MTTR_T$ and A_T, you can compute total MTBF:

$$MTBF_T = \left(\frac{A_T}{1 - A_T}\right) MTTR_T.$$

For an Any R-of-N configuration, the entire connection is operational provided R units of the N available are working. Assume all units have identical MTBF and MTTR values, which are indicated with the subscript i. Use the following formulas to calculate total MTBF and MTTR:

$$MTTR_T = \frac{MTTR_i}{N - R + 1}$$

$$MTBF_T = MTBF_i \left(\frac{MTBF_i}{MTTR_i}\right)^{N-R} \left[\frac{(N-R)!\ (R-1)!}{N!}\right]$$

These two equations are derived with the assumption that $MTTR_i$ is much smaller than $MTBF_i$, as is generally the case. Once $MTTR_T$ and $MTBF_T$ have been computed, solve for total availability:

$$A_T = \frac{MTBF_T}{MTBF_T + MTTR_T}$$

The equations for reliability and failure probabilities—R(t) and P(n)—are the same for all three models. For reliability during t,

$$R(t) = e^{-t/MTBF_T}.$$

For the probability of n failures in a time period t,

$$P(n) = \frac{e^{-\bar{N}} N^n}{n!},$$

where

$$N = \frac{t}{MTBF_T}$$

represents the average number of failures during time period t.

To appreciate the effects various configurations have on dependability parameters, examine the reliability bubble diagrams and results for a set of 12 illustrative examples tabularized in Table 1. The period during which reliability and failure probabilities were calculated was arbitrarily selected as one year of continuous operation (365 days×24 hours/day=8760 hours).

Both examples 1 and 2 may be thought of as either Series or Parallel configurations, consisting of only one element. In both cases, availability exceeds 99%. On the average, these elements will be up (available) more than 99 hours out of every 100 operating hours. However, this is not always a good indicator of dependability. The reliability and failure probabilities in Example 1 are almost zero—i.e., this element is certain to have more than two failures per year.

The element in the second example is much more dependable. It has a 37.78% probability of zero failure, a 36.77% probability of one failure, and a 17.90% probability of two failures per year. Failure probabilities higher than two can also be determined, by entering n, then depressing the key labeled P(n).

Units are strung together in Examples 3, 4, and 5. Where as the Series model may always be applied to such string connections, Any N of N can only be used when all components are identical. Series dependability is always less than for the weakest link in the chain. Observe that the resulting dependability parameters in Example 5 are all less than their respective counterparts in Example 1.

Paralleled elements are represented in Examples 6, 7, 8, and 10. If they aren't all identical, you must use the Parallel model. When they are, either the Parallel or Any 1-of-N models apply. The dependability of a network of parallel elements is always greater than any one of them.

The resulting $MTBF_T$ in Example 6 is 4.8 years. In Example 7, it is 538 years. Yet, in both illustrations, the primary elements have an MTBF of only 500 hours.

Examples 9 and 11 have been computed with the Any R-of-N model. Compare Example 9 with Example 8 and note the decreased dependability. This occurs in Example 9 because two elements are required to be operational instead of only one. The network in Example 11 is a second-level subsystem configured with the first-level subsystems of Example 5.

The reliability diagram in Example 12 is a third-level subsystem. As an exercise, you should redraw it with 17 primary elements.

Occasionally, you will encounter a configuration that cannot be accurately represented by any of the models. Consider an Any Two Required of Three Available condition, where all three elements are different. When such situations occur, the models can always be applied to determine upper and lower bounds on dependability. One assumption will be optimistic, the other, pessimistic. Carrying both limits through the remaining calculations establishes two sets of results. The system's actual dependability lies somewhere between these values.

To code the SR52 calculator, see Table 2. With this coding procedure, you can obtain a program card that, when inserted into the calculator, prepares it to perform the computations described in Table 3.

Ten user-defined functions can be executed by pressing the top row of keys on the SR52 calculator. A great deal of thought has gone into their assignment to provide for maximum user convenience. In this article user function keys are named and referenced as follows:

Key	*Name*	*Key*	*Name*
A	$MTBF_s$	A′	START
B	$MTTR_s$	B′	-
C	$MTTR_p$	C′	A
D	$MTBF_p$	D′	R(t)
E	ANY	E′	P(n)

Program the TI-58 or TI-59 calculators with the keystroke modifications listed in Table 4.■■

Table 4. Keystroke modifications for TI-58 and TI-59 calculators

SR52 Coding	TI-58, TI-59 KEY
rtn	R/S
HLT	R/S
x!	Implemented in master library module Pgm. 16